# Maths Assignment

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## Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

## Solution

Let $a$ denote the first term of the geometric progression and $r$ the common ratio. The sum of a geometric progression with $n$ terms can be calculated using the formula:

$$S_n = a\frac{r^n - 1}{r - 1}$$

The nth term $(x(n))$ in the series starting from $n = 0$ in the geometric progression is given by:

$$x(n) = a \cdot \left(\frac{5 + \sqrt{21}}{2}\right)^{\frac{n}{2}} \times (u(n))$$

The series starting from $n = 0$ in the geometric progression is represented as:

$$\sum_{n=0}^{\infty} x(n) = a + a \cdot r + a \cdot r^2 + \cdots = \frac{a}{1 - r} \times (u(n))$$

where $r = \left(\frac{5+\sqrt{21}}{2}\right)^{\frac{1}{2}}$

The sum of terms occupying odd places (i.e., at positions 1, 3, 5, ...) in a geometric progression can be represented as:

$$S_{\text{odd}} = a\frac{r^{(n/2)} - 1}{r - 1}$$

Given that the sum of all terms $(S_{2n})$ is 5 times the sum of terms occupying odd places $(S_{\text{odd}})$, we can set up the equation:

$$a\frac{r^{2n} - 1}{r - 1} = 5 \cdot a\frac{r^n - 1}{r - 1}$$

Simplifying by canceling out the common term $a$ and dividing both sides by $r-1$:

$$r^{2n} - 1 = 5 \cdot r^n - 5$$

$$r^{2n} - 5 \cdot r^n + 1 = 0$$

Let $x = r^n$, then the equation becomes a quadratic equation in terms of $x$:

$$x^2 - 5x + 1 = 0$$

Solving this quadratic equation for $x$, we can find $r$ as $x^{1/n}$. Using the quadratic formula:

$$x = \frac{5 \pm \sqrt{21}}{2}$$

Since $x = r^n$, $r = x^{1/n}$, and considering $n$ is a positive even number, we take the positive root:

$$r = \left(\frac{5 + \sqrt{21}}{2}\right)^{1/n}$$

This gives the common ratio $r$ in terms of $n$, the number of terms in the geometric progression. Let's assume a sequence $x(n)$ given by $x(n) = a^n u(n)$, where $a$ is a constant and $u(n)$ is the unit step function.

The Z-transform is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1 - az^{-1}}$$

This represents the Z-transform for the given sequence $x(n) = a^n u(n)$.

## Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = a^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \frac{1}{1 - az^{-1}}$$

To obtain the desired sum, let's perform the inverse Z-transform by expressing $X(z)$ in partial fractions:

$$X(z) = \frac{1}{1 - az^{-1}} = \frac{A}{1 - az^{-1}}$$

To find $A$, multiply both sides by the denominator:

$$1 = A(1 - az^{-1})$$

$$A = 1$$

Therefore, the partial sum using $X(z)$ is $x(n) = a^n u(n)$.

## Input Parameters

| Parameter | Description |
| --- | --- |
| $n$ | Number of terms in the G.P (positive even integer) |