CS182 – Foundations of Computer Science

Homework 7 (100 points)

Due: Thursday, April 16, 2020, 11:59pm

Upload the homework to Gradescope. DO NOT SUBMIT TO BLACKBOARD LEARN. No late submissions accepted. Only typed solutions will be graded.

Remark: Provide a brief justification for each of your answers (no more than five lines), explaining which counting rules you used and what your thought process was. Feel free to have expressions of the form $3 \cdot 6^{10}$, etc. in your final answers; no need to use calculators to compute such powers.

Problem 1. (45=5*9 points)
Suppose you have 18 objects (10 of type A, 5 of type B, and 3 of type C). Objects of type A are indistinguishable from each other; objects of type B are indistinguishable from each other; and objects of type C are indistinguishable from each other. In how many ways can you:

1. Put the 18 objects in a row?

2. Pick 3 of the 18 objects (order does not matter)?

3. Pick 4 of the 18 objects (order does not matter)?

4. Pick 5 of the 18 objects (order does not matter)?

5. Pick nine objects out of the 18 objects so that exactly three objects are of type A and exactly two objects are of type B (order does not matter)?

Problem 2. (27 = 3*9 points)
A movie theater can play 30 westerns, 15 science fiction movies, and 10 horror movies (all movies are distinct from each other). Its standard daily program typically consists of a western followed by a science fiction movie, and then a horror movie.

1. How many different programs can it play?

2. How many different programs are there if the three movies can be played in any order? How does this number compare to the previous number and why?

3. How many different three-movie programs are there if there are absolutely no restrictions (e.g., the same movie can be played twice, movies can be played in any order, categories do not matter, etc.)? How does this number compare to the previous numbers and why?

Problem 3. (16 = 2*8 points)
You have 10 of each of the following type of objects: A, B, C, and D. The objects of each type are distinguishable (e.g., the 10 objects of type A are different from each other, think of them as $A_1 \ldots A_{10}$; same for the other three types).

1. In how many ways can you arrange all objects in a row?

2. In how many ways can you choose a set S of 10 objects?

Problem 4. (12 = 2*6 points)

1. What is the coefficient of $x^8 y^4$ in the expansion of $(4x - 4y)^{28}$?

2. Prove that
$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}.$$