

Maths Assignment

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Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Parameter	Description
n	Number of terms in the G.P (positive even integer)
$x(0)$	first term in the G.P
r	common ratio in the G.P
$x(n)$	nth term in the G.P
$X(z)$	Z transform of X(n)

Let $x(0)$ denote the first term and r the common ratio. The sum of a geometric progression with n terms:

$$x(n) = x(0)r^n \quad (1)$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad (2)$$

$$S(z) = X(z)U(z) \quad (3)$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \quad (4)$$

$$= \frac{x(0)(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}})}{(r-1)} \quad (5)$$

The inverse of $S(z)$ is $s(n)$ which is:

$$s(n) = x(0)(\frac{r^{n+1} - 1}{r - 1})u(n) \quad (6)$$

The sum of terms in odd places:

$$X_o(z) = \frac{x(0)}{1 - r^2 z^{-1}} \quad (7)$$

$$S_o(z) = X_o(z)U(z) \quad (8)$$

$$= \frac{x(0)}{(1 - r^2 z^{-1})(1 - z^{-1})} \quad |z| > |r| \quad (9)$$

$$= \frac{x(0) \left(\frac{r}{1 - r^2 z^{-1}} - \frac{1}{1 - z^{-1}} \right)}{(r^2 - 1)} \quad (10)$$

The inverse of $S_o(z)$ is $s_o(n)$ which is:

$$s_o(n) = x(0) \left(\frac{r^{n+1} - 1}{r^2 - 1} \right) u(n) \quad (11)$$

Then from (6) and (11)

$$x(0) \left(\frac{r^{n+1} - 1}{r - 1} \right) u(n) = 5x(0) \left(\frac{r^{n+1} - 1}{r^2 - 1} \right) u(n) \quad (12)$$

$$r^2 - 5r + 4 = 0 \quad (13)$$

$$r = 1 \text{ or } r = 4 \quad (14)$$

$v(n) = x(2m)$, where m is an integer.

substitute $n = 2m$ into the z -transform of $x(n)$:

$$V(z) = \sum_{n=-\infty}^{\infty} v(n)z^{-n} \quad (15)$$

$$= \sum_{n=-\infty}^{\infty} x(2m)z^{-2m} \quad (16)$$

$$V(z) = \sum_{m=-\infty}^{\infty} x(2m)z^{-2m} \quad (17)$$

$$= \sum_{m=-\infty}^{\infty} x(n)z^{-n} \Big|_{n=2m} \quad (18)$$

$$= \sum_{m=-\infty}^{\infty} X(z) \Big|_{z^{-n} \rightarrow z^{-2m}} \quad (19)$$

$$= \sum_{m=-\infty}^{\infty} X(z)z^{-2m} \quad (20)$$

$V(z) = X(z)$ as $\sum_{m=-\infty}^{\infty} X(z)z^{-2m}$.

Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = x(0)^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \frac{1}{1 - x(0)z^{-1}} \quad (21)$$

$$X(z) = \frac{1}{1 - x(0)z^{-1}} = \frac{A}{1 - x(0)z^{-1}} \quad (22)$$

$$1 = A(1 - x(0)z^{-1}) \quad (23)$$

$$A = 1 \quad (24)$$

Therefore, the partial sum using $X(z)$ is $x(n) = x(0)^n u(n)$.