

Name:

USC ID:

Notes:

- Write your name and ID number in the spaces above.
- No books, cell phones or other notes are permitted. Only one letter size cheat sheet (back and front) and a calculator are allowed.
- Problems are not sorted in terms of difficulty. Please avoid guess work and long and irrelevant answers.
- Show all your work and your final answer. Simplify your answer as much as you can.
- Open your exam only when you are instructed to do so.
- The exam has 5 questions, 9 pages, and 20 points extra credit. However, your grade cannot exceed 100/100.

Problem	Score	Earned
1	25	
2	30	
3	25	
4	20	
5	20	
Total	120	

1. For the simple regression model

$$Y = \beta_0 + \beta_1 X + \epsilon$$

the R^2 was calculated to be 25%, from a sample size of $n = 15$. Assuming ϵ is a zero-mean Gaussian, determine whether or not β_1 is statistically significant, when $\alpha = 0.01$. (Important note: you may feel that the instructor has not provided enough information to solve this problem, but that is not correct. Everything you need to solve this problem is on this exam.)

2. In a classification problem with three classes and three features, the distribution of each feature in each class is Gaussian. The mean of feature j in class k is $\mu_{jk} = jk$ and the standard deviation of feature j in class k is $\sigma_{jk} = k$, when $j, k \in \{1, 2, 3\}$. If the Naïve Bayes assumption holds and the classes have equal prior probabilities, in what class the feature vector $(X_1, X_2, X_3) = (1, 5, 9)$ is classified?

3. In an unusual logistic regression problem,

$$\Pr(Y = 1|X_1, X_2) = \frac{X_1^2 X_2^2}{1 + X_1^2 X_2^2}$$

- (a) Write down the model for the odds of $Y = 1$ given X_1 and X_2 .
- (b) Determine the equation for the decision boundary between classes $Y = 1$ and $Y = 0$ and sketch it in the X_1, X_2 plane. Show the regions for each class.

4. Consider the training set $\{1, 2, 3\}$. There are 10 *distinct* bootstrap samples with the same size that can be drawn from this training set. (For example, $\{1, 2, 2\}$ is not distinct from $\{2, 2, 1\}$). Using those bootstrap samples, build an 80% bootstrap confidence interval for the mean.

5. Consider the one-dimensional data set shown below.

- (a) Classify the test data points $x_1 = 5.0, y_1 = +$ and $x_2 = 4.0, y_2 = -$ according to their 1-, 3-, 5-, and 9-nearest neighbors (using majority vote). Each point is excluded from its nearest neighbors.
- (b) Which k or k 's yield the best test results?

x	0.5	3.0	4.5	4.6	4.9	5.2	5.3	5.5	7.0	9.5
y	−	−	+	+	+	−	−	+	−	−

Scratch paper

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F Table for $\alpha = 0.01$

	DF1	$\alpha = 0.01$																	
DF2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	Inf
1	4052.2	4999.5	5403.4	5624.6	5763.7	5859	5928.4	5981.1	6022.5	6055.8	6106.3	6157.3	6208.7	6234.6	6260.6	6286.8	6313	6339.4	6365.9
2	98.503	99	99.166	99.249	99.299	99.333	99.356	99.374	99.388	99.399	99.416	99.433	99.449	99.458	99.466	99.474	99.482	99.491	99.499
3	34.116	30.817	29.457	28.71	28.237	27.911	27.672	27.489	27.345	27.229	27.052	26.872	26.69	26.598	26.505	26.411	26.316	26.221	26.129
4	21.198	18	16.694	15.977	15.522	15.207	14.976	14.799	14.659	14.546	14.374	14.198	14.02	13.929	13.838	13.745	13.652	13.558	13.463
5	16.258	13.274	12.06	11.392	10.967	10.672	10.456	10.289	10.158	10.051	9.888	9.722	9.553	9.466	9.379	9.291	9.202	9.112	9.02
6	13.745	10.925	9.78	9.148	8.746	8.466	8.26	8.102	7.976	7.874	7.718	7.559	7.396	7.313	7.229	7.143	7.057	6.969	6.88
7	12.246	9.547	8.451	7.847	7.46	7.191	6.993	6.84	6.719	6.62	6.469	6.314	6.155	6.074	5.992	5.908	5.824	5.737	5.65
8	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.667	5.515	5.359	5.279	5.198	5.116	5.032	4.946	4.859
9	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.111	4.962	4.808	4.729	4.649	4.567	4.483	4.398	4.31
10	10.044	7.559	6.552	5.994	5.636	5.386	5.2	5.057	4.942	4.849	4.706	4.558	4.405	4.327	4.247	4.165	4.082	3.996	3.909
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.397	4.251	4.099	4.021	3.941	3.86	3.776	3.69	3.60
12	9.33	6.927	5.953	5.412	5.064	4.821	4.64	4.499	4.388	4.296	4.155	4.01	3.858	3.78	3.701	3.619	3.535	3.449	3.36
13	9.074	6.701	5.739	5.205	4.862	4.62	4.441	4.302	4.191	4.1	3.96	3.815	3.665	3.587	3.507	3.425	3.341	3.255	3.16
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.14	4.03	3.939	3.8	3.656	3.505	3.427	3.348	3.266	3.181	3.094	3.00
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.666	3.522	3.372	3.294	3.214	3.132	3.047	2.959	2.86
16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.89	3.78	3.691	3.553	3.409	3.259	3.181	3.101	3.018	2.933	2.845	2.75
17	8.4	6.112	5.185	4.669	4.336	4.102	3.927	3.791	3.682	3.593	3.455	3.312	3.162	3.084	3.003	2.92	2.835	2.746	2.65
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508	3.371	3.227	3.077	2.999	2.919	2.835	2.749	2.66	2.56
19	8.185	5.926	5.01	4.5	4.171	3.939	3.765	3.631	3.523	3.434	3.297	3.153	3.003	2.925	2.844	2.761	2.674	2.584	2.48
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.231	3.088	2.938	2.859	2.778	2.695	2.608	2.517	2.42
21	8.017	5.78	4.874	4.369	4.042	3.812	3.64	3.506	3.398	3.31	3.173	3.03	2.88	2.801	2.72	2.636	2.548	2.457	2.36
22	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258	3.121	2.978	2.827	2.749	2.667	2.583	2.495	2.403	2.30
23	7.881	5.664	4.765	4.264	3.939	3.71	3.539	3.406	3.299	3.211	3.074	2.931	2.781	2.702	2.62	2.535	2.447	2.354	2.25
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168	3.032	2.889	2.738	2.659	2.577	2.492	2.403	2.31	2.21
25	7.77	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	2.993	2.85	2.699	2.62	2.538	2.453	2.364	2.27	2.16
26	7.721	5.526	4.637	4.14	3.818	3.591	3.421	3.288	3.182	3.094	2.958	2.815	2.664	2.585	2.503	2.417	2.327	2.233	2.13
27	7.677	5.488	4.601	4.106	3.785	3.558	3.388	3.256	3.149	3.062	2.926	2.783	2.632	2.552	2.47	2.384	2.294	2.198	2.09
28	7.636	5.453	4.568	4.074	3.754	3.528	3.358	3.226	3.12	3.032	2.896	2.753	2.602	2.522	2.44	2.354	2.263	2.167	2.06
29	7.598	5.42	4.538	4.045	3.725	3.499	3.33	3.198	3.092	3.005	2.868	2.726	2.574	2.495	2.412	2.325	2.234	2.138	2.03
30	7.562	5.39	4.51	4.018	3.699	3.473	3.304	3.173	3.067	2.979	2.843	2.7	2.549	2.469	2.386	2.299	2.208	2.111	2.00
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.665	2.522	2.369	2.288	2.203	2.114	2.019	1.917	1.80
60	7.077	4.977	4.126	3.649	3.339	3.119	2.953	2.823	2.718	2.632	2.496	2.352	2.198	2.115	2.028	1.936	1.836	1.726	1.60
120	6.851	4.787	3.949	3.48	3.174	2.956	2.792	2.663	2.559	2.472	2.336	2.192	2.035	1.95	1.86	1.763	1.656	1.533	1.38
Inf	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321	2.185	2.039	1.878	1.791	1.696	1.592	1.473	1.325	