Maths Assignment

Abhignya Gogula
EE23BTECH11023

January 29, 2024

Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Parameter	Description
n	Number of terms in the G.P (positive even integer)
$x(0)$	first term in the G.P
r	common ratio in the G.P
$x(n)$	nth term in the G.P
$X(z)$	Z transform of X(n)

$$x(n) = x(0)r^n \tag{1}$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \tag{2}$$

$$S(z) = X(z)U(z) \tag{3}$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \tag{4}$$

$$= \frac{x(0)(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}})}{(r - 1)} \tag{5}$$

The inverse of S(z) is s(n) which is:

$$s(n) = x(0)(\frac{r^{n+1} - 1}{r - 1})u(n) \tag{6}$$

$v(n) = x(2m)$,$v(0) = x(0)$, where m is an integer.

$$\begin{aligned}
v(2m) &= v(0)r^{2m} && (7)\\
V(z) &= \sum_{n=-\infty}^{\infty} v(n)z^{-n} && (8)\\
X(z) &= \sum_{m=-\infty}^{\infty} v(2m)z^{-2m} && (9)\\
&= \sum_{m=-\infty}^{\infty} v(n)z^{-n}\Big|_{n=2m} && (10)\\
&= \sum_{m=-\infty}^{\infty} V(z)\Big|_{z^{-n}\to z^{-2m}} && (11)\\
&= \sum_{m=-\infty}^{\infty} V(z)z^{-2m} && (12)\\
X(z) &= V(z) \quad as \quad \sum_{m=-\infty}^{\infty} V(z)z^{-2m}. && (13)
\end{aligned}$$

The sum of terms in odd places:

$$\begin{aligned}
V_o(z) &= \frac{v(0)}{1-r^2z^{-1}} && (14)\\
S_o(z) &= V_o(z)U(z) && (15)\\
&= \frac{v(0)}{(1-r^2z^{-1})(1-z^{-1})} \quad |z| > |r| && (16)\\
&= \frac{v(0)\left(\frac{r}{1-r^2z^{-1}} - \frac{1}{1-z^{-1}}\right)}{(r^2-1)} && (17)
\end{aligned}$$

The inverse of $S_o(z)$ is $s_o(n)$ which is:

$$s_o(n) = v(0)\left(\frac{r^{n+1}-1}{r^2-1}\right)u(n) \tag{18}$$

Then from (6) and (18)

$$\begin{aligned}
v(0)(\frac{r^{n+1}-1}{r-1})u(n) = 5v(0)(\frac{r^{n+1}-1}{r^2-1})u(n) && (19)\\
r^2 - 5r + 4 = 0 && (20)\\
r = 1 \quad \text{or} \quad r = 4 && (21)
\end{aligned}$$

Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = x(0)^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \frac{1}{1 - x(0)z^{-1}} \tag{22}$$

$$X(z) = \frac{1}{1 - x(0)z^{-1}} = \frac{A}{1 - x(0)z^{-1}} \tag{23}$$

$$1 = A(1 - x(0)z^{-1}) \tag{24}$$

$$A = 1 \tag{25}$$

Therefore, the partial sum using $X(z)$ is $x(n) = x(0)^n u(n)$.