Homework 8 (100 points)

Due: Thursday, April 23, 2020, 11:59pm

Upload the homework to Gradescope. DO NOT SUBMIT TO BLACKBOARD LEARN. No late submissions accepted. Only typed solutions will be graded.

Remark: Provide a brief justification for each of your answers (no more than five lines), explaining which counting rules you used and what your thought process was. Feel free to have expressions of the form $3 \cdot 5^{26}$, etc. in your final answers; no need to use calculators to compute such powers.

Problem 1. (30=3*10 points)
Suppose you have a set S of 40 objects, split into four classes (C_1, C_2, C_3, and C_4) of 10 objects each. Objects within a class are identical.

- You pick one object from S at random. What is the probability that the object belongs to the class C_4?

- You pick three objects at random, one at a time, *with replacement*. What is the probability that all three belong to the class C_4?

- You pick three objects at random, one at a time, *without replacement*. What is the probability that all three belong to the class C_1?

Problem 2. (40=5*8 points)
What is the probability of each of the following events:

- What is the probability that a card chosen from an ordinary deck of 52 cards belongs to a red suit?

- What is the probability that the sum of two randomly selected integers chosen from the first 200 positive integers $1, 2, 3, \ldots, 200$ is even?

- What is the probability that the sum of the numbers on four dice rolled is even?

- What is the probability that a fair coin lands heads exactly 15 times out of 18 flips?

- If a biased coin has a probability of heads that is twice the probability of tails and is flipped 100 times, what is the probability of exactly ten heads?

Problem 3. (30=3*10 points)

1. A group of 15 undergraduate and 15 graduate students are in a room. If 10 of the 30 students are selected at random and called sequentially by the instructor (order matters), what is the probability that all 10 are graduate students or all 10 are undergraduate students?

2. Select uniformly at random a permutation of $\{1, 2, ..., 100\}$. What is the probability that in the selected permutation all numbers are in ascending order?

3. A group of 40 women and 40 men are in a room. A committee of 15 is chosen at random. Find the probability that the committee consists only of women.