10.5.4-3

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EE23BTECH11023-ABHIGNYA GOGULA

Question:

A ladder has rungs 25cm apart. The rungs decrease uniformly in length from 45cm at the bottom to 25cm at the top. If the top and bottom rungs are 2 and 1/2 meter apart, what is the length of wood required for the rungs?

	parameter	value
	n	10
	x(0)	45
	x(10)	25
TADIE		

DESCRIPTION OF PARAMETERS

SOLUTION

Total number of rungs:

$$\frac{\langle \frac{5}{2} \rangle 100}{25} + 1 = 11 \tag{1}$$

As the length of rungs decreases uniformly, it is in A.P:

$$x(n) = \langle x(0) + nd \rangle u(n) \tag{2}$$

$$25 = 45 + 10d \tag{3}$$

$$d = -2 \tag{4}$$

The sum of the lengths of all rungs gives the total length of wood required. So, finding the sum of A.P using Z-transform:

$$X(z) = \frac{45}{1 - z^{-1}} + \frac{-2z^{-1}}{\langle 1 - z^{-1} \rangle^2}$$
 (5)

$$y(n) = x(n) * u(n)$$
 (6)

$$Y(z) = X(z)U(z) \tag{7}$$

$$Y(z) = \frac{45}{\langle 1 - z^{-1} \rangle^2} + \frac{-2z^{-1}}{\langle 1 - z^{-1} \rangle^3}$$
 (8)

Taking the inverse of the Z-transform using counterintegration

From (??),

$$y(10) = 45\langle\langle 10 + 1 \rangle\rangle + \frac{-2}{2}\langle 10\langle 10 + 1 \rangle\rangle \tag{9}$$

$$y(10) = 385 \tag{10}$$

The length of wood required for the rungs is 385 cm.