CS763 - Assignment 1

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Problem 3

On the hyperbola $y = 1/x$, denote any point in terms of a parameter t as $p_t = (t, 1/t), \forall t \in R, t \neq 0$.
Note that this parametrisation covers all the points on the hyperbola by covering the range $t \in R - \{0\}$.
Now, let p_t^{HC} denote the homogenous representation of this generic point p_t on the hyperbola.
Then, $p_t^{HC} = (t, 1/t, 1)$. Now use the given projective transformation matrix M.
Using this, we get $q_t^{HC} = Mp_t^{HC}$. Here, q denotes the transformed version of p.

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \text{t} \\ \text{1/t} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \text{1/t} \\ \text{t} \end{pmatrix}$$

Using the above, $q_t^{HC} = (1, 1/t, t)$. Now we bring this point back to the original euclidean coordinate system.
Using $(u, v, w) \longrightarrow (u/w, v/w, 1)$ for $w \neq 0$, we get $q_t = (1/t, 1/t^2)$. Note that $t \in R - \{0\}$.
Now, changing the parametrisation using $w = 1/t$, we get $q_w = (w, w^2)$. Note that the range of w will also be $R - \{0\}$.
Hence, the transformed curve is a parabola $y = x^2$ with the point $(0, 0)$ excluded.

Problem 4

Let us first assume that all the intrinsic parameters of the two cameras are known. This means we know f_p, f_q, s_p, s_q. Let the optical centre of camera P be (o_{px}, o_{py}) and of camera Q be (o_{qx}, o_{qy}) in terms of the pixel units, for each of their pixel coordinate systems. Let us assume these centres are also known. Thus the x-coordinate of direction of first vanishing point p_1 with respect to the camera P will be proportional to $(p_{1x} - o_{px})$. Similarly for y. The aspect ratio is 1 so no relative scaling in x and y. The z-coordinate will be given by the focal length of the camera which is f_p. The resolution comes into picture here as being multiplied by x and y coordinates. So the direction vector corresponding to point p1 is given by $a_1 = (s_p(p_{1x} - o_{px}), s_p(p_{1x} - o_{px}), f_p)$, for point p2 is given by $a_2 = (s_p(p_{2x} - o_{px}), s_p(p_{2x} - o_{px}), f_p)$, and for point p3 is given by $a_3 = (s_p(p_{3x} - o_{px}), s_p(p_{3x} - o_{px}), f_p)$. Similarly for points in the image by second camera, the direction vectors are $b_1 = (s_q(q_{1x} - o_{qx}), s_q(q_{1x} - o_{qx}), f_q)$, $b_2 = (s_q(q_{2x} - o_{qx}), s_q(q_{2x} - o_{qx}), f_q)$, $b_3 = (s_q(q_{3x} - o_{qx}), s_q(q_{3x} - o_{qx}), f_q)$. These direction vectors can further be normalized to be made of unit magnitude. Now since they are simply directions and the cameras are said to be related by a rotation matrix R and translation **t**, these direction vectors will also be related by the just the rotation R (since they are directions) (because translation does not change the vanishing points) viz. the equation : $[a_1|a_2|a_3] = R[b_1|b_2|b_3]$ where the vectors are unit-normalized and written as columns of the 3X3 matrix. From here R can be found easily by matrix inversion given that the matrix is invertible, i.e the vectors are linearly independent. Translation vector t cannot be determined as the vanishing points direction remain same irrespective of the translation.

To find the intrinsic parameters of the cameras

We can find the optical centres (o_{px}, o_{py}) and (o_{qx}, o_{qy}) of the cameras easily as the orthocentre of the three vanishing points corresponding to the three mutually perpendicular lines. Also asserting that the lines are perpendicular i.e $a_1.a_2 = 0 \implies s_{p2}(p_{1x} - o_{px})(p_{1x} - o_{px}) + s_{p2}(p_{1x} - o_{px})(p_{1x} - o_{px}) + f_{p2} = 0$ which gives us an equation in s_p/f_p. Similarly s_q/f_q can be found. There is no way to explicitly find s_p and f_p but this ratio is sufficient for getting the direction of the three lines mentioned above.

CONCLUSION

We inferred the rotation matrix R, and the ratio sp/fp and sq/fq . Translation t could not be inferred by the given information.

Problem 5

(a) We use our knowledge of geometry to recall that any line in R^3 can be represented by using a point on the line, and a vector that is parallel to the line. That is, any line $L \in R^3$ can be represented using $s = (s_1, s_2, s_3)$ (a point on the line) and a vector $v = (v_1, v_2, v_3)$ (direction of the line), such that any point on the line can be recovered using $p = s + \alpha v$ for some $\alpha \in R$.

Now, let the point on L_1 have the representation $a = (a_1, a_2, a_3)$, and similarly let the point on L_2 be $b = (b_1, b_2, b_3)$. Since the lines are parallel, let the common vector of direction be $v = (v_1, v_2, v_3)$.
So now, any generic point p_1 on line L_1 is $p_1 = a + \alpha v$. Similarly a point on L_2 is p_2, given by $p_2 = b + \beta v$.

Now, we use the pinhole camera model, assuming that the pinhole is at the origin. By the property of similar triangles (as demonstrated in the lectures), the transformation is given by $(X, Y, Z) \longrightarrow (cX/Z, cY/Z)$. Note that this also assumes that the image plane is parallel to the XY plane. Here, c is the focal length (a constant).
Using this, point p_1 on L_1 transforms to $p_1^t = [c(a_1 + \alpha v_1)/(a_3 + \alpha v_3),\ c(a_2 + \alpha v_2)/(a_3 + \alpha v_3)]$.
Similarly, point p_2 on L_2 transforms to $p_2^t = [c(b_1 + \beta v_1)/(b_3 + \beta v_3),\ c(b_2 + \beta v_2)/(b_3 + \beta v_3)]$.

Now, the points at infinity on L_1 and L_2 correspond to $\alpha, \beta \to \pm\infty$. Applying this limit on the above expressions of p_1^t and p_2^t, the points of L_1 will converge at $z_1 = (cv_1/v_3, cv_2/v_3)$ and those of L_2 will converge at $z_2 = (cv_1/v_3, cv_2/v_3)$. Clearly we see that $z_1 = z_2$, which means that the image of the points at infinity on both the lines is the same. This concludes that the two lines L_1 and L_2 indeed do intersect at infinity, and the intersection point is therefore the vanishing point (since the point at infinity of either line is termed as the vanishing point).

Note : The above proof relies on the fact that $v_3 \neq 0$, which means that the vector v is not parallel to the plane of the image (because observe that the XY plane is parallel to the plane of the image when we use the $(X, Y, Z) \longrightarrow (cX/Z, cY/Z)$ transformation). And this observation is in accordance with the fact that when $v_3 = 0$, it means that the two parallel lines never intersect in the image plane since they don't have a vanishing point (because each line is parallel to the image plane).

(b) Using the above part of the question, we can infer that for a line $L \in R^3$ parallel to a vector $v = (v_1, v_2, v_3)$ with $v_3 \neq 0$, the vanishing point is given by $(cv_1/v_3, cv_2/v_3)$ in the image plane.

So now, let the three sets of parallel lines be given by the vectors $p = (p_1, p_2, p_3), q = (q_1, q_2, q_3)$ and $r = (r_1, r_2, r_3)$. Then, the three vanishing points are given by $p_v = (cp_1/p_3, cp_2/p_3)$, $q_v = (cq_1/q_3, cq_2/q_3)$ and $r_v = (cr_1/r_3, cr_2/r_3)$.

We need to show that p_v, q_v and r_v are collinear. For that, we need to show that there exists constants f, g, h such that $fx + gy + h = 0$ holds for all the three substitutions of (x, y) as p_v, q_v, r_v.

Substitute $(x, y) = p_v = (cp_1/p_3, cp_2/p_3)$ in the above equation of the line, to get $fcp_1 + gcp_2 + hp_3 = 0$. Note that $fcp_1 + gcp_2 + hp_3 = 0$ is the same as saying $[f, g, h/c].[p_1, p_2, p_3]^T = 0$ or $[f, g, h/c].p^T = 0$.
(Here we use that $c \neq 0$, which is true since the image plane cannot coincide with the plane of the pinhole.)
Let $h' = h/c$. Then, we essentially need to find a vector $s = [f, g, h']$ such that $s.p^T = 0$. Using a Similar argument for the other two substitutions of $(x, y) = q_v$ and $(x, y) = r_v$, we conclude that to show collinearity of p_v, q_v and r_v; we need to show the existence of a 3-dim vector s such that $s.p^T = s.q^T = s.r^T = 0$.

Now, we use the coplanarity of the three sets of lines, which says that the vectors p, q, r lie on a plane. Then, it is immediate that a vector s will exist such that the dot-product of s with all p, q, r will be zero, and that s will be given by the vector normal to the plane that carries all p, q, r.

Hence, it is clear that we can find the parameters of a line such that all the three vanishing points lie on that line. Hence, the three points are collinear.

Note : To actually find the line, we can find the vector s using the cross product of any two of the three given vectors p, q, r. That is, $p \times q, q \times r, p \times r$ all are valid choices for s.

Problem 6

Consider points A_1', B_1', C_1' and A_2', B_2', C_2' as labelled in Figure 1 and let corresponding actual points be A_1, B_1, C_1 and A_2, B_2, C_2 respectively (also lying on vertical lines). The fourth point D_i and D_i' can be taken to be at infinity which means $A_iD_i = B_iD_i$ and $A_i'D_i' = B_i'D_i'$. Also, in actuality the line joining the feet of the two people would be parallel to the horizon which implies $B_1C_1 = B_2C_2$. Therefore, by preservation of cross ratios, we must have

$$\frac{A_1'C_1'}{B_1'C_1'} = \frac{A_1C_1}{B_1C_1}$$

Similarly we have

$$\frac{A_2'C_2'}{B_2'C_2'} = \frac{A_2C_2}{B_2C_2}$$

Using `imtool` in Matlab we found the measurements (in terms of number of pixels, can be seen in Figure 2) and we are given $A_1C_1 = 180$ and also $B_2C_2 = B_1C_1$ as argued above. From measurements, $A_1'C_1' = 253$, $B_1'C_1' = 105$, $A_2'C_2' = 578$, $B_2'C_2' = 216$ pixels.

Thus we finally get $A_2C_2 = \frac{578*180*105}{216*253} = 199.9 \approx 200$. Hence the height of the man is about 200 cm in actual life.

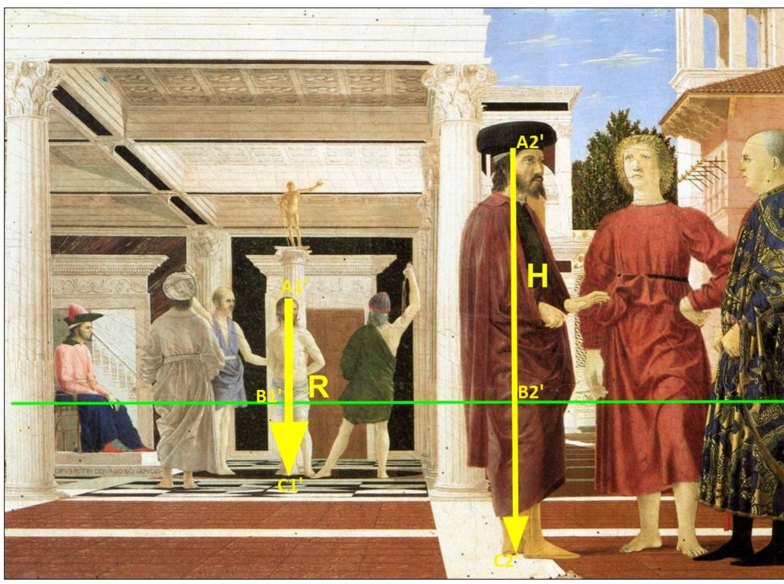

Figure 1: Flagellation of Christ

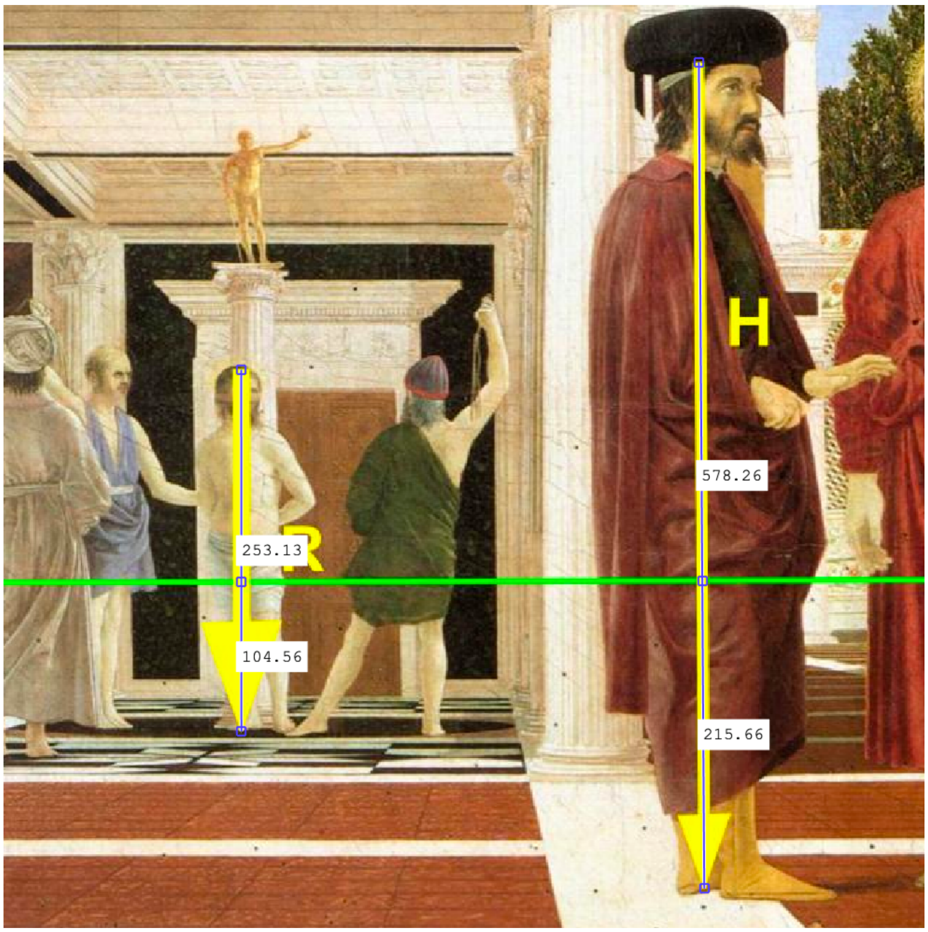

Figure 2: Measurements