ASSIGNMENT 4

This assignment involves writing a MATLAB program to simulate the motion of two-particle semidefinite system connected by a linear spring located in a 2-D free-space.

This is a group assignment – each of you will be allocated to a group comprising of 10 students, and assessment will be carried out through live demonstrations.

The system shown in Figure 1 depicts point masses m_1 and m_2 connected by a spring of constant k and natural length l_0. At time $t = 0$, the initial co-ordinates of mass m_1 are given by x_1, y_1 while those of mass m_2 are given by x_2, y_2 . Note the origin of the co-ordinate system as shown in Fig. 1.

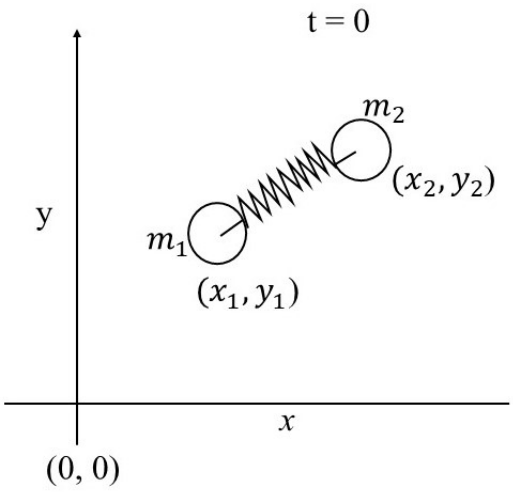

Figure 1

Do the following:

(a) In the Cartesian frame of reference shown in Fig.1, write down the system of equations that govern the motion of these particles. Is the system of equations non-linear?
(b) Convert this system of equation into a set of **coupled** first-order ODEs. How many initial conditions do you require to solve the system? Can you solve it analytically?
(c) Invoke the function ode45 solver in MATLAB to simulate the time-domain motion of these particles subject to the following set of initial conditions:

$$
\begin{aligned}
&x_1(0) = \alpha, \\
&x_2(0) = \alpha + l_0 + 0.25l_0, \\
&y_1(0) = y_2(0) = 0, \\
&u_1(0) = u_2(0) = v_1(0) = v_2(0).
\end{aligned}
\qquad (1)
$$

$$
(2)\quad
\begin{aligned}
&x_1(0)=\alpha,\\
&x_2(0)=\alpha+l_0,\\
&y_1(0)=y_2(0)=0,\\
&u_1(0)=\beta,\ u_2(0)=\beta,\ v_1(0)=v_2(0).
\end{aligned}
$$

$$
(3)\quad
\begin{aligned}
&x_1(0)=\alpha,\\
&x_2(0)=\alpha+l_0+0.25l_0,\\
&y_1(0)=y_2(0)=0,\\
&u_1(0)=\beta,\ u_2(0)=\beta,\ v_1(0)=v_2(0).
\end{aligned}
$$

$$
(4)\quad
\begin{aligned}
&x_1(0)=\alpha,\\
&x_2(0)=\alpha+l_0+0.25l_0,\\
&y_1(0)=y_2(0)=0,\\
&u_1(0)=0,\ u_2(0)=0,\ v_1(0)=-\beta,\ v_2(0)=\beta.
\end{aligned}
$$

$$
(5)\quad
\begin{aligned}
&x_1(0)=\alpha,\\
&x_2(0)=\alpha+l_0,\\
&y_1(0)=y_2(0)=0,\\
&u_1(0)=0,\ u_2(0)=0,\ v_1(0)=-\beta,\ v_2(0)=\beta.
\end{aligned}
$$

Take $\alpha=0.1,\ \beta=1, l_0=1$ and carry out the time-domain simulations. Comment on the nature of each type of motion that you observe.

Part-B <u>(Bonus problem)</u>

Now consider a system shown in Figure 2 below. It is similar to the system shown in Figure 1 except that now there are two particles which are confined in a rigid box and undergo elastic collisions. Assume suitable dimensions of the box, and certain initial conditions for each of the two-particle systems, simulate the motion.

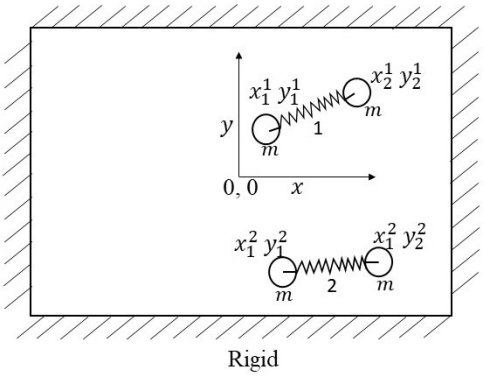

Figure 2

You have until the 22nd April 2022 to demonstrate the results.