1 Chapter Exercises:

1.1 Combinators:

1. $\lambda x.xxx$ -> Combinator
2. $\lambda xy.zx$ -> Not Combinator
3. $\lambda xyz.xy(zx)$ -> Combinator
4. $\lambda xyz.xy(zxy)$ -> Combinator
5. $\lambda xy.xy(zxy)$ -> Not Combinator

1.2 Normal form or diverge

1. $\lambda x.xxx$ -> No further reductions possible so this is already in normal form
2. $(\lambda z.zz)(\lambda y.yy)$ $[z := (\lambda y.yy)]$ $(\lambda y.yy)(\lambda y.yy)$ $[y := (\lambda y.yy)]$ $(\lambda y.yy)(\lambda y.yy)$ This will no longer reduce and will continue infinitely. So the expression is divergent
3. $(\lambda x.xxx)z$ $[x := z]$ zzz This has been reduced to normal form

1.3 Beta reduction

1. $(\lambda abc.cba)zz(\lambda wv.w)$
   $(\lambda a.\lambda b.\lambda c.cba)zz(\lambda wv.w)$
   $[a := z]$
   $(\lambda b.\lambda c.cbz)z(\lambda wv.w)$
   $[b := z]$
   $(\lambda c.czz)(\lambda wv.w)$
   $[c := (\lambda wv.w)]$
   $(\lambda wv.w)zz$
   $[w := z]$
   $(\lambda v.z)$
   $[v := z]$
   z
2. $(\lambda x.\lambda y.xyy)(\lambda a.a)b$
   $[x := (\lambda a.a)]$
   $(\lambda y(\lambda a.a)yy)b$

$[y := (\lambda b.b)]$
$(\lambda a.a)bb$
$[a := b]$
bb

3. $(\lambda y.y)(\lambda x.xx)(\lambda z.zq)$
   $[y := (\lambda x.xx)]$
   $(\lambda x.xx)(\lambda z.zq)$
   $[x := (\lambda z.zq)]$
   $(\lambda z.zq)(\lambda z.zq)$
   $[z := (\lambda z.zq)]$
   $(\lambda z.zq)q$
   $[z := q]$
   qq

4. $(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$
   $(\lambda z_1.z_1)(\lambda z_2.z_2z_2)(\lambda z_3.z_3y)$
   $[z_1 := (\lambda z_2.z_2z_2)]$
   $(\lambda z_2.z_2z_2)(\lambda z_3.z_3y)$
   $[z_2 := (\lambda z_3.z_3y)]$
   $(\lambda z_3.z_3y)(\lambda z_3.z_3y)$
   $[z_3 := (\lambda z_3.z_3y)]$
   $(\lambda z_3.z_3y)y$
   $[z_3 := y]$
   yy

5. $(\lambda x.\lambda y.xyy)(\lambda y.y)y$
   $(\lambda x.\lambda y_1.xy_1y_1)(\lambda y_2.y_2)y_3$
   $[x := (\lambda y_2.y_2)]$
   $(\lambda y_1.(\lambda y_2.y_2)y_1y_1)y_3$
   $[y_1 := y_3]$
   $(\lambda y_2y_2)y_3y_3$
   $[y_2 := y_3]$
   y_3y_3

6. $(\lambda a.aa)(\lambda b.ba)c$
   $[a := (\lambda b.ba)]$
   $(\lambda b.ba)(\lambda b.ba)c$
   $[b := (\lambda b.ba)]$
   $(\lambda b.ba)ac$

$[b := a]$
aac

7. $(\lambda xyz.xz(yz))(\lambda x.z)(\lambda x.a)$
$(\lambda x_1 y_1 z_1.x_1 z_1(y_1 z_1))(\lambda x_2.z_2)(\lambda x_3.a)$
$[x_1 := (\lambda x_2.z_2)]$
$(\lambda y_1 z_1.(\lambda x_2 z_2) z_1(y_1 z_1))(\lambda x_3.a)$
$[y_1 := (\lambda x_3.a)]$
$(\lambda z_1.(\lambda x_2.z_2))z_1((\lambda x_3.a)z_1)$
$[x_2 := z_1]$
$\lambda z_1.z_2((\lambda x_3.a)z_1)$
$[x_3 := z_1]$
$\lambda z_1.z_2 a$