CS182 – Foundations of Computer Science

PSO sessions 1 and 2, week of January 27, 2020

PSO 1

Task 1. Review rules of inference (Modus ponens and modus tollens from Table 1, Table 2, page 76 of Rosen's book, 7th edition).

Problem 1. Suppose you are allowed to give either a direct proof or a proof by contraposition of the following:
If $3n + 5$ is even, then n is odd.
Which type of proof would be easier to give? Explain why.

Problem 2. What is wrong with the following "proof" that $-3 = 3$, using backward reasoning?

Proof: "Assume that $-3 = 3$. Squaring both sides yields $(-3)^2 = 3^2$, or $9 = 9$. Therefore $-3 = 3$."

Problem 3. Prove that the following is true for all positive integers n: n is even if and only if $3n^2 + 8$ is even.

Problem 4. Prove that the following two statements about positive integers n are equivalent: (a) n is even; (b) $n^3 + 1$ is odd.

Task 2. Use any remaining time as office hours or going over clicker questions.

PSO 2

Problem 1. Use proof by cases to show that $|xy| = |x||y|$ for all real numbers x and y.

Problem 2. Prove that if x is irrational and $x \geq 0$ then $\sqrt{x}$ is irrational.

Problem 3. Prove that there exists an integer m such that $m^2 \geq 10^{1000}$. Is your proof constructive or non-constructive?

Problem 4. Given two positive real numbers x and y, their arithmetic mean is $(x+y)/2$ and their geometric mean is $\sqrt{xy}$. Prove that $(x+y)/2 > \sqrt{xy}$.

Task 1. Use any remaining time as office hours or going over clicker questions.