Solutions to Tutorial Sheet-6 IEC102

Q1 Determine the characteristic equation of each of the following differential equations.

a) $5v + 14\frac{dv}{dt} = 0$

b) $\frac{di}{dt} + 18i + \frac{R}{B}i = 0$; where R, B are constants

Sol.

a) The solution to the differential equation $5v + 14\frac{dv}{dt} = 0$ will be of the form $Ae^{S_1 t}$, where A is a constant.

Since the solution when pulgged into equation, has to satisfy the equation

$$5(Ae^{S_1 t}) + 14\frac{d}{dt}(Ae^{S_1 t}) = 0$$

$$\Rightarrow 5Ae^{S_1 t} + 14AS_1 e^{S_1 t} = 0$$

$$\Rightarrow Ae^{S_1 t}(5 + 14S_1) = 0$$

$\therefore$ The characteristic equation of the d-E is $(5 + 14S_1) = 0$

b) $\frac{di}{dt} + 18i + \frac{R}{B}i = 0$

The solution to the above 1st order differential equation will be of the form $Ae^{S_1 t}$

$$\therefore \frac{d}{dt}(Ae^{S_1 t}) + 18(Ae^{S_1 t}) + \frac{R}{B}(Ae^{S_1 t}) = 0$$

$$\Rightarrow AS_1 e^{S_1 t} + 18Ae^{S_1 t} + \frac{R}{B}Ae^{S_1 t} = 0$$

$$\Rightarrow Ae^{S_1 t}\left[S_1 + 18 + \frac{R}{B}\right] = 0$$

PTO

$\therefore$ The characteristic equation is

$$\left[S_1 + \left(18 + \frac{R}{B} \right) \right] = 0$$

(Q2) The switch in the network shown in Fig. Q2 moves from position 1 to position 2 at $t=0$. Find $v_o(t)$ for $t>0$.

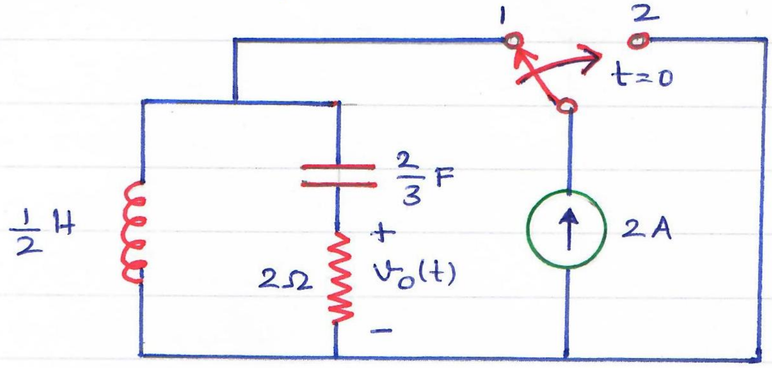
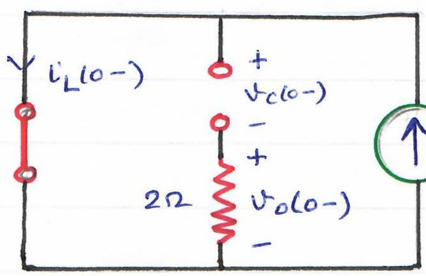


Fig. Q2

Assume that the circuit is in steady state at $t=0-$.

Sol.

Circuit at $t=0-$ (Since the circuit is in steady state, capacitor acts as open circuit and inductor acts as short circuit)

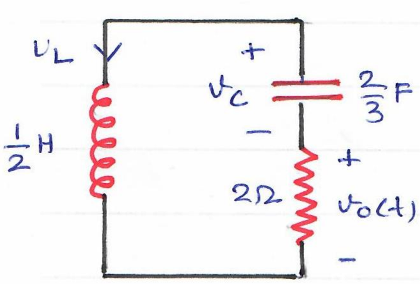


$v_o(0-) = 0$

$i_L(0-) = 2 = i_L(0) = i_L(0+)$

$v_c(0-) = 0 = v_c(0) = v_c(0+)$

Circuit at $t=0$



It is a series RLC circuit

$$L\frac{di_L}{dt} - v_o - v_c = 0$$

but $v_o = -i_L R = -2i_L$

$$\therefore L\frac{di_L}{dt} + 2i_L - v_c = 0$$

$$\therefore L\frac{di_L(0)}{dt} = -2i_L(0) + v_c(0) = 0$$

$$\Rightarrow \frac{di_L(0)}{dt} = \frac{1}{L}\left(-2i_L(0) + v_c(0)\right)$$

$$= \frac{1}{(1/2)}(-2\times 2 + 0)$$

$$\Rightarrow 8 \quad \frac{di_L(0)}{dt} = -8$$

It is a series RLC ckt (without any source)

$$\alpha = \frac{R}{2L} \quad \text{and} \quad w_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore \alpha = \frac{2}{2 \times \frac{1}{2}} = 2 \qquad w_0 = \frac{1}{\sqrt{\frac{1}{2} \times \frac{2}{3}}} = \sqrt{3}$$

$\alpha > w_0$ $\qquad s_1, s_2 = -\alpha \mp \sqrt{\alpha^2 - w_0^2} = -2 \mp \sqrt{4-3} = -3, -1$

$\therefore$ The circuit is overdamped (with no source)

$\therefore$ The solution will be of the form

$$v_o(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

with $s_1 = -3$ and $s_2 = -1$

$$\therefore v_o(t) = A_1 e^{-3t} + A_2 e^{-t}$$

$$v_o(0) = -i_L(0) \times 2 = -4V$$

$$\frac{dv_o}{dt} = R \frac{di_o}{dt} = -R \frac{di_L}{dt}$$

$$\frac{dv_o(0)}{dt} = -2 \times -8 = 16$$

$$\boxed{v_o(0) = -4} \qquad \boxed{\frac{dv_o(0)}{dt} = 16}$$

$$v_o(t) = A_1 e^{-3t} + A_2 e^{-t}$$

$$\frac{dv_o(t)}{dt} = -3A_1 e^{-3t} - A_2 e^{-t}$$

$v_o(0) = A_1 + A_2 = -4 \quad \cdots (A)$

$\frac{dv_o(0)}{dt} = -3A_1 - A_2 = 16$

Solving (A) and (B), $A_1 = -6$ and $A_2 = 2$

$$\boxed{\therefore v_o(t) = -6e^{-3t} + 2e^{-t} = 2(e^{-t} - 3e^{-3t})}$$

Q. Find the expression for $v_C(t)$ and $i_L(t)$ for $t > 0$ in the circuit shown in Fig. Q3

Given that $v_C(0-) = 10\,V$ and $i_L(0-) = 0\,A$

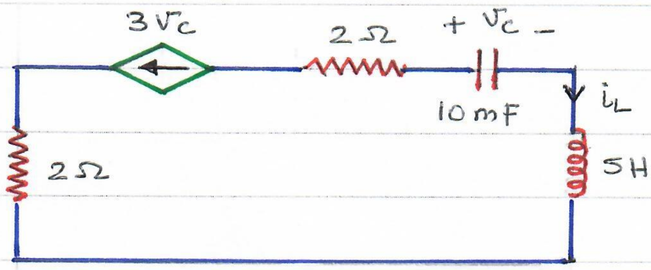


Fig. Q3

Sol.

$$C\frac{dv_C}{dt} = -3v_C \qquad v_C(0-) = v_C(0) = 10\,V$$

$$\Rightarrow \quad 10\times10^{-3}\frac{dv_C}{dt} = -3v_C$$

$$\Rightarrow \quad \frac{dv_C}{dt} + 300v_C = 0$$

$$v_C(t) = v_C(0)\,e^{-300t}$$

$$\Rightarrow \boxed{v_C(t) = 10\,e^{-300t}\ V}$$

$$i_L(t) = \frac{C\,dv_C(t)}{dt} = 10\times10^{-3}\frac{d}{dt}\left(10\,e^{-300t}\right)$$

$$10^{-1}\times -300\,e^{-300t}$$

$$\Rightarrow \boxed{i_L(t) = -30\,e^{-300t}\ A}$$

(Q4) Find $v(t)$ and $i(t)$ for $t>0$ in the circuit shown in Fig. Q4. Given that $v(0) = i(0) = 0$

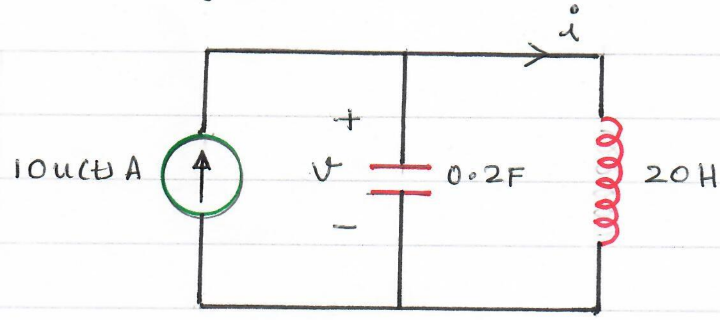


Fig. Q4

Sol. The circuit is a forced LC circuit

$$\alpha = 0 ; \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times0{\cdot}2}} = \frac{1}{\sqrt{4}} = \frac{1}{2} \text{ rad/s}$$

$$i(t) = K + A\cos(\omega_0 t)$$

$$i(0) = 0; \quad K = 10$$

$$i(0) = 0 = 10 + A \Rightarrow A = -10$$

$$\therefore i(t) = 10 - 10\cos\left(\tfrac{1}{2}t\right) \text{ A}$$

$$v = L\frac{di}{dt} = 20\frac{di}{dt} = 20\frac{d}{dt}\left[10 - 10\cos\left(\tfrac{1}{2}t\right)\right]$$

$$= 20 \times -10 \times \tfrac{1}{2} \times -\sin\left(\tfrac{1}{2}t\right)$$

$$\Rightarrow v = 100\sin\left(\tfrac{1}{2}t\right)$$