

**Example 1:** In how many ways can we select two books from different subjects among five CS books, three Math and two Art books?

$$(5 \times 1) + (3 \times 2) + (5 \times 2) \\ 15 + 6 + 10 = 31 \text{ ways}$$

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Select CS+Math, CS+Art, Math+Art:

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**Example 3:** There are  $n$  senators. In how many ways can subcommittees be formed that consist of at least 2 and at most  $n-1$  senators?

$$2^n - 1 - 1 - n$$

subset:  $\emptyset, h$

$$S = \{1, 2, 3, \dots, n-1, n\}$$

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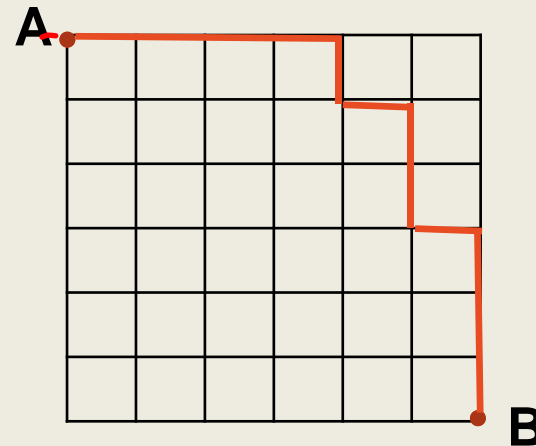
$$2^n - n - 1 - 1 \text{ (don't forget the empty set)}$$

# Combinations

**Example:** A robot moves in a 6 by 6 grid from A to B.

How many distinct ways are there for a robot to move from the upper left corner A to the lower right corner B when movement is right or down?

$$\binom{12}{6} = \frac{12!}{6!6!}$$



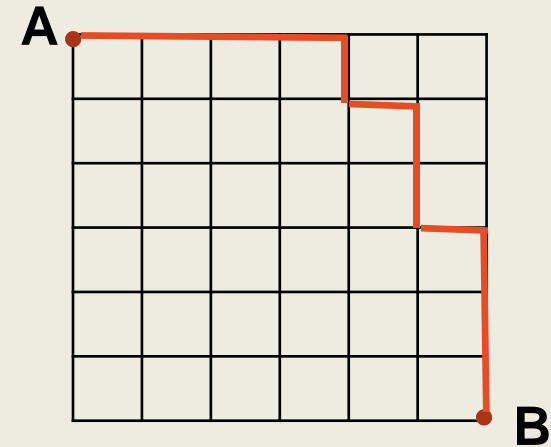
# Combinations

**Example:** How many distinct ways are there for moving from A to B travelling right and down?

**Solution:**

Every path consists 6 right moves and 6 down moves.

A path is a string of length 12:  
RRRRDRDDRDDD



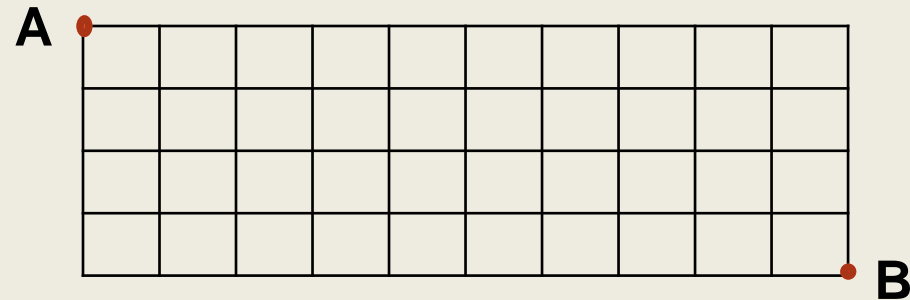
The total number of paths is the number of 6-combinations of a set with 12 elements

$$C(12,6) = \frac{12!}{6! 6!}$$

# Combinations

**Example:** A robot moves in a 4 by 10 grid from A to B.

How many distinct ways are there for a robot to move from the upper left corner A to the lower right corner B when movement is right or down?



A.  $C(14, 10)$

~~B.  $C(10, 4)$~~

C.  $C(14, 4)$

~~D.  $C(18, 10)$~~

~~E.  $14!$~~

$$\binom{n}{r} = \binom{n}{n-r}$$

## Example: Hands with a Full House

A Full House is a hand with three cards of one rank and 2 cards of another rank.

In a 52 card deck,  
how many Full house  
hands are there?

**Solution:**

$$13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2}$$



1 2 3 4 5 ..... 13





## Example: Hands with a Full House

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In a 52 card deck,  
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**Solution:**

- Rank of triple can be chosen in 13 ways
- Suit of triple can be selected in  $\binom{4}{3}$  ways
- Rank of pair can be chosen in 12 ways
- Suit of pair can be selected in  $\binom{4}{2}$  ways

Number of Full Houses is  $13 \times \binom{4}{3} \times 12 \times \binom{4}{2} = 3,744$