Spring-2020-CS-18200-LE1 Homework 1

Abhi Gunasekar

TOTAL POINTS

99 / 100

QUESTION 1

1 1.1 9 / 9

✓ **- 0 pts Correct**

- 1 pts 1.1.a. Minor mistake (used AND, incorrect implication, etc.)

- 2 pts 1.1.a. Significant error (didn't use disjunctive form at all, etc.)

- 3 pts 1.1.a. Blank answer

- 1 pts 1.1.b.i. Minor mistake (incorrect answer with correct reasoning, correct answer without any reasoning, etc.)

- 2 pts 1.1.b.i. Significant error

- 3 pts 1.1.b.i. Blank answer

- 1 pts 1.1.b.ii. Minor mistake (incorrect answer with correct reasoning, correct answer without any reasoning, etc.)

- 2 pts 1.1.b.ii. Significant error

- 3 pts 1.1.b.ii. Blank answer

QUESTION 2

2 1.2 9 / 9

✓ **- 0 pts Correct / Correct answer with minor mistakes in reasoning**

- 1 pts 1.2.a. Significant error (minor mistakes while following the correct reasoning, correct answer without reasoning)

- 3 pts 1.2.a. Blank answer

- 1 pts 1.2.b. Significant error (minor mistakes while following the correct reasoning, correct answer without reasoning)

- 3 pts 1.2.b. Blank answer

- 1 pts 1.2.c. Significant error (minor mistakes while following the correct reasoning, correct answer without reasoning)

- 3 pts 1.2.c. Blank answer

QUESTION 3

3 1.3 12 / 12

✓ **- 0 pts Correct**

- 1 pts 24. Significant error (incorrect answer with correct reasoning, correct answer without reasoning)

- 3 pts 24. Blank answer

- 1 pts 26. Significant error

- 3 pts 26. Blank answer

- 1 pts 28. Significant error

- 3 pts 28. Blank answer

- 1 pts 30. Significant error

- 3 pts 30. Blank answer

QUESTION 4

4 1.4 4 / 4

✓ **- 0 pts Correct**

- 1 pts Minor error (incorrect distributive laws, incorrect disjunctive form of implication, etc)

- 2 pts Significant error (answer without reasoning, etc)

- 4 pts Blank answer

QUESTION 5

5 1.5 3 / 3

✓ **- 0 pts Correct**

- 1 pts Minor error (e.g. negation is outside of the parenthesis, correct law applied but had incorrect outcome, etc.)

- 2 pts Significant error (e.g. used implication, didn't solve nested parenthesis, etc.)

- 3 pts Blank answer

QUESTION 6

6 1.6 6 / 6

✓ **- 0 pts Correct**

- 1 pts 1.6.a. Incorrect answer, Correct answer

without the case where it's satisfiable

- **3 pts** 1.6.a. Blank answer
- **1 pts** 1.6.b. Incorrect answer, Correct answer without reasoning
- **3 pts** 1.6.b. Blank answer

QUESTION 7

7 2.1 **11 / 12**

- **0 pts** Correct
- **1 pts** 2.1.a. Minor errors (AND vs OR, parenthesis, missing FORALL symbol, etc.)
- **3 pts** 2.1.a. Significant errors (expressed the domain, not the statement)
- **6 pts** 2.1.a. Blank answer

✓ **- 1 pts** **2.1.b. One incorrect item**

- **2 pts** 2.1.b. Two incorrect items
- **3 pts** 2.1.b. Didn't evaluate validity, but evaluated truth value
- **6 pts** 2.1.b. Blank answer
- **3 pts** 2.1.b (i) Blank
- **3 pts** 2.1.b (ii) Blank

QUESTION 8

8 2.2 **10 / 10**

✓ **- 0 pts** **Correct**

- **1 pts** 1 incorrect
- **2 pts** 2 incorrect
- **3 pts** 3 incorrect
- **4 pts** 4 incorrect
- **5 pts** 5 incorrect
- **10 pts** Blank
- **4 pts** 2 blank
- **2 pts** 1 blank

QUESTION 9

9 2.3 **18 / 18**

✓ **- 0 pts** **Correct**

- **1 pts** 2.3.a.i. Incorrect answer (e.g. incorrect use of implication, quantifier, or disjunction)
- **1 pts** 2.3.a.ii. Incorrect answer (e.g. incorrect use of implication, quantifier, or disjunction)
- **1 pts** 2.3.a.iii. Incorrect answer (e.g. incorrect use of implication, quantifier, or disjunction)
- **1 pts** 2.3.a.iv. Incorrect answer (e.g. incorrect use of implication, quantifier, or disjunction)
- **1 pts** 2.3.b.i. Incorrect answer, Correct answer without reasoning
- **1 pts** 2.3.b.ii. Incorrect answer, Correct answer without reasoning
- **18 pts** blank
- **4.5 pts** 2.3.b.i blank
- **4.5 pts** 2.3.b.ii blank
- **9 pts** 2.3. b blank

QUESTION 10

10 2.4 **10 / 10**

✓ **- 0 pts** **Correct**

- **1 pts** 1 incorrect
- **2 pts** 2 incorrect
- **3 pts** 3 incorrect
- **4 pts** 4 incorrect
- **5 pts** 5 incorrect
- **10 pts** blank
- **2 pts** 1 blank

QUESTION 11

11 2.5 **7 / 7**

✓ **- 0 pts** **Correct**

- **2 pts** Minor issues (e.g. Used propositions and logical expressions, but failed to generalize all cases with even number: 2k / odd number: 2k + 1)
- **3 pts** Major issues (e.g. Just showing some examples/counter-examples without generalization)
- **5 pts** Failed to prove
- **7 pts** Blank answer

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

Abhishek Gunasekar
01/26/2020

Homework 1 Answers

1 Propositional Logic

1.1 (9 Points)

a) $(p \vee q) \rightarrow r$

b) $T \vee q \rightarrow r$

i. $T \rightarrow r$:
By implication logic, given that the left hand side is true, and that the entire statement is true, ***r must be true.***

ii. $(\neg r \rightarrow \neg p \wedge \neg q) \equiv (p \vee q) \rightarrow r$
Given the contrapositive above, and given that the entire proposition is true that means $\neg p \wedge \neg q$ must be true, which essentially means that **$\neg p$ can be logically concluded to be true.**

1 1.1 **9 / 9**

✓ **- 0 pts** **Correct**

- 1 pts 1.1.a. Minor mistake (used AND, incorrect implication, etc.)

- 2 pts 1.1.a. Significant error (didn't use disjunctive form at all, etc.)

- 3 pts 1.1.a. Blank answer

- 1 pts 1.1.b.i. Minor mistake (incorrect answer with correct reasoning, correct answer without any reasoning, etc.)

- 2 pts 1.1.b.i. Significant error

- 3 pts 1.1.b.i. Blank answer

- 1 pts 1.1.b.ii. Minor mistake (incorrect answer with correct reasoning, correct answer without any reasoning, etc.)

- 2 pts 1.1.b.ii. Significant error

- 3 pts 1.1.b.ii. Blank answer

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv$

1.2 (9 Points)

a) $(p \vee (\neg q \vee r)) \wedge \neg (r \rightarrow \neg q)$
$p = T, q = F, r = T$
$(T \vee (T \vee T)) \wedge \neg (T \rightarrow T)$
$T \wedge F$
False

b) $(p \vee (\neg q \vee r)) \wedge \neg (r \rightarrow \neg q)$
$p = F, q = T$
$(p \vee ((\neg q \vee r)) \wedge \neg (\neg r \vee \neg q)$
$(p \vee ((\neg q \vee r)) \wedge (r \wedge q)$
$(F \vee (F \vee r)) \wedge (r \wedge T)$
$r \wedge r$
r

c) $(p \vee (\neg q \vee r)) \wedge \neg (r \rightarrow \neg q)$
$q = F$
$(p \vee (T \vee r)) \wedge (r \wedge F)$
$T \wedge F$
False

2 1.2 9 / 9

✓ **- 0 pts Correct / Correct answer with minor mistakes in reasoning**

- 1 pts 1.2.a. Significant error (minor mistakes while following the correct reasoning, correct answer without reasoning)

- 3 pts 1.2.a. Blank answer

- 1 pts 1.2.b. Significant error (minor mistakes while following the correct reasoning, correct answer without reasoning)

- 3 pts 1.2.b. Blank answer

- 1 pts 1.2.c. Significant error (minor mistakes while following the correct reasoning, correct answer without reasoning)

- 3 pts 1.2.c. Blank answer

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

1.3 (12 Points)

24) p → *A is a knight and* q → B is a knight
Answer: A is a knave and b is a knight.
Reasoning: If A is a knight then B should be a knight as well, which means that he should be telling the truth. However, B says that A is a knave, which would mean the proposition p ∧ q ≡ False. However, if A is knave, then it would mean that B is a knight and B's statement of A being a knave would be true.

26) p → *A is a knight and* q → B is a knight
Answer: Inconclusive
Reasoning: We cannot conclude anything here. Each of the 4 permutations is possible. If A is a knight, that doesn't tell us anything about B. If A is a knave, that too doesn't imply anything about B, and the same logic can also be applied when we try to deduce anything about A from B.

28)
Unique Solution: A is the knight, B is the Spy, C is the Knave.
Reasoning: If A is a knight, then C must be the knave, which means he's not the spy, pointing that B has to be the spy, given that there are exactly 1 spy, 1 knight, 1 knave.

30)
No Solution Exists
Reasoning: If either A, B, or C is considered as the knave, and given that there exists 1 knave, 1 knight, and 1 spy, the other 2 people must be a knight or a spy, which clearly doesn't match the assertion made by the other 2 people. Therefore, no solution exists.

3 1.3 12 / 12

✓ **- 0 pts** **Correct**

- 1 pts 24. Significant error (incorrect answer with correct reasoning, correct answer without reasoning)

- 3 pts 24. Blank answer

- 1 pts 26. Significant error

- 3 pts 26. Blank answer

- 1 pts 28. Significant error

- 3 pts 28. Blank answer

- 1 pts 30. Significant error

- 3 pts 30. Blank answer

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

1.4 (4 points)

(p → *q)* ∧ (p → *r) ?*≡ p → *(q* ∨ r)
L.H.S R.H.S
(¬ p ∨ (q ∧ r)) (¬ p ∨ (q ∨ r))
Step 2 in L.H.S not logically equivalent to that of R.H.S. Therefore both the propositions are said to be **not logically equivalent to each other.**

4 1.4 4 / 4

✓ **- 0 pts** **Correct**

- 1 pts Minor error (incorrect distributive laws, incorrect disjunctive form of implication, etc)

- 2 pts Significant error (answer without reasoning, etc)

- 4 pts Blank answer

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

1.5 (3 points)

¬ (p ∨ ¬ q) → *(¬* r ∧ (p → *r))*

(p ∨ ¬ q) ∨ (¬ r ∧ (¬ p ∨ r))

(p ∨ ¬ q) ∨ ((¬ r ∧ r) ∨ (¬ r ∧ ¬p)

(p ∨ ¬ q) ∨ (F ∨ (¬ r ∧ ¬p))

(¬(¬p ∧ q)) ∨ ((¬ r ∧ ¬p)

5 1.5 **3 / 3**

✓ **- 0 pts** **Correct**

- 1 pts Minor error (e.g. negation is outside of the parenthesis, correct law applied but had incorrect outcome, etc.)

- 2 pts Significant error (e.g. used implication, didn't solve nested parenthesis, etc.)

- 3 pts Blank answer

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

1.6 (6 points)

a) (p ∨ q ∨ ¬ r) ∧ (p ∨ ¬ q ∨ ¬ s) ∧ (¬p ∨ r)
The above proposition, expressed in conjunctive normal form, implies that each of the sub-propositions must be true. Therefore, the above proposition is **satisfiable given that p = True, q = True, and r = True**

b) p ∧ (¬ p ∨ r) ∧ ¬ r ∧ (¬ p ∨ ¬ r ∨ q)

p	q	r	Proposition
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	T	T	F
F	f	f	F

Unsatisfiable because none of the 8 possible permutations in the truth table above would result in the proposition to be true.

6 1.6 6 / 6

✓ **- 0 pts** **Correct**

- 1 pts 1.6.a. Incorrect answer, Correct answer without the case where it's satisfiable

- 3 pts 1.6.a. Blank answer

- 1 pts 1.6.b. Incorrect answer, Correct answer without reasoning

- 3 pts 1.6.b. Blank answer

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

2 Predicate Logic

2.1 (12 points)

a) Answers Below:
   i. $\forall x(R(x))$
   ii. $\forall x(Q(x) \rightarrow R(x))$
   iii. $\forall x(P(x) \wedge Q(x) \rightarrow R(x))$

b) a → *a given number is a prime number greater than 3.*
   b → equal to a multiple of six, plus 1 or minus 1.
   Statement 1: a → b
   i. b → *a*
      Knowing that statement 1 is true, the converse is not logically true.
      Therefore, ***it is invalid.***
   ii. ¬ b → ¬ a
      Knowing that statement 1 is true, the contrapositive is logically equivalent.
      Therefore, **it is valid.**

7 2.1 11 / 12

- **0 pts** Correct
- **1 pts** 2.1.a. Minor errors (AND vs OR, parenthesis, missing FORALL symbol, etc.)
- **3 pts** 2.1.a. Significant errors (expressed the domain, not the statement)
- **6 pts** 2.1.a. Blank answer

✓ **- 1 pts** **2.1.b. One incorrect item**

- **2 pts** 2.1.b. Two incorrect items
- **3 pts** 2.1.b. Didn't evaluate validity, but evaluated truth value
- **6 pts** 2.1.b. Blank answer
- **3 pts** 2.1.b (i) Blank
- **3 pts** 2.1.b (ii) Blank

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

2.2 (10 points)

De Morgan's Laws can be applied to solving each part of this problem

(a) ¬ ∃*y* ∃*xP(x,y)* ≡ **∀*y*∀*x*¬P(x,y)**

(b) ¬∀x∃P(x,y) ≡ **∃x∀y ¬ P(x,y)**

(c) ¬ ∃y(P(y) ∧ ∀x ¬R(x,y)) ≡ **∀y¬ P(y) ∨ ∃xR(x,y)**

(d) ¬ ∃y(∃xR(x,y) ∨ ∀x S(x,y)) ≡ **∀y(∀x ¬R(x,y) ∧ ∃ x ¬ S(x,y))**

(e) ¬ ∃ y(∀x∃z T(x,y,z) ∨ ∃x∀z U(x,y,z) ≡ **∀y (∃x ∀z¬T(x,y,z) ∧ (∀x∃z ¬ U(x,y,z)))**

8 2.2 **10 / 10**

✓ **- 0 pts Correct**

- 1 pts 1 incorrect

- 2 pts 2 incorrect

- 3 pts 3 incorrect

- 4 pts 4 incorrect

- 5 pts 5 incorrect

- 10 pts Blank

- 4 pts 2 blank

- 2 pts 1 blank

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

2.3 (18 points)

a) A answers below.

i. ¬ ∃xQ(x) ∧ P(x) ≡ **∀x(¬Q(x) ∨ ¬ P(x))**

ii. **∃xQ(x) ∧ S(x)**

iii. **∀xS(x) → *R(x)***

iv. ¬ ∃x(P(x) ∧ Q(x)) ∧ R(x) ≡ **∀x (¬P(x) ∨ ¬Q(x)) ∨¬S(x)**

b) B answers below

i. **Valid** because if (ii) is true then S(x) must be true, and given that (iii) is true, R(x) must also be true, which implies that some kangaroos can leap tall buildings in a single bound.

ii. Given that i, iii, and iv is true, the statement can be translated as:
P(x) ∧ Q(x) → ¬ R(x)
If either P(x) or Q(x) is true as given by the previous proposition, and we know that R(x) is false, that would be translated as:
T ∧ T → *T*
Therefore, **the statement is valid**

9 2.3 **18 / 18**

✓ **- 0 pts Correct**

- 1 pts 2.3.a.i. Incorrect answer (e.g. incorrect use of implication, quantifier, or disjunction)

- 1 pts 2.3.a.ii. Incorrect answer (e.g. incorrect use of implication, quantifier, or disjunction)

- 1 pts 2.3.a.iii. Incorrect answer (e.g. incorrect use of implication, quantifier, or disjunction)

- 1 pts 2.3.a.iv. Incorrect answer (e.g. incorrect use of implication, quantifier, or disjunction)

- 1 pts 2.3.b.i. Incorrect answer, Correct answer without reasoning

- 1 pts 2.3.b.ii. Incorrect answer, Correct answer without reasoning

- 18 pts blank

- 4.5 pts 2.3.b.i blank

- 4.5 pts 2.3.b.ii blank

- 9 pts 2.3. b blank

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

2.4 (10 points)

a) **F(Marlene, Kevin)**
b) **∀y¬F(Kevin, y)**
c) **∀y F(lena, y)**
d) **∀y∃xF(x,y)**
e) **¬∃xF(x,x)**

10 2.4 **10 / 10**

✓ **- 0 pts Correct**

- 1 pts 1 incorrect

- 2 pts 2 incorrect

- 3 pts 3 incorrect

- 4 pts 4 incorrect

- 5 pts 5 incorrect

- 10 pts blank

- 2 pts 1 blank

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡

2.5 (7 Points)

If we know that mn is even, there can be 1 of 3 possible combinations of m and n.

Permutation 1: m is even and n is odd
$m = 2k_1$ $n = 2k_2 + 1$
$mn = 4k_1k_2 + 2k_1 = 2(K_1K_2 + K_1) = 2(\text{integer})$, implying that mn is even

Permutation 2: m is even and n is even
$m = 2k_1$ $n = 2k_2$
$mn = 4k_1k_2 = 2(K_1K_2) = 2(\text{integer})$, implying that mn is even

Permutation 3: m is odd and n is even
$m = 2k_1 + 1$ $n = 2k_2$
$mn = 4k_1k_2 + 2k_2 = 2(k_1k_2 + k_2) = 2(\text{integer})$, implying that mn is even.

Therefore, given that m and n integers, and that mn is even, either m or n is even.

11 2.5 **7 / 7**

✓ **- 0 pts** **Correct**

- 2 pts Minor issues (e.g. Used propositions and logical expressions, but failed to generalize all cases with even number: 2k / odd number: 2k + 1)

- 3 pts Major issues (e.g. Just showing some examples/counter-examples without generalization)

- 5 pts Failed to prove

- 7 pts Blank answer