Q4.

Let us first assume that all the intrinsic parameters of the two cameras are known. This means we know f_p, f_q, s_p, s_q. Let the optical centre of camera P be (o_{px}, o_{py}) and of camera Q be (o_{qx}, o_{qy}) in terms of the pixel units, for each of their pixel coordinate systems. Let us assume these centres are also known. Thus the x-coordinate of direction of first vanishing point p_1 with respect to the camera P will be proportional to (p_{1x}-o_{px}). Similarly for y. The aspect ratio is 1 so no relative scaling in x and y. The z-coordinate will be given by the focal length of the camera which is f_p. The resolution comes into picture here as being multiplied by x and y coordinates. So the direction vector corresponding to point p1 is given by $a_1=(s_p(p_{1x}-o_{px}), s_p(p_{1x}-o_{px}), f_p)$, for point p2 is given by $a_2=(s_p(p_{2x}-o_{px}), s_p(p_{2x}-o_{px}), f_p)$, and for point p3 is given by $a_3=(s_p(p_{3x}-o_{px}), s_p(p_{3x}-o_{px}), f_p)$.

Similarly for points in the image by second camera, the direction vectors are

$b_1=(s_q(q_{1x}-o_{qx}), s_q(q_{1x}-o_{qx}), f_q)$, $b_2=(s_q(q_{2x}-o_{qx}), s_q(q_{2x}-o_{qx}), f_q)$, $b_3=(s_q(q_{3x}-o_{qx}), s_q(q_{3x}-o_{qx}), f_q)$.

These direction vectors can further be normalized to be made of unit magnitude. Now since they are simply directions and the cameras are said to be related by a rotation matrix R and translation t, these direction vectors will also be related by the just the rotation R (since they are directions) (because translation does not change the vanishing points) viz. the equation :

$[a_1 \mid a_2 \mid a_3] = R[b_1 \mid b_2 \mid b_3]$

where the vectors are unit-normalized and written as columns of the 3X3 matrix. From here R can be found easily by matrix inversion given that the matrix is invertible, i.e the vectors are linearly independent. Translation vector **t** cannot be determined as the vanishing points direction remain same irrespective of the translation.

<u>To find the intrinsic parameters of the cameras:</u>

We can find the optical centres (o_{px}, o_{py}) and (o_{qx}, o_{qy}) of the cameras easily as the orthocentre of the three vanishing points corresponding to the three mutually perpendicular lines. Also asserting that the lines are perpendicular i.e

$a_1.a_2 = 0 \Rightarrow s_p^2(p_{1x}-o_{px})(p_{1x}-o_{px}) + s_p^2(p_{1x}-o_{px})(p_{1x}-o_{px}) + f_p^2 = 0$

which gives us an equation in s_p/f_p. Similarly s_q/f_q can be found. There is no way to explicitly find s_p and f_p but this ratio is sufficient for getting the direction of the three lines mentioned above.

<u>CONCLUSION:</u>

We inferred the rotation matrix R, and the ratio s_p/f_p and s_q/f_q . Translation **t** could not be inferred by the given information.