Pattern Recognition

Assignment – 1

Bayes classifier

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1 Objective:

- 1. To build a Bayes Classifier, and use it classify:
 - a. Linearly Separable 2D Dataset
 - b. Non Linearly Separable 2D Dataset
 - c. Real world 2D Dataset
- 2. Plot Decision Regions for all pairs of classes, and one for all classes combined.
- 3. Plot Contour Regions for all pairs of classes, and one for all classes combined.
- 4. Calculate Accuracy, Precision, Mean Recall, F-measure and confusion matrix.

2 Procedure:

- 1. Each dataset was partitioned into 75% training data and 25% testing data.
- 2. Data from each set was assumed to come from Gaussian distribution.
- 3. In Case 1 ($\Sigma = \sigma^2 I$), mean of all the covariance matrices was calculated, and the off-diagonal terms for the resulting matrix were assumed 0.
- 4. In Case 2 ($\sum_{i} = \sum$), mean of the covariance matrices of all classes was calculated, and used for further calculations.
- 5. In Case 3 (\sum_i is a diagonal matrix), off-diagonal terms of covariance matrices of all classes were considered 0.
- 6. In Case $4(\sum_i$ is unique), no assumptions were made.
- 7. Based on assumptions for each case, the discriminant function $(g_i(x))$ was calculated for each class, using which, the Decision region and Contour plot were made.
- 8. The remaining 25% data was used for analysis on each case.

3 Observations:

 $3.1 \text{ Case } 1 - \sum = \sigma^2 I$

3.1.1 Linear Data:

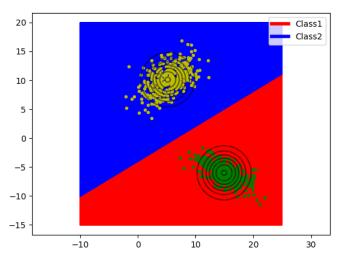


Figure 1. Decision Boundary and Contour Plot for Case 1 with Linear Data for class1 and class2

Accuracy = 100%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

	class1	class2
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

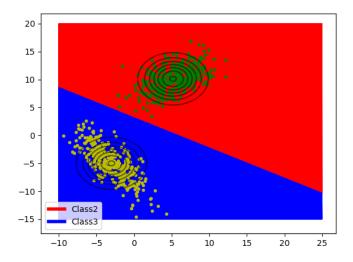


Figure 2. Decision Boundary and Contour Plot for Case 1 with Linear Data for class2 and class3

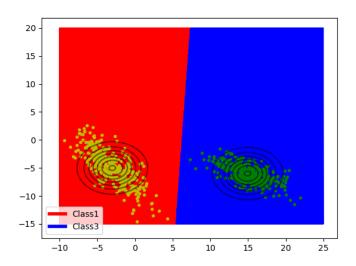
Accuracy = 100%

Confusion Matrix

	class2	class3
class2	125	0
class3	0	125

<u>Analysis</u>

	class2	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5



<u>Figure 3. Decision Boundary and Contour Plot for Case 1 with Linear Data for class1 and class3</u>

Accuracy = 100%

Confusion Matrix

	class1	class3
class1	125	0
class3	0	125

	class1	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

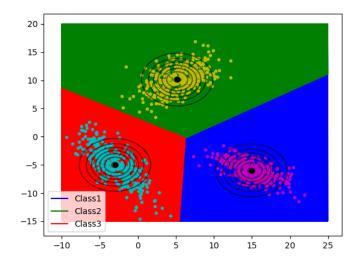


Figure 4. Decision Boundary and Contour Plot for Case 1 with Linear Data for all classes

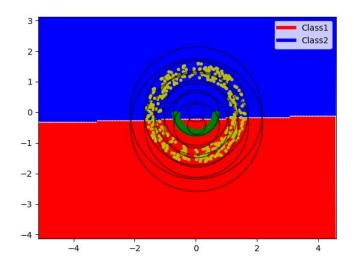
Accuracy = 100%

	class1	class2	class3
class1	125	0	0
class2	0	125	0
class3	0	0	125

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	class1	class2	class3
Precision	1.0	1.0	1.0
Recall	1.0	1.0	1.0
F-measure	0.5	0.5	0.5

3.1.2 Non Linear Data:



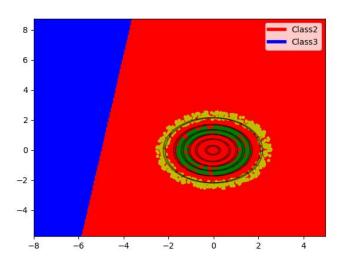
<u>Figure 5. Decision Boundary and Contour Plot for Case 1 with Non Linear Data for class1 and class2</u>

Accuracy = 62.8%

	class1	class2
class1	87	38
class2	55	70

Analysis

	class1	class2
Precision	0.6127	0.6482
Recall	0.696	0.56
F-measure	0.3258	0.3004



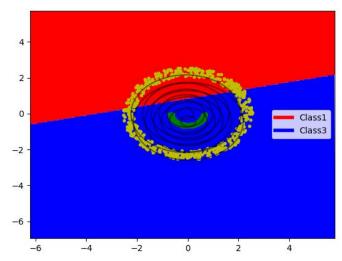
<u>Figure 6. Decision Boundary and Contour Plot for Case 1 with Non Linear Data for class2 and class3</u>

Accuracy = 41.67%

Confusion Matrix

	class2	class3
class2	125	0
class3	175	0

	class2	class3
Precision	0.4167	0.0
Recall	1.0	0.0
F-measure	0.2941	0.0



<u>Figure 7. Decision Boundary and Contour Plot for Case 1 with Non Linear Data for class1 and class3</u>

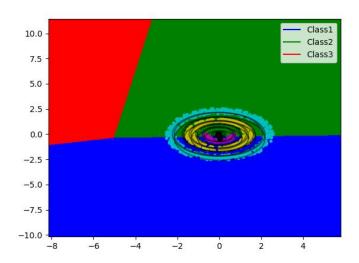
Accuracy = 66.33%

Confusion Matrix

	class1	class3
class1	125	0
class3	101	74

Analysis

	class1	class3
Precision	0.5531	1.0
Recall	1.0	0.4229
F-measure	0.3561	0.2972



<u>Figure 8. Decision Boundary and Contour Plot for Case 1 with Non Linear Data for all classes</u>

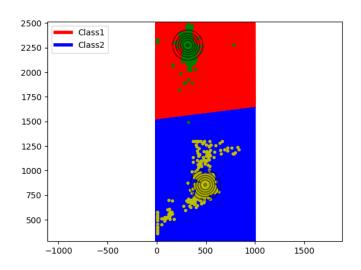
Accuracy = 36.94%

	class1	class2	class3
class1	87	38	0
class2	55	70	0
class3	71	104	0

Analysis

	class1	class2	class3
Precision	0.4084	0.3302	0.0
Recall	0.696	0.56	0.0
F-measure	0.2574	0.2077	0.0

3.1.3 Real World Data:



<u>Figure 9. Decision Boundary and Contour Plot for Case 1 with Real World Data for class1 and class2</u>

Accuracy = 99.67%

Confusion Matrix

	class1	class2
class1	593	4
class2	0	614

	class1	class2
Precision	1.0	0.9935
Recall	0.9933	1.0
F-measure	0.4983	0.4984

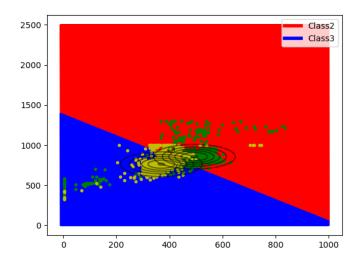


Figure 10. Decision Boundary and Contour Plot for Case 1 with Real World Data for class2 and class3

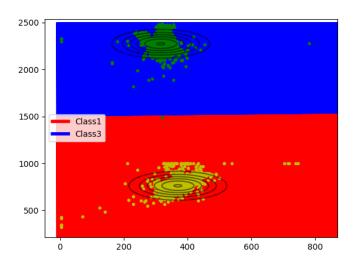
Accuracy = 82.28%

Confusion Matrix

	class2	class3
class2	445	169
class3	50	572

Analysis

	class2	class3
Precision	0.8990	0.7719
Recall	0.7248	0.9196
F-measure	0.4013	0.4197



<u>Figure 11. Decision Boundary and Contour Plot for Case 1 with Real World Data for class1 and class3</u>

Accuracy = 99.75%

	class1	class3
class1	594	3
class3	0	622

<u>Analysis</u>

	class1	class3
Precision	1.0	0.9952
Recall	0.9950	1.0
F-measure	0.4987	0.4988

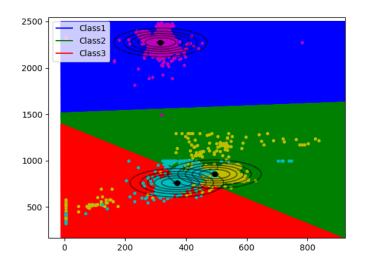


Figure 12. Decision Boundary and Contour Plot for Case 1 with Real World Data for all classes

Accuracy = 87.83%

Confusion Matrix

	class1	class2	class3
class1	593	1	3
class2	0	445	169
class3	0	50	572

Analysis

	class1	class2	class3
Precision	1.0	0.8972	0.7688
Recall	0.9933	0.7248	0.9196
F-measure	0.4983	0.4009	0.4187

3.1.4 Inferences:

- 1. The Decision Surface is linear in nature, as can be seen from the plots.
- 2. The assumption in this case works well for linearly separable data, but gives poor results for non-linearly separable data and the real world data.
- 3. The nature of the contour is circular, since we have taken $\sum = \sigma^2 I$

3.2 Case 2 - $\sum_{i} = \sum_{i}$ 3.2.1 Linear Data:

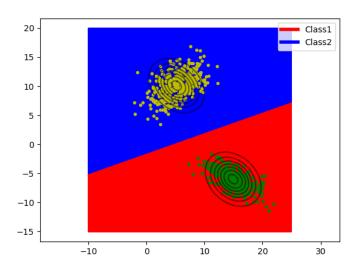


Figure 13. Decision Boundary and Contour Plot for Case 2 with Linear Data for class1 and class2

Accuracy = 100%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

	class1	class2
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

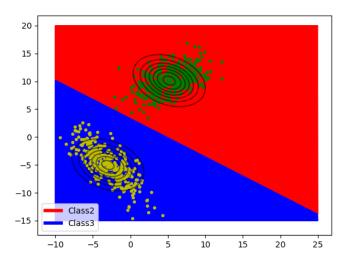


Figure 14. Decision Boundary and Contour Plot for Case 2 with Linear Data for class2 and class3

Accuracy = 100%

Confusion Matrix

	class2	class3
class2	125	0
class3	0	125

<u>Analysis</u>

	class2	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

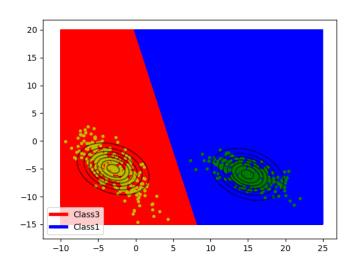


Figure 15. Decision Boundary and Contour Plot for Case 2 with Linear Data for class1 and class3

Accuracy = 100%

Confusion Matrix

	class1	class3
class1	125	0
class3	0	125

	class1	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

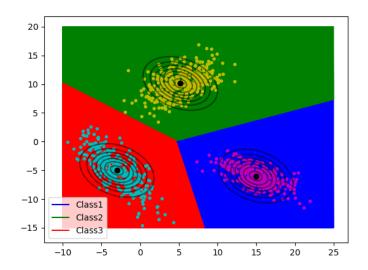


Figure 16. Decision Boundary and Contour Plot for Case 2 with Linear Data for all <u>classes</u>

Accuracy = 100%

Confusion Matrix

	class1	class2	class3
class1	125	0	0
class2	0	125	0
class3	0	0	125

<u>Analysis</u>

	class1	class2	class3
Precision	1.0	1.0	1.0
Recall	1.0	1.0	1.0
F-measure	0.5	0.5	0.5

3.2.2 Non Linear Data:

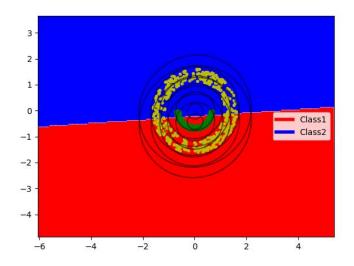
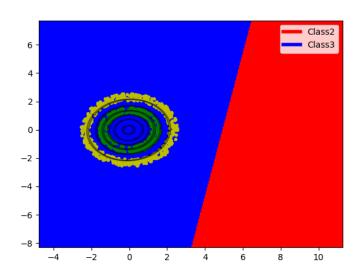


Figure 17. Decision Boundary and Contour Plot for Case 2 with Non Linear Data for class1 and class2

	class1	class2
class1	89	36
class2	55	70

Analysis

	class1	class2
Precision	0.6181	0.6604
Recall	0.712	0.56
F-measure	0.3309	0.3030



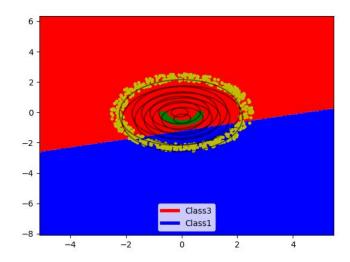
<u>Figure 18. Decision Boundary and Contour Plot for Case 2 with Non Linear Data for class2 and class3</u>

Accuracy = 58.33%

Confusion Matrix

	class2	class3
class2	0	125
class3	0	175

	class2	class3
Precision	0.0	0.5833
Recall	0.0	1.0
F-measure	0.0	0.3684



<u>Figure 19. Decision Boundary and Contour Plot for Case 2 with Non Linear Data for class1 and class3</u>

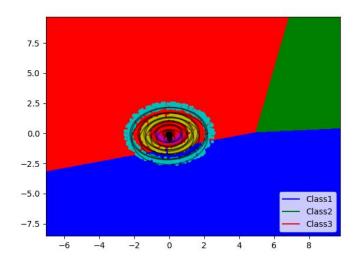
Accuracy = 40.67%

Confusion Matrix

	class1	class3
class1	0	125
class3	53	122

Analysis

	class1	class3
Precision	0.0	0.4939
Recall	0.0	0.6971
F-measure	0.0	0.2891



<u>Figure 20. Decision Boundary and Contour Plot for Case 2 with Non Linear Data for all classes</u>

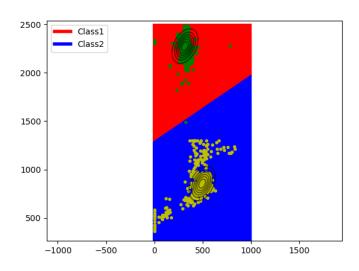
Accuracy = 28.71%

	class1	class2	class3
class1	0	0	125
class2	10	0	115
class3	53	0	122

Analysis

	class1	class2	class3
Precision	0.0	0.0	0.3370
Recall	0.0	0.0	0.6971
F-measure	0.0	0.0	0.2272

3.2.3 Real World Data:



<u>Figure 21. Decision Boundary and Contour Plot for Case 2 with Real World Data for</u> class1 and class2

Accuracy = 98.68%

Confusion Matrix

	class1	class2
class1	581	16
class2	0	614

	class1	class2
Precision	1.0	0.9746
Recall	0.9732	1.0
F-measure	0.4932	0.4936

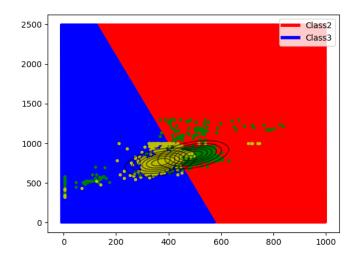


Figure 22. Decision Boundary and Contour Plot for Case 2 with Real World Data for class2 and class3

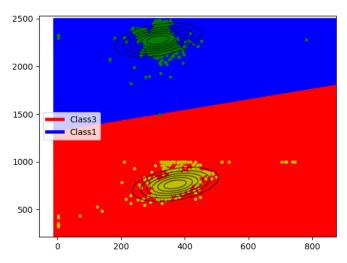
Accuracy = 82.12%

Confusion Matrix

	class2	class3
class2	423	191
class3	30	592

Analysis

	class2	class3
Precision	0.9338	0.7561
Recall	0.6889	0.9518
F-measure	0.3964	0.4214



<u>Figure 23. Decision Boundary and Contour Plot for Case 2 with Real World Data for class1 and class3</u>

Accuracy = 98.93%

	class1	class3
class1	584	13
class3	0	622

<u>Analysis</u>

	class1	class3
Precision	1.0	0.9795
Recall	0.9782	1.0
F-measure	0.4945	0.4948

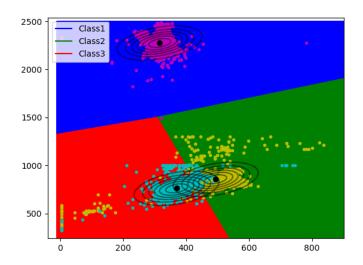


Figure 24. Decision Boundary and Contour Plot for Case 2 with Real World

Data for all classes

Accuracy = 87.07%

Confusion Matrix

	class1	class2	class3
class1	581	13	3
class2	0	423	191
class3	0	30	592

Analysis

	class1	class2	class3
Precision	1.0	0.9077	0.7532
Recall	0.9732	0.6889	0.9518
F-measure	0.4932	0.3917	0.4204

3.2.4 Inferences:

- 1. The Decision Surface is linear in nature, as can be seen from the plots.
- 2. The assumption in this case works well for linearly separable data, but gives poor results for non-linearly separable data and real world data.
- 3. The nature of the contour is elliptical, since we have taken $\sum_{i} = \sum$ (where \sum is found by taking mean of the \sum of the three classes.

3.3 Case 3 - $\sum_{i} = \sum_{i}$ 3.3.1 Linear Data:

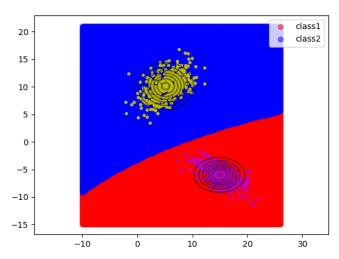


Figure 25. Decision Boundary and Contour Plot for Case 3 with Linear Data for class1 and class2

Accuracy = 100%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

	class1	class2
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

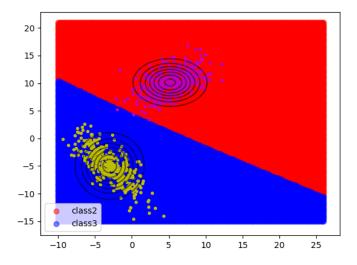


Figure 26. Decision Boundary and Contour Plot for Case 3 with Linear Data for class2 and class3

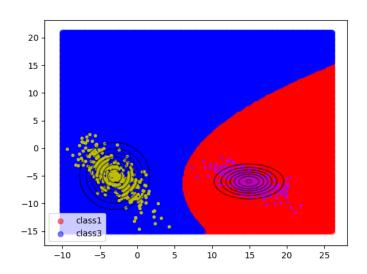
Accuracy = 100%

Confusion Matrix

	class2	class3
class2	125	0
class3	0	125

Analysis

	class2	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5



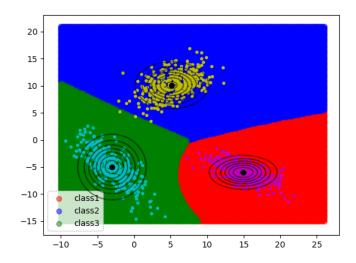
<u>Figure 27. Decision Boundary and Contour Plot for Case 3 with Linear Data for class1</u> and class3

Accuracy = 100%

Confusion Matrix

	class1	class3
class1	125	0
class3	0	125

	class1	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5



<u>Figure 28. Decision Boundary and Contour Plot for Case 3 with Linear Data for all classes</u>

Accuracy = 100%

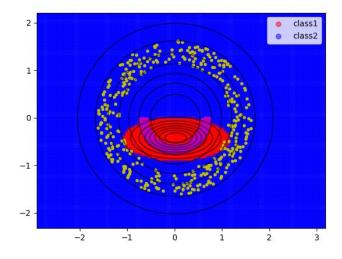
Confusion Matrix

	class1	class2	class3
class1	125	0	0
class2	0	125	0
class3	0	0	125

Analysis

	class1	class2	class3
Precision	1.0	1.0	1.0
Recall	1.0	1.0	1.0
F-measure	0.5	0.5	0.5

3.3.2 Non Linear Data:

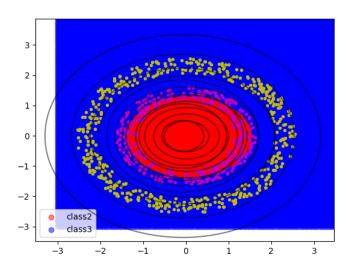


<u>Figure 29. Decision Boundary and Contour Plot for Case 3 with Non Linear Data for class1 and class2</u>

	class1	class2
class1	120	5
class2	2	123

<u>Analysis</u>

	class1	class2
Precision	0.9836	0.9609
Recall	0.96	0.984
F-measure	0.4858	0.4862



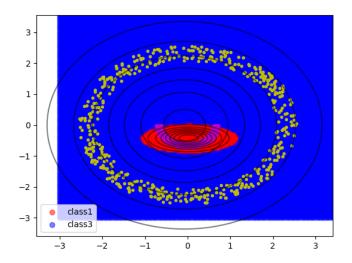
<u>Figure 30. Decision Boundary and Contour Plot for Case 3 with Non Linear Data for class2 and class3</u>

Accuracy = 83.33%

Confusion Matrix

	class2	class3
class2	75	50
class3	0	175

	class2	class3
Precision	1.0	0.7778
Recall	0.6	1.0
F-measure	0.375	0.4375



<u>Figure 31. Decision Boundary and Contour Plot for Case 3 with Non Linear Data for class1 and class3</u>

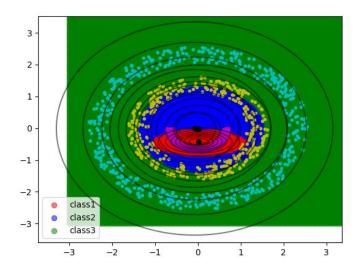
Accuracy = 100%

Confusion Matrix

	class1	class3
class1	125	0
class3	0	175

<u>Analysis</u>

	class1	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5



<u>Figure 32. Decision Boundary and Contour Plot for Case 3 with Non Linear Data for all classes</u>

Accuracy = 86.59%

	class1	class2	class3
class1	120	5	0
class2	2	73	50
class3	0	0	175

Analysis

	class1	class2	class3
Precision	0.9836	0.9359	0.7778
Recall	0.96	0.584	1.0
F-measure	0.4858	0.3596	0.4375

3.3.3 Real World Data:

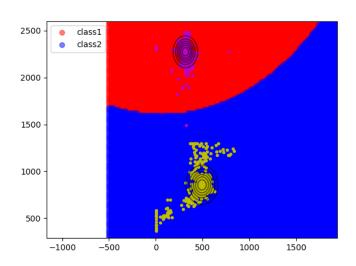


Figure 33. Decision Boundary and Contour Plot for Case 3 with Real World Data for class1 and class2

Accuracy = 99.67%

Confusion Matrix

	class1	class2
class1	594	3
class2	1	613

	class1	class2
Precision	0.9983	0.9951
Recall	0.9950	0.9984
F-measure	0.4983	0.4984

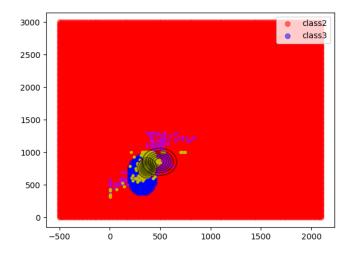


Figure 34. Decision Boundary and Contour Plot for Case 3 with Real World Data for class2 and class3

Accuracy = 86.33%

Confusion Matrix

	class2	class3
class2	505	109
class3	60	562

Analysis

	class2	class3
Precision	0.8938	0.8376
Recall	0.8225	0.9035
F-measure	0.4283	0.4346

2500 2000 1500 1000 500 - class1 class3

Figure 35. Decision Boundary and Contour Plot for Case 3 with Real World Data for class1 and class3

750

1000

1250

500

Accuracy = 99.75%

-250

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250

	class1	class3
class1	594	3
class3	0	622

<u>Analysis</u>

	class1	class3
Precision	1.0	0.9952
Recall	0.9950	1.0
F-measure	0.4987	0.4988

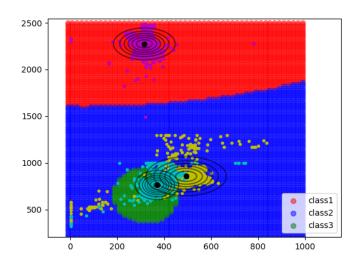


Figure 36. Decision Boundary and Contour Plot for Case 3 with Real World

Data for all classes

Accuracy = 90.56%

Confusion Matrix

	class1	class2	class3
class1	594	0	3
class2	1	504	109
class3	0	60	562

Analysis

	class1	class2	class3
Precision	0.9983	0.8936	0.8338
Recall	0.9950	0.8208	0.9035
F-measure	0.4983	0.4278	0.4336

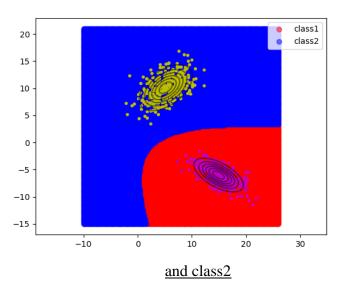
3.3.4 Inferences:

- 1. The Decision Surface is non-linear in nature, as can be seen from the plots.
- 2. The assumption in this case works well for linearly separable data, and good (but not as good as linearly separable data) for other sorts of data.
- 3. The nature of the contour is elliptical and is different for each class, but is oriented horizontally.

3.4 Case 4 - \sum_i is unique

3.4.1 Linear Data:

Figure 37. Decision Boundary and Contour Plot for Case 4 with Linear Data for class1



Accuracy = 100%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

<u>Analysis</u>

	class1	class2
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

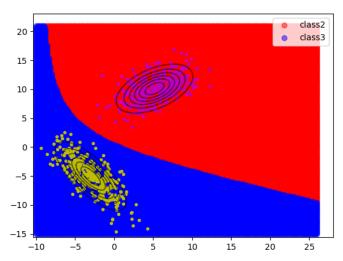
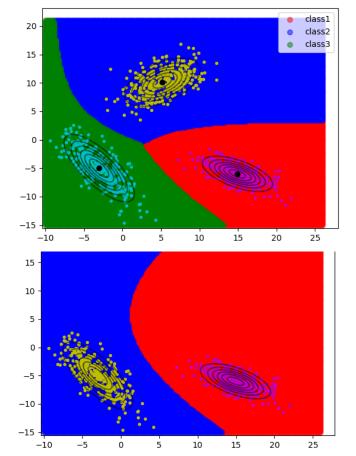


Figure 38. Decision Boundary and Contour Plot for Case 4 with Linear Data for class2 and class3

Accuracy = 100%



ss2	class3	
.0	1.0	
.0	1.0	
.5	0.5	

 class2
 class3

 class2
 125
 0

 class3
 0
 125

Analysis

	class1	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

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Figure 39. Decision Boundary and Contour Plot for Case 4 with Linear Data for class1 and class3

Accuracy = 100%

Confusion Matrix

	class1	class3
class1	125	0
class3	0	125

Figure 40. Decision Boundary and Contour Plot for Case 4 with Linear Data for all classes

Accuracy = 100%

Confusion Matrix

	class1	class2	class3
class1	125	0	0
class2	0	125	0
class3	0	0	125

<u>Analysis</u>

	class1	class2	class3
Precision	1.0	1.0	1.0
Recall	1.0	1.0	1.0
F-measure	0.5	0.5	0.5

3.4.2 Non Linear Data:

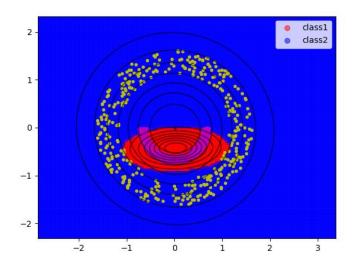
Figure 41. Decision Boundary and Contour Plot for Case 4 with Non Linear Data for class1 and class2

Accuracy = 97.2%

Confusion Matrix

	class1	class2
class1	120	5
class2	2	123

<u>Analysis</u>



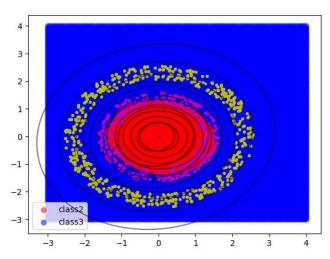
	class1	class2
Precision	0.9836	0.9609
Recall	0.96	0.984
F-measure	0.4858	0.4862

<u>Figure 42. Decision Boundary and Contour Plot for Case 4 with Non Linear Data for class2 and class3</u>

Accuracy = 83.33%

Confusion Matrix

<u>Analysis</u>



	class2	class3
Precision	1.0	0.7778
Recall	0.6	1.0
F-measure	0.375	0.4375

	class2	class3
class2	75	50
class3	0	175

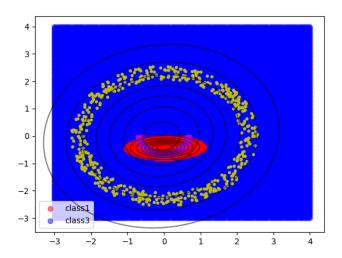


Figure 43. Decision Boundary and Contour Plot for Case 4 with Non Linear Data for class1 and class3

Accuracy = 100%

Confusion Matrix

	class1	class3
class1	125	0
class3	0	175

Analysis

	class1	class3
Precision	1.0	1.0
Recall	1.0	1.0
F-measure	0.5	0.5

4-3-2-1-0--1--2--3 -2 -1 0 1 2 3 4

<u>Figure 44. Decision Boundary and Contour Plot for Case 4 with Non Linear Data for all classes</u>

Accuracy = 86.59%

	class1	class2	class3
class1	120	5	0
class2	2	73	50
class3	0	0	175

<u>Analysis</u>

	class1	class2	class3
Precision	0.9836	0.9359	0.7778
Recall	0.96	0.584	1.0
F-measure	0.4858	0.3596	0.4375

3.4.3 Real World Data:

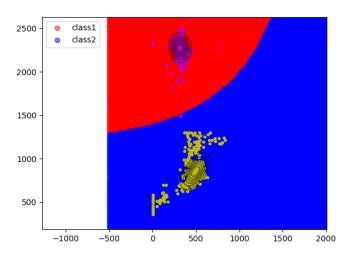


Figure 45. Decision Boundary and Contour Plot for Case 4 with Real World Data for class1 and class2

Accuracy = 99.67%

Confusion Matrix

	class1	class2
class1	594	3
class2	1	613

	class1	class2
Precision	0.9983	0.9951
Recall	0.9950	0.9984
F-measure	0.4983	0.4983

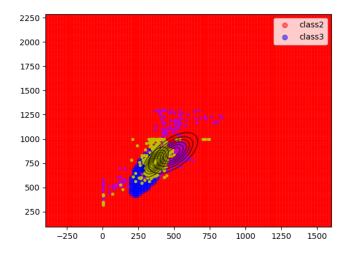


Figure 46. Decision Boundary and Contour Plot for Case 4 with Real World Data for class2 and class3

Accuracy = 86.33%

Confusion Matrix

	class2	class3
class2	505	109
class3	60	562

Analysis

	class2	class3
Precision	0.8938	0.8376
Recall	0.8225	0.9035
F-measure	0.4283	0.4346

<u>Figure 47. Decision Boundary and Contour Plot for Case 4 with Real World Data for class1 and class3</u>

Accuracy = 99.75%

	class1	class3
class1	594	3
class3	0	622

<u>Analysis</u>

	class1	class3	
Precision	1.0	0.9952	
Recall	0.9950	1.0	
F-measure	0.4987	0.4988	

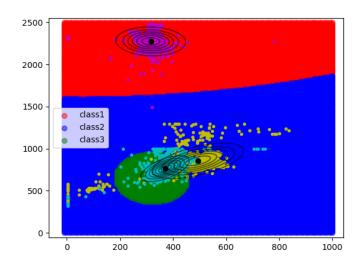


Figure 48. Decision Boundary and Contour Plot for Case 4 with Real World

Data for all classes

Accuracy = 90.56%

Confusion Matrix

	class1	class2	class3
class1	594	0	3
class2	1	504	109
class3	0	60	562

Analysis

	class1	class2	class3
Precision	0.9983	0.8936	0.8338
Recall	0.9950	0.8208	0.9035
F-measure	0.4983	0.4278	0.4336

3.4.4 Inferences:

- 1. The Decision Surface is non-linear in nature, as can be seen from the plots.
- 2. The assumption in this case works very well for linearly separable data, but gives not so good results for other sorts of data.
- 3. The nature of the contour is elliptical, different for each class, and oriented randomly.

4. Conclusion

- 1. Bayes Classifier works well for Linearly separable data in all cases with high accuracy but fails for Non linearly separable data with poor accuracy.
- 2. In case of Real World Data, the data is overlapping and thus results in lesser accuracy.
- 3. In first two cases, the decision boundary comes out to be straight line whereas in last two cases, the decision boundary is quadratic due to different covariance matrix chosen.