GATE 2023 EC

EE23BTECH11023-ABHIGNYA GOGULA

Question28:

The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2}\,dy = \sqrt{\pi}$

A) $\sqrt{\pi}e^{\frac{\omega^2}{2}}$

B) $\frac{e^{\frac{-\omega^2}{4}}}{2\sqrt{\pi}}$

C) $\sqrt{\pi}e^{\frac{-\omega^2}{4}}$

D) $\sqrt{\pi}e^{\frac{-\omega^2}{2}}$

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Solution

$$x'(t) = -2te^{-t^2} \tag{1}$$

$$x'(t) = -2tx(t) \tag{2}$$

doing laplace transform

$$j\omega X(\omega) = -2j\frac{dX(\omega)}{d\omega} \tag{3}$$

$$\int_0^{\omega} \frac{dX(\omega)}{X(\omega)} = \int_0^{\omega} \frac{\omega d\omega}{-2} \tag{4}$$

$$\frac{X(\omega)}{X(0)} = e^{\frac{-\omega^2}{4}} \tag{5}$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\,dt = \sqrt{\pi} \tag{6}$$

$$X(\omega) = \sqrt{\pi}e^{\frac{-\omega^2}{4}} \tag{7}$$