Name:

USC ID:

Notes:

- Write your name and ID number in the solution you submit.
- No books, cell phones or other notes are permitted. Only one letter size cheat sheet (back and front), a tablet for writing down your solutions, and and a calculator are allowed.
- Problems are not sorted in terms of difficulty. Please avoid guess work and long and irrelevant answers.
- Show all your work and your final answer. Simplify your answer as much as you can.
- Open your exam only when you are instructed to do so.
- The exam has 5 questions, 11 pages, and 13 points extra credit.

Problem	Score	Earned
1	22	
2	25	
3	22	
4	22	
5	22	
Total	113	

- 1. As director of the local tourist board, you are interested in determining the factors that influence the hotel occupancy rate in your city each month. Hotel occupancy can be measured as the percentage of available hotel rooms that are occupied by paying customers. You develop the following model: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \epsilon$, where Y is the hotel occupancy rate, X_1 is the total number of passengers arriving at the airport, X_2 is a price index of local hotel room rates, X_3 is the consumer confidence index, and X_4 is a dummy variable that shows being in the months of June, July, and August. You look at data from the past 35 months and obtain the following results: $y = 67.1 + 0.02x_1 0.055x_2 + 0.08x_3 + 12.3x_4$. Also, assume that the standard errors of $\hat{\beta}_i$'s are calculated as $SE(\hat{\beta}_0) = 58.3$, $SE(\hat{\beta}_1) = 0.008$, $SE(\hat{\beta}_2) = 0.01$, $SE(\hat{\beta}_3) = 0.06$, $SE(\hat{\beta}_4) = 4.7$. Moreover, the regression sum of squares is RegSS = 1, 169.45, and the residual (error) sum of squares is RSS = 576.
 - (a) Interpret the estimated regression coefficient $\hat{\beta}_2$.
 - (b) Interpret the estimated regression coefficient $\hat{\beta}_4$.
 - (c) Test the hypothesis H_0 : $\beta_1 = 0$ at $\alpha_1 = 0.05$ and $\alpha_2 = 0.01$ and interpret your results. Based on your results, explain why level of significance has to be determined *before* testing a hypothesis.
 - (d) Test the hypothesis $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ at $\alpha = 0.05$. Interpret your results.
 - (e) Build a 98% confidence interval for β_3 . What happens to the confidence interval if the level of confidence is increased? Is it desirable to have a very high level of confidence?

- 2. Choose either T (True) or F (False) (no need to explain why):
 - (a) When the assumption of conditional independence of features holds, the Naïve Bayes' classifier provides the best accuracy among all possible classifiers. T F
 - (b) The F1 score is not an appropriate measure for evaluating binary classifiers when data are not imbalanced. T F
 - (c) Leave-One-Out Cross Validation has less bias in estimating the error of a classifier for a large data set than 5 fold cross validation. T F
 - (d) When classifying imbalanced data into two classes, we can decrease the threshold on class conditional probability $\Pr(Y = k | X_1 = x_1, \dots, X_p = x_p]$ to increase the true positive rate at the expense of increasing the false negative rate. T F
 - (e) Logistic regression assumes that the conditional odds of the outcome Y given the features, $\mathbb{O}[Y=k|X_1=x_1,\ldots,X_p=x_p]$, is a logistic function of the features. T.F.

3. Assume that we have a binary classification problem with only one feature in which the conditional distribution of the feature in class k=1 is a normal with mean $\mu_1=1$ and standard deviation $\sigma_1=1$ and the conditional distribution of the feature in class k=2 is normal with mean $\mu_2=2$ and $\sigma_2=1$. Determine the values of x that are classified in each class by the Bayes' optimal classifier. Assume the classes are balanced, i.e. $\pi_1=\pi_2=0.5$.

Important note: you must *derive* the decision rule from scratch, i.e. you must write down the posterior probabilities.

4. Consider the logistic regression method for binary classification (Y = 0 or Y = 1) with two features $\mathbf{X} = (X_1, X_2)$, formulated by

$$P(Y = 1|\mathbf{X}) = \frac{e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2}}{1 + e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2}}$$

Assume that using a dataset of 200 observations, we obtained the following estimates:

	Coefficient	Standard Error
β_0	1	0.2
β_1	2	0.1
β_2	1	s

- (a) Find all values of s that makes the coefficient β_2 statistically insignificant. You can consider the significance level to be $\alpha=0.05$.
- (b) Determine the equation for the decision boundary for this classifier.
- (c) In what class will $\mathbf{X} = (-1, 1)$ will be classified?

- 5. In a weird simulated world, we have three types of creatures. Each creature has between 1 to 100 legs, 1 to 100 teeth, and 1 to 100 noses. The fraction of creatures type-1, type-2, and type-3 are respectively l/(l+t+n), t/(l+t+n), and n/(l+t+n), where l,t,n are respectively the number of legs, teeth, and noses of a creature.
 - (a) If a creature has 10 teeth, 25 legs, and 30 noses, what is your best guess about the type of the creature?
 - (b) What type of supervised learning problem are you solving in this question? Explain.

Scratch paper

Name:

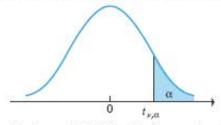
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Upper Critical Values of Student's t Distribution with ν Degrees of Freedom



For selected probabilities, α , the table shows the values $t_{\nu,\alpha}$ such that $P(t_{\nu} > t_{\nu,\alpha}) = \alpha$, where t_{ν} is a Student's t random variable with ν degrees of freedom. For example, the probability is .10 that a Student's t random variable with 10 degrees of freedom exceeds 1.372.

		PROBABILE	ty of Exceeding th	E CRITICAL VALUE		
ν	0.10	0.05	0.025	0.01	0.005	0.001
1	3.078	6.314	12.706	31.821	63.657	318.313
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.20
7	1.415	1.895	2.365	2.998	3.499	4.782
8	1.397	1.860	2.306	2.896	3.355	4,49
9	1.383	1.833	2.262	2.821	3.250	4.29
10	1.372	1.812	2.228	2.764	3.169	4.14
11	1.363	1.796	2.201	2.718	3.106	4.02
12	1.356	1.782	2.179	2.681	3.055	3.92
13	1.350	1.771	2.160	2.650	3.012	3.85
14	1.345	1.761	2.145	2.624	2.977	3.78
15	1.341	1.753	2.131	2.602	2.947	3.73
16	1.337	1.746	2.120	2.583	2.921	3.68
17	1.333	1.740	2.110	2.567	2.898	3.64
18	1.330	1.734	2.101	2.552	2.878	3.61
19	1,328	1.729	2.093	2.539	2.861	3.57
20	1.325	1.725	2.086	2.528	2.845	3.55
21	1.323	1.721	2.080	2.518	2.831	3.52
22	1,321	1.717	2.074	2.508	2.819	3.50
23	1.319	1.714	2.069	2.500	2.807	3.48
24	1.318	1.711	2.064	2.492	2.797	3.46
25	1.316	1.708	2.060	2.485	2.787	3.45
26	1.315	1.706	2.056	2.479	2.779	3.43
27	1.314	1.703	2.052	2.473	2.771	3.42
28	1.313	1.701	2.048	2.467	2.763	3.40
29	1.311	1.699	2.045	2.462	2.756	3.39
30	1.310	1.697	2.042	2.457	2.750	3.38
40	1.303	1.684	2.021	2.423	2.704	3.30
60	1.296	1.671	2.000	2.390	2.660	3.23
100	1.290	1.660	1.984	2.364	2.626	3.17
09	1.282	1.645	1.960	2.326	2.576	3.09
ν	0.10	0.05	0.025	0.01	0.005	0.001

F - Distribution (α = 0.05 in the Right Tail)

161,45 199,50 215,71 224,58 230,16 233,99 236,77 238,88 240 238,18 238,18	۲	df,		,	, .	Numerator Degrees of Freedom	Degrees	of Freedo	Ĕ,	×	0
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4 7,7086 ⋅9,9443 6.5914 6.3882 6.2561 6.1631 6.0942 6.0410 5 6,6079 5.7861 5.4995 5.1922 5.0503 4.9503 4.8759 4.8183 6 5.9874 5.1433 4.7571 4.3877 4.3874 4.2839 4.2067 4.1468 9 5.51174 4.4590 4.0662 3.8379 3.6875 3.5866 3.7807 3.1358 10 4.9646 4.1028 3.7083 3.4780 3.2586 3.5005 3.4817 11 4.8443 3.9823 3.8417 3.2039 3.095 3.2940 3.1328 3.2172 3.1358 3.2040 3.4817 12 4.9640 4.0602 3.8373 3.2392 3.1059 2.9961 2.9137 2.3480 13 4.6672 3.8853 3.4903 3.2592 3.1059 2.9961 2.9134 2.8486 14 4.6001 3.8833 3.4903 3.2592 3.1059	3		10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123
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12 4.7472 3.8853 3.4903 3.2592 3.1059 2.961 2.9134 2.8486 13 4.6672 3.8056 3.4105 3.1791 3.0254 2.9153 2.8321 2.7669 14 4.6001 3.7389 3.1122 2.9582 2.8477 2.7642 2.6987 15 4.64940 3.7389 3.1122 2.9582 2.8477 2.7642 2.6987 17 4.4940 3.6337 3.2384 3.0699 2.8524 2.7413 2.6987 2.6408 17 4.44513 3.5516 3.1596 2.9277 2.7413 2.6432 2.4471 19 4.3807 3.5219 3.1274 2.8951 2.7401 2.6933 2.5480 2.4471 20 4.3212 3.4908 2.9671 2.7109 2.5990 2.5140 2.4471 21 4.3248 3.0725 2.8401 2.6848 2.5727 2.4408 22 4.2797 3.4028 2.7957	=		4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962
13 4,6672 3,8056 3,4105 3,1791 3,0254 2,9153 2,8321 2,7669 14 4,6001 3,7389 3,3439 3,1122 2,9582 2,8477 2,7642 2,6987 15 4,5431 3,6823 3,2874 3,0556 2,9013 2,7905 2,7066 2,6408 16 4,4940 3,6337 3,2389 3,0669 2,8524 2,7413 2,6572 2,5911 17 4,4513 3,5316 3,1588 2,9647 2,8100 2,6987 2,6143 2,5480 20 4,43807 3,5219 3,1274 2,8951 2,7401 2,6987 2,6132 2,5410 21 4,43807 3,6984 2,8641 2,7401 2,6843 2,5446 2,3663 22 4,3099 3,4454 3,0491 2,8617 2,6613 2,5435 2,4406 23 4,3099 3,4454 3,0491 2,8617 2,6613 2,5491 2,4406 24	:re		4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964
14 4,6001 3.7389 3.3439 3.1122 2.9582 2.8477 2.7642 2.6987 15 4,5431 3,6823 3.2874 3,0556 2.9013 2.7905 2.7066 2,6408 16 4,4940 3,6337 3,2389 3,0069 2,8524 2.7413 2,6572 2,5911 17 4,44513 3,5346 3,1968 2,9647 2,8100 2,6987 2,6143 2,5480 19 4,44139 3,5346 3,1594 2,9647 2,8100 2,6987 2,6143 2,5480 20 4,4139 3,5219 3,1274 2,8951 2,7401 2,6837 2,6478 2,5476 2,4768 21 4,438 3,4084 2,8641 2,7109 2,5990 2,5140 2,4471 22 4,3208 3,4084 2,8641 2,7401 2,6283 2,3405 23 4,2293 3,468 3,0491 2,817 2,6283 2,4436 24 4,2294	<u>"</u> t t	,,,	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144
15 4.5431 3.6823 3.2874 3.0556 2.9013 2.7905 2.7066 2.6408 16 4.4940 3.6337 3.2884 3.0669 2.8524 2.7413 2.6572 2.5911 17 4.4513 3.5316 3.1968 2.9647 2.8100 2.6987 2.6143 2.5480 18 4.4139 3.5516 3.1599 2.9277 2.7729 2.6613 2.5435 2.4768 20 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 2.5435 2.4768 21 4.3807 3.668 3.0725 2.8401 2.6848 2.5727 2.4876 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5727 2.4876 2.4205 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5727 2.4422 2.3748 24 4.2793 3.4028 2.7763 2.5082 2.4024 2.4422 2.3468	0 8		4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458
16 4,4940 3,6337 3,2389 3,0069 2,8524 2,7413 2,6572 2,5911 17 4,4513 3,5915 3,1968 2,9647 2,8100 2,6987 2,6143 2,5480 18 4,4513 3,5246 3,1599 2,9277 2,7129 2,6987 2,6143 2,5480 20 4,3807 3,5219 3,1274 2,8951 2,7109 2,6987 2,6143 2,5480 21 4,3807 3,5219 3,1274 2,8951 2,7401 2,6983 2,5435 2,4768 22 4,3009 3,4434 3,0994 2,8661 2,7109 2,5990 2,5140 2,4471 23 4,3049 3,0280 2,7955 2,6400 2,5327 2,4421 2,4471 24 4,2793 3,4434 3,0984 2,7643 2,5641 2,4438 2,3468 2,3468 24 4,2793 3,4434 3,088 2,7763 2,6400 2,5443 2,348	96		4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876
17 4.4513 3.5915 3.1968 2.9647 2.8100 2.6987 2.6143 2.5480 18 4.4139 3.5546 3.1599 2.9277 2.7729 2.6613 2.5435 2.5480 20 4.3807 3.5219 3.1274 2.8951 2.7729 2.6613 2.5767 2.5102 21 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 2.5435 2.4768 22 4.3009 3.4434 3.0491 2.8661 2.7109 2.5990 2.5140 2.4716 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5777 2.4287 2.3065 24 4.2793 3.4221 3.0280 2.7763 2.6037 2.5082 2.4226 2.3748 25 4.2417 3.3852 2.9912 2.7783 2.6077 2.5082 2.4226 2.3363 26 4.2252 3.3690 2.9726 2.7426 2.5888 2.4741 2.3883 <th>3u</th> <th></th> <th>4.4940</th> <th>3.6337</th> <th>3.2389</th> <th>3.0069</th> <th>2.8524</th> <th>2.7413</th> <th>2.6572</th> <th>2.5911</th> <th>2.5377</th>	3u		4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377
18 4.4139 3.5546 3.1599 2.9277 2.7729 2.6613 2.5767 2.5102 20 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 2.5435 2.4768 20 4.3807 3.5219 3.1274 2.8961 2.7401 2.6283 2.5435 2.4768 21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5496 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6400 2.5491 2.4876 2.4205 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4628 2.3365 24 4.2597 3.4028 2.7763 2.6007 2.5082 2.4226 2.3371 25 4.2417 3.3852 2.9912 2.7763 2.6007 2.4491 2.3463 2.3148 27 4.2252 3.3690 2.9752 2.7426 2.5868 2.4471 2.3883 2.2913	:: G	-	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943
19 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 2.5435 2.4768 20 4.3512 3.4928 3.0984 2.8661 2.7109 2.5990 2.5140 2.4471 21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5727 2.4876 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5727 2.4876 2.4471 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2793 3.4028 2.7763 2.6030 2.4904 2.4422 2.3748 25 4.2217 3.3852 2.9912 2.7783 2.6030 2.4904 2.4226 2.3551 26 4.2252 3.3690 2.9723 2.7426 2.5868 2.4741 2.3833 2.3053 28 4.1960 3.3404 2.9644 2.77426 2.5368 2.4453 2.3463 2.2463 <th><u>≃</u> . ق</th> <th>~</th> <th>4.4139</th> <th>3.5546</th> <th>3.1599</th> <th>2.9277</th> <th>2.7729</th> <th>2.6613</th> <th>2.5767</th> <th>2.5102</th> <th>2.4563</th>	<u>≃</u> . ق	~	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563
20 4.3512 3.4928 3.0984 2.8661 2.7109 2.5990 2.5140 2.4471 21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5727 2.4876 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5491 2.4638 2.3965 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2597 3.4028 2.7763 2.6207 2.582 2.4226 2.3748 25 4.2257 3.4028 2.7763 2.6207 2.4904 2.4047 2.3371 26 4.2257 3.3852 2.9912 2.7786 2.588 2.4741 2.3883 2.3205 27 4.2100 3.3541 2.9467 2.7141 2.581 2.4453 2.363 2.2913 29 4.1830 3.3158 2.9223 2.6896 2.534 2.4324 2.3463 2.2183		^	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227
21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5727 2.4876 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5491 2.4638 2.3965 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2597 3.4028 3.0088 2.7763 2.6207 2.5082 2.4226 2.3551 25 4.2417 3.3852 2.9912 2.7763 2.6030 2.4904 2.4404 2.3478 26 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3053 27 4.1960 3.3541 2.9604 2.7778 2.5719 2.4591 2.3732 2.2913 28 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3463 2.2783 29 4.1830 3.3277 2.9340 2.7014 2.5454 2.4205 2.3490 <th></th> <th>_</th> <th>4.3512</th> <th>3.4928</th> <th>3.0984</th> <th>2.8661</th> <th>2.7109</th> <th>2.5990</th> <th>2.5140</th> <th>2.4471</th> <th>2.3928</th>		_	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928
22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5491 2.4638 2.3965 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2597 3.4028 3.0088 2.7763 2.6207 2.5082 2.4226 2.3748 25 4.2417 3.3852 2.9912 2.7587 2.6030 2.4904 2.4047 2.3371 26 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3205 27 4.2100 3.3541 2.9604 2.7278 2.5719 2.4591 2.3732 2.3053 28 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3593 2.2913 29 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3343 2.2662 20 4.0012 3.3504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 20 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 2.9340 2.3719 2.3719 2.2899 2.1750 2.0868 2.0164 2.9400 2.3415 2.957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384		_	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660
23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2597 3.4028 3.0088 2.7763 2.6207 2.5082 2.4226 2.3551 25 4.2597 3.4028 3.0088 2.7763 2.6030 2.4904 2.4047 2.3371 26 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3205 27 4.2100 3.3541 2.9604 2.7278 2.5719 2.4591 2.3732 2.3053 28 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3593 2.2913 29 4.1830 3.3277 2.9340 2.7014 2.5454 2.4453 2.3463 2.2783 40 4.0847 3.2317 2.9340 2.7014 2.5454 2.4453 2.3463 2.2490 2.1802 40 4.0847 3.2317 2.8387 2.6060 2.4495 2.3463 <th></th> <th>63</th> <th>4.3009</th> <th>3,4434</th> <th>3.0491</th> <th>2.8167</th> <th>2.6613</th> <th>2.5491</th> <th>2.4638</th> <th>2.3965</th> <th>2.3419</th>		63	4.3009	3,4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419
24 4.2597 3.4028 2.7763 2.6207 2.5082 2.4226 2.3551 25 4.2417 3.3852 2.9912 2.7587 2.6030 2.4904 2.4047 2.3371 26 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3205 27 4.2100 3.3541 2.9604 2.7278 2.5719 2.4591 2.3732 2.3053 28 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3593 2.2913 29 4.1830 3.3277 2.9340 2.7014 2.5454 2.4324 2.3463 2.2783 30 4.1709 3.3158 2.9223 2.6896 2.5336 2.4329 2.2490 2.1802 40 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 20 3.3201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 20 3.38415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384		~	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201
4.2417 3.3852 2.9912 2.7587 2.6030 2.4904 2.4047 2.3371 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3205 4.2100 3.3541 2.9604 2.7278 2.5719 2.4591 2.3732 2.3053 4.1960 3.3404 2.9467 2.7141 2.5581 2.4591 2.3732 2.2913 4.1830 3.3277 2.9340 2.7014 2.5454 2.4453 2.3593 2.2913 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3463 2.2662 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384		**	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002
4,2252 3,3690 2,9752 2,7426 2,5868 2,4741 2,3883 2,3205 4,2100 3,3541 2,9604 2,7278 2,5719 2,4591 2,3732 2,3053 4,1960 3,3404 2,9467 2,7141 2,5581 2,4453 2,3593 2,2913 4,1830 3,3277 2,9340 2,7014 2,5454 2,4324 2,3463 2,2783 4,1709 3,3158 2,9223 2,6896 2,5336 2,4205 2,3463 2,1802 4,0847 3,2317 2,8387 2,6060 2,4495 2,3359 2,2490 2,1802 4,0847 3,1504 2,7581 2,5252 2,3683 2,2541 2,1665 2,0970 3,9201 3,0718 2,6802 2,4472 2,2899 2,1750 2,0868 2,0164 3,8415 2,9957 2,6049 2,3719 2,2141 2,0986 2,0096 1,9384	25	10	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821
4,2100 3,3541 2,9604 2,7278 2,5719 2,4591 2,3732 2,3053 4,1960 3,3404 2,9467 2,7141 2,5581 2,4453 2,3593 2,2913 4,1830 3,3277 2,9340 2,7014 2,5584 2,4324 2,3463 2,2783 4,1709 3,3158 2,9223 2,6896 2,5336 2,4205 2,3343 2,2662 4,0847 3,2317 2,8387 2,6060 2,4495 2,3359 2,2490 2,1802 4,0012 3,1504 2,7581 2,5252 2,3683 2,2541 2,1665 2,0970 3,9201 3,0718 2,6802 2,4472 2,2899 2,1750 2,0868 2,0164 3,8415 2,9957 2,6049 2,3719 2,2141 2,0986 2,0096 1,9384	2	100	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655
4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3593 2.2913 4.1830 3.3277 2.9340 2.7014 2.5454 2.4324 2.3463 2.2783 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3343 2.2662 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384	2.7	_	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501
4.1830 3.3277 2.9340 2.7014 2.5454 2.4324 2.3463 2.2783 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3343 2.2662 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384	25	00	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360
4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3343 2.2662 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384	22	0	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229
4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384	3	-	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107
4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384	4	-	4.0847	3.2317	2.8387	2.6060	2,4495	2.3359	2.2490	2.1802	2.1240
3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384	3	0	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401
3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384	121	0	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588
	8	_	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799

Cumulative Distribution Function, F(z), of the Standard Normal Distribution Table

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
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Cumulative Distribution Function, F(z), of the Standard Normal Distribution Table