

GATE 2023 EC

EE23BTECH11023-ABHIGNYA GOGULA

Question28:

The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$

A) $\sqrt{\pi} e^{\frac{\omega^2}{2}}$

B) $\frac{e^{-\frac{\omega^2}{4}}}{2\sqrt{\pi}}$

C) $\sqrt{\pi} e^{-\frac{\omega^2}{4}}$

D) $\sqrt{\pi} e^{-\frac{\omega^2}{2}}$

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Solution

$$x'(t) = -2te^{-t^2} \quad (1)$$

$$x'(t) = -2tx(t) \quad (2)$$

doing fourier transform

$$j\omega X(\omega) = -2j \frac{dX(\omega)}{d\omega} \quad (3)$$

$$\int_0^{\omega} \frac{dX(\omega)}{X(\omega)} = \int_0^{\omega} \frac{\omega d\omega}{-2} \quad (4)$$

$$\frac{X(\omega)}{X(0)} = e^{-\frac{\omega^2}{4}} \quad (5)$$

$$X(0) = \int_{-\infty}^{\infty} x(t) dt = \sqrt{\pi} \quad (6)$$

$$X(\omega) = \sqrt{\pi} e^{-\frac{\omega^2}{4}} \quad (7)$$