Maths Assignment

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January 6, 2024

Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Let a denote the first term of the geometric progression and r the common ratio. The sum of a geometric progression with n terms can be calculated using the formula:

$$S_n = a\frac{r^n - 1}{r - 1}$$

The sum of terms occupying odd places (i.e., at positions 1, 3, 5, ...) in a geometric progression can be represented as:

$$S_{\text{odd}} = a\frac{r^{(n/2)} - 1}{r - 1}$$

Given that the sum of all terms (S_{2n}) is 5 times the sum of terms occupying odd places (S_{odd}), we can set up the equation:

$$a\frac{r^{2n} - 1}{r - 1} = 5 \cdot a\frac{r^n - 1}{r - 1}$$

Simplifying by canceling out the common term a and dividing both sides by $r - 1$:

$$r^{2n} - 1 = 5 \cdot r^n - 5$$

$$r^{2n} - 5 \cdot r^n + 1 = 0$$

Let $x = r^n$, then the equation becomes a quadratic equation in terms of x:

$$x^2 - 5x + 1 = 0$$

Solving this quadratic equation for x, we can find r as $x^{1/n}$. Using the quadratic formula:

$$x = \frac{5 \pm \sqrt{21}}{2}$$

Since $x = r^n$, $r = x^{1/n}$, and considering n is a positive even number, we take the positive root:

$$r = \left(\frac{5 + \sqrt{21}}{2}\right)^{1/n}$$

This gives the common ratio r in terms of n, the number of terms in the geometric progression.

Input Parameters

Parameter	Description
n	Number of terms in the G.P (positive even integer)