PSO sessions 1 and 2, week of March 23, 2020

PSO 1

Problem 1. Find the best (i.e., asymptotically smallest) big-Oh function for $\frac{x^3+7x}{3x+1}$.

Problem 2. Prove that $n^3 + 3n^2 + 2n$ is a multiple of three for all $n \geq 1$.

PSO 2

Problem 1. Prove that $n^2 + 3n$ is a multiple of two for all $n \geq 1$.

Problem 2. Prove that 6 is a factor of $7^n - 1$ for all positive integers n.

Problem 3. What is wrong with the following proof by induction?

<u>Theorem:</u> For every non-negative integer n, $5n = 0$.

<u>Basis Step:</u> $5 \cdot 0 = 0$.

<u>Induction hypothesis:</u> $5 \cdot j = 0$ for all non-negative integers $j = 0, 1, \ldots, k$.

<u>Will prove:</u> $5 \cdot (k + 1) = 0$. Write $k + 1 = i + j$, where i and j are natural numbers strictly less than $k + 1$. By the inductive hypothesis, $5 \cdot i = 5 \cdot j = 0$ and thus

$$5 \cdot (k + 1) = 5 \cdot (i + j) = 5i + 5j = 0.$$