Question 2

November 8, 2019

1 Dust Particle Trajectory

First, let us find the equation of the particle. We know that x and y should be dependent on time t, and since it is following a helical path with reducing radius, we can assume that:

$$x(t) = f(t)\cos(t), y(t) = f(t)\sin(t)$$

such that $f(T) \to 0$ where T is the time the particle takes to reach the bottom.

Now, we also know that the particle's angular velocity is constant, hence we can say:

$$\omega = \frac{v}{r} = \frac{\sqrt{v_x^2 + v_y^2}}{x^2 + y^2} = \omega_0 \text{ (a constant)}$$

Now, let us replace $x(t), y(t), v_x(t) = x'(t),$ and $v_y(t) = y'(t)$ into the equation. This yields:

$$\frac{\sqrt{f(t)^2 + f'(t)^2}}{f(t)} = \omega_0$$

On squaring both sides and doing some manipulations, we arrive at:

$$f'(t) = \pm\sqrt{\omega_0 - 1}\,|f(t)|$$

When we solve the above differential equation, we get:

$$f(t) = Ae^{Kt}, \text{ where } K = \pm\sqrt{\omega_0 - 1}$$

As the radius is decreasing with time, $K = -\sqrt{\omega_0 - 1}$.

Thus, we choose the equation of the curve to be:

$$(x, y, z) = \left(kr_0 e^{-\frac{t}{2k}}\cos(t), kr_0 e^{-\frac{t}{2k}}\sin(t), r_0 - \frac{t}{k}\right), t \in [0, kr_0]$$

[1]:
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r_0 = 10
k=3
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plobject = parametric_plot3d((lambda t: (k*r_0*exp(-t/(2*k)))*cos(t), lambda t:
    (k*r_0*exp(-t/(2*k)))*sin(t), lambda t: r_0-t/k),(0,k*r_0))
plobject.show(aspect_ratio=[2,2,2])
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Graphics3d Object
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