

GATE 2023 EC

EE23BTECH11023-ABHIGNYA GOGULA

Question28:

The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$

A) $\sqrt{\pi} e^{\frac{\omega^2}{2}}$

B) $\frac{e^{-\frac{\omega^2}{4}}}{2\sqrt{\pi}}$

C) $\sqrt{\pi} e^{-\frac{\omega^2}{4}}$

D) $\sqrt{\pi} e^{-\frac{\omega^2}{2}}$

Gate 2023 EC Question 28

Solution

$$x'(t) = -2te^{-t^2} \quad (1)$$

$$x'(t) = -2tx(t) \quad (2)$$

doing laplace transform

$$sX(s) = 2 \frac{dX(s)}{ds} \quad (3)$$

$$\int_0^s \frac{dX(s)}{X(s)} = \int_0^s \frac{s ds}{2} \quad (4)$$

$$\frac{X(s)}{X(0)} = e^{\frac{s^2}{4}} \quad (5)$$

$$x(0) = \int_{-\infty}^{\infty} x(t) dy = \sqrt{\pi} \quad (6)$$

$$X(s) = \sqrt{\pi} e^{\frac{s^2}{4}} \quad (7)$$

$$s = j\omega \quad (8)$$

$$X(j\omega) = \sqrt{\pi} e^{-\frac{\omega^2}{4}} \quad (9)$$