Spring-2020-CS-18200-LE1 Homework 4

Abhi Gunasekar

TOTAL POINTS

98 / 100

QUESTION 1

1 Problem 1 20 / 20

✓ **- 0 pts** Correct

- 2 pts minor mistake in one proof

- 4 pts minor mistakes in two proofs

- 5 pts major mistake in one proof (did not state P=Q)

- 10 pts major mistake in two proofs (did not state P=Q)

- 10 pts included just one proof

- 20 pts blank

QUESTION 2

2 Problem 2 10 / 10

✓ **- 0 pts** Correct

- 2 pts minor mistake in proof

- 5 pts major mistake in proof

- 10 pts blank

QUESTION 3

3 Problem 3 10 / 10

✓ **- 0 pts** Correct

- 10 pts blank/hand written solution

- 1.5 pts one incorrect/incomplete (say only about either onto or one to one)

- 3.5 pts two incorrects/incomplete (say only about either onto or one to one)

- 4.5 pts three incorrects/incomplete (say only about either onto or one to one)

- 3 pts did not say about Onto.

- 3.3 pts 1 blank

- 6.6 pts 2 blanks

QUESTION 4

4 Problem 4 9 / 9

✓ **- 0 pts** Correct

- 2 pts 4.1 not correct

- 2 pts 4.2 not correct

- 2 pts 4.3 not correct

- 3 pts 4.1 Blank

- 3 pts 4.2 Blank

- 3 pts 4.3 Blank

- 2 pts should give formulas

- 1 pts 4.3 almost correct. notice that it's n(n+1)/2 instead of 2n(n+1).

QUESTION 5

5 Problem 5 8 / 8

- 4 pts 5.1 missing

- 4 pts 5.2 missing

- 1 pts 5.1 correct answer but not proper explanation or incorrect answer

- 1 pts 5.2 correct answer but not proper explanation or incorrect answer

- 3 pts 5.1 Only answer reported

- 3 pts 5.2 Only answer reported

✓ **- 0 pts** Correct

QUESTION 6

6 Problem 6 9 / 9

✓ **- 0 pts** Correct

- 2 pts 6.1 is wrong

- 2 pts 6.2 is wrong

- 2 pts 6.3 is wrong

- 9 pts blank

- 3 pts blank item

- 1 pts minor mistake in 6.1

- 1 pts minor mistake in 6.2

QUESTION 7

7 Problem 7 8 / 8

✓ - **0 pts** **Correct**

- **1 pts** 7.1 incorrect but work/explanation given
- **1 pts** 7.2 incorrect but work/explanation given
- **2 pts** 7.1 incorrect with no work/explanation given
- **2 pts** 7.2 incorrect with no work/explanation given
- **4 pts** 7.1 missing
- **4 pts** 7.2 missing
- **8 pts** Blank

QUESTION 8

8 Problem 8 **8 / 10**

- **0 pts** Correct

✓ - **2 pts** **Incorrect answer but contains some sort of explanation/work**

- **4 pts** Incorrect answer, no work/explanation
- **10 pts** Blank

QUESTION 9

9 Problem 9 **6 / 6**

✓ - **0 pts** **Correct**

- **3 pts** One incorrect
- **6 pts** Both incorrect
- **4 pts** No reasoning
- **6 pts** Blank

QUESTION 10

10 Problem 10 **10 / 10**

- **3 pts** minor mistake
- **10 pts** Blank

✓ - **0 pts** **Correct**

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅ ⊙ ⊕

Abhishek Gunasekar
03/02/2020

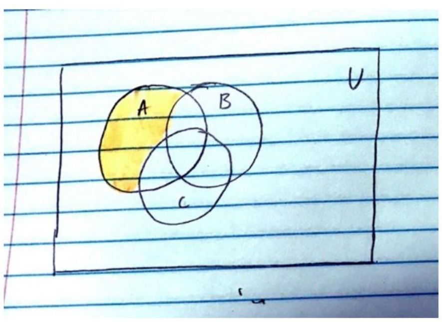
CS 182: Homework 4

1. Problem 1: (20 Points)

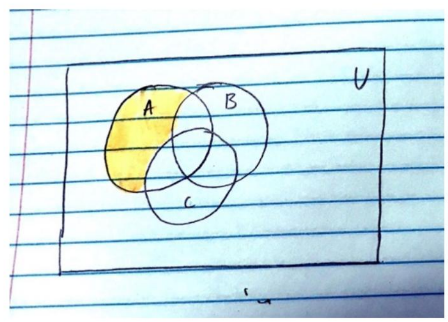
Consider the sets P = (A − B) − C and Q = (A − C) − (B − C). Determine which relationship (⊆, =, ⊇) holds between the two sets P and Q. Your answer will be either P ⊆ Q, or P = Q, or P ⊇ Q. Justify your answer in two ways:

1.1 Drawing a Venn Diagram

P = (A – B) – C



Q = (A – C) – (B – C)



P = Q since P and Q span the same set in the Venn Diagrams shown above.

1.2 Constructing a membership Table

A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	(A – B)	(A – B) – C	(A – C)	(B – C)	$\overline{(B - C)}$	(A – C) – (B – C)
1	1	1	0	0	0	0	0	0	0	1	0
1	1	0	0	0	1	0	0	1	1	0	0
1	0	1	0	1	0	1	0	0	0	1	0
1	0	0	0	1	1	1	1	1	0	1	1
0	1	1	1	0	0	0	0	0	0	1	0
0	1	0	1	0	1	0	0	0	1	0	0
0	0	1	1	1	0	0	0	0	0	1	0
0	0	0	1	1	1	0	0	0	0	1	0

P = Q because they have the same values in the membership table above.

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A}\ \overline{B}\,\overline{C} \emptyset \odot \oplus$

2. Problem 2: (10 Points)

For this problem we want you to see if A ⊕ (B ⊕ C) = (A ⊕ B) ⊕ C has a symmetric difference that is associative, where A, B, and C are sets.

A	B	C	B ⊕ C	A ⊕ (B ⊕ C)	A ⊕ B	(A ⊕ B) ⊕ C
T	T	T	F	T	F	T
T	T	F	T	F	F	F
T	F	T	T	F	T	F
T	F	F	F	T	T	T
F	T	T	F	F	T	F
F	T	F	T	T	T	T
F	F	T	T	T	F	T
F	F	F	F	F	F	F

A ⊕ (B ⊕ C) = (A ⊕ B) ⊕ C because both sides of the theorem have the same values in the truth table above.

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A}\ \overline{B}\,\overline{C} \emptyset \odot \oplus$

3. Problem 3: (10 Points)

For this problem we want you to determine whether each of these functions are one to one and onto. Assume that the domain is **R** and the Codomain is also **R.**

• 3.1 $f(n) = n - 1$

Onto? Yes, because all the elements of the codomain (R) are images of the elements in the domain (R).

One-to-One? Suppose that x and y are real numbers with $f(x) = f(y)$ so that $x - 1 = y - 1$, this means that $x = y$. Hence, $f(n) = n - 1$ is a one-to-one function in R to R.

$\therefore$ $f(n) = n - 1$ is a **bijective** function.

• 3.2 $f(n) = n^2 + 1$

Onto? No, the function $f(n) = n^2 + 1$ is not onto because there's no real number x with $f(x) = x^2 + 1 = -2$ for instance.

One-to-One? We also know that the function is also not one to one because $f(-1) = f(1)$, for instance, serving as a counter-example.

$\therefore$ $f(n) = n^2 + 1$ is **not a bijective** function.

• 3.3 $f(n) = n^3$

Onto? Yes, because all the elements of the codomain (R) are images of the elements in the domain (R).

One-to-One? Suppose that x and y are real numbers with $f(x) = f(y)$ so that $x^3 = y^3$, this means that $x = y$. Hence, $f(n) = n^3$ is a one-to-one function in R to R.

$\therefore$ $f(n) = n^3$ is a **bijective** function.

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A}\ \overline{B}\,\overline{C}\ \emptyset \odot \oplus$

4. Problem 4: (9 Points)

For this problem we want you to find the solution to each of these recurrence relations with the given initial conditions. Use an iterative approach.

- **4.1 $a_n = a_{n-1}$, $a_0 = 5$**

5, 5, 5, 5, 5, ...

Solution: $\{ a_n \} = 5$ where n is a non-negative integer.

- **4.2 $a_n = a_{n-1} + 3$, $a_0 = 1$**

1, 4, 7, 10, 13, ...

Solution: $\{ a_n \} = 1 + 3n$ where n is a non-negative integer.

- **4.3 $a_n = a_{n-1} - n$, $a_0 = 4$**

a0 = 4

a1 = 4 – 1 = 3

a2 = (4 – 1) – 2 = 1

a3 = ((4 – 1) – 2) – 3 = -2

a4 = ((((4 – 1) – 2) – 3) – 4) = - 6

Solution: $\{ a_n \} = -\left(\frac{(n(n+1))}{2} - 4\right)$

where n is a non-negative integer.

Operators to pick: ∨ ∧ ¬ → ∀ ∃ ≡ ∴ ∪ ∩ $\overline{A}$ $\overline{B}$ $\overline{C}$ ∅ ⊙ ⊕

5. Problem 5: (8 Points)

- **5.1:** $\boldsymbol{\sum_{i=1}^{8} 2^j}$

  $= 2^1 + 2^2 + 2^3 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8$
  $= 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256$
  $= \mathbf{510}$

- **5.2:** $\boldsymbol{\sum_{i=2}^{8} (-3)^j}$

  $= \sum_{i=2}^{8} (-1)^n (3)^j$
  $= 3^2 - 3^3 + 3^4 - 3^5 + 3^6 - 3^7 + 3^8$
  $= 9 - 27 + 81 - 243 + 729 - 2187 + 6561$
  $= \mathbf{4923}$

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A}\ \overline{B}\,\overline{C}\ \emptyset \odot \oplus$

6 Problem 6:(9 Points)

Give an example of two uncountable sets A and B such that A ∩ B is:

- **6.1: finite**

  If A was the set of all real numbers between 0 and 1 and B was the set of all real numbers between 2 and 3, then A ∩ B is the empty set ∅, which is a finite set.

- **6.2: countably infinite**

  Let A be the set of all positive real numbers. Let B be the set of all negative real numbers and all integers, then A ∩ B $= \boldsymbol{Z}^{+}$which is a countably infinite set.

- **6.3: uncountable**

  Let A and B both be the set of all real numbers between 0 and 1, then A ∩ B is the set of all real numbers between 0 and 1, which is an uncountable set.

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A} \overline{B} \overline{C} \emptyset \odot \oplus$

7. Problem 7: (8 Points)

- **7.1 A ∧ B**

$$\begin{matrix} 1 \wedge 0 & 0 \wedge 1 & 1 \wedge 1 \\ 1 \wedge 1 & 1 \wedge 0 & 0 \wedge 1 \\ 0 \wedge 1 & 0 \wedge 0 & 1 \wedge 1 \end{matrix} = \begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$$

- **7.2 A ⊙ B**

$$\mathrm{A} \odot \mathrm{B} = \begin{matrix} 0 \vee 0 \vee 1 & 1 \vee 0 \vee 0 & 1 \vee 0 \vee 1 \\ 0 \vee 1 \vee 0 & 1 \vee 0 \vee 0 & 1 \vee 1 \vee 0 \\ 0 \vee 0 \vee 1 & 0 \vee 0 \vee 0 & 0 \vee 0 \vee 1 \end{matrix}$$

$$= \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{matrix}$$

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A} \overline{B} \overline{C} \emptyset \odot \oplus$

8. Problem 8: (10 Points)

How many comparisons does the insertion sort use to sort the list n, n−1 ,..., 2, 1?

Insertion sort compares the second element of a set with the first and sorts them, then it takes the next element of the set and does the same.

Therefore, the insertion sort takes **n – 1 comparisons**.

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists \equiv \therefore \cup \cap \overline{A}\ \overline{B}\,\overline{C} \emptyset \odot \oplus$

9. Problem 9: (6 Points)

Use the greedy algorithm to make change using quarters, dimes, nickels, and pennies for

9.1: 51 cents

$51 \text{ cents} = (2 * 0.25) + (1 * 0.01) = 0.51$

(2 quarters and a penny)

9.2: 69 cents

$69 \text{ cents} = (2 * 0.25) + (1 * 0.10) + (1 * 0.05) + (4 * 0.01) = 0.69$

(2 quarters, 1 dime, 1 nickel and 4 pennies).

Operators to pick: $\vee \wedge \neg \rightarrow \forall \exists\ \equiv \therefore\ \cup\ \cap\ \overline{A}\ \overline{B}\,\overline{C}\ \emptyset \odot \oplus$

10 Problem 10: (10 Points)

$(\log n)^3$ $\sqrt{n}\log n$ $n^{99} + n^{98}$ n^{100} $(1.5)^n$ 10^n $(n!)^2$