Maths Assignment

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Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Parameter	Description
n	Number of terms in the G.P (positive even integer)
$x(0)$	first term in the G.P
r	common ratio in the G.P
$x(n)$	nth term in the G.P
$X(z)$	Z transform of X(n)

Let $x(0)$ denote the first term of the geometric progression and r the common ratio. The sum of a geometric progression with n terms can be calculated using

the formula:

$$S_n = x(0)\frac{r^n - 1}{r - 1} \tag{1}$$

$$X(z) = x(0)\left(\frac{1}{1 - rz^{-1}}\right) \tag{2}$$

$$X(z) = 5X_o(z) \tag{3}$$

$$X_o(z) = x(0)\left(\frac{1}{1 - r^2z^{-2}}\right) \tag{4}$$

$$x(0)\left(\frac{1}{1 - rz^{-1}}\right) = 5x(0)\left(\frac{1}{1 - r^2z^{-2}}\right) \tag{5}$$

$$\frac{1 - rz^{-1}}{1 - r^2z^{-2}} = 5\frac{1 - r^2z^{-2}}{1 - rz^{-1}} \tag{6}$$

$$(1 - r^2z^{-2}) = 5(1 - rz^{-1}) \tag{7}$$

$$1 - r^2z^{-2} = 5 - 5rz^{-1} \tag{8}$$

$$4 = 5rz^{-1} - r^2z^{-2} \tag{9}$$

$$4z^2 = 5rz - r^2 \tag{10}$$

$$r^2 - 5rz + 4z^2 = 0 \tag{11}$$

$$r = \frac{5z \pm \sqrt{25z^2 - 16z^2}}{2} \tag{12}$$

$$r = 4zorr = z \tag{13}$$

Let's assume a sequence $x(n)$ given by $x(n) = x(0)^n u(n)$, where $x(0)$ is a constant and $u(n)$ is the unit step function.

The Z-transform is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty}(x(0)z^{-1})^n = \frac{1}{1 - x(0)z^{-1}} \tag{14}$$

This represents the Z-transform for the given sequence $x(n) = x(0)^n u(n)$.

Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = x(0)^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \frac{1}{1 - az^{-1}} \tag{15}$$

To obtain the desired sum, let's perform the inverse Z-transform by expressing $X(z)$ in partial fractions:

$$X(z) = \frac{1}{1 - x(0)z^{-1}} = \frac{A}{1 - az^{-1}} \tag{16}$$

To find A, multiply both sides by the denominator:

$$1 = A(1 - x(0)z^{-1}) \tag{17}$$

$$A = 1 \tag{18}$$

Therefore, the partial sum using $X(z)$ is $x(n) = x(0)^n u(n)$.