

CS 182: Homework 6

Problem 1. (100 = $10 \cdot 10$ points)

In this problem, a word is any string of 12 numbers (e.g., 002342393765 is a codeword but 01245 is not, since it consists of only five numbers).

Provide a brief justification for each of your answers (no more than five lines or so), explaining which counting rules you used and what your thought process was. Feel free to give expressions in the form $2 \cdot 4^{10}$, etc. in your final answers; no need to use calculators to compute such powers.

1. How many codewords are there?

Total number of possible numbers = 10

Total Numbers in a string = 12

Therefore, by the Product Rule, there are 10^{12} *codewords*.

2. How many codewords end with 45?

Since there are a total of 12 characters in the string and 10 possible places where you could have the total number of possibilities (i.e. 10 possibilities for each digit), **by the Product Rule**, we know that there are:

$$10^{10} * 1 * 1 = 10^{10} \text{ *such codewords.*}$$

3. How many codewords begin with 1 and end with 1?

Since, there are 10 places, other than the first and last char, where you could have the total number of possibilities, **by the Product Rule**, we know that there are:

$$= 1 * 10^{10} * 1 \text{ codewords}$$

$$= \mathbf{10^{10} such codewords.}$$

4. How many codewords begin with 52 or 62?

This is a Disjoint Event.

= Codewords that begin with 52 + Codeword that begin with 62.

- I. Codewords that begin with 52: $1 * 1 * 10^{10} = 10^{10}$ possibilities (**Product Rule**)
- II. Codewords that begin with 62: $1 * 1 * 10^{10} = 10^{10}$ possibilities (**Product Rule**)
- III. Total = $10^{10} + 10^{10}$ (**Sum Rule**) = $2 * 10^{10}$ such codewords

5. How many codewords begin with 00 or end with 00?

= *codewords that begin with 00* + *codewords that end with 00*
– (*codewords that begin with 00 and end with 00*). [***Principle of inclusion***
– ***exclusion***)

- I. Codewords that begin with 00: $1 * 1 * 10^{10} = 10^{10}$ possibilities (***Product Rule***)
- II. Codewords that end with 00: $1 * 1 * 10^{10} = 10^{10}$ possibilities (***Product rule***)
- III. Codewords that begin with 00 and end with 00: $1 * 1 * 10^8 * 1 * 1 = 10^8$ (***Product Rule***)
- IV. Total = $[(10^{10} + 10^{10}) - 10^8] = 2 * 10^{10} - 10^8$ (***Sum and Product Rule***)

6. How many codewords begin with 22 or 12 and end with 0 or 1?

4 Cases in total:

Case 1: Total Number of codewords beginning with 22 and ending with 0

= 9 numbers have 10 choices each = 10^9 possibilities (**Product Rule**)

Case 2: By similar logic, total number of words beginning with 22 and ending with 1
= 10^9 possibilities (**Product Rule**)

Case 3: By similar logic, total number of words beginning with 12 and ending with 0
= 10^9 possibilities (**Product Rule**)

Case 4: By similar logic, total number of words beginning with 1 and ending with 1
= 10^9 possibilities (**Product Rule**)

Total (Sum Rule) = $4 * 10^9$ such codewords

7. How many codewords begin with a number strictly smaller than 5 and end with a number strictly larger than 5?

First Digit: can be 1 of 5 choices of numbers (0,1,2,3,4)

Last Digit: can be 1 of 4 choices of numbers (6,7,8,9)

Remaining 10 digits: Can be any of the 10 possibilities of numbers (0,1, ..., 9)

Total possibilities (Product Rule) = $5 * 10 * \dots * 10 * 4 = 20 * 10^{10} = 2 * 10^{11}$ such codewords

8. How many codewords have their first three numbers be all strictly smaller than 6?
For example, 03242393765 and 01342393765 are such codewords but
02742393765 is not.

Possibilities for each of first three digits: 6

Possibilities for each of the remaining nine digits = 10

Total Possibilities (Product Rule) = $6 * 6 * 6 * 10 * \dots * 10 = 216 * 10^9$ such codewords

9. How many codewords have no zeros and no ones and no twos?

Possibilities for each digit: 7 since each digit cannot have 0,1,or 2

Total possibilities for 12 digits (Product Rule) = 7^{12} *such codewords*

10. How many codewords have no ones or no fives?

Application of the Principle of Inclusion-Exclusion

- I. Codewords with no ones **[Product Rule]:** $9 * 9 * \dots * 9 = 9^{12}$ *possibilities*
- II. Codewords with no fives **[Product Rule]:** $9 * 9 * \dots * 9 = 9^{12}$ *possibilities*
- III. Codewords with no ones and no fives **[Product Rule]:** $8 * \dots * 8 = 8^{12}$ *possibilities*

Total Codewords [Sum Rule]: $9^{12} + 9^{12} - 8^{12} = 2 * 9^{12} - 8^{12}$ *such codewords*