Spring-2020-CS-18200-LE1 Homework 2

Abhi Gunasekar

TOTAL POINTS

100 / 100

QUESTION 1

1 Problem 1 16 / 16

✓ - **0 pts** Correct

- **16 pts** Blank
- **3 pts** Missing at most 2 steps of reasoning
- **6 pts** Missing at most 3 steps of reasoning
- **9 pts** Missing at most 4 steps of reasoning
- **12 pts** Major errors in reasoning

QUESTION 2

2 Problem 2 24 / 24

✓ - **0 pts** Correct

- **24 pts** Blank Answer
- **6 pts** 3 or less missing or incorrect steps
- **12 pts** 4 to 6 missing or incorrect steps
- **18 pts** 7 or more missing or incorrect steps

QUESTION 3

3 Problem 3 20 / 20

✓ - **0 pts** Correct

- **2 pts** minor mistake
- **4 pts** major mistake/did not show how to obtain/did not mention rule of inference/2 or more wrong conclusion
- **20 pts** blank
- **3 pts** 1 conclusion missing/wrong

QUESTION 4

4 Problem 4 16 / 16

✓ - **0 pts** Correct

- **2 pts** minor mistake in proof by contraposition (not p --> not q instead of not q --> not p, etc)
- **2 pts** minor mistake in proof by contradiction (assumed 3n+2 is odd and n is even, etc)
- **4 pts** major mistake in proof by contraposition(did not assume n is odd, etc)
- **4 pts** major mistake in proof by contradiction (did not use proof by contradiction, etc)
- **8 pts** Proof by Contraposition blank
- **8 pts** Proof by Contradiction blank

QUESTION 5

5 Problem 5 14 / 14

✓ - **0 pts** Correct

- **1 pts** Minor mistake in argument
- **3 pts** Major mistake in argument
- **4 pts** Missing case/equivalency not fully established
- **14 pts** Blank
- **7 pts** Insufficient proof

QUESTION 6

6 Problem 6 10 / 10

✓ - **0 pts** Correct

- **7 pts** failed to prove
- **0 pts** In this problem, giving one example would be sufficient.
- **10 pts** Blank
- **1 pts** Minor mistake
- **0 pts** 1/2 is a rational number.
- **0 pts** actually xy can be rational if x is zero.

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴

Abhishek Gunasekar
02/05/2020

Homework 2 Answers

1 Problem 1: (16 Points)

$\forall x[(P(x) \land Q(x))] \land (\exists x)(Q(x)) \rightarrow (\exists x)(P(x))$

1.	$\forall x[(P(x) \land Q(x))]$	**Premise 1**
2.	$P(c) \land Q(c)$	**Universal Instantiation on 1.**
3.	$\exists x(Q(x))$	**Premise 2**
4.	$Q(c)$	**Existential Instantiation on 2.**
5.	$P(c)$	**Simplification on 2.**
6.	$\exists x (P(x))$	**Existential Generalization on 5.**

∴ QED

1 Problem 1 **16 / 16**

- ✓ **- 0 pts** Correct
- **- 16 pts** Blank
- **- 3 pts** Missing at most 2 steps of reasoning
- **- 6 pts** Missing at most 3 steps of reasoning
- **- 9 pts** Missing at most 4 steps of reasoning
- **- 12 pts** Major errors in reasoning

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore$

2 Problem 2: (24 Points)

$\forall x(P(x) \rightarrow (Q(x) \wedge S(x)) \wedge \forall x(P(x) \wedge R(x)) \rightarrow \forall x(R(x) \wedge S(x))$

1.	$\forall x(P(x) \rightarrow (Q(x) \wedge S(x))$	**Premise 1**
2.	$P(c) \rightarrow Q(c) \wedge S(c)$	**Universal Instantiation on 1.**
3.	$\forall x(P(x) \wedge R(x))$	**Premise 2.**
4.	$P(c) \wedge R(c)$	**Universal Instantiation on 3.**
5.	$P(c)$	**Simplification on 4.**
6.	$Q(c) \wedge S(c)$	**Modus Ponens on 2.**
7.	$S(c)$	**Simplification on 6.**
8.	$R(c)$	**Simplification on 4.**
9.	$S(c) \wedge R(c)$	**Conjunction on 7 and 8.**
10.	$\forall x(R(x) \wedge S(x))$	**Universal Generalization on 9.**

2 Problem 2 **24 / 24**

- ✓ **- 0 pts** Correct
- **- 24 pts** Blank Answer
- **- 6 pts** 3 or less missing or incorrect steps
- **- 12 pts** 4 to 6 missing or incorrect steps
- **- 18 pts** 7 or more missing or incorrect steps

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore$

3 Problem 3: (20 Points)

The predicates needed for this question can be defined as follows:

- x is an insect can be represented by the predicate I(x).
- x has six legs can be represented by the predicate L(x).
- x eats y can be represented by the predicate E(x,y)

Given the above declaration, the premises as given in the handout can be described as:

- Premise 1: $\forall x(I(x) \rightarrow L(x))$.
- Premise 2: $I(\text{dragonfly})$
- Premise 3: $\neg\, L(\text{spider})$
- Premise 4: $E(\text{spider, dragonfly})$

And from the premises, we can draw three conclusions.

1. Dragonflies have six legs : L(dragonfly)
2. Spider is not an insect ($\neg I(\text{spider})$)
3. There exists some insect that is eaten by a spider: $\exists y(E(x,y) \wedge I(y))$

The conclusions above can be derived using rules of inference as follows:

- **Conclusion 1:**

1. $\forall x(I(x) \rightarrow L(x))$	**Premise 1**
2. $I(\text{dragonfly})$	**Premise 2**
3. $I(\text{dragonfly}) \rightarrow L(\text{dragonfly})$	**Universal instantiation on 1**
4. $L(\text{dragonfly})$	**Modus Ponens on 3**

- **Conclusion 2:**

1. $\forall x(I(x) \rightarrow L(x))$	**Premise 1**
2. $\neg L(\text{Spider})$	**Premise 3**
3. $I(\text{Spider}) \rightarrow L(\text{Spider})$	**Universal Instantiation on 1**
4. $\neg I(\text{Spider})$	**Modus tollens on 2 and 3**

- **Conclusion 3:**

1. E(Spider, Dragonfly)	**Premise 1**
2. I(dragonfly)	**Premise 2**
3. E(spider, dragonfly)	**Conjunction on 1 and 3**
4. $\exists y(E(x,y) \wedge I(y))$	**Existential Generalization**

3 Problem 3 **20 / 20**

- ✓ **- 0 pts** **Correct**
- **- 2 pts** minor mistake
- **- 4 pts** major mistake/did not show how to obtain/did not mention rule of inference/2 or more wrong conclusion
- **- 20 pts** blank
- **- 3 pts** 1 conclusion missing/wrong

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore$

4 Problem 4 (16 Points)

Prove that if n is an integer and 3n + 2 is even, then n is even using

- <u>A proof by contraposition</u>

  If 3n + 2 is even, then n is even is of the form $p \rightarrow q$.
  It's contrapositive would be $\neg q \rightarrow \neg p$
  So, the contrapositive can be translated as if n is odd, then $3n + 2$ is odd.
  Let's assume an odd number n to be $2k + 1$ where k is some real integer, then
  $3n + 2 = 3(2k + 1) + 2$
  $= 6k + 3 + 2$
  $= 6k + 4 + 1$
  $= 2(3k + 2) + 1$
  $= 2(k') + 1$ where k' is another real integer, implying that $3n + 2$ is odd.
  $\therefore$ If 3n+2 is even, then n is even because the contrapositive is proven to be true above.

  $\therefore$ QED.

- <u>A proof by contradiction</u>

  Assume $3n + 2$ is even and n is odd,
  then given that n is odd, $n = 2k + 1$ where k is some real integer.
  Then $3n + 2 = 3(2k + 1) + 2$
  $= 6k + 5$
  $= 2(3k + 2) + 1$
  $= 2(k') + 1$ where k' is another real integer, implying that $3n + 2$ is odd.
  Therefore $3n + 2$ is odd. This is a clear contradiction to the original statement $3n + 2$ is even.

4 Problem 4 **16 / 16**

✓ **- 0 pts** Correct

- 2 pts minor mistake in proof by contraposition (not p --> not q instead of not q --> not p, etc)

- 2 pts minor mistake in proof by contradiction (assumed 3n+2 is odd and n is even, etc)

- 4 pts major mistake in proof by contraposition(did not assume n is odd, etc)

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- 8 pts Proof by Contraposition blank

- 8 pts Proof by Contradiction blank

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴

5 Problem 5: (14 Points)

Let 5.1, 5.2, 5.3 represent p1, p2, and p3 respectively, then to prove that they are logically equivalent we need to show p1 → p2, p2 → p3, and p3 → p1

<u>Definition of a Rational Number:</u> The real number r is rational if there exists integers p and q with $q \neq 0$ such that $r = p / q$

1. Proving p1 → p2
   If x is irrational, then 3x + 2 is irrational.
   We can prove this using contraposition:
   Contrapositive would be ¬p2 → ¬p1
   If 3x + 2 is rational, then x is rational.
   $3x + 2 = p/q$ **Definition**
   $x = (1/3)\ ((p/q)\ -2)$
   $x = (1/3)\ ((p - 2q)/q)$
   $x = (p - 2q)/(3q)$
   $x = (p')/(q')$ where p' and q' are integers.
   ∴ x is rational and p1 → p2 is proven through contrapositive.

2. Proving p2 → p3
   If 3x + 2 is irrational, then x/2 is irrational.
   We can prove this using contraposition:
   Contrapositive would be ¬p3 → ¬p2
   If x/2 is rational, then 3x + 2 is rational.
   $(x/2) = p/q$ **Definition**
   $x = (2p)/q$
   $3x + 2 = 3(2p/q) + 2$
   $3x + 2 = (6p/q) + 2$
   $3x + 2 = (6p + 2q)/(q)$
   $3x + 2 = (p')/(q')$ where p' and q' are integers.
   ∴ 3x + 2 is rational and p2 → p3 is proven through contrapositive.

3. Proving p3 → p1
   If (x/2) is irrational, then x is irrational.
   We can prove this using contraposition:
   Contrapositive would be ¬p1 → ¬p3
   If x is rational, then x/2 is rational
   $x = p/q$ **Definition**
   $(x/2) = (1/2)(p/q)$
   $(x/2) = (p/2q)$
   $(x /2) = (p'/q')$ where p' and q' are integers.
   ∴ x/2 is rational and p3 → p1 is proven through contrapositive.

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴

Since the propositions p1 → p2, p2 → p3, p3 → p1 are each proven using contraposition, the propositions about the real number x represented by 5.1, 5.2, 5.3 are said to be logically equivalent to each other.

6 Problem 6: (10 Points)

Prove? There is a rational number x and an irrational number y such that xy is irrational.

Let's assume x to be 1 and y to be $\sqrt{2}$, which we clearly know to be irrational as mentioned by the proof in slide 14 in lecture slides Chapter1p3_5. Therefore the product of x and y, xy in particular would be $\sqrt{2}$ as well, implying that it is also irrational.

∴ It is proven that there exists a rational and an irrational number whose product is irrational.

∴ QED.

5 Problem 5 14 / 14

- ✓ **- 0 pts** **Correct**
- **- 1 pts** Minor mistake in argument
- **- 3 pts** Major mistake in argument
- **- 4 pts** Missing case/equivalency not fully established
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∴ It is proven that there exists a rational and an irrational number whose product is irrational.

∴ QED.

6 Problem 6 **10 / 10**

- ✓ **- 0 pts** **Correct**
- **- 7 pts** failed to prove
- **- 0 pts** In this problem, giving one example would be sufficient.
- **- 10 pts** Blank
- **- 1 pts** Minor mistake
- **- 0 pts** 1/2 is a rational number.
- **- 0 pts** actually xy can be rational if x is zero.