

Isomorphism

Iso - morph
"same" , "shape"/"form"

Def. An isomorphism between V and W is an invertible linear transformation $T: V \rightarrow W$.

Slogan: Isomorphisms let you translate anything you can do with V into an analogous thing with W .

(Slogan II: Isomorphic vector space are mathematically "the same.").

Key example: V is an n -dimensional vector space, B is a basis.

$$V \xrightleftharpoons[T_B^P]{T_B^C} \mathbb{R}^n \text{ are inverse isomorphisms between } V \text{ and } \mathbb{R}^n.$$

\hookrightarrow Any n -dimensional vector space is isomorphic to \mathbb{R}^n !

⚠ You get an isomorphism from any choice of basis, but different bases give you different isomorphisms.

Optional: different notions of isomorphism

A function between sets $f: S \rightarrow T$
... is just a function,

An isomorphism of sets is an invertible function.

There are isomorphisms $\mathbb{R}^1 \rightarrow \mathbb{R}^2$ as sets,
but not as vector spaces.

ex. Peano space-filling curve

