10.5.4-3

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EE23BTECH11023-ABHIGNYA GOGULA

Question:

A ladder has rungs 25cm apart. The rungs decrease uniformly in length from 45cm at the bottom to 25cm at the top. If the top and bottom rungs are 2 and 1/2 meter apart, what is the length of wood required for the rungs?

Parameter	Value
n	10
x(0)	45
x(10)	25
TABLE 0	

DESCRIPTION OF PARAMETERS

Solution

Total number of rungs:

$$\frac{\langle \frac{5}{2} \rangle 100}{25} + 1 = 11 \tag{1}$$

As the length of rungs decreases uniformly, it is in A.P:

$$x(n) = \langle x(0) + nd \rangle u(n) \tag{2}$$

$$25 = 45 + 10d \tag{3}$$

$$d = -2 \tag{4}$$

The sum of the lengths of all rungs gives the total length of wood required. So, finding the sum of A.P using Z-transform:

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{\langle 1 - z^{-1} \rangle^2}$$
 (5)

Taking the inverse of the Z-transform using counter-integration:

$$y(n) = x(0)\langle\langle n+1\rangle u(n)\rangle + \frac{d}{2}\langle n\langle n+1\rangle u(n)\rangle \qquad (6)$$

$$y(n) = \frac{n+1}{2} \langle 2x(0) + nd \rangle u(n)$$
 (7)

$$y(10) = 385$$
 (8)

The length of wood required for the rungs is 385 cm.