Indian Institute of Technology Kanpur
Department of Mechanical Engineering
ME 341A, Heat and Mass Transfer
Assignment – 3

Note: Problem 1 is an assignment to be submitted. Please submit it as a small report (about 5-6 pages). The report is expected to include methodology, plots and program listing. You can use any programming language. Problem-9 has to be submitted too. Utilize Mid-semester recess.

1. Blasius equation for flow over a flat plate must be iterated to find the correct value of $f''(0)$ which causes $f'(\infty)$ to equal 1.0. Use Runge-Kutta method in combination with any root finding method to find (u/U_∞) as a function of η. The parameter $\eta = y/\sqrt{\nu x/U_\infty}$. Having solved the velocity boundary layer equation, solve the thermal boundary layer equation

$$\theta'' + \frac{Pr}{2} f\theta' - \lambda Pr f'\theta = 0$$

The plate is at a constant temperature T_w and the free stream temperature is T_∞. The non-dimensional temperature is given by $\theta = (T - T_\infty)/(T_w - T_\infty)$. Find θ versus η for different values of Pr. The available boundary condition are: at $\eta = 0, \theta = 1$ and at $\eta = \infty, \theta = 0$. The correct value of $\theta'\,(0)$ must be obtained to find the variation of θ with η. Write a computer code that is capable of solving the problem for any given value of Prandtl number of the flowing fluid. Plot θ versus η for Pr = 0.7, 0.8, 1.0 and 7.0 [12]

2. For the case of flow over a flat heated plate, the thermal boundary layer equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right)$$

with

u= $U_\infty f'(\eta)$, $v = -\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}[f(\eta) - \eta f'(\eta)]$, $\theta = (T - T_\infty)/(T_w - T_\infty)$, $\eta = y/\sqrt{\frac{\nu x}{U_\infty}}$

In this problem wall temperature is a function of x while the non-dimensional temperature field and the velocity field are the functions of similarity parameter η. On substitution of the velocity and temperatures in terms of f and θ in the governing equations, separate the groups that are functions of x and η. Show that the thermal boundary layer equation can be written as

$$\theta'' + \frac{Pr}{2} f\theta' - \lambda Pr f'\theta = 0$$

3. After applying the separation of variables technique specified in Problem 2 consider the part that is function of x. We get $(T_w - T_\infty) = Cx^\lambda$. Start with the definition of heat flux at the wall and determine the value of λ that makes the condition of constant wall heat flux on the wall. Use any relation that you have used in Problem 2.
4. The thermal boundary layer equation for constant wall temperature, using similarity variables, can be expressed as

$$\theta'' + \frac{Pr}{2} f\theta' = 0$$

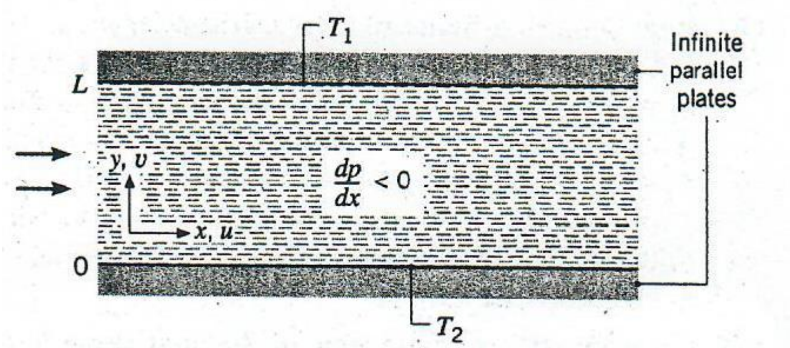
The boundary conditions are $\theta(0) = 1$ and $\theta(\infty) = 0$. Find the solution as $\theta(\eta)$ using appropriate analytical technique.

5. Consider the thermal boundary layer equation for flow over a flat plate as

$$\rho c_p(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = k(\frac{\partial^2 T}{\partial y^2})$$

The plate temperature is T_w (uniform) and the incoming fluid temperature is T_∞. Find the expression for energy integral equation (thermal). Explain how the partial derivative of x is written as the ordinary derivative in the final form of energy integral equation.

6. Consider the problem of steady, incompressible laminar flow between two stationary, infinite parallel plates maintained at different temperatures.



Referred to as Poiseuille flow with heat transfer, this special case of parallel flow is one for which the x velocity component is finite, but the y and z components (v and w) are zero.
(a) What is the form of the continuity equation for this case? In what way is the flow fully developed?
(b) What forms do the x and y-momentum equations take? What is the form of the velocity profile? Note that, unlike Couette flow, fluid motion between the plates is now sustained by a finite pressure gradient. How is this pressure gradient related to the maximum fluid velocity?
(c) Assuming viscous dissipation to be neglected and recognizing that conditions must be thermally fully developed, what is the appropriate form of the energy equation? Solve this equation for the temperature distribution. The frame of reference may be taken at the middle of the channel. You may like to assume $T_1 = T_2 = T_w$

7. In a hydrodynamically developed and thermally developed flow through a tube of radius r_o, constant wall heat flux boundary condition is applied. The governing equation is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})$$

It is known that $\frac{dT_m}{dx} = \frac{q_w''}{\dot{m}C_p}P$ where T_m is the bulk mean temperature, q_w'' is the wall heat flux. P is the perimeter of the duct, $\dot{m}$ is the mass flow rate and C_p is the specific heat at constant pressure. It is also known that at $r = 0$, T is finite and at $r = r_O$, $T = T_w$. Show that the Nusselt number based on the duct diameter at any x is 4.36
Furthermore, show that the temperature distribution is given by

$$\mathrm{T}(r) = T_m + \frac{q_w r_O}{k}\left[\left(\frac{r}{r_O}\right)^2 - \frac{1}{4}\left(\frac{r}{r_O}\right)^4 - \frac{7}{24}\right]$$

8. Slug flow is an idealized pipe flow for which velocity profile does not change in the flow direction and it is uniform over the entire pipe cross-section. For the case of a laminar slug flow through a pipe with the uniform heat flux boundary condition, determine the temperature profile $T(r)$ of the fluid in a pipe with radius R when the flow is thermally fully developed. Also find the Nusselt number based on the pipe diameter Nu_D.
9. Consider a cylindrical rod (heating element) of length L and diameter D that is enclosed with a concentric tube. Water flows through the annular region between the rod and the tube at a rate $\dot{m}$. The outer surface of the tube is insulated. Heat generation occurs within the rod, and the volumetric generation rate is known to vary with the distance along the element. The variation is given by

$$q^*(x) = q_0^*\left(x / L\right)^2$$

Where q_0^* (W / m^3) is a constant

A convection coefficient h exists between the surface of the rod and the water.

(a) Obtain an expression for the local heat flux, $q''(x)$ and the total heat transfer, Q from the heating element to the water.
(b) Obtain an expression for the variation of bulk mean temperature, $T_m(x)$ of the water with distance x along the tube.
(c) Obtain an expression for the variation of the surface temperature of the heating element, $T_w(x)$ with distance x along the tube.

[3]