1 Chapter Exercises:

1.1 Combinators:

1. $\lambda x.xxx$ -> Combinator
2. $\lambda xy.zx$ -> Not Combinator
3. $\lambda xyz.xy(zx)$ -> Combinator
4. $\lambda xyz.xy(zxy)$ -> Combinator
5. $\lambda xy.xy(zxy)$ -> Not Combinator

1.2 Normal form or diverge

1. $\lambda x.xxx$ -> No further reductions possible so this is already in normal form
2. $(\lambda z.zz)(\lambda y.yy)$ $[z := (\lambda y.yy)]$ $(\lambda y.yy)(\lambda y.yy)$ $[y := (\lambda y.yy)]$ $(\lambda y.yy)(\lambda y.yy)$ This will no longer reduce and will continue infinitely. So the expression is divergent
3. $(\lambda x.xxx)z$ $[x := z]$ zzz This has been reduced to normal form

1.3 Beta reduction

1. $(\lambda abc.cba)zz(\lambda wv.w)$ $(\lambda a.\lambda b.\lambda c.cba)zz(\lambda wv.w)$ $[a := z]$ $(\lambda b.\lambda c.cbz)z(\lambda wv.w)$ $[b := z]$ $(\lambda c.czz)(\lambda wv.w)$ $[c := (\lambda wv.w)]$ $(\lambda wv.w)zz$ $[w := z]$ $(\lambda v.z)$ $[v := z]$ z
2. $(\lambda x.\lambda y.xyy)(\lambda a.a)b$ $[x := (\lambda a.a)]$ $(\lambda y(\lambda a.a)yy)b$ $[y := (\lambda b.b)]$ $(\lambda a.a)bb$ $[a := b]$ bb
3. $(\lambda y.y)(\lambda x.xx)(\lambda z.zq)$ $[y := (\lambda x.xx)]$ $(\lambda x.xx)(\lambda z.zq)$ $[x := (\lambda z.zq)]$ $(\lambda z.zq)(\lambda z.zq)$ $[z := (\lambda z.zq)]$ $(\lambda z.zq)q$ $[z := q]$ qq
4. $(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$ $(\lambda z_1.z_1)(\lambda z_2.z_2z_2)(\lambda z_3.z_3y)$ $[z_1 := (\lambda z_2.z_2z_2)]$ $(\lambda z_2.z_2z_2)(\lambda z_3.z_3y)$ $[z_2 := (\lambda z_3.z_3y)]$ $(\lambda z_3.z_3y)(\lambda z_3.z_3y)$ $[z_3 := (\lambda z_3.z_3y)]$ $(\lambda z_3.z_3y)y$ $[z_3 := y]$ yy
5. $(\lambda x.\lambda y.xyy)(\lambda y.y)y$ $(\lambda x.\lambda y_1.xy_1y_1)(\lambda y_2.y_2)y_3$ $[x := (\lambda y_2.y_2)]$ $(\lambda y_1.(\lambda y_2.y_2)y_1y_1)y_3$ $[y_1 := y_3]$ $(\lambda y_2y_2)y_3y_3$ $[y_2 := y_3]$ y_3y_3

6. $(\lambda a.aa)(\lambda b.ba)c$ $[a := (\lambda b.ba)]$ $(\lambda b.ba)(\lambda b.ba)c$ $[b := (\lambda b.ba)]$ $(\lambda b.ba)ac$ $[b := a]$ aac

7. $(\lambda xyz.xz(yz))(\lambda x.z)(\lambda x.a)$ $(\lambda x_1 y_1 z_1.x_1 z_1(y_1 z_1))(\lambda x_2.z_2)(\lambda x_3.a)$ $[x_1 := (\lambda x_2.z_2)]$ $(\lambda y_1 z_1.(\lambda x_2 z_2)z_1(y_1 z_1))(\lambda x_3.a)$ $[y_1 := (\lambda x_3.a)]$ $(\lambda z_1.(\lambda x_2.z_2))z_1((\lambda x_3.a)z_1)$ $[x_2 := z_1]$ $\lambda z_1.z_2((\lambda x_3.a)z_1)$ $[x_3 := z_1]$ $\lambda z_1.z_2 a$