GATE 2023 EC

EE23BTECH11023-ABHIGNYA GOGULA

Question28:

The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2} \, dy = \sqrt{\pi}$

A) $\sqrt{\pi} e^{\frac{\omega^2}{2}}$

B) $\frac{e^{\frac{-\omega^2}{4}}}{2\sqrt{\pi}}$

C) $\sqrt{\pi} e^{\frac{-\omega^2}{4}}$

D) $\sqrt{\pi} e^{\frac{-\omega^2}{2}}$

Gate 2023 EC Question 28

Solution

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt \tag{1}$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-t^2} e^{-j\omega t} \, dt \tag{2}$$

$$t^2 + j\omega t = \langle t + \frac{j\omega}{2} \rangle^2 + \frac{\omega^2}{4} \tag{3}$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-\langle t + \frac{j\omega}{2} \rangle^2 - \frac{\omega^2}{4}} \, dt \tag{4}$$

$$u = t + \frac{j\omega}{2} \tag{5}$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-u^2} e^{\frac{-\omega^2}{4}} \, du \tag{6}$$

$$X(\omega) = e^{\frac{-\omega^2}{4}} \int_{-\infty}^{\infty} e^{-u^2} \, du \tag{7}$$

$$X(\omega) = \sqrt{\pi} e^{\frac{-\omega^2}{4}} \tag{8}$$