

PSO 1

Problem 1. Prove (from scratch) that $\sqrt{3}$ is irrational.

Important hint: you will need to prove that for any positive integer n , if $n^2 = 3k$ then $n = 3\ell$ (for some positive integers k and ℓ). Prove this from scratch via contraposition: if $n \neq 3\ell$, then, for some integer ℓ' , $n = 3\ell' + 1$ or $n = 3\ell' + 2$. Then, for each of those two cases, prove that n^2 cannot be a multiple of three.

Problem 2. Give a direct proof of the following: “If x is an odd integer and y is an even integer, then $x + y$ is odd”.

Problem 3. Give a proof by contradiction of the following: “If n is an odd integer, then n^2 is odd”.

Problem 4. Consider the following theorem: “if x and y are odd integers, then $x + y$ is even”. Give a direct proof of this theorem.

Task 1. Use any remaining time as office hours.

PSO 2

Problem 1. Consider the following theorem: “if x and y are odd integers, then $x + y$ is even”. Give a proof by contradiction of this theorem.

Problem 2. Give a proof by contradiction of the following: “If x and y are even integers, then xy is even”.

Problem 3. Give a proof by cases that $x \leq |x|$ for all real numbers x .

Problem 4. Given any 40 people, prove that at least four of them were born in the same month of the year.

Task 1. Use any remaining time as office hours.