Maths Assignment

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Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Let $x(0)$ denote the first term of the geometric progression and r the common ratio. The sum of a geometric progression with n terms can be calculated using the formula:

$$S_n = x(0)\frac{r^n - 1}{r - 1}$$

The given equations are as follows:

$$S_n = x(0)\frac{r^n - 1}{r - 1}, \quad S_{\text{odd}} = x(0)(1 + r^2 + r^4 + \ldots + r^{n-2})$$

The condition is: $S_n = 5S_{\text{odd}}$.

Now, we'll express these equations in terms of Z-transforms:

$$X(z) = x(0)\frac{1 - r^n z^{-1}}{1 - rz^{-1}}, \quad X_{\text{odd}}(z) = x(0)(1+r^2 z^{-2}+r^4 z^{-4}+\ldots+r^{n-2} z^{-(n-2)})$$

To equate S_n and $5S_{\text{odd}}$ using Z-transforms, we manipulate the Z-transforms to find a relationship involving z and the common ratio r:

$$x(0)\frac{r^n - 1}{r - 1} = 5x(0)(1 + r^2 z^{-2} + r^4 z^{-4} + \ldots + r^{n-2} z^{-(n-2)})$$

Simplifying further:

$$\frac{r^n - 1}{r - 1} = 5(1 + r^2 z^{-2} + r^4 z^{-4} + \ldots + r^{n-2} z^{-(n-2)})$$

By considering the geometric series in the parentheses:

$$1 + r^2 z^{-2} + r^4 z^{-4} + \ldots + r^{n-2} z^{-(n-2)} = \frac{1 - r^n z^{-n}}{1 - r^2 z^{-2}}$$

Substituting this back:

$$\frac{r^n - 1}{r - 1} = 5\frac{1 - r^n z^{-n}}{1 - r^2 z^{-2}}$$

Expanding and rearranging:

$$r^n - 5r + r^{n+2} z^{-2} - r^2 z^{-2} + 6 = 0$$

Let's assume a sequence $x(n)$ given by $x(n) = x(0)^n u(n)$, where $x(0)$ is a constant and $u(n)$ is the unit step function.

The Z-transform is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} (x(0) z^{-1})^n = \frac{1}{1 - x(0) z^{-1}}$$

This represents the Z-transform for the given sequence $x(n) = x(0)^n u(n)$.

Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = x(0)^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \frac{1}{1 - a z^{-1}}$$

To obtain the desired sum, let's perform the inverse Z-transform by expressing $X(z)$ in partial fractions:

$$X(z) = \frac{1}{1 - x(0) z^{-1}} = \frac{A}{1 - a z^{-1}}$$

To find A, multiply both sides by the denominator:

$$1 = A(1 - x(0) z^{-1})$$

$$A = 1$$

Therefore, the partial sum using $X(z)$ is $x(n) = x(0)^n u(n)$.

Parameter	Description
n	Number of terms in the G.P (positive even integer)
$x(0)$	first term in the G.P
r	common ratio in the G.P
$x(n)$	nth term in the G.P
$X(z)$	Z transform of X(n)