CS182 – Foundations of Computer Science

Homework 6 (100 points)

Due: Thursday, March XX, 2020, 11:59pm

Upload the homework to Gradescope. No late submissions accepted. Only typed solutions will be graded.

Problem 1. (100 = 10*10 points)
In this problem, a word is any string of 12 numbers (e.g., 002342393765 is a codeword but 01245 is not, since it consists of only five numbers).

Provide a *brief* justification for each of your answers (no more than five lines or so), explaining which counting rules you used and what your thought process was. Feel free to give expressions in the form $2 \cdot 4^{10}$, etc. in your final answers; no need to use calculators to compute such powers.

1. How many codewords are there?
   - **Ans:** 10^{12}
   - Each bit of codewords has 10 possible cases, which is from number 0 to number 9. Consider a task of assigning a number to a bit. Assigning numbers to all bits in the codeword is done sequentially, therefore we apply the product rule. Each task has 10 ways, and we have 12 sequential tasks, so the answer is 10^{12}

2. How many codewords end with 45?
   - **Ans:** 10^{10}
   - In this case, we fix the last two bits as 45. Therefore, our task is to assign bits to 10 positions except for the last two. Each task has 10 ways as before, and we have 10 sequential tasks, so the answer is 10^{10}

3. How many codewords begin with 1 and end with 1?
   - **Ans:** 10^{10}
   - Similar to the previous case, we fixt the first and the last bit as 1 and 1. Then our task is to assign bits to 10 positions except for the first and the last. Each task has 10 ways as before, and we have 10 sequential tasks, so the answer is 10^{10}

4. How many codewords begin with 52 or 62?
   - **Ans:** 2×10^{10}
   - Think about the codewords that begin with 52 first. This is a similar to Prob2, and the answer is 10^{10}. Now we can think about the codewords that begin with 62, and the answer for this case is also 10^{10}. All codewords generated from this second case is not overlapping with the first case where the codewords started with 52. So we apply the sum rule to get the final answer. The answer is therefore $10^{10} + 10^{10} = 2 \times 10^{10}$

5. How many codewords begin with 00 or end with 00?
   - **Ans:** $2 \times 10^{10} - 10^{8}$
   - Similar to previous problem, we first get the answer for the codewords begin with 00–10^{10}. The number of codewords end with 00 is also 10^{10}. These two cases are not sequential, thus we apply the sum rule, and we get 2×10^{10}. Now think of cases where the codewords begin with 00 AND end with 00. We added up the number of this case twice–once when we calculated the codewords begin with 00, and twice when we calculated the codewords end with 00. So we need to subtract

the number of codewords begin with and end with 00, which is 10^8 (we've fixed the first two and the last two bits, so we have 8 remaining bits with 10 possible assignment). Therefor the answer is $2 \times 10^{10} - 10^8$

6. How many codewords begin with 22 or 12 and end with 0 or 1?
   - **Ans:** 4×10^9
   - For this problem, we fix the first two digits and the last digit. Then we have 9 remaining bits, and the number of ways to assign numbers to these bits is 10^9. Now we think of all cases for the first two bits and the last bit. We have four cases in total – (22 XXXXXXXXX 1) (22 XXXXXXXXX 0) (12 XXXXXXXXX 1) (12 XXXXXXXXX 0). These cases are not overlapping. Therefore, the answer is 4×10^9

7. How many codewords begin with a number strictly smaller than 5 and end with a number strictly larger than 5?
   - **Ans:** 20×10^{10}
   - For this problem, we fix the first and the last digit. Then we have 10 remaining bits, and the number of ways to assign numbers to these bits is 10^{10}. Now we think of all cases for the first and the last bit. We have five possible assignments (from 0 to 4) to the first bit, and four possible assignments (from 6 to 9) to the last bit. The number of sequentially assigning these two bits is thus $5 \times 4 = 20$. Therefore, the answer is 20×10^{10}

8. How many codewords have their first three numbers be all strictly smaller than 6? For example, 03242393765 and 01342393765 are such codewords but 02742393765 is not.
   - **Ans:** $6^3 \times 10^9$
   - For this problem, we fix the first three bits. Then we have 9 remaining bits, and the number of ways to assign numbers to these bits is 10^9. Now we think of all cases for the first three bits. We have six possible assignments (from 0 to 5) to the first three bits. , and four possible assignments (from 6 to 9) to the last bit. The number of sequentially assigning these three bits is thus 6^3. Therefore, the answer is $6^3 \times 10^9$

9. How many codewords have no zeros *and* no ones *and* no twos?
   - **Ans:** 7^{12}
   - For this problem, we can simply exclude 0, 1, 2 from our choice of numbers when assigning them to each bit. Then we have seven possible ways to assign a number to each bit. We still have 12 bits to assign, therefore the answer is 7^{12}

10. How many codewords have no ones *or* no fives?
    - **Ans:** $2 \times 9^{12} - 8^{12}$
    - Similar to the previous problem, the number of codewords without using 1 is 9^{12} because we have nine possible ways to assign. The number of codewords without using 5 is also 9^{12}. We then apply the sum rule to count the case of no 1 or no 5, which gives us 2×9^{12}. Now consider the case where we didn't use 1 AND 5 for all bits in the codeword. The number of this case was added twice, thus we need to subtract the number once. The number of codewords without using 1 AND 5 is 8^{12}. Therefore the answer is $2 \times 9^{12} - 8^{12}$