# First Midterm Exam MA 35100

February 26, 2020

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☐ Section 111
TTh 10:30 am
Prof. Patzt

☐ Section 112
TTh 9:00 am
Prof. Patzt

☐ Section 126
TTh 7:30 am
Prof. VanKoughnett

☐ Section 127
TTh 9:00 am
Prof. VanKoughnett

DO NOT TURN THE PAGE UNTIL INSTRUCTED!

Rules:

- Put the answers for Multiple Choice Questions in the answer box.
- Do not write on the backside of any pages.
- Every problem is worth 10 points.
- The first two problems have to be answered in a written response and a complete argument has to be presented.
- For the following 8 multiple choice problems.
  - 10 points are achieved if the correct response is indicated in the answer box.
  - Up to 5 points can be achieved if a written response contains essential ideas how to solve the problem.

1. Give the definition of linear independence.

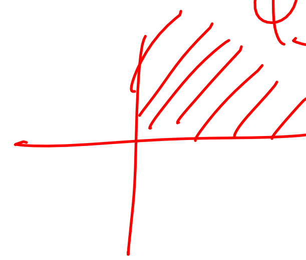
A set of vectors $S = \{x_1, x_2, \cdots, x_n\}$ is said to be linearly independent iff none of the elements of S can be expressed as a linear combination of any of the other vectors.

In other words, all $c_i$'s in the linear dependency equation: $c_1 x_1 + c_2 x_2 + \cdots + c_i x_i = 0$ are zero, proving linear indepen

Continued Answer for Problem 1

2. Let $V$ be the subset of $\mathbb{R}^2$ consisting of all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ such that $x, y \geq 0$. Is $V$ a subspace of $\mathbb{R}^2$? Support your answer by checking every part of the definition of "subspace".

QI



A subset V is is said to be a subspace if it satisfies the three conditions below:

$\boxed{1}$ $0 \in V$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, when $x, y = 0$ $\in V$, Done

$\boxed{2}$ Closed under vector Addition

Let $X_1, X_2 \in V$ : $X_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$, $X_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

then, $X_1 + X_2 = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix} \in V$, Done

$\boxed{3}$ Closed under scaler multiplication

if $c < 0$, like $c = -1$ for instance,

$c \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix} \notin V$

V is not closed under scaler multiplication.

$\therefore$ V cannot be a subspace in $\mathbb{R}^2$ !!!

Continued Answer for Problem 2

3. What is true about the following linear system?

$$
\begin{aligned}
4x + 2y + 4z &= 6 \\
x + 5y - 2z &= 0 \\
3x + 6y + 2z &= 1
\end{aligned}
$$

~~(a) There is no solution.~~

(b) There is exactly one solution and $x = 3$.

~~(c) There is exactly one solution and $y = 1$.~~

~~(d) There are infinitely many solutions and $x = 3z$.~~

~~(e) There are infinitely many solutions and $y = 2 - 2z$.~~

Answer: B

$$\left[\begin{array}{ccc:c} 4 & 2 & 4 & 6 \\ 1 & 5 & -2 & 0 \\ 3 & 6 & 2 & 1 \end{array}\right] \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$x = 3$

$y = -1$

$z = -1$

Continued Answer for Problem 3

4. What is the RREF of the matrix

$$A = \begin{bmatrix} 3 & 5 & -2 & 3 \\ 0 & 1 & 2 & 0 \\ 1 & 1 & -2 & 1 \end{bmatrix}?$$

~~(a)~~ $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

~~(b)~~ $\begin{bmatrix} 1 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

~~(c)~~ $\begin{bmatrix} 3 & 5 & -2 & 3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

~~(d)~~ $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & -4 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Answer: E

Continued Answer for Problem 4

5. Which of the following vectors is in span $\left(\begin{bmatrix}1\\0\\3\\0\end{bmatrix}, \begin{bmatrix}0\\1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\2\\1\end{bmatrix}\right)$?

(a) $\begin{bmatrix}2\\1\\3\\-1\end{bmatrix}$ (b) $\cancel{\begin{bmatrix}3\\0\\3\\-2\end{bmatrix}}$ (c) $\cancel{\begin{bmatrix}1\\2\\0\\1\end{bmatrix}}$ (d) $\cancel{\begin{bmatrix}0\\2\\6\\3\end{bmatrix}}$ (e) $\cancel{\begin{bmatrix}1\\5\\-2\\1\end{bmatrix}}$

Answer: A

inconsistency

Continued Answer for Problem 5

6. Which of the following sets of vectors is **not** linearly independent?

(a) $\begin{bmatrix}1\\1\\1\end{bmatrix}$, (b) $\begin{bmatrix}1\\2\\4\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}$, (c) $\begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$, (d) $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\1\end{bmatrix}$,

(e) $\begin{bmatrix}2\\2\\2\end{bmatrix}, \begin{bmatrix}1\\2\\-1\end{bmatrix}, \begin{bmatrix}1\\4\\-5\end{bmatrix}$ RREF $\rightarrow$ $\begin{bmatrix}1 & 0 & -1\\0 & 1 & 3\\0 & 0 & 0\end{bmatrix}$

Answer:

Continued Answer for Problem 6

7. Let $S$ be the subspace of $\mathbb{R}^4$ consisting of all vectors of the form

$$\begin{bmatrix} a+b+d+e \\ 0 \\ 2a+b+c+3e \\ 3b-3c+6d-3e \end{bmatrix}$$

where $a, b, c, d, e$ are real numbers. What is the dimension of $S$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

Answer: B

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 3 \\ 0 & 3 & -3 & 6 & -3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

$$\Big\downarrow \text{RREF}$$

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 2 \\ 0 & 1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Continued Answer for Problem 7

8. Which of the following is a basis of the nullspace of

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}?$$

(a) $\cancel{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}$, (b) $\cancel{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}}$ (c) $\cancel{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}$, (d) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 0 \\ -2 \\ 1 \end{bmatrix}$ (e) $\cancel{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \\ 2 \\ 1 \end{bmatrix}}$

Answer: D

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$x_1 = c$

$x_2 = -3x_5$

$x_3 = s$

$x_4 = -2x_5$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = c\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -3 \\ 0 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Continued Answer for Problem 8

9. Which of the following statements is true?

(a) If a $3 \times 4$ matrix has nullity 1, then its columns span $\mathbb{R}^3$. ✓ $3 = 4 - 1$

(b) ~~The rank of a matrix is the dimension of the nullspace.~~

(c) ~~If a $4 \times 5$ matrix has rank 2, then its nullity is also 2.~~ $5 - 2 = 3$

(d) ~~The nullity of a matrix is the number of zero rows in its RREF.~~

(e) ~~The rank of a matrix is greater than or equal to its nullity.~~

$\boxed{\text{Answer: } A}$

$$\text{Rank}(A) + \text{nullity}(A) = n$$

$$\Downarrow$$

$$\dim(\text{colspace}) / \dim(\text{rowspace})$$

Continued Answer for Problem 9

10. Which of the following statements makes sense? (We are not asking about the truth of the statements.)

   (a) ~~The vector $v_1, v_2, v_3 \in V$ are inconsistent.~~

   (b) The columns of the matrix $A$ are linearly independent. ✓

   (c) ~~The rank of the matrix $B$ equals its column space.~~

   (d) The vector space of polynomials ~~is linearly independent of $\mathbb{R}^4$.~~

   (e) ~~The dimension of the matrix $A$ is 3.~~

   | Answer: B |
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Continued Answer for Problem 10