CS182 – Foundations of Computer Science

PSO sessions 1 and 2, week of March 30, 2020

PSO 1

Problem 1. Consider the sequence $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, and, for $n > 3$,

$$a_n = a_{n-1} + a_{n-2} + a_{n-3}.$$

Prove that $a_n < 2^n$ for $n \geq 1$. What kind of induction did you use.

Problem 2. Let $x_1 = 1$ and $x_{n+1} = \sqrt{1 + 2x_n}$ for $n \geq 1$. Prove that $x_n < 4$ for $n \geq 1$.

Problem 3. Let $f(n) = f(n/2) + 4n$ for $n = 2^k$ and $k \geq 1$; $f(1) = 1$. Prove that $f(n) \leq 8n$.

Problem 4. Which of the following recursive functions are well-defined for integer $n \geq 0$:

$$f(n) = \begin{cases} 1 & , n = 0 \\ 2f(n-1) & , n > 0 \end{cases}$$

$$f(n) = \begin{cases} 1 & , n = 0 \\ f(n+1) - 1 & , n > 0 \end{cases}$$

$$f(n) = \begin{cases} 1 & , n = 0 \\ nf(n-1) & , n > 0 \end{cases}$$

$$f(n) = \begin{cases} 1 & , n = 0 \\ f(n-2) + 2 & , n > 0 \end{cases}$$

PSO 2

Problem 1. The three recursive functions appear only slightly different. In each case guess a non-recursive formula for $f(n)$ and prove your guess by induction.

- $f(0) = 0$; $f(n) = 2 + f(n-1)$, for integer $n > 0$.
- $f(0) = 0$; $f(n) = 2f(n-1)$, for integer $n > 0$.
- $f(0) = 1$; $f(n) = 2f(n-1)$, for integer $n > 0$.

Problem 2. Consider the following pseudocode:

```
Function Big(n)
if n = 0 then
    return(1)
else
    return(2 × Big(n − 1))
end if
```

Prove by induction that the output of `Big(n)` is 2^n.
Prove by induction that the running time of `Big(n)` is $O(n)$.