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MA351 Final Practice Exam

Q1

$M_{T,B} = C_B M_T P_B$ $\qquad M(m,n) \longrightarrow mn \times mn \; M_{T,B}$

$T(X_1) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$T(X_2) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$T(X_3) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$T(X_4) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$T(X_5) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} =$

$T(X_6) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$\boxed{M_{T,B} = \begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \; (6\times 6).}$$

Q2

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$

$a+b=2$ $\quad \boxed{b=-1}$ $\qquad 2a=6$ $\quad \boxed{a=3}$

$c+d=2$ $\qquad 2c=0$

$\boxed{d=2}$ $\qquad \boxed{c=0}$

$$\boxed{A = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}}$$

Q3

$A = (\lambda-2)^2(\lambda)$

$= (\lambda^2 - 4\lambda + 4)(\lambda)$

$= \lambda^3 - 4\lambda^2 + 4\lambda$

$\boxed{C}$

eigenvalue: 0, 2, 2.

Fact: If one of the eigenvalues is 0, the $\det(A)=0$, A is not invertible.

Rank$(A) = \dim(V) - \dim E(0,T)$ $\quad E := $ Eigenspace

$= 3 - 1 = \boxed{2}$

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

not 3 distinct eigenvalues of multiplicity 1

~~A. A is diagonalizable~~

~~B. A has determinant 2~~ $\det(A)=0$

C. A has rank 2

~~D. A is invertible~~ if one of $\lambda_i = 0$, then A not invertible.

~~E. The entries of A are all positive~~ "doesn't have to be."

!!!Focus!!!

Q4

Let B be the matrix $\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}$. Which of the following is an eigenvalue of B?

A. $\frac{1+\sqrt{2}}{2}$

B. $\frac{1-\sqrt{2}}{2}$

C. 0

D. 1 $\boxed{E}$

E. 5

$P_\lambda(A) = |A - \lambda I| = 0$

$\begin{vmatrix} 4-\lambda & -1 \\ -2 & 3-\lambda \end{vmatrix}$

$(4-\lambda)(3-\lambda) - 2$

$12 - 7\lambda + \lambda^2 - 2$

$\lambda^2 - 7\lambda + 10$

$\frac{7 \pm \sqrt{49-40}}{2}$

$\frac{7 \pm 3}{2}$ $\quad \frac{10}{2}$

5

Q5

Which of the following matrices has determinant 12?

~~A.~~ $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 1 & 2 & -1 & -3 \end{bmatrix}$ $\det(A) = 0$

~~B.~~ $\begin{bmatrix} 3 & 6 & 9 \\ -1 & -1 & -1 \end{bmatrix}$ $\det(A) = \text{DNE}$

~~C.~~ $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ $\det(A) = 1 \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1$

~~D.~~ $\begin{bmatrix} -5 & 3 \\ 4 & 7 \end{bmatrix}$ $\det(A) = -47$

~~E.~~ $\begin{bmatrix} 1 & 9 & 1 \\ 0 & 1 & 0 \\ 7/4 & 9/4 & 5/4 \end{bmatrix}$ $\det(A) = \begin{vmatrix} 1 & 1 \\ 7/4 & 5/4 \end{vmatrix} = \frac{5}{4} - \frac{7}{4} = -\frac{1}{2}$

Q6].

$\det(B) = -\det A = -2 \rightarrow \boxed{-2} \rightarrow 2 \rightarrow 2$

swap add swap subtract

$\boxed{C}$

2

Q7) The set $\{x, y, z\}$ is a set of linearly independent vectors in some vector space. Which of the following sets is also linearly independent

~~A.~~ $\{2x-2y, x+y+3z, x-3y-3z\}$ L.D. by Dependency equation.

B. $\{x+y+z, y+z, y-z\}$ L.I by Dependency equation.

$\boxed{B}$

C. $\{x, y, x+y\}$

D. $\{2x-y, 4x-2y, 2z\}$

E. $\{x+y-z, -2x+y+z, x+4y-2z\}$

Q8] Which of the following is W a subspace of V?

$\boxed{A}$

line with origin, in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.

A. $V=\mathbb{R}^2$; $W=\{\begin{pmatrix} x \\ y \end{pmatrix}: 2x+y=0\}$.

No Zero Matrix.

~~B.~~ $V=M(3,3)$; W is the set of permutation matrices

~~C.~~ $V=\mathbb{R}^2$; $W=\mathbb{R}^3$ Not a Subset!

D. $V=\mathbb{R}^3$; $W=\{(x,y,z)^T : z = xy\}$ Can you explain why this is not a subspace.

~~E.~~ V is the set of continuous functions $\mathbb{R} \rightarrow \mathbb{R}$; W is the set of functions $f(x)$ such that $f(1)=1$.

$0 \notin W$

Q9] Which of the following is the inverse of $\begin{bmatrix} 1 & 1 & 1 \\ -4 & -3 & -1 \\ 9 & 8 & 7 \end{bmatrix}$?

A. $\begin{bmatrix} -13 & 1 & 2 \\ 19 & -2 & -3 \\ -5 & 1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 3 & -2 & 12 \\ 7 & 0 & -4 \\ 5 & 2 & 1 \end{bmatrix}$

$\boxed{A}$

B. $\begin{bmatrix} -1 & -1 & -2 \\ 0 & -2 & 0 \\ -3 & 0 & -1 \end{bmatrix}$

E. $\begin{bmatrix} -3 & 7 & 4/5 \\ 1/5 & -2/5 & 0 \\ -3/5 & 2/5 & -1/5 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 5 & -1 \\ -9 & 3 & 1 \\ 7 & -5 & 1 \end{bmatrix}$

Q10 which of the following matrices has $\begin{bmatrix}0\\1\\0\end{bmatrix}$ as an eigenvector?

~~A~~. $\begin{bmatrix}1 & 2 & 3\\ -1 & -4 & -7\\ 0 & 6 & 3\end{bmatrix}$

~~B~~. $\begin{bmatrix}-3 & 1 & 3\\ 0 & 4 & 5\\ -1 & -2 & 7\end{bmatrix}$

C. $\begin{bmatrix}3 & 0 & 2\\ -1 & -1 & -1\\ 5 & 0 & -3\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = -1\begin{bmatrix}0\\1\\0\end{bmatrix}$

~~D~~. $\begin{bmatrix}1 & 2 & -2\\ 0 & 3 & -4\\ 0 & 0 & -6\end{bmatrix}$

~~E~~. $\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$