# FOUNDATIONS OF COMPUTER SCIENCE: REVIEW

# Logic

- Propositional variables, compound propositions
- Implication $p \rightarrow q$ and its converse, contrapositive, and inverse
- De Morgan's Laws ($\neg(p \wedge q) \equiv \neg p \vee \neg q$ and more)
- Basic logical equivalences (shown in tables in KR)
- Constructing new logical equivalences
- Disjunctive and Conjunctive Normal Forms; satisfiability
- Predicates and Quantiers: $\forall x\, P(x)$, $\exists x\, P(x)$
  - Precedence, negating quantifiers, nesting quantifiers
- Translating English sentences

# Proofs

- Rules of inference
- Using rules of inference to build correct arguments
- Common types of proof
  - Direct proofs
  - Proofs by contraposition
  - Proofs by contradiction
  - Bidirectional statements
  - Existence proofs
  - Nonexistence proofs
  - Proofs by counterexample
  - Proof by cases
  - Proof by induction (mathematical, structural)
  - Proof by counting

# Sets, functions, and sums

- Set operations, identities, de Morgan’s law
- Functions
  - Injections, surjections, bijections
- Sequences
  - Arithmetic and geometric progressions
  - recurrence relations
- Summations
  - General rules of summation (and products)
  - Manipulating and evaluating sums, bounding sums
  - Important summations

# Big-O, algorithms, asymptotic growth

- Analyzing algorithms: Binary search, Bubble sort, Insertion sort
- Big O-notation
  - f(x) is O(g(x)) if there are constants C and k such that **$|f(x)| \leq C|g(x)|$** for all integers $x>k$.
  - Bounding loops
  - Showing equivalence (e.g., $(2n)!$ is not $O(n!)$, but $2n! = O(n!)$)

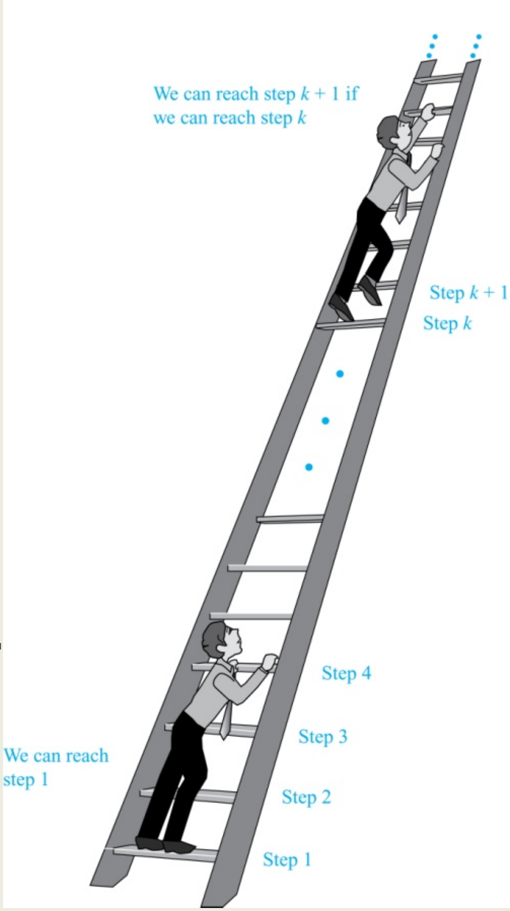
# Principle of Mathematical Induction

**Climbing an Infinite Ladder**:

- Basis Step:
  By (1), we can reach rung 1.
- Inductive Step:
  Assume we can reach rung $k$.
  Then by (2), we can reach rung $k$+1.

Hence, $P(k) \rightarrow P(k+1)$ is true for
all positive integers $k$.

We can reach every rung on the ladder.



# Induction and recursion

- many sums including $1 + 3 + 5 + \cdots + (2k - 1) = k^2$

- $n < 2^n$, $2^n < n!$, $f_n > \alpha^{n-2}$, $\alpha = (1 + \sqrt{5})/2$.
- $n^3 - n$ is divisible by 3
- every set with $k$ elements has $2^k$ subsets
- Structural inductions (e.g., tilings, tree properties)
- Weak and strong induction

# Recursion

Recurrence relations

- $a_n = a_{n-1} + 3$, $a_n = a_{n-1} – a_{n-2}$
- $f_n = f_{n-1} + f_{n-2}$
- $t(n) = t(n/2) + 3$ with $t(1) = 1$

Recursive definitions

- Factorial, Fibonacci numbers
- Tree properties (e.g., height)

Recursive algorithms

- Computing $a^n$
- Binary search
- Functions on rooted trees

# Counting

Basic rules: product, sum, subtraction, division

Pigeonhole principle

Permutations, r-permutations

- Counting ordered arrangements
- Distinguishable and indistinguishable objects
- Permutations with repetitions

Combinations and r-combinations

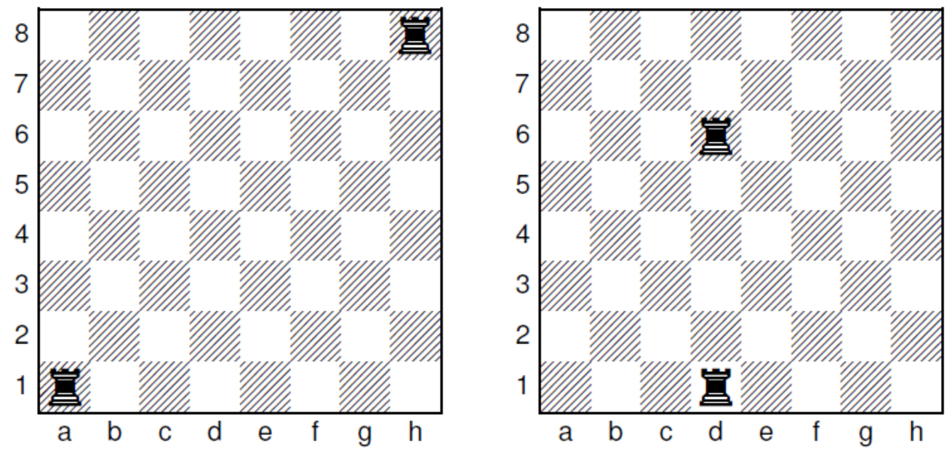
- Counting unordered selections
- Combinations with repetitions

Binomial coefficients, Binomial theorem

In how many ways can 12 identical CS books be distributed among 4 students?

A. 12!/8!
B. C(15,3)
C. C(12,4)
D. C(16,4)

In how many ways can two identical rooks be placed on a 8 by 8 chessboard when they can't share a row or a column?




(a) valid (b) invalid

A. $8 \times 7 \times 6 \times 5$
B. $8^2 7^2$
C. $(8^2 7^2)/2$
D. $C(16,4)$

## Example: Martians, Vesuvians and Jovians

In how many ways can five distinct Martians, 6 distinct Vesuvians, and four distinct Jovians wait in line if no two Martians stand together?

M1 M2 M3 M4 M5

V1 V2 V3 V4 V5 V6

J1 J2 J3 J4

V1 V2 M1 V3 V4 M2 V5 J1 M3 J2 M4 J3 J4 V6 M5

## Example: Martians, Vesuvians and Jovians

In how many ways can five distinct Martians, six distinct Vesuvians, and four distinct Jovians wait in line if no two Martians stand together?

V1 V2 M1 V3 V4 M2 V5 J1 M3 J2 M4 J3 J4 V6 M5

**Solution**:

Arrange Vesuvians and Jovians in line. You can arrange them in 10! ways.

As no two Martians can stand together, each Martian should stand in the 11 spaces in between Vesuvians and Jovians.

The number of ways Martians can be arranged is P(11,5).
Total number of ways is $10! \times P(11,5)$

# Introduction to Probability

Probability of an event E: $p(E) = |E| / |S|$

- Involves counting

Probability of Complements and Unions of Events

Conditional probability: $p(E|F) = \dfrac{p(E \cap F)}{p(F)}$

Independence of events: $p(E \cap F) = p(E)p(F)$.

Bernoulli Trials: probability of k successes is $C(n,k)p^k q^{n-k}$.

Alice flips a biased coin 12 times.
The probability of heads is 3/4 and the probability of tails is 1/4.

What is the probability of getting exactly nine heads?

A. $(3/4)^9$
B. $(3/4)^9(1/4)^3$
C. $C(12,9)(3/4)^9(1/4)^3$
D. $C(12,3)(3/4)^3(1/4)^9$

# Graphs and Trees

Terminology

- undirected, directed, simple, degrees
- representations

Properties

- Connected, connected components
- Paths and circuits
- Bipartite graphs, Euler circuits
- Special graphs
- Handshaking theorem

Trees

- Tree properties