1.Let X be a random variable with PDF given by

$$f(x) = \begin{cases} cx^2 & |x| \le 1 \\ 0 & \text{Otherwise} \end{cases}$$

1. Find the constant c.
2. Find E[X] and Var(X)
3. Find $P(X \ge 1/2)$.

2.

The discrete random variable X can take only the values 1, 2 and 3. For these values cumulative distribution function is defined by

$$F(x) = \frac{x^3 + k}{40} \qquad x = 1, 2, 3$$

(a) Show that $k = 13$

(b) Find the probability distribution of X.

Given that $\mathrm{Var}(X) = \dfrac{259}{320}$

(c) find the exact value of $\mathrm{Var}(4X - 5)$.

3. A dice is thrown, until it for the first time shows a six. Find the conditional probability that the first six occurs in the 2nd throw, given that the first six occurs in a throw of even number.

4. An information channel can transmit 0s and 1s, though some errors may occur. One expect that a sent 0 is changed with the probability $\frac{1}{5}$ to a 1, and that a sent 1 is changed with the probability $\frac{1}{6}$ to a 0. It is also given that in mean $\frac{2}{3}$ of all signals are 0s. Assuming that we receive a 0, what is the probability that a 0 was sent?

5. Find the constant c such that the function

$$f(x)=\begin{cases} cx^2 & 0<x<3 \\ 0 & \text{otherwise} \end{cases}$$

is a density function, and (b) compute $P(1<X<2)$

6. A random variable X has the density function

$$f(x) = c/(x^2 + 1) \quad -\inf < x < \inf$$

a. Find the value of the constant c.
b. Find the probability that x^2 lies between 1/3 and 1.

7. The distribution (CDF) for a random variable is

$$F(x) =\begin{cases} 1-e^{-2x} & x>=0 \\ 0 & x<0 \end{cases}$$

Find

a. The density function
b. The probability that $x>2$
c. The probability that $-3<x<=4$

8. Find whether the following functions are cdf's.

a. F1(x) =

$$\begin{cases} 0 & x<=0 \\ 0.5x & 0<x<=1 \\ 0.25+0.25x & 1<x<=3 \\ 1 & x>3 \end{cases}$$

B. F2(x) =

$$\begin{cases} 0.5 & x<1 \\ 0.75 & 1<=x<3 \\ 1 & x>3 \end{cases}$$

9. Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for

each white ball selected. Let X denote our winnings. What are the possible values of X, and what are the probabilities associated with each value?

10. Five distinct numbers are randomly distributed to players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let X denote the number of times player 1 is a winner. Find P{X = i}, i = 0, 1, 2, 3, 4.

11. A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.

12. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability 0.2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding, what is the probability that the message will be wrong when decoded?
What independence assumptions are you making?

13. Five men and 5 women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman. (For instance, X = 1 if the top-ranked person is female.)
Find P{X = i},i = 1, 2, 3, . . . , 8, 9, 10.

14. A group of 6 men and 6 women is randomly divided into 2 groups of size 6 each. What is the
probability that both groups will have the same number of men?

15. Elvis Presley had a twin brother who died at birth. What is the probability that Elvis was an identical twin? (Twins are estimated to be approximately 1.9% of the world population, with monozygotic twins making up 0.2% of the total---and 8% of all twins.)

16. Let X and Y be two jointly continuous random variables with joint PDF as

fXY(x,y) = {

2 $y + x \leq 1$, $x > 0$, $y > 0$

0 otherwise

}

Find Cov(X,Y).

17. What is the Probability of ‘X’ being true given X = “probability of getting a head and getting a tail in a coin toss is equal to ½ ”. (valid assumptions are allowed).