Spring-2020-CS-18200-LE1 Homework 3

Abhi Gunasekar

TOTAL POINTS

100 / 100

QUESTION 1

1 1 **32 / 32**

✓ **- 0 pts** Correct

QUESTION 2

2 2 **24 / 24**

✓ **- 0 pts** Correct

QUESTION 3

3 3 **24 / 24**

✓ **- 0 pts** Correct

QUESTION 4

4 4 **20 / 20**

✓ **- 0 pts** Correct

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅

Abhishek Gunasekar
02/13/2020

CS 182: Homework 3

1

a) (10 points)

Let A = {0, 3, 6, 9, 12}, B = {−2, 0, 2, 4, 6, 8, 10, 12}, and C = {4, 5, 6, 7, 8, 9, 10}. Determine the following sets:

i. (A ∩ B) – C
≡ (A ∩ B) ∩ $\overline{C}$ **By Definition of Difference**
= {0, 6, 12} ∩ {-2, 0, 2, 3, 12}
= **{0, 12}**

ii. (A – B) ∪ (B – C)
≡ (A ∩ $\overline{B}$) ∪ (B ∩ $\overline{C}$) **By Definition of Difference**
= {3, 9} ∪ {-2, 0, 2, 12}
= **{-2, 0, 2, 3, 9, 12}**

b) (22 Points)

Given that A, B, and C are sets, determine if each statement below is true or false. Prove your answer using set builder notation and logical equivalences and/or giving a counterexample.

i. If A ∪ C = B ∪ C, then A = B **Conjecture**
False:
Counterexample: If A and B are subsets of C, then A ∪ C = B ∪ C, but A and B don't have to be the same subset. For example, A could be {0}, B could be {0, 1}, and C could be {0, 1, 2}. In this, A ∪ C = {0, 1, 2} and B ∪ C = {0, 1, 2}, but A is not equal to B.

∴ Disproven **using Counterexample**

ii. If A = B ∪ C, then (A – C) ∪ (B ∩ C) = B **Conjecture**

Using Logical Equivalences and Set Builder Notation

L.H.S
≡ {x | x ∈ ((B ∪ C) – C) ∨ x ∈ (B ∩ C)} **By Definition of Union**
≡ {x | x ∈ ((B ∪ C) ∩ $\overline{C}$) ∨ x ∈ (B ∩ C)} **By Definition of Difference**
≡ {x | x ∈ ((B ∩ $\overline{C}$) ∪ (C ∩ $\overline{C}$)) ∨ x ∈ (B ∩ C)} **Distributive Law**
≡ {x | x ∈ (B ∩ $\overline{C}$ ∪ ∅) ∨ x ∈ (B ∩ C)} **Complement Law**

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅

$\equiv \{x \mid x \in (B \cap \overline{C}) \vee x \in (B \cap C)\}$	**Identity Law**
$\equiv \{x \mid x \in (B \cup (C \cap \overline{C})\}$	**Distributive Law and Definition of Union**
$\equiv \{x \mid x \in (B \cup \emptyset)\}$	**Complement Law**
$\equiv \{x \mid x \in B\}$	**Identity Law**
$\equiv B$	

Since **L.H.S = R.H.S**, both the propositions are said to be **logically equivalent to each other**

1 1 **32 / 32**

✓ **- 0 pts** Correct

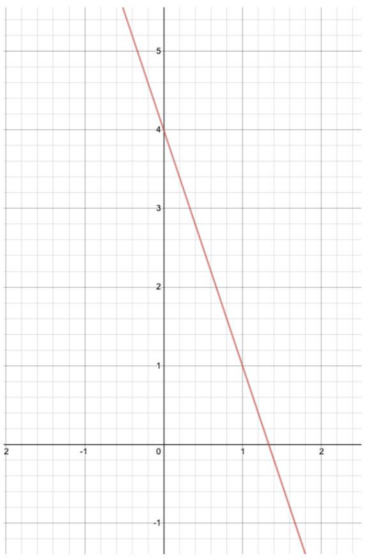
Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A}\ \overline{B}\,\overline{C}\ \emptyset$

2. **(24 Points)**

Determine if each function is a bijection from R to R. Explain your reasoning.

Definition of a Bijection: A function f is a bijection if it is both one-to-one and onto (i.e. both surjective and injective)

a. $f(x) = -3x+4$



Domain: R (the set of all real numbers)

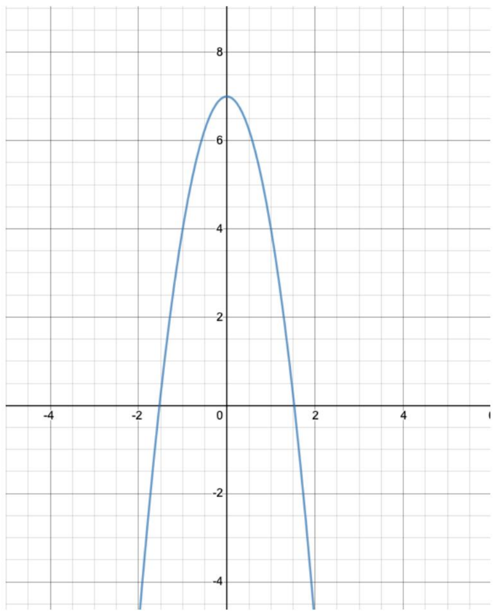
Codomain: R (the set of all real numbers)

- **Onto? Yes**, because all the elements of the codomain (R) are images of the elements in the domain (R).
- **One-To-One?** Suppose that x and y are real numbers with $f(x) = f(y)$, so that $-3x + 4 = -3y + 4$; this means that $x = y$. Hence, $f(x) = -3x + 4$ is a **one to one function** in R to R.

$\therefore$ Since $f(x) = -3x + 4$ is both onto and one to one, we can conclude that **it is a bijection** from R to R.

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅

b. $f(x) = -3x^2 + 7$



Domain: R (the set of all real numbers)

Codomain: R (the set of all real numbers)

Onto? No, the function $f(x) = -3x^2 + 7$ is not onto because there's no real number x with $f(x) = -3x^2 + 7 = 10$ for instance.

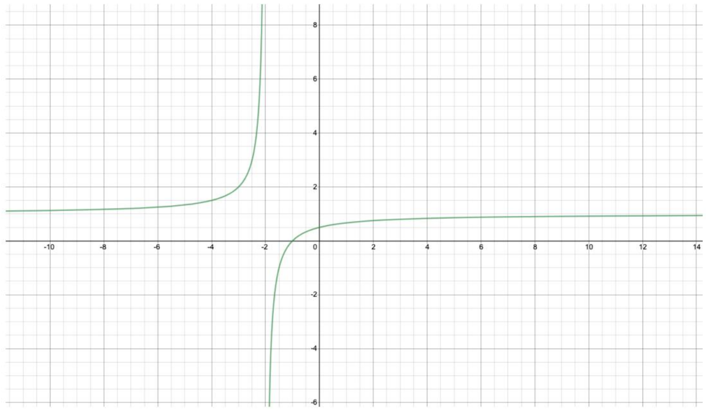
Since the function is not onto, we don't even need to check if its one-to-one.

One-to-One? We also know that the function is also not one to one because f(-1) = f(1) for instance, serving as a counter example.

∴ The function $f(x) = -3x^2 + 7$ is **not a bijection** from R to R.

c. f(x) = (x+1)/(x+2)

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅

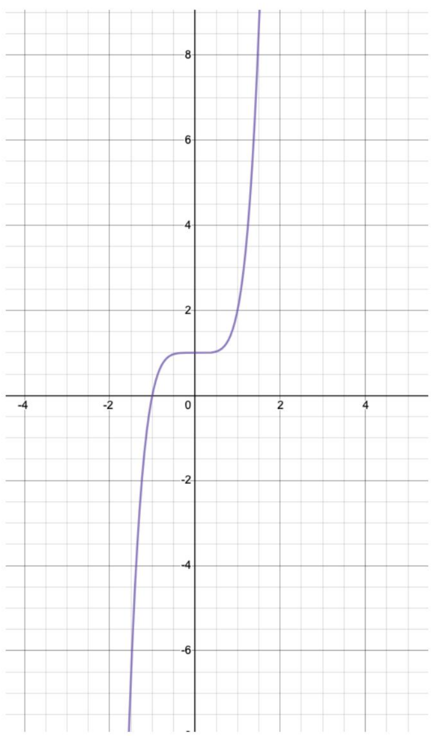


Domain: R (the set of all real numbers)

Codomain: R (the set of all real numbers)

∴ The function f(x) = (x+1)/(x+2) is not a function from R to R because it is not defined at -2, due to a presence of a vertical asymptote. Therefore f is **not a bijection** from R to R.

d. $f(x) = x^5 + 1$



Domain: R (the set of all real numbers)

Codomain: R (the set of all real numbers)

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A}\ \overline{B}\,\overline{C}\ \emptyset$

- **Onto? Yes**, because all the elements of the codomain (R) are images of the elements in the domain (R).
- **One-To-One?** Suppose that x and y are real numbers with $f(y) = f(x)$, so that $y^5 + 1 = x^5 + 1$; this means that $x = y$. Hence, $f(x) = x^5 + 1$ is a one to one function in R to R.

$\therefore$ Since $f(x) = x^5 + 1$ is both onto and one to one, we can conclude that it **is a bijection** from R to R.

2 2 **24 / 24**

✓ **- 0 pts** Correct

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅

3. **(24 Points)**

Compute these sums. Show your work.

a. $\sum_{i=1}^{3}(\sum_{i=0}^{4}(ij)^2) = \sum_{i=1}^{3}(\sum_{i=0}^{4}(i)^2(j^2))$

$= \sum_{i=1}^{3}(0 + i^2 + 4i^2 + 9i^2 + 16i^2)$

$= \sum_{i=1}^{3} 30i^2$

$= 30\sum_{i=1}^{3} i^2$

$= 30 * (1 + 4 + 9)$

$= 30 * (14)$

$=$ **420**

b. $\sum_{i=1}^{125} 3k^2$

$= 3\sum_{i=1}^{125} k^2$

$= 3\,\frac{n(n+1)(2n+1)}{6}$, where n $= 125$ **Closed Form**

$= 3\,\frac{125(126)(251)}{6}$

$=$ **1,976,625**

c. $\sum_{i=0}^{100}(4n^3 + \frac{n}{2})$

$= ((4 * 0) + (0 / 2)) + 4\sum_{i=1}^{100} n^3 + (\frac{1}{2})\sum_{i=1}^{100} n$

$= 0 + 4\sum_{i=1}^{100} n^3 + (\frac{1}{2})\sum_{i=1}^{100} n$

$= 4\,(\frac{n^2(n+1)^2}{4}) + \frac{1}{2}\,(\frac{n(n+1)}{2})$, where n $= 100$ **Closed Form**

$= 4\,(\frac{100^2(101)^2}{4}) + \frac{1}{2}\,(\frac{100(101)}{2})$

$= 102{,}010{,}000 + 2525$

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅

= **102,012,525**

4. **(20 Points)**

Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set. For those that are finite or uncountable, explain your reasoning.

a. Integers that are divisible by 7 or divisible by 10
   - **Finite or Infinite?** All the integers that are divisible by 7 or by 10 are basically nothing but multiples of 7 or 10. Since there's no upper or lower bound, we know that **the set is infinite**.

   - **Countable or Uncountable?** The set of integers that are divisible by 7 or divisible by 10 is a subset of the set of all integers. Since we know that integers, by definition, are countable, this particular **set is also countable**.

   - **Conclusion?** This set can be classified as **countably infinite**.

   - **One to One Correspondence?**
     - The function that represents a one to one correspondence would be
     - F(x) : $S1 = \{x : x = 7^a\, 10^b;\ x \in \boldsymbol{Z}^+;\ a, b \geq 0\} \rightarrow 2k,\ k \geq 1,\ k \in N$
       : $S2 = \{x : x = -7^a\, 10^b;\ x \in \boldsymbol{Z}^+;\ a, b \geq 0\} \rightarrow 2k - 1,\ k \geq 1,\ k \in N$
       : $S3 = \{0\}$ for $k = 0$

     Where S1, S2, S3 are mutually disjoint sets that span the function F(x).
     This function was derived using Cantor's Diagonality

b. The integers less than 100
   - **Finite or Infinite?** The set of integers less than 100 is **infinite** because there's an upper bound of 100 but there's no lower bound. Therefore, the set of all negative integers will also belong to this set.

   - **Countable or Uncountable?** This set is countable because this set is a subset of all integers, and since we know that integers, by definition, are countable, this set must also be **countable.**

   - **Conclusion?** This set can be classified as **countably infinite.**

   - **One to One Correspondence?**
     - Let $f(n) = 100 - n$ be a one-to-one correspondence function in $\boldsymbol{Z}^+$.

c. The real numbers between 1 and 2

3 3 **24 / 24**

✓ **- 0 pts** Correct

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅

$= \mathbf{102{,}012{,}525}$

4. (20 Points)

Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set. For those that are finite or uncountable, explain your reasoning.

a. Integers that are divisible by 7 or divisible by 10
   - **Finite or Infinite?** All the integers that are divisible by 7 or by 10 are basically nothing but multiples of 7 or 10. Since there's no upper or lower bound, we know that **the set is infinite**.

   - **Countable or Uncountable?** The set of integers that are divisible by 7 or divisible by 10 is a subset of the set of all integers. Since we know that integers, by definition, are countable, this particular **set is also countable**.

   - **Conclusion?** This set can be classified as **countably infinite**.

   - **One to One Correspondence?**
     - The function that represents a one to one correspondence would be
     - $F(x) : S1 = \{x : x = 7^a 10^b;\ x \in \boldsymbol{Z}^+;\ a, b \geq 0\} \rightarrow 2k,\ k \geq 1,\ k \in N$

       $: S2 = \{x : x = -7^a 10^b;\ x \in \boldsymbol{Z}^+;\ a, b \geq 0\} \rightarrow 2k - 1,\ k \geq 1,\ k \in N$

       $: S3 = \{0\}$ for $k = 0$

     Where S1, S2, S3 are mutually disjoint sets that span the function F(x). This function was derived using Cantor's Diagonality

b. The integers less than 100
   - **Finite or Infinite?** The set of integers less than 100 is **infinite** because there's an upper bound of 100 but there's no lower bound. Therefore, the set of all negative integers will also belong to this set.

   - **Countable or Uncountable?** This set is countable because this set is a subset of all integers, and since we know that integers, by definition, are countable, this set must also be **countable.**

   - **Conclusion?** This set can be classified as **countably infinite.**

   - **One to One Correspondence?**
     - Let $f(n) = 100 - n$ be a one-to-one correspondence function in $\boldsymbol{Z}^+$.

c. The real numbers between 1 and 2

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅

- **Finite or Infinite?** The set corresponding to the real number between 1 and 2 is **infinite** because there are infinite real numbers between 1 and 2.

- **Countable or Uncountable?** The set is uncountable because real numbers that are irrational such as $\sqrt{2}$ cannot be represented with a bounded number of digits. Therefore, by counterexample, we know that this is an **uncountable** set.

- **Conclusion?** This set can be classified as **uncountable**.

- **One to One Correspondence?** We don't need to check one-to-one correspondence in this case.

d. The prime numbers

- **Finite or Infinite?** The set of prime numbers is infinite because this set is a subset of the set of all integers, which are infinite. Therefore, this set, which is a subset is also **infinite.**

- **Countable or Uncountable?** Since prime numbers are a subset of integers, and we know that integers, by definition are countable, we can conclude that prime numbers are **countable.**

- **Conclusion?** This set can be classified as **countably infinite.**

- **One to One Correspondence?** If the function that represents the set of positive prime numbers exist, then let's suppose that there are two positive integers x and y such that the function with x as an input maps to the same value as the function with y as an input. Therefore, we know that x = y. Therefore, the respective function, which represents the given set, is a **one to one correspondence function in $\boldsymbol{Z^+}$.**

e. Integers that are divisible by 3 but not by 6

- **Finite or Infinite?** This set is a subset of the set of integers, which are known to be infinite by definition. Therefore, the given set, which is a subset, is also **infinite.**

- **Countable or Uncountable?** Since this set is a subset of the set of integers, and since integers are countable, we know that this set is also **countable**.

- **Conclusion?** This set can be classified as **countably infinite.**

- **One to One Correspondence?**
  - Let f(n) =$\{\ 3n\ (n \,\%\, 2 == 1);\ -3(\mathrm{n}-1)\ \ (n \,\%\, 2 ==\ 0)\}$ be a one-to-one correspondence function in $\boldsymbol{Z^+}$.

4 4 **20 / 20**

✓ **- 0 pts** Correct