CS182 – Foundations of Computer Science

**Homework 5 (100 points)**

**Due:** Thursday, April 2, 2020, 11:59pm

**Upload the homework to Gradescope. DO NOT SUBMIT TO BLACKBOARD LEARN.** No late submissions accepted. Only typed solutions will be graded.

**Problem 1. (20 points)**
Prove or disprove that for any positive integer $n \geq 0$, $n^3 + 2n + 3$ is divisible by 3.

**Problem 2. (20 points)**
Prove or disprove that for all $n \geq 1$

$$\sum_{i=1}^{n} i2^i \leq 2^{n+1}n.$$

**Problem 3. (20 points)**
Let

$$H_n = \sum_{i=1}^{n} \frac{1}{i}$$

for any $n \geq 1$. Prove or disprove that for any $n \geq 1$

$$H_{2^n} \geq \frac{n}{4}.$$

**Problem 4. (20 points)**
Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Prove or disprove that $a_n \leq 3^n$ for all positive integers $n$.

**Problem 5. (20 points)**
Describe the most efficient algorithm to compute $5^{\log_2 m}$ where $m$ is a nonnegative integer. Prove its correctness.