Maths Assignment

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Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Parameter	Description
n	Number of terms in the G.P (positive even integer)
$x(0)$	first term in the G.P
r	common ratio in the G.P
$x(n)$	nth term in the G.P
$X(z)$	Z transform of X(n)

Let $x(0)$ denote the first term and r the common ratio. The sum of a geometric progression with n terms:

$$x(n) = x(0)r^n \tag{1}$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \tag{2}$$

$$S(z) = X(z)U(z) \tag{3}$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \tag{4}$$

$$= \frac{x(0)(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}})}{(r-1)} \tag{5}$$

The inverse of S(z) is s(n) which is:

$$s(n) = x(0)(\frac{r^{n+1} - 1}{r - 1})u(n) \tag{6}$$

The sum of terms in odd places:

$$X_o(z) = \frac{x(0)}{1 - r^2 z^{-1}} \tag{7}$$

$$S_o(z) = X_o(z)U(z) \tag{8}$$

$$= \frac{x(0)}{(1 - r^2 z^{-1})(1 - z^{-1})} \quad |z| > |r| \tag{9}$$

$$= \frac{x(0)\left(\frac{r}{1-r^2z^{-1}} - \frac{1}{1-z^{-1}}\right)}{(r^2 - 1)} \tag{10}$$

The inverse of $S_o(z)$ is $s_o(n)$ which is:

$$s_o(n) = x(0)\left(\frac{r^{2n+2} - 1}{r^2 - 1}\right)u(n) \tag{11}$$

Then from (6) and (11)

$$x(0)(\frac{r^{n+1} - 1}{r - 1})u(n) = 5x(0)(\frac{r^{2n+2} - 1}{r^2 - 1})u(n) \tag{12}$$

$$5r^{2n+2} - r^{n+1} + r - r^{n+2} - 4 = 0 \tag{13}$$

$v(n) = x(2m)$, where m is an integer.
substitute $n = 2m$ into the z-transform of $x(n)$:

$$V(z) = \sum_{n=-\infty}^{\infty} v(n)z^{-n} \tag{14}$$

$$= \sum_{n=-\infty}^{\infty} x(2m)z^{-2m} \tag{15}$$

$$V(z) = \sum_{m=-\infty}^{\infty} x(2m)z^{-2m} \tag{16}$$

$$= \sum_{m=-\infty}^{\infty} x(n)z^{-n}\Bigg|_{n=2m} \tag{17}$$

$$= \sum_{m=-\infty}^{\infty} X(z)\Bigg|_{z^{-n}\to z^{-2m}} \tag{18}$$

$$= \sum_{m=-\infty}^{\infty} X(z)z^{-2m} \tag{19}$$

$V(z) = X(z)$ as $\sum_{m=-\infty}^{\infty} X(z)z^{-2m}$.

Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = x(0)^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \frac{1}{1 - x(0)z^{-1}} \tag{20}$$

$$X(z) = \frac{1}{1 - x(0)z^{-1}} = \frac{A}{1 - x(0)z^{-1}} \tag{21}$$

$$1 = A(1 - x(0)z^{-1}) \tag{22}$$

$$A = 1 \tag{23}$$

Therefore, the partial sum using $X(z)$ is $x(n) = x(0)^n u(n)$.