10.5.4-3

EE23BTECH11023-ABHIGNYA GOGULA

Question:
A ladder has rungs 25cm apart. The rungs decrease uniformly in length from 45cm at the bottom to 25cm at the top. If the top and bottom rungs are 2 and 1/2 meter apart, what is the length of wood required for the rungs?

Parameter	Value
n	10
$x(0)$	45
$x(10)$	25

TABLE 0
DESCRIPTION OF PARAMETERS

SOLUTION

Total number of rungs:

$$\frac{\langle\frac{5}{2}\rangle 100}{25} + 1 = 11 \tag{1}$$

As the length of rungs decreases uniformly, it is in A.P:

$$x(n) = \langle x(0) + nd\rangle u(n) \tag{2}$$
$$25 = 45 + 10d \tag{3}$$
$$d = -2 \tag{4}$$

The sum of the lengths of all rungs gives the total length of wood required. So, finding the sum of A.P using Z-transform:

$$X(z) = \frac{45}{1 - z^{-1}} + \frac{-2z^{-1}}{\langle 1 - z^{-1}\rangle^2} \tag{5}$$
$$y(n) = x(n) * u(n) \tag{6}$$
$$Y(z) = X(z)U(z) \tag{7}$$
$$Y(z) = \frac{45}{\langle 1 - z^{-1}\rangle^2} + \frac{-2z^{-1}}{\langle 1 - z^{-1}\rangle^3} \tag{8}$$

Taking the inverse of the Z-transform using counter-integration:

$$y(n) = x(0)\langle\langle n + 1\rangle u(n)\rangle + \frac{d}{2}\langle n\langle n + 1\rangle u(n)\rangle \tag{9}$$
$$y(n) = \frac{n + 1}{2}\langle 2x(0) + nd\rangle u(n) \tag{10}$$
$$y(10) = 385 \tag{11}$$

The length of wood required for the rungs is 385 cm.