Introduction to Game Theory
Dominant Strategy Equilibrium

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Agenda

- Recap

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- What is Strategic Form/Normal Form/ Matrix Game
    - Dominant Strategy
    - Weakly Dominant Strategy
    - Dominant Strategy Equilibrium

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Intelligent, Rational Players and Common Knowledge

Last time we skipped explanation of rationality and common knowledge

[1] gametheory.net

Intelligent, Rational Players and Common Knowledge

- **Intelligent**: Can take into account all the information that is available
- **Rationality**[1] In its mildest form, rationality implies that every player is motivated by maximizing his own payoff/objectives
- Intelligent+Rational: implies that every player always maximizes his utility, thus being able to perfectly calculate the probabilistic result of every action.

[1] `gametheory.net`

Common Knowledge

Myerson's Book:

Common Knowledge (Aumann 1976)

Every Player knows it. Every players knows that every player knows it. Every Player knows that every player knows that every player knows it. (Every player knows that)^k every player knows it is true for all $k = 1, 2, \ldots$.

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- Island with two water streams and all the humans can't speak
- If a person knows he has blue mark on forehead drinks water only from far away stream
- Group of 5 most intelligent and rational humans
- One fine day, one visitor, knowing the above fact, shouts at them, "why a person with blue mark drinking water here?"

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(! a game exactly....but interesting puzzle. Optional Exercise)

- Someone imagined two positive whole numbers. Both numbers are greater than 1, and less than 21.
- That person tells the sum of those two numbers to mathematician A
- and the product of those two numbers to mathematician B.
- Couple of days later, A and B talk to each other:

  A: There is no way for you to find the sum.
  B: But I know the sum now!
  A: And now I know the product.

Strategic Form Games (Normal form Games)

Normal Form Game: $\Gamma = < N, (S_i)_{i \in N}, (u_i)_{i \in N} >$

N : Set of players
$N = \{1, 2, \ldots, n\}$

S_1 : Strategies available to player 1
S_2 : Strategies available to player 2
$\vdots$
S_n : Strategies available to player n
$S = S_1 \times S_2 \times \ldots \times S_n$

Strategy space of all the players

$u_1 : S \to \mathbb{R}$
$u_2 : S \to \mathbb{R}$
$\vdots$
$u_n : S \to \mathbb{R}$

Utility or Payoff Functions

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Utility or Payoff Functions

- This is also known as **matrix form** games

Dominant Strategies

Strongly Dominated Strategy

- Given a game $\Gamma =< N, (S_i), (u_i) >$, a strategy $s_i \in S_i$ is said to be strongly dominated if there exists another strategy $s_i' \in S_i$ such that

$$u_i(s_i, s_{-i}) < u_i(s_i', s_{-i}) \forall s_{-i} \in S_{-i}$$

In such a case, we say strategy s_i' strongly dominates strategy s_i.

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Strongly Dominant Strategy

- A strategy $s_i^* \in S_i$ is said to be a *strongly dominant strategy*, for player i if it strongly dominates every other strategy $s_i \in S_i$. That is, $\forall s_i \neq s_i^*$,

$$u_i(s_i, s_{-i}) < u_i(s_i^*, s_i) \forall s_{-i} \in S_{-i}$$

.

Weakly Dominated Strategy

- Given a game $\Gamma =< N, (S_i), (u_i) >$, a strategy $s_i \in S_i$ is said to be weakly dominated if there exists another strategy $s_i' \in S_i$ such that

$$u_i(s_i, s_{-i}) \leq u_i(s_i', s_{-i}) \forall s_{-i} \in S_{-i}$$

with strict inequality for at least one s_{-i}. In such a case, we say strategy s_i' weakly dominates strategy s_i.

Weakly Dominant Strategy

- A strategy $s_i^* \in S_i$ is said to be a *weakly dominant strategy*, for player i if it weakly dominates every other strategy $s_i \in S_i$. That is, $\forall s_i \neq s_i$,

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with strict inequality for at least one s_{-i}.

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Example: Prisoner's Dilemma

	No Confess NC	Confess C
No Confess NC	- 2, - 2	- 10, - 1
Confess C	-1, - 10	- 5, - 5

- Is there any strongly dominated strategy?
- Is there any strongly dominant strategy?
- Is there any weakly dominant strategy?

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Example: Prisoner's Dilemma

Consider the following variation in Prisoner's Dilemma:

	C	NC
C	(-5,-5)	(-2,-10)
NC	(-10,-1)	(-2,-2)

- Is there any strongly dominated strategy?
- Is there any strongly dominant strategy?
- Is there any weakly dominated/dominant strategy?

Example: Another Game

Consider the following game (just row player's utility are shown):

	a	b	c
A	5	5	5
B	4	5	5
C	4	4	4

- Is the information sufficient to analyze any kind of dominance for Player 1?

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- Is the information sufficient to analyze any kind of dominance for Player 1?
- Is there any strongly dominated strategy?
- Is there any strongly dominant strategy?
- Is there any weakly dominated/dominant strategy?
- What can we say when $u_1(B, a) = 5$?

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Strongly (Weakly) Dominant Strategy Equilibrium A profile of strategies $(s_1^*, s_2^*, \ldots, s_n^*)$ is called a *strongly dominant strategy equilibrium* of the game $\Gamma =< N, (S_i), (U_i) >$ if $\forall i = 1, 2, \ldots, n$, the strategy s_i^* is a strongly dominating strategy for player i.

Example: Prisoner's Dilemna

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Example: Prisoner's Dilemna

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- Possible strategies: (NC,NC), (C,NC), (NC,C) and (C,C)
- C is best response if other player is playing C
- Note: C is best response even other player is playing NC
- C is dominant strategy for both the players
- (C,C) is Dominant Strategy Equilibrium

Example: Tragedy of Commons

Garrett Hardin: dilemma occurring in the situation when multiple agents act rationally in self-interest and ultimately deplete a shared limited resource

Image Credits: Wikipedia

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Image Credits: Wikipedia

- Each farmer either can allow his cow to graze or does not keep a cow
- If he allows, say he receives benefit of unit 1
- However, damage to environment is 3 for each cow.
- Total damage equally shared by all the farmers

Tragedy of Commons Contd...

N	Set of farmers $=\{1,2,\ldots,n\}$
S_i	$\{0,1\}$ $\forall i$ Strategy for each farmer
$u_i(s_1, s_2, \ldots, s_n)$	$s_i - \frac{3(s_1+s_2\ldots+s_n)}{n}$

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- If $n > 3$, each farmer keeps cow
- If $n < 3$, each farmer prefers not to keep cow
- Suppose, Government puts environment tax of 3 to those who keep cow
- $u_i(s_1,s_2,\ldots,s_n) = s_i - 3 * s_i - \frac{3(s_1+s_2\ldots+s_n)}{n}$

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Tragedy of Commons Contd...

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- Each farmer prefers not to keep cow

Meeting at the Cafe Game

(Also called Co-ordination Game, Battle of Sexes game)

- Two Friends (**A,B**)
- Agreed to meet in the canteen after lecture
- Two options: Library canteen, Gymkhana canteen
- Enjoy time spent with each other, hence utility 1
- **A** enjoys Library canteen (utility 1) where as B enjoys Gymkhana canteen (utility 1)
- Each Player obtains utility of 0.5 if goes to favorite canteen without friend.

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L	(2,1)	(0.5,0.5)
G	(0,0)	(1,2)

Does Dominant Strategy Equilibrium Always Guaranteed?

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What is dominant strategy equilibrium?

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No Dominant Strategy for any of the player.

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Further Reading

- **Game Theory and Mechanism Design**, Y Narahari. World Scientific Publishing Company, 2014.
- **Multiagent systems: Algorithmic, game-theoretic, and logical foundations**, Shoham, Yoav, and Kevin Leyton-Brown. Cambridge University Press, 2008. (Free download).
- **Game Theory** by Roger Myerson. Harvard University press, 2013.
- **Algorithmic Game Theory**, edited by Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay Vazerani. (Non-printable version available online).

`http://gametheory.net/`

`http://lcm.csa.iisc.ernet.in/gametheory/lecture.html`

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