CS 182 Midterm
March 11, 2020

Name:	
PUID:	
Purdue email:	

Instructions

1. Please read and follow all instructions carefully.
2. Put your name, PUID, and Purdue e-mail address in the spaces provided above.
3. Put your name on each page of the exam.
4. All answers and explanations should go in the boxes provided. Things written in the margins may not be visible after scanning, and therefore may not be graded.
5. If you need extra space, you may use the extra pages provided. Be sure to indicate that you have used the extra pages and number your work accordingly.
6. Write clearly and with a dark pen or pencil. Illegible answers will not receive credit.
7. You may not use any resources other than the sheet of notes indicated in the syllabus.
8. You must show your work and/or explain your reasoning on every question to receive full credit!
9. When appropriate, make sure it is very clear what your final answer is by putting a box around it.

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Question 1: Logical Equivalence (4 + 6 points)

(a) Determine if the following logical expressions are equivalent or not and justify your answer by filling in the truth table provided below.

Expression 1: $(\boldsymbol{p} \wedge \boldsymbol{q}) \vee \boldsymbol{r}$
Expression 2: $(\neg \boldsymbol{p} \rightarrow \boldsymbol{r}) \wedge (\neg \boldsymbol{q} \rightarrow \boldsymbol{r})$

p	q	r	$(\boldsymbol{p} \wedge \boldsymbol{q}) \vee \boldsymbol{r}$	$(\neg \boldsymbol{p} \rightarrow \boldsymbol{r}) \wedge (\neg \boldsymbol{q} \rightarrow \boldsymbol{r})$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

(b) Determine if the following expressions are logically equivalent or not. If they are, show their equivalence by applying logical equivalences. Be sure to name the equivalences used. If they are not equivalent, explain why by giving a counterexample.

Expression 1: $\boldsymbol{p} \vee \neg(\boldsymbol{p} \vee \boldsymbol{q})$
Expression 2: $\neg \boldsymbol{p} \rightarrow \boldsymbol{q}$

Question 2: Predicate Logic and Inferences (5 + 5 points)

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(a) Translate each English statement into logic according to the definitions below.

$x, y \in U$ **where** U **is the set of all people**

$A(x, y) =$ "x **is taller than** y"

$B(x, y) =$ "x **weighs more than** y"

Statement 1: Someone who is taller than Bob weighs less than Bob.
Statement 2: Someone is taller and weighs more than everyone.

(b) Derive the statement "No cats are dogs" **from the premises below by translating them into logic and then showing the argument using inferences. Name the inference rules used in the argument, and use the definitions defined below.**

Let *C(x) = "x is a cat," F(x) = "x is a feline,"* **and** *D(x) = "x is a dog"* where the domain of *x* is all animals.

Premise 1: All cats are felines.
Premise 2: No dogs are felines.

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Question 3: Sets and Functions (5 + 5 points)

(a) Prove the following statement using set notation and logical equivalences.

$(\boldsymbol{B} \cup \boldsymbol{C}) - \boldsymbol{A} = (\boldsymbol{B} - \boldsymbol{A}) \cup (\boldsymbol{C} - \boldsymbol{A})$

(b) Determine if each of the functions below is a bijection or not. If it is, give the inverse. If it isn't, explain why it isn't. Assume that function f maps from the set of reals to positive reals, and function g maps from the set of positive integers to reals.

(i) $f(x) = 4x^2 + 2$

(ii) $\boldsymbol{g(x) = \frac{x^2+8}{3}}$

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Question 4: Proof Techniques (6 + 4 points)

(a) Prove that if n is an integer and $3n + 1$ is even then n is odd.

(b) Prove or disprove the following – " every positive integer can be written as the sum of squares of three integers".

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Question 5. Order Notion and Analysis of Algorithms (6 + 4 points)

(a) What is the order of growth in Big-Theta notation for each of the functions below?

(i) $\boldsymbol{f(x) = 3x^2\, log_2(x^4) + 3^x + x^3}$

(ii) $\boldsymbol{g(x) = 3x log_2(x^4) + 5x^2 + 8}$

(b) What is the runtime in Big-Theta notation for the number of times the underlined command executes in the following pseudocode?

```
count := 0
for i := 0 to n-1
   for j := 0 to i
      for k := 1 to 10
          count := count + 1
```

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Question 6: Number Theory (2 + 2 + 2 + 2 + 2 points)

(a) Determine the values of the following and explain your reasoning.

(i) $-45 \ mod \ 7$

(ii) $(1234567890 \times 987654321) \ mod \ 5$

(b) Are the following statements of congruency true or false? Explain your reasoning.

(i) $45 \equiv 53 \ mod \ 7$

(ii) $-5 \equiv (1234567890 + 234567895) \ mod \ 5$

(c) Convert the following numbers from base 10 into base 2. Show your work.

(i) 33

(ii) 41

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(d) Convert the following numbers from base 2 to base 10.

(i) 10101

(ii) 100111

(e) Convert the following numbers from base 2 to base 8.

(i) 101011010010000101

(ii) 1101010010101010