Practice Midterm 2 MA 35100

1. Let $\mathcal{P}_3$ be the vector space of cubic polynomials, with the ordered basis

$$\mathcal{B} = \{(x-1)^3, (x-1)^2, (x-1), 1\}.$$

Let $\mathcal{P}_2$ be the vector space of quadratic polynomials, with the ordered basis

$$\overline{\mathcal{B}} = \{x^2, x, 1\}.$$

Let $D : \mathcal{P}_3 \to \mathcal{P}_2$ be the linear transformation $D(f) = f'$. Write the matrix for D in terms of the two bases given.

2. Suppose that A has the LU factorization

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}.$$

Solve the system of equations

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

3. Let A be an $n \times n$ square matrix. Which of the following *does not* imply that A is invertible?

(a) A has n linearly independent rows.

(b) The system of equations $AX = 0$ has a unique solution.

(c) For any B in $\mathbb{R}^n$, the system of equations $AX = B$ has at least one solution.

(d) A is in reduced row echelon form.

(e) A is a permutation matrix.

4. Suppose that A is a 3×4 matrix of rank 2, and B is a 4×5 matrix of rank 3. Which of the following *could be* true about AB?

    (a) AB has rank 1.

    (b) AB has rank 3.

    (c) AB is invertible.

    (d) AB is a permutation matrix.

    (e) AB is not defined.

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by rotating $\pi/4$ radians counterclockwise about the origin, and then applying the matrix $\left(\begin{smallmatrix}1 & 0\\0 & 0\end{smallmatrix}\right)$. Which of the following is a basis for the space of vectors X such that $T(X) = 0$?

(a) $\left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)$

(b) $\left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)$

(c) $\left(\begin{smallmatrix}1\\1\end{smallmatrix}\right)$

(d) $\left(\begin{smallmatrix}1\\-1\end{smallmatrix}\right)$

(e) $\left(\begin{smallmatrix}1\\\sqrt{3}\end{smallmatrix}\right)$

6. Which of the following matrices *does not* have an *LU* factorization?

(a) $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 0 & 1 \\ 1 & 1 & 4 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -2 & 4 & 1 \\ 2 & 3 & 1 & 3 \\ -5 & 1 & 4 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$

(e) All of the above matrices have an *LU* factorization.

7. A certain matrix A has
$$A^{-1} = \begin{pmatrix} 3 & 0 & 3 \\ 1 & 2 & -1 \\ 2 & 3 & 0 \end{pmatrix}$$

Which of the following is the solution to
$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ?$$

(a) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
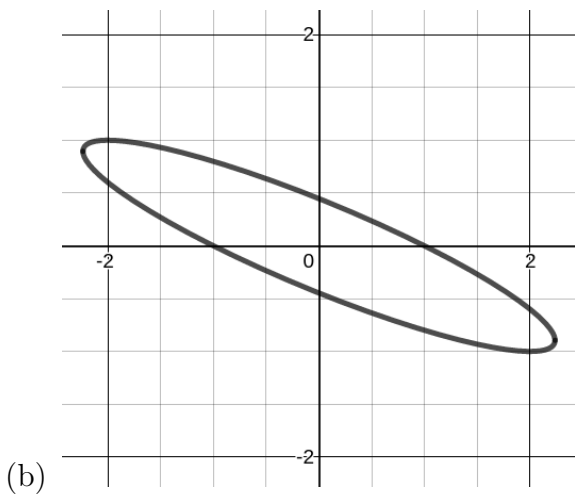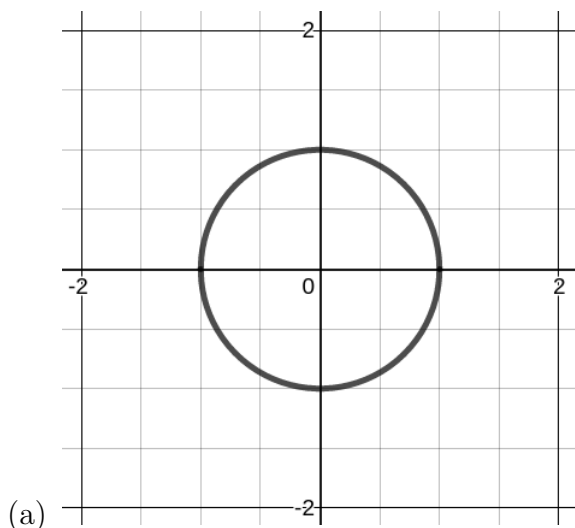
(b) $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

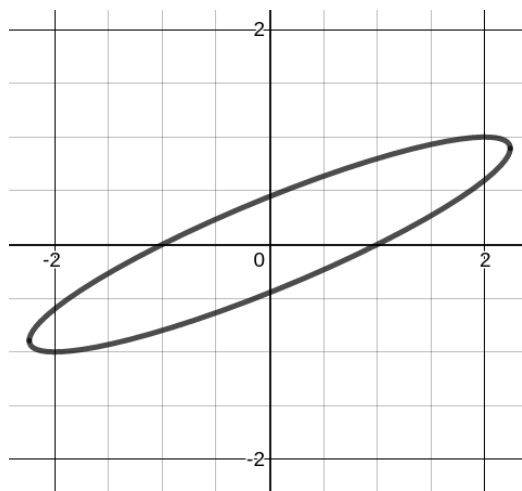(c) $\begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix}$

(d) $\begin{pmatrix} 1 \\ -1/3 \\ -2/3 \end{pmatrix}$
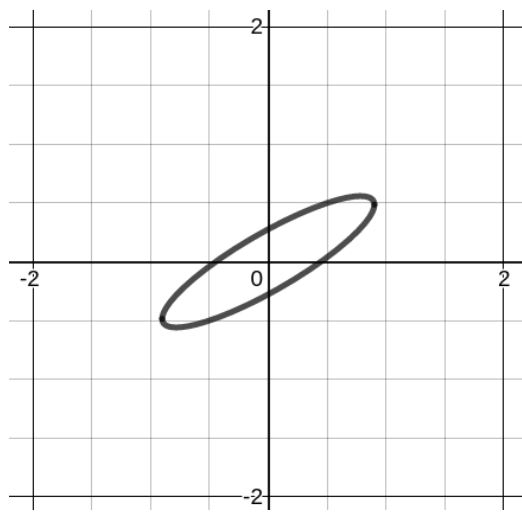
(e) No solution exists.

8. Which of the following is the image of the unit circle, $x^2 + y^2 = 1$, under the linear transformation given by the matrix
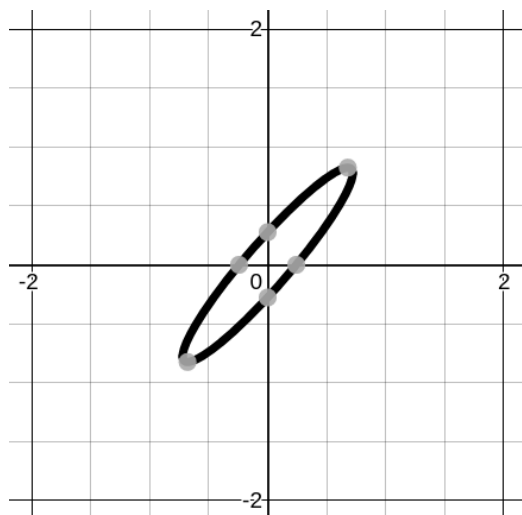
$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}?$$

(a)

(b)

(c)

(d)

(e)

9. Suppose that A is a 3×4 matrix. Let A' be the matrix by doing the following row operation to A:

$$\text{add } (-2) \text{ times row 2 to row 1.}$$

Which of the following is true about the matrix B such that $BA = A'$?

(a) B is lower triangular.

(b) B is upper triangular.

(c) B is a permutation matrix.

(d) B is a diagonal matrix.

(e) No such B exists.

10. Which of the following matrices, viewed as a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$, sends some rectangle to a line segment?

(a) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 3 \\ 3 & 6 \end{pmatrix}$ (**Note:** This originally said $\begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$.)