Introduction to Game Theory
Extensive Form and Normal Form Games

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Agenda

- Recap

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- What is Game Theory
  - Elements of a Game
  - Extensive Form Games and Strategic Form Games
  - Dominant Strategy Equilibrium

Introduction to Game Theory

Game Theory

- Analysis of conflict
- According to Myerson

Definition (Game Theory)

Game theory is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers.

- Analyses and predicts the behaviour of strategic agents (players) with conflicting interests
- Suggests the strategies to play

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What are Elements of A Game?

[1] Credits Prof Boi Faltings

What are Elements of A Game?

Elements of a game[1]:

- **Players**: the agents playing the game.
- **States** of the game.
- **Actions**: that change the state of the game.
- **Knowledge** (beliefs) of the state and actions.
- **Outcome** of the players actions, in particular payoffs for each player.
- **Payoff or Utility** That each player derives from the outcome
- **Assumptions** (We will come back to these)
  - Every player acts rationally so as to maximize its own payoff.
  - Information about game is **common knowledge**

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Prisoner's Dilemma

Recall Prisoner's Dilemma

- Two friends (A,B) caught for cheating in exams
- Invigilator takes both of them to two different rooms
- Offers A: If you be confess cheating and B does not, I will let you go with only 1 point deduction and deduct 10 points from B.
- If both of you confess, I deduct 5 points from each
- Similarly if you dont confess, but he confesses, I will deduct 10 points from you
- In the absence of nobody confessing, I will deduct 2 points each

Extensive Form Games

Prisoner's Dilemma

- Who are the players?

Extensive Form Games

Prisoner's Dilemma

- 2 players, Player A and B.

Extensive Form Games

Prisoner's Dilemma

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- States:

Extensive Form Games

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- What are actions?

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- Knowledge: A and B both know the game and rules. However both do not know the others choice.

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- Actions: player A and B choose between C and NC.
- Knowledge: A and B both know the game and rules. However both do not know the others choice.
- Outcomes: 4 possible outcomes
- Utilities: depend on the outcome

Extensive Form of a game is a graphical representation of the game.

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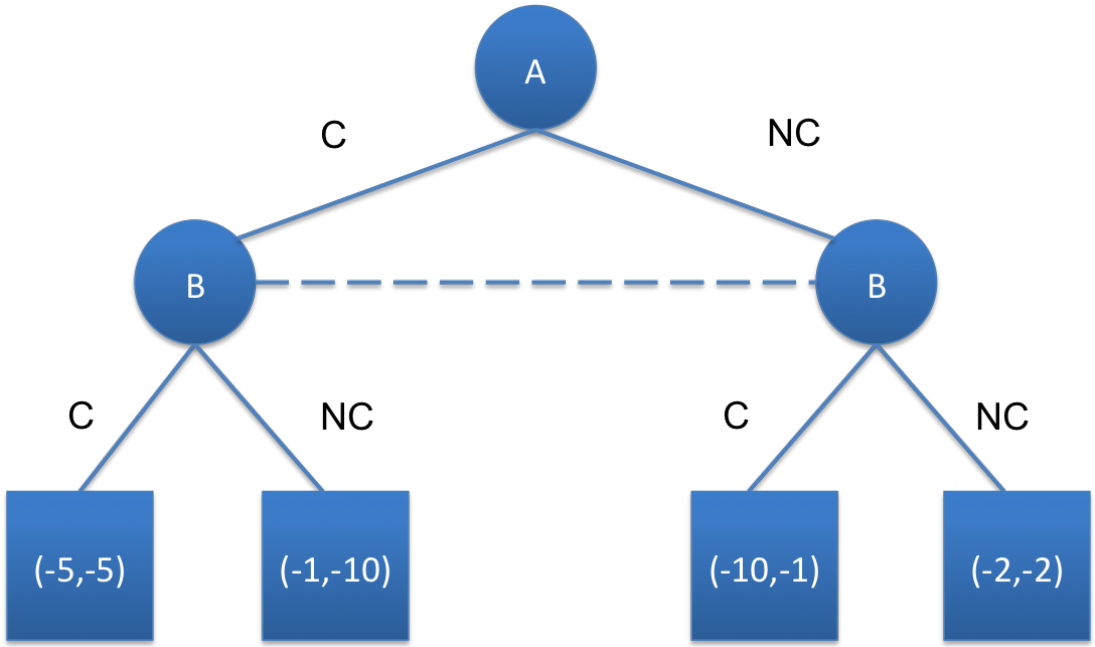
Extensive Form Games



- Dotted line indicates Player B does not know in which state she is.
- Such sets are called as **Information sSets**.

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An Entry Game

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What is extensive form game representation of the above game?

Extensive Form Game: Definition

Finite Extensive Form Game is defined as tuple
$\Gamma =< N, T, Z, o(), A(), s(), (u_i())_{i \in N}, \mathcal{H} >$

- $N = \{1, 2, \ldots, n\}$ Set of players
- T: Tree representation of the game
- Z: Leaf nodes of T
- $o(t)$: Owner of node t (the player who acts at node t)
- $A(t)$: Actions available to $o(t)$ at node t
- $s(t, a)$: Successor of node t resulting from action $a \in A(t)$
- $u_i(z)$: utility to player i at terminal node $z \in Z$
- $\mathcal{H}$: partition of $T \setminus Z$ into information sets
  If $t, t' \in h$ where $h \in \mathcal{H}$, (i) $o(t) = o(t')$, (ii) $A(t) = A(t')$ and (iii) $h(t) = h(t')$ where $h(t)$ denotes all possible nodes for $o(t)$ at t

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- If all information sets are singleton, then its called a game with **perfect information**
- Is **Chess** a game with perfect information or imperfect information?
- What about prisoner's dilemma? Cards?

- Recall: Game theory analyses and predicts the behaviour of strategic agents (players) with conflicting interests.
- So far no analysis or prediction
- Strategic agents will strategize
- What is strategy?

Strategies

- **Strategy**: is an algorithm or rule by which each player chooses an action
  - a complete contingent plan explaining what a player will do in every situation (state)

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Pure: not mixed or adulterated with any other substance or material

Strategies Continued...

- $\forall i \in N$, S_i be the space of strategies available to player i
- Extensive Form Games: $H_i = \{h \in \mathcal{H} : t \in h \text{ with } o(t) = i\}$ and A_i set of actions available to i on any of his information set
- $s_i : H_i \rightarrow A_i$ with $s_i(h) \in A_i(h)\ \forall h \in H_i$
- $S = S_1 \times S_2 \times \ldots S_n$ is space of strategy profiles that all the players can play
- We write $s = (s_i, s_{-i})$, where s_{-i} is strategy followed by all the players except player i

- x ....still we are not analyzing...not predicting
- Easier representation of games?

Strategic Form Games (Normal form Games)

N : Set of players
$N = \{1, 2, \ldots, n\}$

S_1 : Strategies available to player 1
S_2 : Strategies available to player 2
$\vdots$
S_n : Strategies available to player n
$S = S_1 \times S_2 \times \ldots \times S_n$
Strategy space of all the players

$u_1 : S \to \mathbb{R}$
$u_2 : S \to \mathbb{R}$
$\vdots$
$u_n : S \to \mathbb{R}$

Utility or Payoff Functions

- This is also known as **matrix form** games

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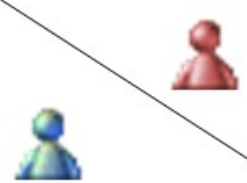
Example: Prisoner's Dilemma

	No Confess **NC**	Confess **C**
No Confess **NC**	**- 2, - 2**	**- 10, - 1**
Confess **C**	**-1, - 10**	**- 5, - 5**

Normal Form Games

How to represent matching coins game in normal form?
With Observation and without observation?

Further Reading

- **Game Theory and Mechanism Design**, Y Narahari. World Scientific Publishing Company, 2014.
- **Multiagent systems: Algorithmic, game-theoretic, and logical foundations**, Shoham, Yoav, and Kevin Leyton-Brown. Cambridge University Press, 2008. (Free download).
- **Game Theory** by Roger Myerson. Harvard University press, 2013.
- **Algorithmic Game Theory**, edited by Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay Vazerani. (Non-printable version available online).

`http://gametheory.net/`

`http://lcm.csa.iisc.ernet.in/gametheory/lecture.html`

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