Divergence Mach Number and Engineering Design

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The primary objective of this experiment is to use given data sets to establish relationships and dependencies on the Drag Divergence Mach number M_{div} and then use the data sets to create interpolated models that can converge to any points within the range. The lab also includes a design problem where for given flight conditions, the model created before should be used to make important engineering decisions. It was found that the linear relationship between the lift coefficient and M_{div} could be used to make the interpolation easier and more accurate. Using build-in functions in the computational tool MATLAB, an interpolated model was created that predicts the Drag Divergence Mach number for given lift coefficient, sweep angle and thickness ratio. For the design problem it was found that for the given flight condition, the lift coefficient was about 0.35 therefore, the sweep angle should range between 30.4-35 degrees and the thickness should be between 10-12%. The biggest takeaway from this experiment is the designing part of the experiment where a range of independent parameter is reverse engineered for an optimal design. The lab experiment advances students toward a better understanding of real world aerodynamics.

I. Nomenclature

ρ	=	Density of air
P_∞	=	Atmospheric pressure
T_∞	=	Ambient temperature
M_{div}	=	Divergence Mach number
V	=	Velocity
S	=	Area
M	=	Mach number
Λ	=	Sweep angle
$\frac{t}{c}$	=	Thickness ratio
C_L	=	Lift Coefficient

II. Introduction

The purpose of this experiment is to utilize the Mach Divergence data provided to generate to working model that predicts interpolated Mach Divergence number outside the elements in the provided data. Further more, the model is also to be used in a engineering design problem that focuses on obtaining a convergent range of aircraft sweep angle and thickness ratio for given flight parameters.

The dataset provided by the TA for this lab experiment is as follows:

The above data set is for a sweep angle $\Lambda = 0°$. Similar sets of data are given for an additional 15, 25 & 35 degrees.

A. Divergence Mach number

It has been explored in the previous experiments that for a given airfoil, the drag and lift exhibit an unexpected trend in their behaviors, that onsets typically around the transonic region of Mach = 0.6. The drag increases almost 10 folds than at low-speeds. Once sonic speed is achieved, the drag starts decreasing again and returning to its original trajectory.

Divergence Mach number typically is a little more than the critical Mach number. Briefly, critical Mach number is the lowest Mach number at which the airflow over some point of the aircraft reaches the speed of sound, but does not exceed it.

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Table 1 Divergence Mach number

C_L	$\frac{t}{c} = 0.08$	$\frac{t}{c} = 0.1$	$\frac{t}{c} = 0.12$
0.2	0.806	0.784	0.762
0.3	0.785	0.764	0.744
0.4	0.763	0.744	0.727
0.5	0.742	0.724	0.722
0.6	0.720	0.703	0.692

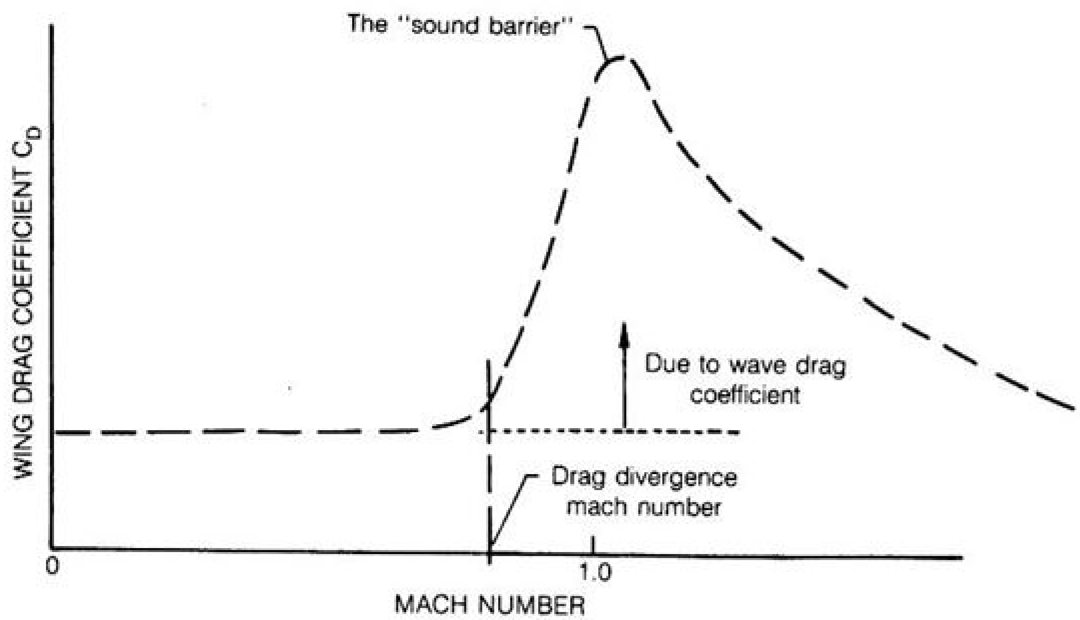

Fig. 1 Drag Divergence Mach number

The lab is performed in 4 distinct parts each helping obtain a better insight from the previous one. A brief introduction is given as follows:

1. Impact factors

This is the introduction part for the students that was to be done before the data was provided. After doing some research on Divergence Mach number, students are asked to make educated/calculated guesses on what factors of an aircraft's parameters and flight regime affect the Divergence Mach number.

2. Measurement requirements

It is difficult to define the onset point which can be defined as the Divergence Mach number. This is where a mathematical definition of "rapidly changing" must be made.

3. Data analysis and interpretation

Plotting the data that would clearly demonstrate the effects of all the impact factors on the Divergence Mach number and defining a working model for interpolation of the data sets.

4. Using data to make engineering decisions

Using the model created and with the knowledge so far, a range of parameters ideal for the given specifications needs to be found.

Specifications of the flight are as follows:

1) Mach = 0.82
2) Nominal altitude = 30,000 ft
3) Nominal cruise loading = 50 N/m^2

III. Experimental Setup, Procedure & Model

Initially, a list of all possible variables that may affect the Divergence Mach Number was created and the following five independent variables were short-listed for investigation.

1) Thickness of the airfoil
   The thickness of the airfoil has a major impact of the flow regime around it and effects the pressure distribution of the surfaces of the airfoil. This directly affects the drag generated
2) Sweep angle of the wings
   According to the sweep angle thoery, having a sweep angle majorly assist in delaying the critical Mach number in a flow feild. This possibly delays the onset of the drag divergence and hence effects M_{div}.
3) Aspect ratio of the wing
   Aspect ratio is known to be a major factor affecting the amount of lift generation on the airfoil and we know that a major portion of drag can be computer by the square of the lift and the aspect ratio as well.

$$C_D = C_{D_0} + \frac{C_L^2}{\pi ARe_o} \tag{1}$$

4) Lift coefficient
   As shown in the equation 1, the lift contributes a majority of the instantaneous drag and definitely needs to be considered.
5) Angle of attack
   During all this time spent studying aerodynamics, it has become self-evident that angle of attack matters almost everywhere and thus should be considered as well.

A. Finding 3 independent variables

During the preliminary research, it was found that divergence Mach number can be defined by something called **Korn's relation** that is defined as follow:

$$M_{div} = K - \frac{C_L}{10} - \frac{t}{c} \tag{2}$$

where K is a constant factor that can be found using CFD.

Clearly, first two variables are the lift coefficient and the thickness ratio.

$$C_L = \frac{L}{0.5\rho V^2 S} = \frac{[MLT^-2]}{[ML^-3][LT^-1]^2[L^2]} = DIMENSIONLESS \tag{3}$$

$$\frac{t}{c} = \frac{[L]}{[L]} = DIMENSIONLESS \tag{4}$$

Now, for the third parameter it is known that the Divergence Mach number almost always lags behind and very close to the critical Mach number. Critical Mach number is defined as follows:

$$M_{crit} = \frac{M_{cr_{\Lambda=0}}}{cos\Lambda} \tag{5}$$

Hence it is only fair to say that the sweep angle of the plane's wings can be the third independent factor in determining M_{div}.

Also,

$$\Lambda = DIMENSIONLESS \tag{6}$$

Hence, the divergence Mach number is given here as a function of the sweep angle, the thickness ratio & the lift coefficient.

$$M_{div} = f(\Lambda, \frac{t}{c}, C_L) \tag{7}$$

B. Range for the independent variables

A good range to test over the three independent variables can be predicted by the following criterion:

1) C_L between 0.1 and 0.8
   This is the typical lift coefficient range for varying the angle of attack between 0 and 7 degrees. Also, it can be seen that the lift coefficient approximately increases linearly until it reaches the value of 1 for NACA airfoils. This should give an optimum range of values to be tested.
2) $\frac{t}{c}$ between 8% and 12%
   Since it is established that the drag divergence occurs mostly in the transonic region, it is a good assumptions to find the range of airfoil thickness on aircraft's flying around that range. Most commercial airliners have a thickness ratio stated above.
3) Λ between 0° to 45°
   This is the typical range that any non-fighter stable aircraft possesses. It cover almost all of the possible ranges of sweep angles and thus should suffice.

C. Define criterion for M_{div}

The drag divergence Mach numbers is defined at the point when the drag coefficient as a function of Mach number starts increasing very rapidly, and has the highest slope in the subsonic region. A decent way to approach this would be to look at the overall dataset and them define a differential $dM = 0.1$ and check for the slope of C_D in the whole trend. This essentially help get an idea of the onset region, which can then be used to predict exactly where the drag divergence starts. The local maxima in the region gives the point location for M_{div}.

This shouldn't be difficult to compute using tools such as MATLAB.

D. Experiment to find M_{div}

The data sets provided contain M_{div} as a function of all the independent variables listed above. It was also found that the M_{div} varies linearly with the lift coefficient. Hence, it makes sense to find two mach numbers for two known datasets of the lift coefficient and then just do a linear fit between the both.

This is the approach that'll be taken. The input lift coefficient decides what two data sets to work with (eg. if input is 0.25, then work with 0.2 and 0.3). After than the Mach number is found for all the sweep and thickness in the lift coefficient of 0.25 and then the matrix is condition to find the mach numbers great than 0.86. This gives the ranges of sweep angle and thickness ratios that are required. Modified code is in the appendix.

IV. Result

It can be seen that, M_{div} is roughly a linear function of C_L at a given $\Lambda \& thickness$; there parallel lines are the evidence of that. Also be seen that some points are not defined in the data but more on this later in the error section.

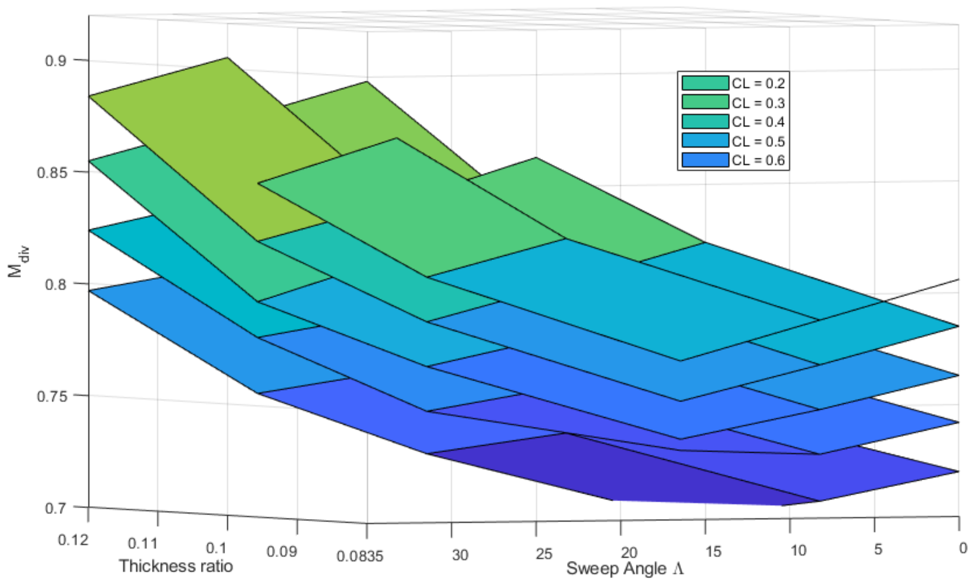

Fig. 2 All provided data in 3D surface form

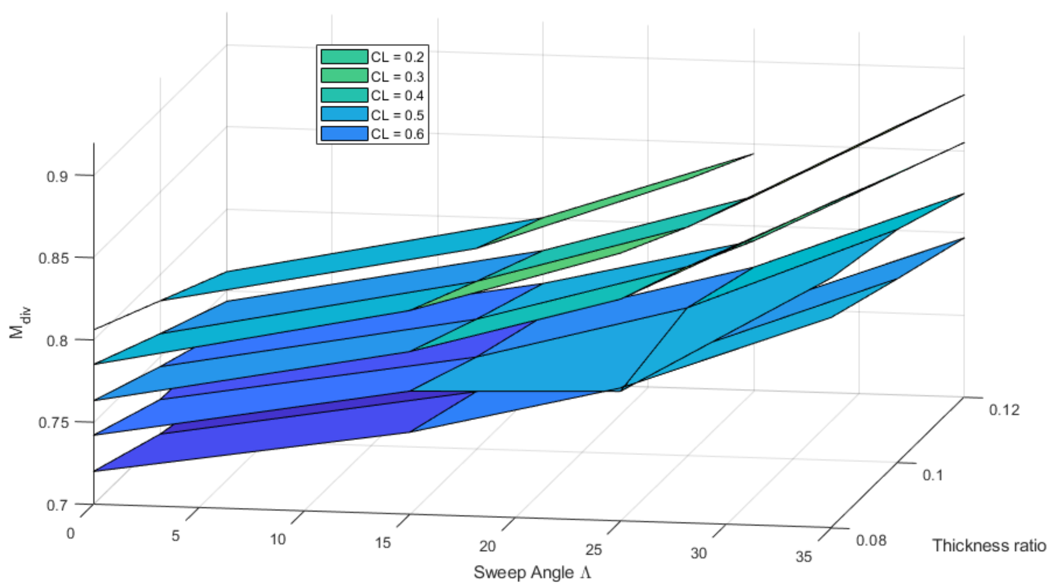

Fig. 3 All provided data in 3D surface form

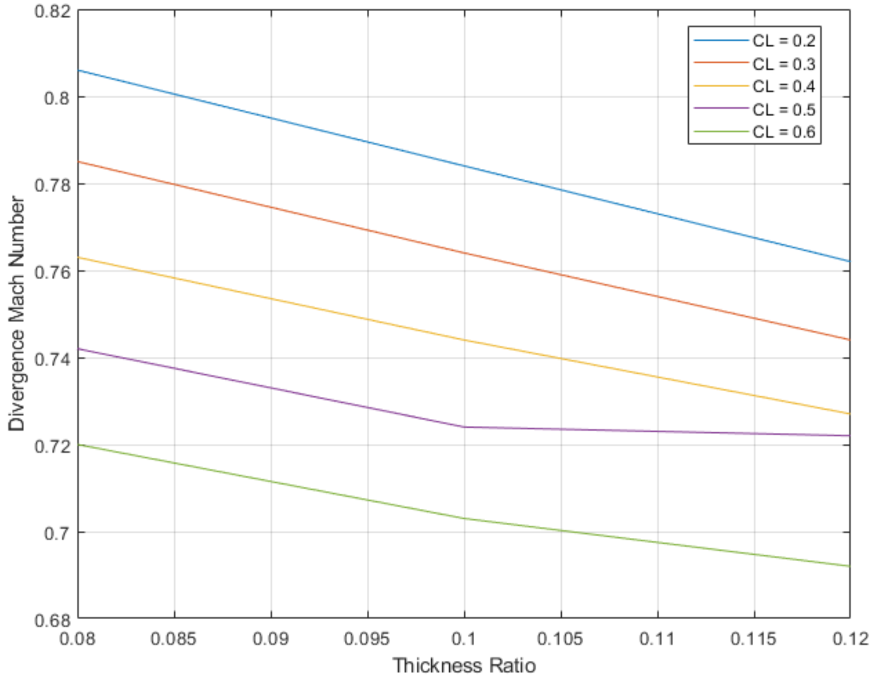

Fig. 4 Divergence Mach number $M_{div} = f(C_L, \frac{t}{c}) at \Lambda = 0°$

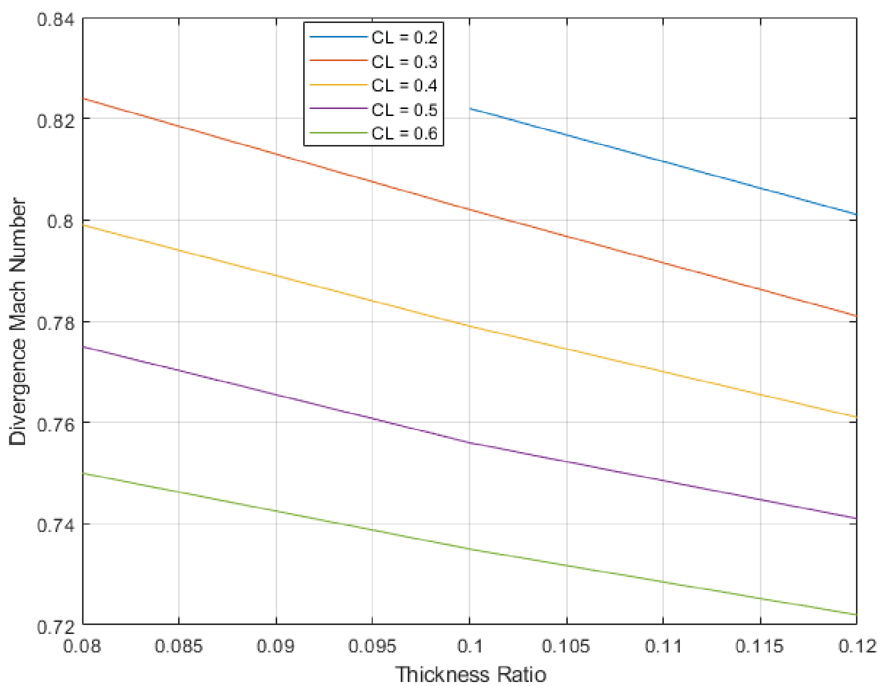

Fig. 5 Divergence Mach number $M_{div} = f(C_L, \frac{t}{c}) at \Lambda = 15°$

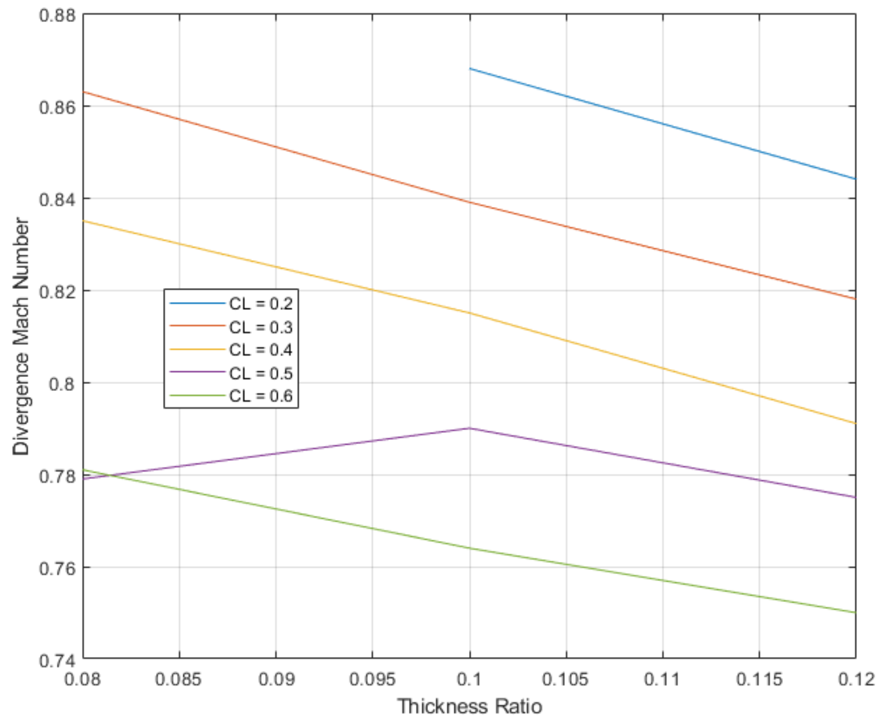

Fig. 6 Divergence Mach number $M_{div} = f(C_L, \frac{t}{c}) at \Lambda = 25°$

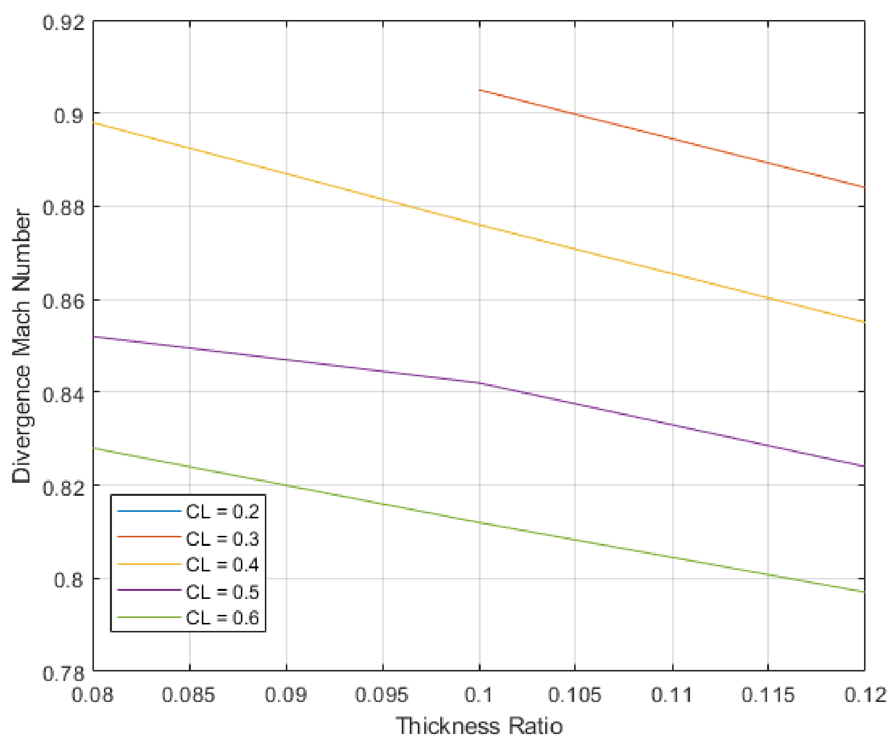

Fig. 7 Divergence Mach number $M_{div} = f(C_L, \frac{t}{c}) at \Lambda = 35°$

A. MATLAB model function

Using the data sets provided a MATLAB function **aarlab3.m** is created that predicts the Drag Divergence Mach number as a function of the three independent variables, $\Lambda, thickness \& C_L$. A "help" description created is shown below:

```
Command Window
>> help aarlab3
  aarlab3  Predicts the Divergence Mach number
  at a given sweep angle, thickness ratio and lift coefficient

  SYNTAX: Mdiv = aarlab3(sweep, thickness ratio, lift coefficient)

  Range of inputs allowed:
                          Lift Coefficient: [0.2, 0.6]
                          Sweep angle:    [0, 35] degrees
                          Thickness ratio: [0.08, 0.12]
```

Fig. 8 Function Syntax and range

As seen from the plots before, M_{div} is roughly linear function for the lift coefficient. Taking this into account, the data sets are transposed such that for each given lift coefficient, there's a 3×4 matrix of M_{div} numbers that are a function of $\Lambda \& thickness$. This makes it easy to program the script and to exploit the linearity observed for lift coefficient.

```
%% Find bounds for lift coefficient
if CL >=0.2 && CL < 0.3
    uppercL = 0.3; lowercL = 0.2;
        elseif CL >=0.3 && CL < 0.4
                uppercL = 0.4; lowercL = 0.3;
        elseif CL >=0.4 && CL < 0.5
                uppercL = 0.5; lowercL = 0.4;
        elseif CL >=0.5 && CL <= 0.6
                uppercL = 0.6; lowercL = 0.5;
        else
            fprintf("error")
            uppercL = NaN;
            lowercL = NaN;
end
idx(1) = find(cL == uppercL);
idx(2) = find(cL == lowercL);
```

Fig. 9 Choose upper and lower lift coefficient

As shown in Fig.9, for a given input of C_L, the code tries to find the two lift coefficient above and below the input. For example, if the input is $C_L = 0.25$, then the script defines two new variables **uppercL** & **lowercL** and finds the index of the datasets for those two lift coefficients, in this case:

$$uppercL = 0.2$$

$$lowercL = 0.3$$

Next step is to interpolate the data for both lift coefficient matrices over a large grid. This essentially means that, the function **griddata** takes all the original input points for a given lift coefficient and then predicts M_{div} for each point in the meshgrid defined.

```
%% Mesh griddata

x = linspace(0, 35, 5001);          % sweep angle grid
y = linspace(0.08, 0.12, 501); % thickness grid
[xq, yq] = meshgrid(x, y);

% Interpolated Matrix- The following contains solutions for respective cL
GridUpper = griddata(sweep, tc, Mach(:,:,idx(1)), xq, yq);
GridLower = griddata(sweep, tc, Mach(:,:,idx(2)), xq, yq);

% Scanning for correct location of requested Sweep and thickness
[c, indS] = min(abs(S-x)); [c, indT] = min(abs(TC-y));
% Mach divergence prediction for upper and lower cL
Md(1) = GridLower(indT, indS); Md(2) = GridUpper(indT, indS);
```

Fig. 10 Interpolation

Once the interpolation is done, now there exists two sets of data sets contained all the possible divergence Mach numbers for the upper and lower lift coefficient matrix. The larger the size of the mesh created, the finer the results shall be. In this case, the mesh matrix is of the dimensions 501×5001 meaning the original 3×4 matrix is now a 501×5001 matrix with all the values of M_{div} defined for any sweep angle and thickness between 0 to 35 degrees and 8% to 12% respectively.

The following figure makes it easier to understand the process.

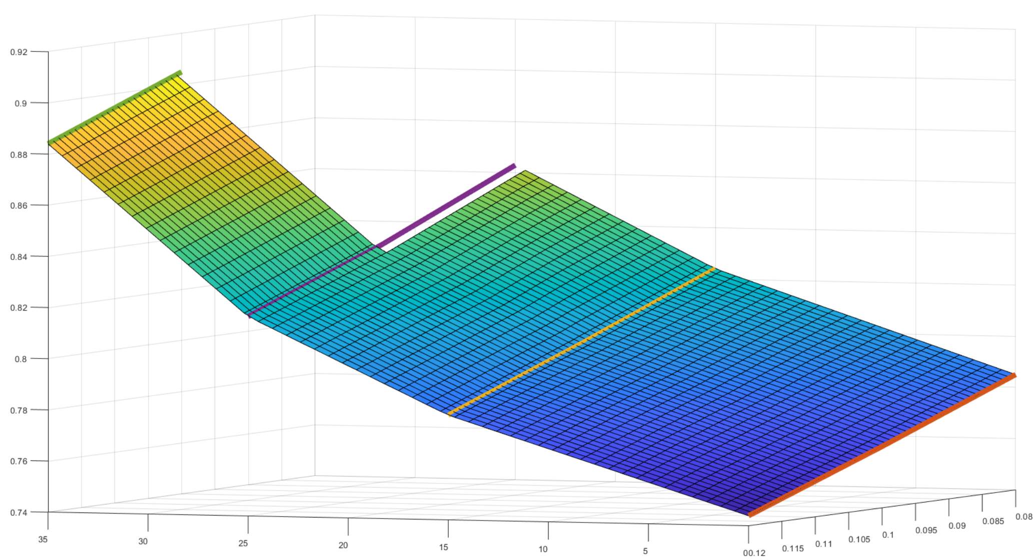

Fig. 11 Original Data line and interpolated grid for $C_L = 0.2$

It can be seen that the bold thick lines represent the original data and each point on the meshed grid represent the prediction made using the interpolated data.

The input sweep angle and thickness are now scanned against the grid to find the index that related to the closest Mach in the interpolated data set.

Finally once two M_{div} is found for both upper and lower lift coefficients, the property established before about the linearity amongst both is used to compute the final prediction of the requred M_{div}.

```
%% Final linear-fit for cL

slope = (Md(2)-Md(1))/(uppercL - lowercL);
intercept = Md(2) - slope*uppercL;

Mdiv = slope*CL + intercept;
```

Fig. 12 Final linear fit

B. MATLAB function verbatim

NOTE: I'm using the official LATEX template from AIAA's website and the formatting for 'verbatim' code is set by the template hence I can't change it. PLEASE do not take points off.

```
    function [Mdiv] = aarlab3(S, TC, CL)

% AARLAB3  Predicts the Divergence Mach number
% at a given sweep angle, thickness ratio and lift coefficient
%
% SYNTAX: Mdiv = AARLAB3(sweep, thickness ratio, lift coefficient)
%
% Range of inputs allowed:
%                                       Lift Coefficient: [0.2, 0.6]
%                                       Sweep angle:    [0, 35] degrees
%                                       Thickness ratio: [0.08, 0.12]

%% Organize dataset

sweep = [0 15 25 35];                         % sweep angle
cL = linspace(0.2,0.6,5);                      % lift coefficient
tc =[0.08 0.1 0.12];                             % thickness

for i = 1:length(sweep)
    % The spreadsheet used here is attached on the CANVAS submission
    % Please use that for the function to work
    Mach(:,:,i) = xlsread('data.xlsx',i);
end
Mach(Mach == 0.11) = NaN;
for i = 1:length(cL)
    Mach1(:, :, i) = Mach(i, :,:);
end
Mach = Mach1;  % for data in Mach, thickness is y-axis, lift is z-axis and sweep is x-axis


%% Find bounds for lift coefficient
if CL >=0.2 && CL < 0.3
    uppercL = 0.3; lowercL = 0.2;
        elseif CL >=0.3 && CL < 0.4
```

```
                uppercL = 0.4; lowercL = 0.3;
        elseif CL >=0.4 && CL < 0.5
                uppercL = 0.5; lowercL = 0.4;
        elseif CL >=0.5 && CL <= 0.6
                uppercL = 0.6; lowercL = 0.5;
        else
            fprintf("error")
            uppercL = NaN;
            lowercL = NaN;
end
idx(1) = find(cL == uppercL);
idx(2) = find(cL == lowercL);

%% Mesh griddata

x = linspace(0, 35, 5001);          % sweep angle grid
y = linspace(0.08, 0.12, 501); % thickness grid
[xq, yq] = meshgrid(x, y);

% Interpolated Matrix- The following contains solutions for respective cL
GridUpper = griddata(sweep, tc, Mach(:,:,idx(1)), xq, yq);
GridLower = griddata(sweep, tc, Mach(:,:,idx(2)), xq, yq);

% Scanning for correct location of requested Sweep and thickness
[c, indS] = min(abs(S-x)); [c, indT] = min(abs(TC-y));
% Mach divergence prediction for upper and lower cL
Md(1) = GridLower(indT, indS); Md(2) = GridUpper(indT, indS);

%% Final linear-fit for cL

slope = (Md(2)-Md(1))/(uppercL - lowercL);
intercept = Md(2) - slope*uppercL;

Mdiv = slope*CL + intercept;
```

C. Proof of model working

The code is tested over the known values and then over unknown points to verify its performance.

Known value used to verify:

Table 2 Divergence Mach number

C_L	$\frac{t}{c} = 0.08$	Λ	M_{div}
0.5	0.08	25	0.779

It can be seen that the predicted value for the known point is **EXACTLY** equal. The other predicted values at unknown points also seem to be correct based on the trend it follows.

```
Command Window
>> Mdiv = aarlab3(25,0.08,0.5)

Mdiv =

    0.7790

>> Mdiv = aarlab3(27,0.09,0.55)

Mdiv =

    0.7905

>> Mdiv = aarlab3(30,0.1,0.6)

Mdiv =

    0.7880
```

Fig. 13 Model working proof

D. Design to make engineering decisions

The function code is not modified to output a range of values for sweep angle and thickness ratio that would work for the given specifications.

First, the design lift coefficient needs to be computed. That goes as follows:

$$C_L = \frac{L}{\frac{1}{2}\gamma P_\infty M_\infty^2 S} \tag{8}$$

But $L = NominalLoading \times S$

$$\therefore C_L = \frac{NominalLoading}{\frac{1}{2}\gamma P_\infty M_\infty^2} \tag{9}$$

$$C_L = \frac{5000}{0.5 * 1.4 * 30100 * 0.82^2}$$

at 30,000 ft, $P_\infty = 30.1kPa$ and $\gamma = 1.4$,

$$\therefore C_L = 0.353$$

Now, since the nominal Mach number of the flight is 0.82, a good design would be where M_{div} is a little greater than that, lets say about 0.86. This ensures that the drag on the flight does not diverge and it stays smooth.

Using this in the modified model, the following ranges are ideal for a good design:

Table 3 Range for good design

	Minimum	Maximum
Thickness $\frac{t}{c}$	10%	12%
Sweep angle Λ	30.38°	35°

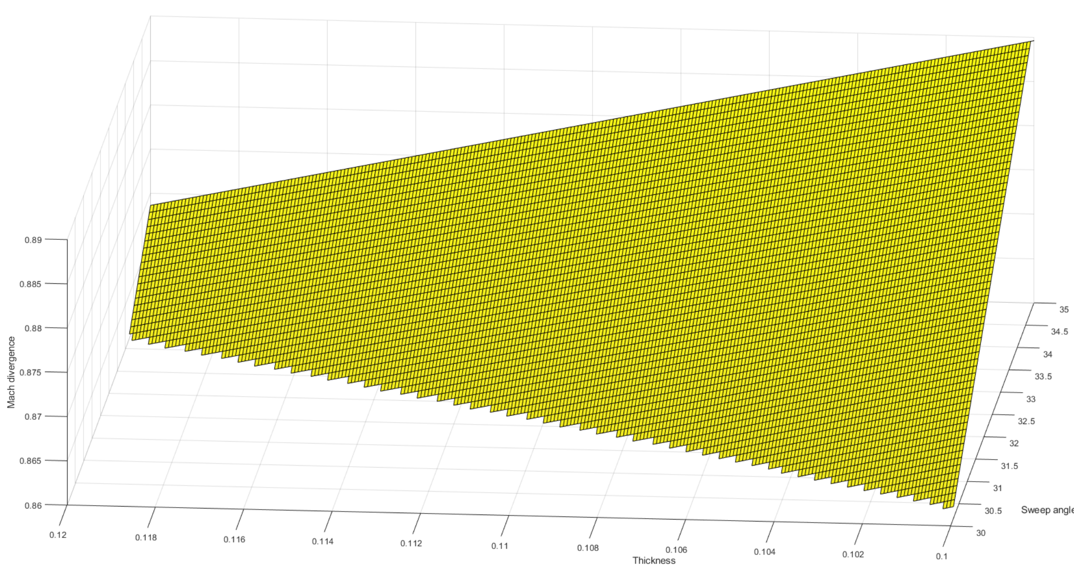

Fig. 14 Design ranges and respective Mach Divergence numbers

V. Conclusion

Following major conclusions can be made from this lab experiment:

1. Importance of Drag divergence Mach number and the variables that affect it

It can be seen from Fig.1 that the drag can increase very rapidly at the drag divergence Mach number and this can have unimaginable repurcussions in real life. If the designer hasn't accounted for these factors then the plane can possible go into early stall or experience high turbulence and possibly crash due to fuel lack. The fact that we were able to quantize and find the independent variables that affect M_{div} tells a lot about how a design problem should be approached and how many parameters need to be accounted for and tested in order to get things just right.

It was seen from the data and the plots above that M_{div} was related to the three factors roughly in the following ways:

1) Thickness Ratio $\frac{t}{c}$: Inversely Proportional
2) Sweep angle Λ: Directly Proportional
3) Lift coefficient C_L: Inversely proportional and roughly linear

The above conclusions make sense from what we have seen about the lift coefficient properties and with the Korn's equation defined in the introduction section.

2. Working with given data and making important engineering decisions with it

The most interesting part of the experiment was the design problem. It was very interesting to first create a model with given data and then modify and use that model to make educated predictions on what ranges and margin should be for a good design.

For the ranges found, a good judgement can be made to move forward to testing mockups in the wind tunnel with the possible sweep angles and thickness ratios. This was perhaps the most interesting lab done so far in the Aerodynamics course.

A. Uncertainties

There is an inherent uncertainty just from not knowing the model which generated the datasets given. From the initial plots of thickness vs Mach divergence numbers for respective lift and sweep angle, it can be seen that there are certain **redbad data points** that do not follow the remaining trend. An example is shown in Fig.15.

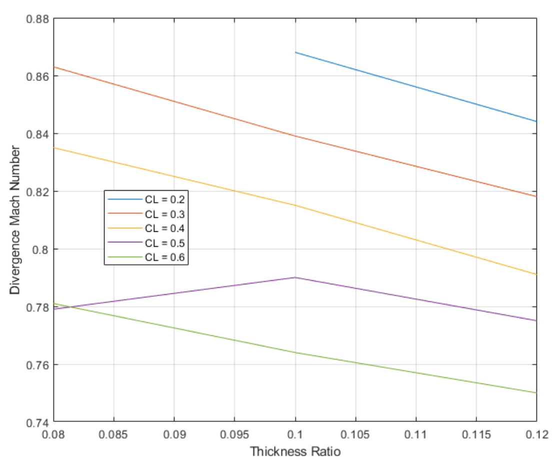

Fig. 15 Bad point in $C_L = 0.5$ line

There are obviously other numerous errors associated with these measurements. For instance, our assumptions may not be completely accurate. Machine errors in the some of the instruments used to get the original data and error in the interpolation algorithm.

On top of that, there's uncertainty associated with other human error, assumptions made, variable parameters, etc.

Detailed equations, tabular data & sample calculations in Appendix

Overall, it was an intriguing experiment that gave the students a better insight into what's studied in theory in class and opened a whole lot of opportunities for questioning the design process in the aerospace world!

Appendix A - Data tables

Table 4 Divergence Mach number for Sweep angle $\Lambda = 0°$

C_L	$\frac{t}{c} = 0.08$	$\frac{t}{c} = 0.1$	$\frac{t}{c} = 0.12$
0.2	0.806	0.784	0.762
0.3	0.785	0.764	0.744
0.4	0.763	0.744	0.727
0.5	0.742	0.724	0.722
0.6	0.720	0.703	0.692

Table 5 Divergence Mach number for Sweep angle $\Lambda = 15°$

C_L	$\frac{t}{c} = 0.08$	$\frac{t}{c} = 0.1$	$\frac{t}{c} = 0.12$
0.200	NaN	0.822	0.801
0.300	0.824	0.802	0.781
0.400	0.799	0.779	0.761
0.500	0.775	0.756	0.741
0.600	0.750	0.735	0.722

Table 6 Divergence Mach number for Sweep angle $\Lambda = 25°$

C_L	$\frac{t}{c} = 0.08$	$\frac{t}{c} = 0.1$	$\frac{t}{c} = 0.12$
0.200	NaN	0.868	0.844
0.300	0.863	0.839	0.818
0.400	0.835	0.815	0.791
0.500	0.779	0.790	0.775
0.600	0.781	0.764	0.750

Table 7 Divergence Mach number for Sweep angle $\Lambda = 35°$

C_L	$\frac{t}{c} = 0.08$	$\frac{t}{c} = 0.1$	$\frac{t}{c} = 0.12$
0.200	NaN	NaN	NaN
0.300	NaN	0.905	0.884
0.400	0.898	0.876	0.855
0.500	0.852	0.842	0.824
0.600	0.828	0.812	0.797

Appendix B - Sample Calculations

```
Command Window
>> Mdiv = aarlab3(25,0.08,0.5)

Mdiv =

    0.7790

>> Mdiv = aarlab3(27,0.09,0.55)

Mdiv =

    0.7905

>> Mdiv = aarlab3(30,0.1,0.6)

Mdiv =

    0.7880
```

Fig. 16 Model working proof

Appendix C - MATLAB Part 3

```
    function [Mdiv] = aarlab3(S, TC, CL)

% AARLAB3  Predicts the Divergence Mach number
% at a given sweep angle, thickness ratio and lift coefficient
%
% SYNTAX: Mdiv = AARLAB3(sweep, thickness ratio, lift coefficient)
%
% Range of inputs allowed:
%                                      Lift Coefficient: [0.2, 0.6]
%                                      Sweep angle:    [0, 35] degrees
%                                      Thickness ratio: [0.08, 0.12]

%% Organize dataset

sweep = [0 15 25 35];                            % sweep angle
cL = linspace(0.2,0.6,5);                         % lift coefficient
tc =[0.08 0.1 0.12];                                % thickness

for i = 1:length(sweep)
    % The spreadsheet used here is attached on the CANVAS submission
    % Please use that for the function to work
    Mach(:,:,i) = xlsread('data.xlsx',i);
end
Mach(Mach == 0.11) = NaN;
for i = 1:length(cL)
    Mach1(:, :, i) = Mach(i, :,:);
end
Mach = Mach1;  % for data in Mach, thickness is y-axis, lift is z-axis and sweep is x-axis


%% Find bounds for lift coefficient
if CL >=0.2 && CL < 0.3
    uppercL = 0.3; lowercL = 0.2;
        elseif CL >=0.3 && CL < 0.4
                uppercL = 0.4; lowercL = 0.3;
        elseif CL >=0.4 && CL < 0.5
                uppercL = 0.5; lowercL = 0.4;
        elseif CL >=0.5 && CL <= 0.6
                uppercL = 0.6; lowercL = 0.5;
        else
            fprintf("error")
            uppercL = NaN;
            lowercL = NaN;
end
idx(1) = find(cL == uppercL);
idx(2) = find(cL == lowercL);

%% Mesh griddata

x = linspace(0, 35, 5001);          % sweep angle grid
y = linspace(0.08, 0.12, 501); % thickness grid
[xq, yq] = meshgrid(x, y);
```

```
% Interpolated Matrix- The following contains solutions for respective cL
GridUpper = griddata(sweep, tc, Mach(:,:,idx(1)), xq, yq);
GridLower = griddata(sweep, tc, Mach(:,:,idx(2)), xq, yq);

% Scanning for correct location of requested Sweep and thickness
[c, indS] = min(abs(S-x)); [c, indT] = min(abs(TC-y));
% Mach divergence prediction for upper and lower cL
Md(1) = GridLower(indT, indS); Md(2) = GridUpper(indT, indS);

%% Final linear-fit for cL

slope = (Md(2)-Md(1))/(uppercL - lowercL);
intercept = Md(2) - slope*uppercL;

Mdiv = slope*CL + intercept;
```

Appendix C - MATLAB Part 4

```
    clear all;
clc

CL = 0.353;

%% Organize dataset

sweep = [0 15 25 35];                               % sweep angle
cL = linspace(0.2,0.6,5);                            % lift coefficient
tc =[0.08 0.1 0.12];                                   % thickness

for i = 1:length(sweep)
    % The spreadsheet used here is attached on the CANVAS submission
    % Please use that for the function to work
    Mach(:,:,i) = xlsread('data.xlsx',i);
end
Mach(Mach == 0.11) = NaN;
for i = 1:length(cL)
    Mach1(:, :, i) = Mach(i, :,:);
end
Mach = Mach1;  % for data in Mach, thickness is y-axis, lift is z-axis and sweep is x-axis


%% Find bounds for lift coefficient
if CL >=0.2 && CL < 0.3
    uppercL = 0.3; lowercL = 0.2;
        elseif CL >=0.3 && CL < 0.4
                uppercL = 0.4; lowercL = 0.3;
        elseif CL >=0.4 && CL < 0.5
                uppercL = 0.5; lowercL = 0.4;
        elseif CL >=0.5 && CL <= 0.6
                uppercL = 0.6; lowercL = 0.5;
        else
            fprintf("error")
            uppercL = NaN;
            lowercL = NaN;
```

```
end
idx(1) = find(cL == uppercL);
idx(2) = find(cL == lowercL);

%% Mesh griddata

x = linspace(0, 35, 501);          % sweep angle grid
y = linspace(0.08, 0.12, 501); % thickness grid
[xq, yq] = meshgrid(x, y);

% Interpolated Matrix- The following contains solutions for respective cL
GridUpper = griddata(sweep, tc, Mach(:,:,idx(1)), xq, yq);
GridLower = griddata(sweep, tc, Mach(:,:,idx(2)), xq, yq);

%% Finding all the Mdiv vectors at given Cl using linear  fit

slope = (GridUpper - GridLower)/(uppercL - lowercL);
intercept = GridUpper - slope*uppercL;

Mdiv = slope*CL + intercept;

%% Design Problem

log = Mdiv>0.86; % logical array
sweep = xq.*log; % required sweep angles
t = yq.*log; % required thickness


% Range of thickness
t(t == 0) = NaN;
tmin = min(min(t))
tmax = max(max(t))

% Range of Sweep angles
sweep(sweep == 0) = NaN;
smin = min(min(sweep(:,:)))
smax = max(max(sweep(:,:)))

%% Plot

surf(sweep,t,Mdiv.*log);hold on; grid on;
xlabel('Sweep angle')
ylabel('Thickness')
zlabel('Mach divergence')
h = colorbar;
set(h, 'ylim', [0.84 1])
```