Maths Assignment

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January 9, 2024

Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Let $x(0)$ denote the first term of the geometric progression and r the common ratio. The sum of a geometric progression with n terms can be calculated using the formula:

$$S_n = x(0)\frac{r^n - 1}{r - 1}$$

The nth term $(x(n))$ in the series starting from $n = 0$ in the geometric progression is given by:

$$x(n) = x(0)\left(\frac{5+\sqrt{21}}{2}\right)^{\frac{n}{2}}(u(n))$$

The series starting from $n = 0$ in the geometric progression is represented as:

$$\sum_{n=0}^{\infty} x(n) = x(0) + x(0)r + ar^2 + \cdots = \frac{x(0)}{1-r}(u(n))$$

where $r = \left(\frac{5+\sqrt{21}}{2}\right)^{\frac{1}{2}}$

The sum of terms occupying odd places (i.e., at positions 1, 3, 5, ...) in a geometric progression can be represented as:

$$S_{\text{odd}} = x(0)\frac{r^{(n/2)} - 1}{r - 1}$$

Given that the sum of all terms (S_{2n}) is 5 times the sum of terms occupying odd places (S_{odd}), we can set up the equation:

$$x(0)\frac{r^{2n} - 1}{r - 1} = 5 \cdot x(0)\frac{r^n - 1}{r - 1}$$

Simplifying by canceling out the common term $x(0)$ and dividing both sides by $r-1$:

$$r^{2n} - 1 = 5r^n - 5$$

$$r^{2n} - 5r^n + 1 = 0$$

Let $x = r^n$, then the equation becomes a quadratic equation in terms of x:

$$x^2 - 5x + 1 = 0$$

Solving this quadratic equation for x, we can find r as $x^{1/n}$. Using the quadratic formula:

$$x = \frac{5 \pm \sqrt{21}}{2}$$

Since $x = r^n$, $r = x^{1/n}$, and considering n is a positive even number, we take the positive root:

$$r = \left(\frac{5 + \sqrt{21}}{2}\right)^{1/n}$$

This gives the common ratio r in terms of n, the number of terms in the geometric progression. Let's assume a sequence $x(n)$ given by $x(n) = x(0)^n u(n)$, where $x(0)$ is a constant and $u(n)$ is the unit step function.

The Z-transform is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} (x(0)z^{-1})^n = \frac{1}{1 - x(0)z^{-1}}$$

This represents the Z-transform for the given sequence $x(n) = x(0)^n u(n)$.

Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = x(0)^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \frac{1}{1 - az^{-1}}$$

To obtain the desired sum, let's perform the inverse Z-transform by expressing $X(z)$ in partial fractions:

$$X(z) = \frac{1}{1 - x(0)z^{-1}} = \frac{A}{1 - az^{-1}}$$

To find A, multiply both sides by the denominator:

$$1 = A(1 - x(0)z^{-1})$$

$$A = 1$$

Therefore, the partial sum using $X(z)$ is $x(n) = x(0)^n u(n)$.

Input Parameters

Parameter	Description
n	Number of terms in the G.P (positive even integer)
$x(0)$	first term in the G.P
r	common ratio in the G.P
$x(n)$	nth term in the G.P
$X(z)$	Z transform of X(n)