Maths Assignment

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January 20, 2024

Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Parameter	Description
n	Number of terms in the G.P (positive even integer)
$x(0)$	first term in the G.P
r	common ratio in the G.P
$x(n)$	nth term in the G.P
$X(z)$	Z transform of X(n)

Let $x(0)$ denote the first term of the geometric progression and r the common ratio. The sum of a geometric progression with n terms can be calculated using the formula:

$$S_n = x(0)\frac{r^n - 1}{r - 1} \tag{1}$$

$$X(z) = x(0)\left(\frac{1}{1 - rz^{-1}}\right) \tag{2}$$

$$X(z) = 5X_o(z) \tag{3}$$

$$X_o(z) = x(0)\left(\frac{1}{1 - r^2z^{-2}}\right) \tag{4}$$

$$\tag{5}$$

By the defination of Z transform

$$X(z) = \sum_{n=-\infty}^{\infty} [x(0)r^n u(n)]z^{-n} \tag{6}$$

as we need only odd terms that is nth terms,let n=2m-1 and m is positive integer.

$$\sum_{i=0}^{\infty}[x(0)r^{2i}z^{-2i}] \tag{7}$$

As this is in infinite GP with common ratio r^2z^{-2}
$X_o(z) = x(0)\left(\frac{1}{1-r^2z^{-2}}\right)$

$$x(0)\left(\frac{1}{1-rz^{-1}}\right) = 5x(0)\left(\frac{1}{1-r^2z^{-2}}\right) \tag{8}$$
$$\frac{1-rz^{-1}}{1-r^2z^{-2}} = 5\frac{1-r^2z^{-2}}{1-rz^{-1}} \tag{9}$$
$$(1-r^2z^{-2}) = 5(1-rz^{-1}) \tag{10}$$
$$1-r^2z^{-2} = 5-5rz^{-1} \tag{11}$$
$$4 = 5rz^{-1} - r^2z^{-2} \tag{12}$$
$$4z^2 = 5rz - r^2 \tag{13}$$
$$r^2 - 5rz + 4z^2 = 0 \tag{14}$$
$$r = \frac{5z \pm \sqrt{25z^2 - 16z^2}}{2} \tag{15}$$
$$r = 4zorr = z \tag{16}$$

Let's assume a sequence $x(n)$ given by $x(n) = x(0)^n u(n)$, where $x(0)$ is a constant and $u(n)$ is the unit step function.

The Z-transform is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty}(x(0)z^{-1})^n = \frac{1}{1-x(0)z^{-1}} \tag{17}$$

This represents the Z-transform for the given sequence $x(n) = x(0)^n u(n)$. If $X(z)$ is the z-transform of $x(n)$, then $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$. Now, let $v(n) = x(2m)$, where m is an integer.

To express $V(z)$ in terms of $X(z)$, we can substitute $n = 2m$ into the z-transform of $x(n)$:

$$V(z) = \sum_{n=-\infty}^{\infty} v(n)z^{-n} \tag{18}$$
$$= \sum_{n=-\infty}^{\infty} x(2m)z^{-2m} \tag{19}$$

Now, we can factor out the constant terms and rewrite it in terms of $X(z)$:

$$V(z) = \sum_{m=-\infty}^{\infty} x(2m)z^{-2m} \tag{20}$$
$$= \sum_{m=-\infty}^{\infty} x(n)z^{-n}\Big|_{n=2m} \tag{21}$$
$$= \sum_{m=-\infty}^{\infty} X(z)\Big|_{z^{-n}\to z^{-2m}} \tag{22}$$
$$= \sum_{m=-\infty}^{\infty} X(z)z^{-2m} \tag{23}$$

So, $V(z)$ can be expressed in terms of $X(z)$ as $\sum_{m=-\infty}^{\infty} X(z)z^{-2m}$.

Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = x(0)^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \frac{1}{1 - x(0)z^{-1}} \tag{24}$$

To obtain the desired sum, let's perform the inverse Z-transform by expressing $X(z)$ in partial fractions:

$$X(z) = \frac{1}{1 - x(0)z^{-1}} = \frac{A}{1 - x(0)z^{-1}} \tag{25}$$

To find A, multiply both sides by the denominator:

$$1 = A(1 - x(0)z^{-1}) \tag{26}$$
$$A = 1 \tag{27}$$

Therefore, the partial sum using $X(z)$ is $x(n) = x(0)^n u(n)$.