Orthogonal Rotation Matrices

November 1, 2019

1 Homework 9 - Q7

1.1 Problem Statement:

Consider the following two matrices:

$$A = \begin{bmatrix} 1.0 & 2.0 & 3.0 \\ 1.1 & 2.1 & 3.1 \\ 2.5 & 1.6 & 3.3 \end{bmatrix} \text{ and } T = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The new matrix B is defined as follows:

$$B = T \times A \times \text{transpose}(T)$$

Evaluate the trace of A and B for different values of θ and comment on your observation.

This is a very simple problem in GNU Octave/MATLAB, as they have very extensive linear algebra support.

[1]:
```
A = [1.0,2.0,3.0;1.1,2.1,3.1;2.5,1.6,3.3]
```

```
A =

   1.0000   2.0000   3.0000
   1.1000   2.1000   3.1000
   2.5000   1.6000   3.3000
```

[2]:
```
theta = [pi/2:pi/2:5*pi]
```

```
theta =

 Columns 1 through 8:

    1.5708    3.1416    4.7124    6.2832    7.8540    9.4248   10.9956   12.5664

 Columns 9 and 10:

   14.1372   15.7080
```

[3]:

```
for i = 1:10
    T = [cos(theta(i)), sin(theta(i)), 0; -1*sin(theta(i)), cos(theta(i)),0;␣
↪0,0,1];
    B = T * A * T';
    disp('trace(A) = '),disp(trace(A))
    disp('trace(B) = '),disp(trace(B))
end;
```

```
trace(A) =
 6.4000
trace(B) =
 6.4000
trace(A) =
 6.4000
trace(B) =
 6.4000
trace(A) =
 6.4000
trace(B) =
 6.4000
trace(A) =
 6.4000
trace(B) =
 6.4000
trace(A) =
 6.4000
trace(B) =
 6.4000
trace(A) =
 6.4000
trace(B) =
 6.4000
trace(A) =
 6.4000
trace(B) =
 6.4000
trace(A) =
 6.4000
trace(B) =
 6.4000
trace(A) =
 6.4000
trace(B) =
```

```
6.4000
```

Thus, the value of `trace(A)` is equal to `trace(B)` whenever we apply this transformation to A.

Basically, the matrix $T = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the inverse of the orthogonal rotation matrix in the z direction, hence, when we multiply it with A, we basically rotate each of its columns by θ in the clockwise direction. Let us call this transformed matrix as A_{rot}.

Now, when we multiply the new matrix, A_{rot} with the transpose of T, we're basically getting a new matrix with the same trace.