

Maths Assignment

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Problem Statement

A G.P consists of an even number of terms. If the sum of all terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Solution

Parameter	Description
n	Number of terms in the G.P (positive even integer)
$x(0)$	first term in the G.P
r	common ratio in the G.P
$x(n)$	nth term in the G.P
$X(z)$	Z transform of X(n)

Let $x(0)$ denote the first term of the geometric progression and r the common ratio. The sum of a geometric progression with n terms can be calculated using the formula:

$$S_n = x(0) \frac{r^n - 1}{r - 1} \quad (1)$$

$$X(z) = x(0) \left(\frac{1}{1 - rz^{-1}} \right) \quad (2)$$

$$X(z) = 5X_o(z) \quad (3)$$

$$X_o(z) = x(0) \left(\frac{1}{1 - r^2 z^{-2}} \right) \quad (4)$$

$$(5)$$

By the definition of Z transform

$$X(z) = \sum_{n=-\infty}^{\infty} [x[n]r^n u(n)]z^{-n} \quad (6)$$

as we need only odd terms

$$x(0)r^0u(0)z^{-0} + x(0)r^2u(2)z^{-2} + \dots \quad (7)$$

As this is in infinite GP with common ratio r^2z^{-2}

$$X_o(z) = x(0) \left(\frac{1}{1-r^2z^{-2}} \right)$$

$$x(0) \left(\frac{1}{1-rz^{-1}} \right) = 5x(0) \left(\frac{1}{1-r^2z^{-2}} \right) \quad (8)$$

$$\frac{1-rz^{-1}}{1-r^2z^{-2}} = 5 \frac{1-r^2z^{-2}}{1-rz^{-1}} \quad (9)$$

$$(1-r^2z^{-2}) = 5(1-rz^{-1}) \quad (10)$$

$$1-r^2z^{-2} = 5-5rz^{-1} \quad (11)$$

$$4 = 5rz^{-1} - r^2z^{-2} \quad (12)$$

$$4z^2 = 5rz - r^2 \quad (13)$$

$$r^2 - 5rz + 4z^2 = 0 \quad (14)$$

$$r = \frac{5z \pm \sqrt{25z^2 - 16z^2}}{2} \quad (15)$$

$$r = 4z \text{ or } r = z \quad (16)$$

Let's assume a sequence $x(n)$ given by $x(n) = x(0)^n u(n)$, where $x(0)$ is a constant and $u(n)$ is the unit step function.

The Z-transform is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} (x(0)z^{-1})^n = \frac{1}{1-x(0)z^{-1}} \quad (17)$$

This represents the Z-transform for the given sequence $x(n) = x(0)^n u(n)$.

Desired Sum Using $X(z)$

The Z-transform of the sequence $x(n) = x(0)^n u(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = \frac{1}{1-az^{-1}} \quad (18)$$

To obtain the desired sum, let's perform the inverse Z-transform by expressing $X(z)$ in partial fractions:

$$X(z) = \frac{1}{1-x(0)z^{-1}} = \frac{A}{1-az^{-1}} \quad (19)$$

To find A , multiply both sides by the denominator:

$$1 = A(1-x(0)z^{-1}) \quad (20)$$

$$A = 1 \quad (21)$$

Therefore, the partial sum using $X(z)$ is $x(n) = x(0)^n u(n)$.