International Institute of Information Technology, Hyderabad

Mathematics III – Monsoon 2017

Mid I Examination

Max. Time: 1.5 Hr **Max. Marks:50**

Special Instructions about the exam

1. Calculators maybe allowed
2. Answer all questions

Additional sheet for rough work is allowed ☐Yes

1) Determine the Principal value of the Argument:

a) $-\pi^2$

b) $(1+i)^{12}$

(4 marks)

2) Find the Square Root of 1) $4i$ 2) $16-30i$ (4 marks)

3) Determine whether the function f(z) is continuous at the origin, give reasons why.

$f(z) = z\frac{Re(z)}{|z|} \quad for\ z \neq 0$

$f(z) = 2 \quad for\ z = 0$

(4 marks)

4) Which of the following functions are Analytic? Give reasons

$f(z) = Im\, z^2$

$f(z) = \frac{1}{1-z^4}$

$f(z) = Arg\ \pi z$

$f(z) = z^2 + \frac{1}{z^2}$

(8 marks)

5) Are the following functions Harmonic? If yes, find the corresponding Harmonic conjugate and the corresponding Analytic function $f(z) = u(x,y) + iv(x,y)$

$u = xy,$

$v = -\frac{y}{x^2+y^2}$

(8 marks)

6) Find the orthogonal trajectories of the family of curves: $x^3 - 3xy^2 = c$ (5 marks)

7) Compute the following Line integrals

a) $\int_C \bar{z}\, dz$ C is the contour along the parabola $y = x^2$ from $-1+i$ to $1+i$

b) $\int_C Im\, z^2\, dz$ counterclockwise around the triangle with vertices $z = 0, 1, i$ (6 marks)

8) Find an upper bound of the absolute value of the integral $\int_C Re\, z\, dz$ where C is the shortest path from 0 to $1 + i$ (3 marks)

9)Integrate f(z) counterclockwise around the unit circle , indicate whether Cauchy's integral theorem applies

a) $f(z) = \sec\left(\frac{z}{3}\right)$

b) $f(z) = \frac{3}{4z-3}$ (8 marks)