ME354: ASSIGNMENT 4

GROUP 7

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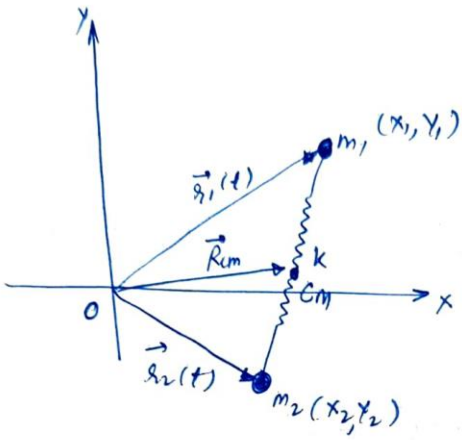
ASSIGNMENT #4
GROUP #07

(a).



let $M = m_1 + m_2$.

$$\vec{r}_1(t) = \vec{R}_{cm}(t) - \frac{m_2}{M}\vec{r}(t)$$

$$\vec{r}_2(t) = \vec{R}_{cm}(t) - \frac{m_1}{M}\vec{r}(t).$$

where

$$\vec{r}(t) = \vec{r}_2(t) - \vec{r}_1(t)$$

$$|\vec{r}(t)| = l_0 + \delta$$

δ → extension in the spring.

Total kinetic energy of the system:

$$T = \sum_{\alpha=1}^{N} \frac{m_\alpha}{2} v_\alpha^2 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$$

Also $V = \frac{1}{2} k \delta^2 = \frac{1}{2} k \left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} - l_0\right)^2.$

The system has four degrees of freedom (x_1, y_1, x_2, y_2).

The Lagrangian for this system can be expressed as,

$$\mathcal{L} = T - V$$

$$\mathcal{L} = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) - \frac{1}{2} k \left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} - l_0\right)^2.$$

$$\delta \mathcal{L} = 0 \quad \text{[variation of Functional]}.$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = 0$$

For x_1:

$$\frac{d}{dt}\left(\frac{1}{2}\cdot m_1 \cdot 2\cdot \dot{x}_1\right) + \frac{1}{2}k\cdot 2\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} - l_0\right)\cdot \frac{2(x_1-x_2)}{2\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = 0$$

$$\Rightarrow m_1\ddot{x}_1 + k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} - l_0\right)\frac{(x_1-x_2)}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = 0 \quad \text{— (1)}$$

we know that, $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} - l_0 = \delta$

Similarly, for y_1:

$$m_1\ddot{y}_1 + k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} - l_0\right)\cdot \frac{(y_1-y_2)}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = 0 \quad \text{— (2)}$$

Since no external force is being applied on the system,

$$\therefore \boxed{\bar{a}_{c.o.m} = 0}$$

$$\Rightarrow m_1\ddot{x}_1 + m_2\ddot{x}_2 = 0 \quad \text{and} \quad m_1\ddot{y}_1 + m_2\ddot{y}_2 = 0$$

$$\Rightarrow \boxed{\ddot{x}_2 = -\frac{m_1}{m_2}\ddot{x}_1} \quad \text{— (3)}$$

and $\boxed{\ddot{y}_2 = -\frac{m_1}{m_2}\ddot{y}_1}$ — (4)

Therefore, the governing equations of motion are:

$$m\ddot{x}_1 + k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}-l_0\right)\frac{\cdot(x_1-x_2)}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = 0 \quad \text{--- (1)}$$

$$m_1\ddot{y}_1 + k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}-l_0\right)\frac{\cdot(y_1-y_2)}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = 0 \quad \text{--- (2)}$$

$$\ddot{x}_2 = -\frac{m_1}{m_2}\ddot{x}_1 \quad \text{--- (3)}$$

$$\ddot{y}_2 = -\frac{m_1}{m_2}\cdot\ddot{y}_1 \quad \text{--- (4)}$$

Yes, the system of equations is NON-LINEAR.

From eqn (1), ∴

$$\underbrace{m\cdot\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\cdot\ddot{x}_1}_{\text{gives rise to NON-LINEARITY.}} + k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}-l_0\right)\cdot(x_1-x_2)=0$$

(b). We have four second-order ODEs.

Therefore $4\times 2 = \boxed{8}$ initial conditions would be required to solve the system of equations.

The numerical solution to the system is possible using ode45 solver.

(c)

Here is the required MATLAB program to solve the system numerically using *ode45* solver, and for simulating the time-domain motion of the particles:

NOTE: We have reduced the value of beta to 1/500, to view the system response for a long time for the defined limit of x-y space.

```
%%% ME354 - ASSIGNMENT 4
%%% GROUP 7
clc;
clear all;
close all;

m1 = 1;
m2 = 1;
k = 1;
l0 = 1;
alpha = 0.1;
beta = 1/500;

%%% INITIAL CONDITIONS
ch = 'case4';
switch ch
    case 'case1'
        % CASE 1
        x1_0 = alpha;
        x2_0 = alpha + l0 + .25*l0;
        y1_0 = 0;
        y2_0 = 0;

        u1_0 = 1/500;
        v1_0 = 1/500;
        u2_0 = 1/500;
        v2_0 = 1/500;
    case 'case2'
        %CASE 2
        x1_0 = alpha;
        x2_0 = alpha + l0;
        y1_0 = 0;
        y2_0 = 0;

        u1_0 = beta;
        v1_0 = 1/500;
        u2_0 = beta;
        v2_0 = 1/500;
    case 'case3'
        %CASE 3
        x1_0 = alpha;
        x2_0 = alpha + l0 + .25*l0;
        y1_0 = 0;
        y2_0 = 0;

        u1_0 = beta;
        v1_0 = 1/500;
        u2_0 = beta;
        v2_0 = 1/500;
    case 'case4'
        %CASE 4;
        x1_0 = alpha;
```

```
        x2_0 = alpha + l0 + .25*l0;
        y1_0 = 0;
        y2_0 = 0;

        u1_0 = 0;
        v1_0 = -beta;
        u2_0 = 0;
        v2_0 = beta;
    case 'case5'
        % CASE 5
        x1_0 = alpha;
        x2_0 = alpha + l0;
        y1_0 = 0;
        y2_0 = 0;

        u1_0 = 0;
        v1_0 = -beta;
        u2_0 = 0;
        v2_0 = beta;
end
tspan = [0,10000];
X0 = [x1_0 y1_0 x2_0 y2_0 u1_0 v1_0 u2_0 v2_0]; %%INITIAL CONDITIONS
[T, X_sol] = ode45(@(t,x)odefun(m1,m2,k,l0, x, t), tspan, X0);


%%% SIMULATION

x1 = X_sol(:,1);
y1 = X_sol(:,2);
x2 = X_sol(:,3);
y2 =  X_sol(:, 4);
%%% we know that Mass = density*volume
%%% therefore, radius = constant*mass^(1/3)
constant = 0.1;
Rad_1 = constant*m1^(1/3);
Rad_2 = constant*m2^(1/3);
theta = linspace(0, 2*pi, 100);
figure(1);
grid on;
grid minor;
daspect([1 1 1]);
xlabel('$\textit{\textbf{x-axis}}$','Interpreter', 'latex', 'FontSize',11)
ylabel('$\textit{\textbf{y-axis}}$','Interpreter', 'latex', 'FontSize',11)
xlim([-5,5]);
ylim([-5,5]);
hold on;
title('$\textbf{ME354 - GROUP \#07}$','Interpreter', 'latex','FontSize',18);
mass_1 = fill(Rad_1*cos(theta)+x1(1), Rad_1*sin(theta)+y1(1), 'R',
DisplayName='Mass m1');
mass_2 = fill(Rad_2*cos(theta)+x2(1), Rad_2*sin(theta)+y2(1), 'B',
DisplayName='Mass m2');
legend;
% v = VideoWriter('Group7.avi');
% open(v);
pause(0.1);
for i = 1:length(T)
%     hold on
    hg = hggroup;
    mass_1.XData = Rad_1*cos(theta) +x1(i);
```

```
    mass_1.YData = Rad_1*sin(theta) + y1(i);
    mass_2.XData = Rad_2*cos(theta) + x2(i);
    mass_2.YData = Rad_2*sin(theta) + y2(i);
    xa = x1(i); ya = y1(i); xb = x2(i); yb = y2(i); ne = 10; a = 1; ro = 0.1;
    [xs_,ys_] = spring(xa,ya,xb,yb,ne,a,ro); plot(xs_,ys_,'LineWidth',0.2,
'Parent', hg, 'Color', 'black');
%     hold off
%     pause(5);
    drawnow limitrate;
%     frame = getframe(gcf);
%     writeVideo(v, frame);
    if i<length(T)
        delete(hg);
    end
end
% close(v);

function [xs ys] = spring(xa,ya,xb,yb,varargin)
persistent ne Li_2 ei b

if nargin > 4 % calculating some fixed spring parameters only once time
    [ne a r0] = varargin{1:3};                  % ne: number of coils - a =
natural length - r0 = natural radius
    Li_2 =  (a/(4*ne))^2 + r0^2;                % (large of a quarter of coil)^2
    ei = 0:(2*ne+1);                            % vector of longitudinal positions
    j = 0:2*ne-1; b = [0 (-ones(1,2*ne)).^j 0]; % vector of transversal positions
end
R = [xb yb] - [xa ya]; mod_R = norm(R); % relative position between "end_B" and
"end_A"
L_2 = (mod_R/(4*ne))^2; % (actual longitudinal extensión of a coil )^2
if L_2 > Li_2
   error('Spring:TooEnlargement', ...
   'Initial conditions cause pulling the spring beyond its maximum large. \n Try
reducing these conditions.')
else
    r = sqrt(Li_2 - L_2);   %actual radius
end
c = r*b;    % vector of transversal positions
u1 = R/mod_R; u2 = [-u1(2) u1(1)]; % unitary longitudinal and transversal vectors
xs = xa + u1(1)*(mod_R/(2*ne+1)).*ei + u2(1)*c; % horizontal coordinates
ys = ya + u1(2)*(mod_R/(2*ne+1)).*ei + u2(2)*c; % vertical coordinates
end

function [x_dot] = odefun(m1, m2, k, l0, X, t)
    x_dot = zeros(8,1);
    x_dot(1,1) = X(5);
    x_dot(2,1) = X(6);
    x_dot(3,1) = X(7);
    x_dot(4,1) = X(8);
    x_dot(5,1) = (1/m1)*k*(sqrt((X(1) - X(3))^2 + (X(2)-X(4))^2) - l0)*(X(3) -
X(1))/(sqrt((X(1) - X(3))^2 + (X(2)-X(4))^2));
    x_dot(6,1) = x_dot(5)*(X(4)-X(2))/(X(3) - X(1));
    x_dot(7,1) = -(m1/m2)*x_dot(5,1);
    x_dot(8,1) = -(m1/m2)*x_dot(6,1);
end

%%% THANK YOU!
```

Comments on the nature of each type of observed motion.

CASE 1:

$$(1)\quad \begin{aligned} &x_1(0)=\alpha,\\ &x_2(0)=\alpha+l_0+0.25l_0,\\ &y_1(0)=y_2(0)=0,\\ &u_1(0)=u_2(0)=v_1(0)=v_2(0). \end{aligned}$$

The motion is the combination of the oscillatory motion of the particles about their C.O.M and the motion of C.O.M, without any rotary motion.

CASE 2:

$$(2)\quad \begin{aligned} &x_1(0)=\alpha,\\ &x_2(0)=\alpha+l_0,\\ &y_1(0)=y_2(0)=0,\\ &u_1(0)=\beta,\ u_2(0)=\beta,\ v_1(0)=v_2(0). \end{aligned}$$

The motion of the system involves the motion of C.O.M only, without any oscillation or rotary motion.

CASE 3:

$$(3)\quad \begin{aligned} &x_1(0)=\alpha,\\ &x_2(0)=\alpha+l_0+0.25l_0,\\ &y_1(0)=y_2(0)=0,\\ &u_1(0)=\beta,\ u_2(0)=\beta,\ v_1(0)=v_2(0). \end{aligned}$$

The motion is the combination of the oscillatory motion of the particles about their C.O.M and the motion of C.O.M, without any rotary motion. The oscillations in this case are more violent than Case1.

CASE 4:

$$(4)\quad \begin{aligned} &x_1(0)=\alpha,\\ &x_2(0)=\alpha+l_0+0.25l_0,\\ &y_1(0)=y_2(0)=0,\\ &u_1(0)=0,\ u_2(0)=0,\ v_1(0)=-\beta,\ v_2(0)=\beta. \end{aligned}$$

The motion is the combination of the oscillatory motion of the particles about their C.O.M, and the rotational motion of the system, and the translatory motion of C.O.M.

CASE 5:

$$(5)\quad \begin{aligned} &x_1(0) = \alpha, \\ &x_2(0) = \alpha + l_0, \\ &y_1(0) = y_2(0) = 0, \\ &u_1(0) = 0,\ u_2(0) = 0,\ v_1(0) = -\beta,\ v_2(0) = \beta. \end{aligned}$$

The system executes only rotational motion about the C.O.M with no oscillations or the translatory motion of the C.O.M.