GATE 2023 EC

EE23BTECH11023-ABHIGNYA GOGULA

Question28:

The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2}\,dy = \sqrt{\pi}$

A) $\sqrt{\pi}e^{\frac{\omega^2}{2}}$

B) $\frac{e^{\frac{-\omega^2}{4}}}{2\sqrt{\pi}}$

C) $\sqrt{\pi}e^{\frac{-\omega^2}{4}}$

D) $\sqrt{\pi}e^{\frac{-\omega^2}{2}}$

Gate 2023 EC Question 28

Solution

$$x'(t) = -2te^{-t^2} \tag{1}$$

$$x'(t) = -2tx(t) \tag{2}$$

doing fourier transform

$$j2\pi fX(f) = -2j\frac{dX(f)}{df} \tag{3}$$

$$\int_0^f \frac{dX(f)}{X(f)} = \int_0^f \frac{2\pi f\,df}{-2} \tag{4}$$

$$\frac{X(f)}{X(0)} = e^{\frac{-\langle 2\pi f\rangle^2}{4}} \tag{5}$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\,dt = \sqrt{\pi} \tag{6}$$

$$X(f) = \sqrt{\pi}e^{\frac{-\langle 2\pi f\rangle^2}{4}} \tag{7}$$

$$X(f) = \sqrt{\pi}e^{-\langle \pi f\rangle^2} \tag{8}$$

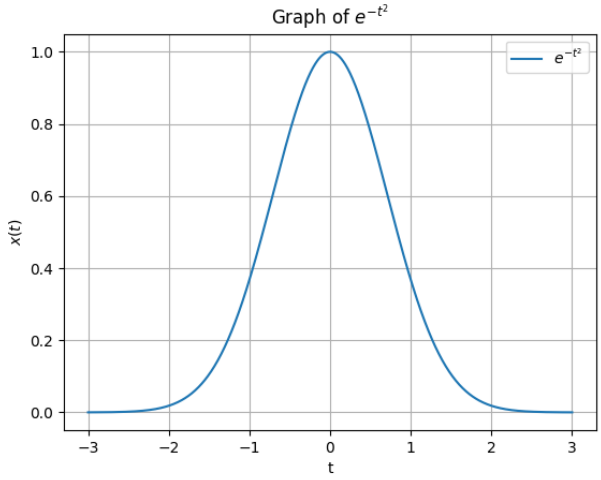

Fig. 0. Graph of e^{-t^2}

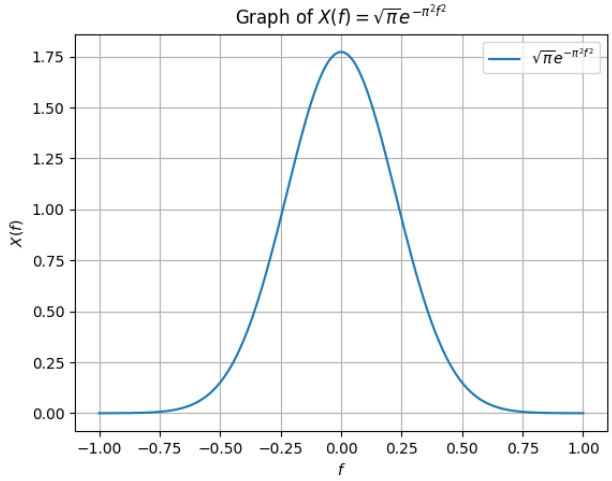

Fig. 0. Graph of $X(f) = \sqrt{\pi}e^{-\pi^2 f^2}$