

Spring-2020-CS-18200-LE1 Exam 1

Abhi Gunasekar

TOTAL POINTS

54 / 60

QUESTION 1

1 10 pts

1.1 4 / 4

✓ - **0 pts** Correct

- **0.5 pts** Missing conclusion(didn't mention they are equivalent)

- **1 pts** Correct conclusion, but some rows in the table are incorrect

- **2 pts** Made an incorrect conclusion because of some incorrect rows in the table

- **3 pts** Conclusion without filling out the truth table

- **4 pts** Blank

- **1 pts** Wrong conclusion

- **0.5 pts** Empty row in the table

1.2 4 / 6

- **0 pts** Correct

- **1 pts** Correct conclusion but not enough explanation(e.g. applying logical equivalence)

✓ - **2 pts** Correct conclusion without counterexample

- **4 pts** Incorrect conclusion

- **6 pts** Blank

- **1 pts** Correct conclusion with wrong counterexample

- **3 pts** No conclusion

- **4 pts** No conclusion with no counterexample

- **0.5 pts** Minor mistake

QUESTION 2

2 10 pts

2.1 5 / 5

- **1 pts** Statement 1 minor mistake: failed to specify

Bob

- **1 pts** Statement 2 minor mistake: incorrect

quantifier order

- **2 pts** Incorrect statement 2

- **5 pts** Blank

✓ - **0 pts** Correct

- **2 pts** Incorrect statement 1

2.2 5 / 5

✓ - **0 pts** Correct

- **1 pts** Did not use the defined functions C, F, D

- **1 pts** Incorrect translation of premises into logical expressions

- **1 pts** Missing inference rule name

- **1 pts** Minor error or leap in reasoning

- **2 pts** Incorrect final expression

- **4 pts** Incorrect/incomplete explanation

- **5 pts** Blank

QUESTION 3

3 10 pts

3.1 5 / 5

✓ - **0 pts** Correct

- **2 pts** Only used Venn diagram without any use of logical expressions

- **4 pts** Incorrect/Incomplete reasoning

- **5 pts** Blank

3.2 3 / 5

- **0 pts** Correct

- **1 pts** Incorrectly shown (ii) is not injective

- **2 pts** Incorrect answer for (i)

✓ - **2 pts** Incorrect answer for (ii)

- **5 pts** Blank

QUESTION 4

4 10 pts

4.1 6 / 6

✓ - 0 pts Correct

- 3 pts Logical errors in proof (e.g. just proved for some examples)

- 6 pts Blank

4.2 3 / 4

- 0 pts Correct

✓ - 1 pts Provided a counterexample, but not enough explanation

- 3 pts Incorrect reasoning

- 4 pts Blank

QUESTION 5

5 10 pts

5.1 6 / 6

✓ - 0 pts Correct

- 1 pts (i). Correct answer without lower bound/upper bound explanation

- 1 pts (ii). Correct answer without lower bound/upper bound explanation

- 2 pts (i). Incorrect answer

- 2 pts (ii). Incorrect answer

- 6 pts Blank

5.2 3 / 4

- 0 pts Correct

✓ - 1 pts Correct answers with imperfect reasoning

- 2 pts Correct answer. Reasoning incorrect or missing.

- 2 pts Incorrect answer with almost correct reasoning

- 3 pts Incorrect

- 4 pts Blank

QUESTION 6

6 10 pts

6.1 2 / 2

✓ - 0 pts Correct

- 0.5 pts (i). Minor mistakes

- 0.5 pts (ii). Minor mistakes

- 1 pts (i). Incorrect

- 1 pts (ii). Incorrect

6.2 2 / 2

- 0.5 pts (i). Minor mistakes

- 0.5 pts (ii). Minor mistakes

- 1 pts (i). Incorrect

- 1 pts (ii). Incorrect

✓ - 0 pts Correct

- 2 pts Blank

6.3 2 / 2

✓ - 0 pts Correct

- 0.5 pts (i). Minor mistakes

- 0.5 pts (ii). Minor mistakes

- 1 pts (i). Incorrect

- 1 pts (ii). Incorrect

6.4 2 / 2

✓ - 0 pts Correct

- 0.5 pts (i). Minor mistakes

- 0.5 pts (ii). Minor mistakes

- 1 pts (i). Incorrect

- 1 pts (ii). Incorrect

6.5 2 / 2

✓ - 0 pts Correct

- 0.5 pts (i). Minor mistakes

- 0.5 pts (ii). Minor mistakes

- 1 pts (i). Incorrect

- 1 pts (ii). Incorrect

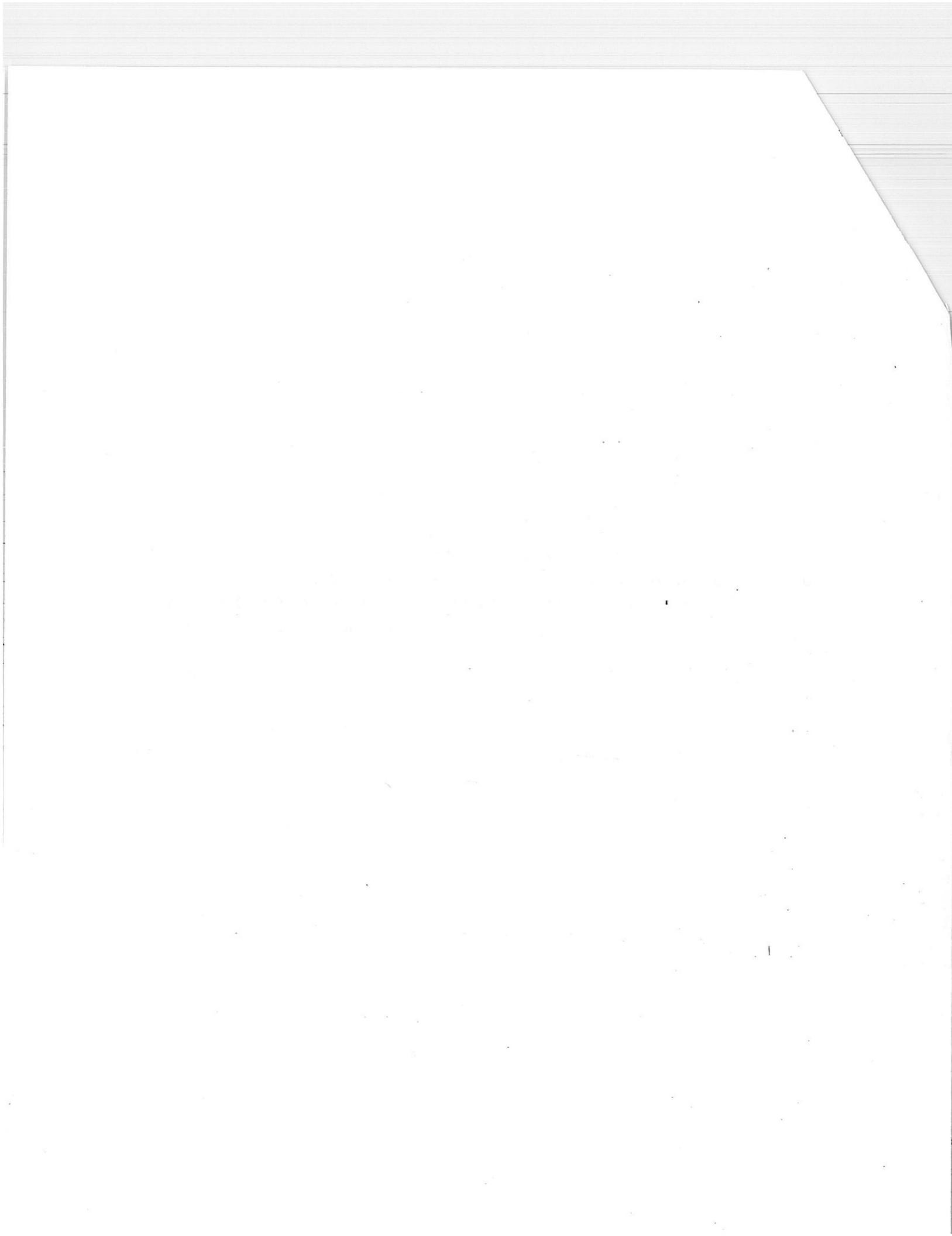
- 2 pts all incorrect/blank

CS 182 Midterm
March 11, 2020

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Instructions

1. Please read and follow all instructions carefully.
2. Put your name, PUID, and Purdue e-mail address in the spaces provided above.
3. Put your name on each page of the exam.
4. All answers and explanations should go in the boxes provided. Things written in the margins may not be visible after scanning, and therefore may not be graded.
5. If you need extra space, you may use the extra pages provided. Be sure to indicate that you have used the extra pages and number your work accordingly.
6. Write clearly and with a dark pen or pencil. Illegible answers will not receive credit.
7. You may not use any resources other than the sheet of notes indicated in the syllabus.
8. You must show your work and/or explain your reasoning on every question to receive full credit!
9. When appropriate, make sure it is very clear what your final answer is by putting a box around it.
10. There are two scratch pages at the end of the exam that you can use if you need extra space.



NAME: Abhishek Bhagat

Question 1: Logical Equivalence (4 + 6 points)

(a) Determine if the following logical expressions are equivalent or not and justify your answer by filling in the truth table provided below.

$$\text{Expression 1: } (p \wedge q) \vee r$$

$$\text{Expression 2: } (\neg p \rightarrow r) \wedge (\neg q \rightarrow r)$$

p	q	r	$(p \wedge q) \vee r$	$(\neg p \rightarrow r) \wedge (\neg q \rightarrow r)$
T	T	T	T	$T \wedge T = T$
T	T	F	T	$T \wedge T = T$
T	F	T	T	$T \wedge T = T$
T	F	F	F	$T \wedge F = F$
F	T	T	T	$T \wedge T = T$
F	T	F	F	$F \wedge T = F$
F	F	T	T	$T \wedge T = T$
F	F	F	F	$F \wedge F = F$

They are equivalent

$$T \\ F \rightarrow T \quad F \rightarrow T$$

$$F \rightarrow F \quad F \rightarrow F$$

$$F \rightarrow T \quad T \rightarrow T$$

$$F \rightarrow F \quad T \rightarrow F$$

$$T \rightarrow T \quad F \rightarrow T$$

$$T \rightarrow F \quad F \rightarrow F$$

$$T \rightarrow T \quad T \rightarrow T$$

$$F \rightarrow F \quad T \rightarrow F$$

(b) Determine if the following expressions are logically equivalent or not. If they are, show their equivalence by applying logical equivalences. Be sure to name the equivalences used. If they are not equivalent, explain why by giving a counterexample.

$$\text{Expression 1: } p \vee \neg(p \vee q)$$

$$\text{Expression 2: } \neg p \rightarrow q$$

$$\text{L.H.S.} \\ p \vee (\neg p \wedge \neg q) \text{ de Morgan}$$

$$(\neg p \vee \neg p) \wedge (\neg p \vee \neg q) \text{ distribution}$$

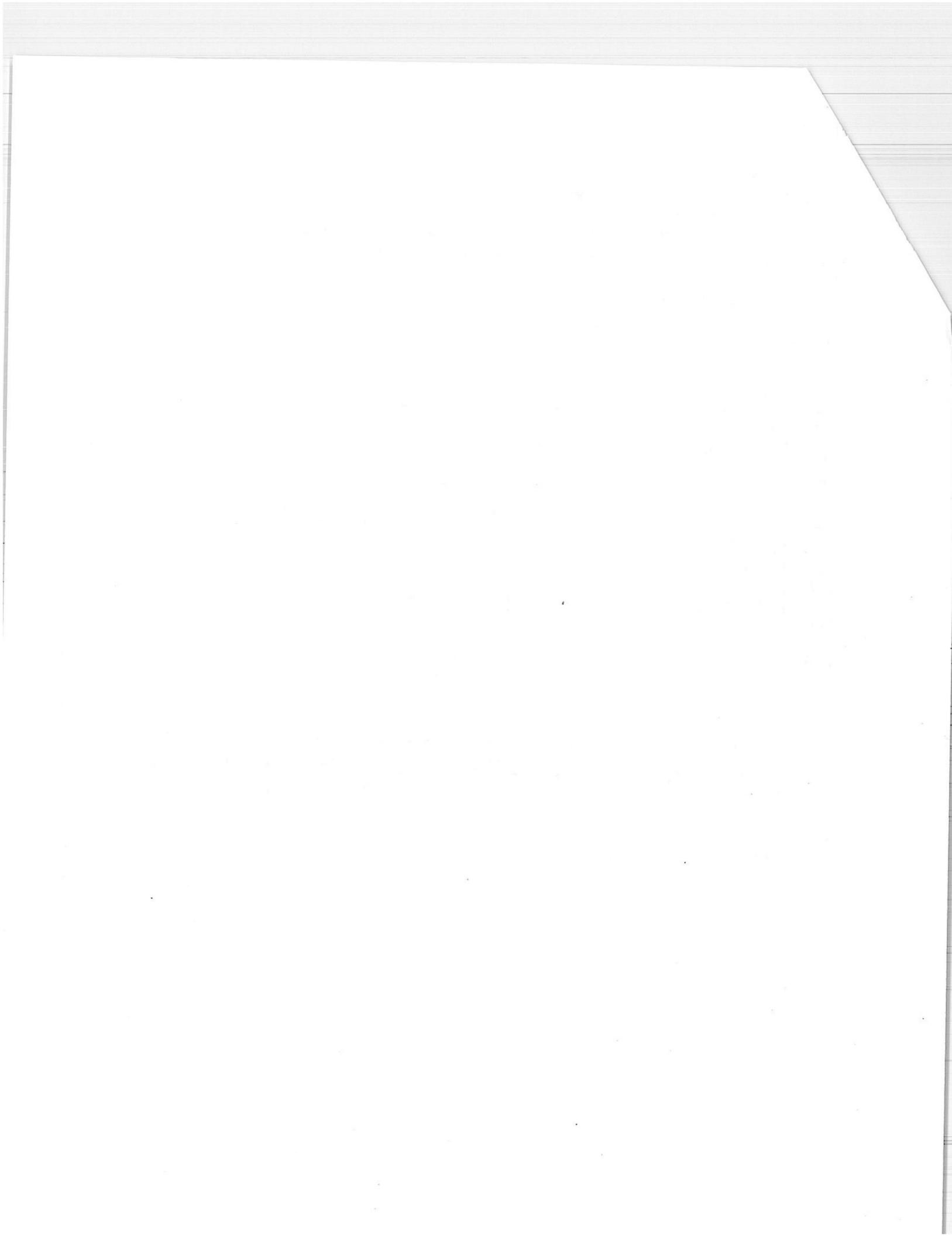
$$T \wedge (\neg p \vee \neg q) \text{ Negate}$$

$$\neg p \rightarrow \neg q \text{ implication as distribution}$$

$$\neg p \rightarrow \neg q \neq \neg p \rightarrow q$$

not equivalent.

Question 2: Predicate Logic and Inferences (5 + 5 points)



NAME: Ashley busy ever

(a) Translate each English statement into logic according to the definitions below.

$x, y \in U$ where U is the set of all people

$\checkmark A(x, y) = "x \text{ is taller than } y"$

$\checkmark B(x, y) = "x \text{ weighs more than } y"$

Statement 1: Someone who is taller than Bob weighs less than Bob.

Statement 2: Someone is taller and weighs more than everyone.

$$S_1: \exists x A(x, \text{Bob}) \wedge (\neg B(x, \text{Bob})).$$

$$S_2: \exists x \forall y A(x, y) \wedge B(x, y).$$

(b) Derive the statement "No cats are dogs" from the premises below by translating them into logic and then showing the argument using inferences. Name the inference rules used in the argument, and use the definitions defined below.

Let $C(x) = "x \text{ is a cat,"}$ $F(x) = "x \text{ is a feline,"}$ and $D(x) = "x \text{ is a dog}"$ where the domain of x is all animals.

Premise 1: All cats are felines.

Premise 2: No dogs are felines.

$$1. \forall x C(x) \rightarrow F(x)$$

Premise 1

$$2. \forall x D(x) \rightarrow \neg F(x)$$

Premise 2

$$3. C(z) \rightarrow F(z)$$

U.I. on 1

$$4. D(z) \rightarrow \neg F(z)$$

U.I. on 2

$$5. F(z) \rightarrow \neg D(z)$$

antonym of 4.

$$6. (\exists z) \rightarrow \neg D(z)$$

hypothetical syllogism on 3, 5.

$$7. \forall x C(x) \rightarrow \neg D(x)$$

U.G. on 6

$$\forall x C(x) \rightarrow F(x)$$

$$\neg \exists x (D(x) \wedge F(x)) = \forall x \neg (D(x) \wedge F(x))$$

Conclusion: $\neg \exists x (C(x) \wedge D(x))$

$$= \forall x \neg (C(x) \vee D(x))$$

$$= \forall x C(x) \rightarrow \neg D(x)$$



NAME: Mishka bawalew

Question 3: Sets and Functions (5 + 5 points)

(a) Prove the following statement using set notation and logical equivalences.

$$(B \cup C) - A = (B - A) \cup (C - A)$$

L.H.S

$$\{x \mid x \in (B \cup C) - A\}$$

$$\{x \mid x \in (B \cup C) \wedge \neg A\}$$

$$\{x \mid x \in B \cup C \wedge x \notin A\}$$

$$\{x \mid x \in B \cup C \wedge x \neq A\}$$

$$\{x \mid (x \in B \vee x \in C) \wedge \neg (x \in A)\}$$

$$\{x \mid [x \in B \wedge \neg (x \in A)] \vee [x \in C \wedge \neg (x \in A)]\}$$

$$\{x \mid (x \in B \wedge \neg (x \in A)) \vee (x \in C \wedge \neg (x \in A))\}$$

$$\{x \mid x \in (B \cap \neg A) \vee (x \in (C \cap \neg A))\}$$

$$\{x \mid x \in (B - A) \vee x \in (C - A)\}$$

$$(B - A) \cup (C - A)$$

Rough Work

L.H.S

$$(B \cup C) \cap \neg$$

$$\neg \in (B \cup C)$$

$$(A \cap B) \cup (\bar{A} \cap C)$$

$$(B - A) \cup (C - A)$$

Distribution

Cleb st not rns.

Cleb of min and intersection.

set difference.

both statements are equivalent.

(b) Determine if each of the functions below is a bijection or not. If it is, give the inverse. If it isn't, explain why it isn't. Assume that function f maps from the set of reals to positive reals, and function g maps from the set of positive integers to reals.

$$(i) f(x) = 4x^2 + 2$$

$$(ii) g(x) = \frac{x^2+8}{3}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}^+$$

$$g: \mathbb{Z}^+ \rightarrow \mathbb{R}$$

$$\cancel{f}$$

$$(i) f(x) = 4x^2 + 2$$

one to one but not onto because $f(-1) = f(1)$ for instance.
Since not one to one, f is not a bijection. Since function must be both one to one and onto to be a bijection.

$$(ii) g(x) = \frac{x^2+8}{3}$$

one to one? Yes. Let $x, y \in \mathbb{Z}^+$, then $g(x) = g(y)$, $\frac{x^2+8}{3} = \frac{y^2+8}{3}$,
implies $x^2 = y^2$. By definition, $\exists x, \exists y$ such that
onto? Yes. Let y be some arbitrary $\in \mathbb{R}$, then if $x \in \mathbb{Z}^+$, $g(x) =$
 $\sqrt{3y-8}$, then $g(x) = y \dots g$ is onto.

Since onto, g is surjective, g is a bijection.

$$g(y) = \pm \sqrt{3x-8} \text{ is the inverse}$$

$\frac{1}{2} \lambda^2$ $\left(\frac{\partial}{\partial \lambda} \right)^2$ \hat{A}

λ^2 $\left(\frac{\partial}{\partial \lambda} \right)^2$ \hat{A}

NAME: Abhishek Bawali

Question 4: Proof Techniques (6 + 4 points)

(a) Prove that if n is an integer and $3n+1$ is even then n is odd.

AHP: "If P : '3n+1 is even', Q: 'n is odd'."
 $\neg Q \rightarrow \neg P \equiv P \rightarrow Q$: If n is even, then $3n+1$ is odd.

Let $n = 2k$ where $k \in \mathbb{Z}$, then
 $3n+1 = 3(2k)+1 = 6k+1 = 2(3k)+1 = 2(k') + 1$ where $k' \in \mathbb{Z}$
∴ $3n+1$ is odd.

Since, we have shown that the contrapositive holds.
∴ by proof of contraposition, we have shown that the original statement holds. Therefore, If $3n+1$ is even, then n is odd.

(b) Prove or disprove the following - "every positive integer can be written as the sum of squares of three integers".

$$21 = 4^2 + 2^2 + 1^2$$

Disprove by counterexample:

Let the integer under consideration by 61, 61 cannot be written as the sum of squares of 3 integers. Note that the perfect squares less than 61 are $(\pm 1)^2, (\pm 2)^2, (\pm 3)^2, (\pm 4)^2, (\pm 5)^2, (\pm 6)^2$.

$61 = 49 + 9 + 3$
69 and 9 are perfect squares of 7 and 3 respectively, but 3 cannot be expressed as a perfect square.

∴ we can disprove by counterexample.



NAME: Abhishek Banerjee

Question 5. Order Notation and Analysis of Algorithms (6 + 4 points)

(a) What is the order of growth in Big-Theta notation for each of the functions below?

$$(i) f(x) = 3x^2 \log_2(x^4) + 3^x + x^3$$

$$(ii) g(x) = 3x \log_2(x^4) + 5x^2 + 8$$

$$\begin{aligned} (i) 3\left(\frac{3^x}{2}\right) + \left(\frac{3^x}{2}\right) + \left(\frac{3^x}{2}\right) &\leq 3(x^2 \log_2(x^4)) + (3^x) + x^3 \leq 12 \cdot (3^x) + 3^x + 3^x \\ \sum \left(\frac{3^x}{2}\right) &\leq 3(x^2 \log_2(x^4)) + 3^x + x^3 \leq 12 \cdot (3^x) \quad \text{for } k > 1 \end{aligned}$$

$$\begin{aligned} (ii) 12\left(\frac{x}{2}\right)^2 + 5\left(\frac{x}{2}\right)^2 + 8(k) &\leq 3x \log_2 x^4 + 5x^2 + 8 \leq 12x^2 + 5x^2 + 8x^2 \\ \frac{25}{4}x^2 &\leq 3x \log_2 x^4 + 5x^2 + 8 \leq 25x^2 \quad \text{for } k > 1 \end{aligned}$$

(b) What is the runtime in Big-Theta notation for the number of times the underlined command executes in the following pseudocode?

```

count := 0
for i := 0 to n-1
    for j := 0 to i
        for k := 1 to 10
            count := count + 1
    
```

total number of iterations: $\frac{n(n+1)}{2} \cdot 10$

total comparisons: $\frac{n(n+1)}{2}$

Runtime = $\Theta(n^2)$

i	j	n-1
0	0	
1	1	
2	2	
3	3	
...	...	
n-1	n-1	

i	j	total
0	0	0
1	1	10
2	2	20
3	3	30
...
n-1	n-1	$10(n-1)$



NAME: Bhushan Bawal

Question 6: Number Theory (2 + 2 + 2 + 2 + 2 points)

(a) Determine the values of the following and explain your reasoning.

- (i) $-45 \bmod 7$
(ii) $(1234567890 \times 987654321) \bmod 5$

$$(i) -45 \bmod 7 = 4$$

$$q = -45 \div 7 = -7 \\ r = -45 \bmod 7 = -45 + 49 = 4$$

$$(ii) (a \cdot b) \bmod 5 = ((a \bmod m) \cdot (b \bmod n)) \bmod 5 \\ = ((1234567890 \bmod 5) \cdot (987654321 \bmod 5)) \bmod 5 \\ = (0 \cdot 1) \bmod 5 \\ = 0$$

(b) Are the following statements of congruency true or false? Explain your reasoning.

$$(i) 45 \equiv 53 \bmod 7$$

$$(ii) -5 \equiv (1234567890 + 234567895) \bmod 5$$

$$(i) 45 \equiv 53 \bmod 7 \quad \text{if } 45 \bmod 7 = 53 \bmod 7 \Rightarrow 3 \neq 4 \therefore$$

$$45 \not\equiv 53 \bmod 7$$

$$(ii) -5 \equiv (1234567890 \bmod 5 + 234567895 \bmod 5) \bmod 5 \\ = (0 + 0) \bmod 5 = 0 \bmod 5$$

$$-5 \bmod 5 = 0 \bmod 5 \quad \therefore -5 \equiv (1234567890 + 234567895) \bmod 5$$

(c) Convert the following numbers from base 10 into base 2. Show your work.

(i) 33
(ii) 41

$$\begin{array}{r} 33/2 & 16 & 1 \\ 16/2 & 8 & 0 \\ 8/2 & 4 & 0 \\ 4/2 & 2 & 0 \\ 2/2 & 1 & 0 \\ 1/2 & 0 & 1 \end{array}$$

$(33)_{10} = (10001)_2$

$$\begin{array}{r} 41/2 & 20 & 1 \\ 20/2 & 10 & 0 \\ 10/2 & 5 & 0 \\ 5/2 & 2 & 1 \\ 2/2 & 1 & 0 \end{array}$$

$$\begin{array}{r} 1/2 & 0 & 1 \\ (41)_{10} & & (101001)_2 \end{array}$$





NAME: Rishabh Gavaskar

Scratch page

$$(B \cup C) - A =$$

L.H.S

$$(B \cup C) \cap \bar{A}$$

$$\bar{A} \cap (B \cup C)$$

$$(\bar{A} \cap B) \cup (\bar{A} \cap C)$$

$$(B - A) \cup (C - A)$$

$$1+4+16$$

$$1+2+4+32$$

$$1+$$

~~$$1+8+32$$~~

29.8

NAME: Mihalee Guravon

Scratch page

$$\begin{aligned} & 12\left(\frac{1}{4}\right)^2 \\ & 5\left(\frac{1}{4}\right)^2 \\ & 8\left(\frac{1}{4}\right)^2 \\ & \cancel{3+\cancel{5}+2} \\ & \cancel{12+8} \end{aligned}$$

$$\begin{aligned} & (B - A) \cup (\neg A) \\ & (B \cap \neg A) \cup (C \cap \neg A) \\ & \text{Let } C = B \cap \neg A, \text{ then} \\ & 2 \cup (C \cap \neg A) \\ & (2 \cup C) \cap (2 \cup \neg A) \\ & [(B \cap \neg A) \cup C] \cap [(B \cap \neg A) \cup \neg A] \\ & [(B \cup C) \cap (C \cap \neg A)] \cap [(B \cup \neg A) \cap \neg A]. \end{aligned}$$

Every positive integer can be written as the sum of squares of three integers (sum of squares) or more integers as $x_1^2 + x_2^2 + x_3^2 + \dots$.

Now sum of
squares of
0 = 0

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

$$10^2 = 100$$

$$\begin{aligned} 62 &= 49 + 9 + 4 \\ 61 &= 49 + 9 + 3 \end{aligned}$$

$$\begin{aligned} y^2 + 8 &= 9 \\ y^2 &= 9 - 8 \\ y^2 &= 1 \\ y &= \pm \sqrt{1} \\ y &= \pm 1 \end{aligned}$$

$$\begin{aligned} y &= x^2 - 3 \\ x &= \sqrt{y^2 + 8} \\ 3x &= y^2 + 8 \\ 3x - 8 &= y^2 \\ y &= \pm \sqrt{3x - 8} \end{aligned}$$

