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CS 182: Homework 8

Problem 1. (30=3*10 points)

Suppose you have a set S of 40 *objects, split into four classes* $(C_1, C_2, C_3, and\ C_4)$ *of* 10 *objects each. Objects within a class are identical*

- You pick one object from S at random. What is the probability that the object belongs to the class C_4?

  $|S| = Total\ Number\ of\ Ways\ of\ Picking\ 1\ Object\ at\ random = C(40,1)$

  $|E| = Total\ Ways\ of\ that\ the\ picked\ object\ belongs\ to\ C_4 = C(10,1)$

  $$\boldsymbol{P(E) = \frac{|E|}{|S|} = \frac{C(10,1)}{C(40,1)} = \frac{1}{4} = 0.25}$$

- You pick three objects at random, one at a time, with replacement. What is the probability that all three belong to the class C_4?

  Picking with Replacement: Yes

  $$\boldsymbol{P\ (All\ Three\ Objects\ Belong\ to\ C_4) = \frac{1}{4} * \frac{1}{4} * \frac{1}{4} = \frac{1}{64}}$$

- You pick three objects at random, one at a time, *without* replacement. What is the probability that all three belong to the class C_1?

  Picking with Replacement: No

  $$\boldsymbol{P(All\ Three\ Objects\ Belong\ to\ C_1) = \left(\frac{10}{40}\right) * \left(\frac{9}{39}\right) * \left(\frac{8}{38}\right) = 0.01215}$$

Problem 2. (40=5*8 points)
What is the probability of each of the following events:

- What is the probability that a card chosen from an ordinary deck of 52 cards belongs to a red suit?

  $P(E) = \frac{|E|}{|S|}$

  $\boldsymbol{P(E) = \frac{26\ (Cards\ in\ Red\ Suit)}{52\ (Total\ Cards)} = \frac{1}{2} = 0.5}$

- What is the probability that the sum of two randomly selected integers chosen from the first 200 positive integers 1,2,3, ...,200 is even?

  $S = Set\ of\ all\ two\ integer\ subsets\ from\ 1-200$

  $E = Set\ of\ two\ integer\ subsets\ whose\ sum\ is\ even$

  $Lemma: a+b\ is\ even\ iff\ a\ \&\ b\ are\ both\ even\ or\ both\ odd$

  $E_1: Choosing\ Odd\ Numbers\ between\ 1-200$

  $E_2: Choosing\ Even\ Numbers\ between\ 1-200$

  $Case\ 1: Even + Even$

  $P(E_1) = \frac{100}{200} * \frac{100}{200} = \frac{1}{4}$

  $Case\ 2: Odd + Odd$

  $P(E_2) = \frac{100}{200} * \frac{100}{200} = \frac{1}{4}$

  $\boldsymbol{P(E) = P(E_1) + P(E_2) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} = 0.5}$

- What is the probability that the sum of the numbers on four dice rolled is even?

  $E: Sum\ of\ numbers\ on\ four\ dice\ rolled\ is\ even$

  $Even\ Sum$: 3 $Possibilities\ in\ Total$: Number of Dices with an Odd number

  $Possibility\ 1: OOOO = \binom{4}{4} = 1$

  $Possibility\ 2: OOEE = \binom{4}{2} = 6$

  $Possibility\ 3: EEEE = \binom{4}{0} = 1$

  $|S| = 2^4 = 16\ Total\ Possibilities$

  $\boldsymbol{P(E) = \frac{|E|}{|S|} = \frac{8}{16} = \frac{1}{2} = 0.5}$

- What is the probability that a fair coin lands heads exactly 15 times out of 18 flips?

  $E: 15\ Heads\ out\ of\ 18\ Flips$

$$\boldsymbol{P(E) = C(18, 15) * \left(\frac{1}{2}\right)^{15} \left(\frac{1}{2}\right)^{3} \ by\ Bernoulli/Binomial\ Distribution}$$

- If a biased coin has a probability of heads that is twice the probability of tails and is flipped 100 times, what is the probability of exactly ten heads?

  $2x + x = 1 => x = \frac{1}{3}. Therefore, P(Heads) = \frac{2}{3}$

  $E: Exactly\ 10\ Heads\ out\ of\ 100\ flips.$

  $$\boldsymbol{P(E) = \ C(100, 10) * \left(\frac{2}{3}\right)^{10} \left(\frac{1}{3}\right)^{90} \ by\ Bernoulli/Binomial\ Distribution}$$

Problem 3. (30=3*10 points)

1. A group of 15 undergraduate and 15 graduate students are in a room. If 10 of the 30 students are selected at random and called sequentially by the instructor (order matters). What is the probability that all 10 are graduate students or all 10 are undergraduate students?

   $E_1: All\ picked\ students\ are\ graduate\ students$
   $E_2: All\ picked\ students\ are\ undergraduate\ students$
   $E: Either\ E_1\ or\ E_2$
   $Therefore, P(E) = P(E_1) + P(E_2) - 0$
   $$P(E) = \frac{\big(C(15,10) * C(15,0)\big)}{C(30,10)} + \left(\frac{C(15,0) * C(15,10)}{C(30,10)}\right)$$
   $$\boldsymbol{P(E) = \left(\frac{2 * C(15,10)}{C(30,10)}\right) = 1.999 * 10^{-4}}$$

2. Select uniformly at random a permutation of {1,2, …, 100}. What is the probability that in the selected permutation all numbers are in ascending order?

   $S: Set\ of\ random\ permutations\ between\ 1 - 100$
   $|S| = 100!$
   $E: The\ Chosen\ permutation\ is\ in\ ascending\ order$
   $|E| = 1$
   $$\boldsymbol{P(E) = \frac{|E|}{|S|} = \frac{1}{100!}}$$

3. A group of 40 women and 40 men are in a room. A committee of 15 is chosen at random. Find the probability that the committee consists only of women.

   $|S| = Total\ Ways\ of\ Selecting\ People = C\ (80,15)$
   $E: Only\ Women\ selected$
   $$\boldsymbol{P(E) = \left(\frac{C(40,15) * C(40,0)}{C(80,15)}\right) = 6.062 * 10^{-6}}$$