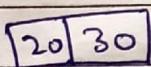


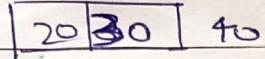
2-3 Tree Construction

Keys $\rightarrow 20, 30, 40, 50, 60, 10, 15, 70, 80, 90$

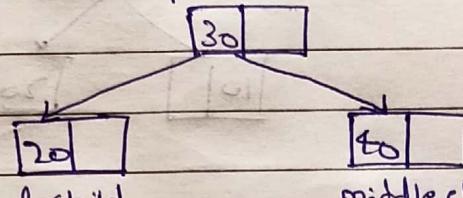
1) Insert 20, 30



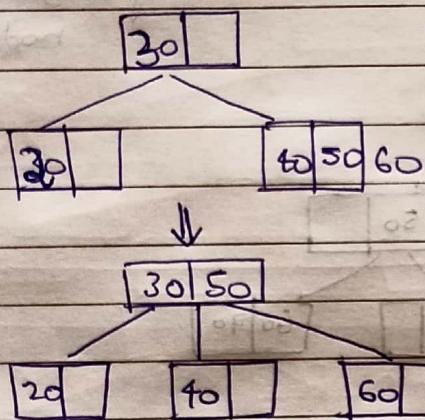
2) Insert 40



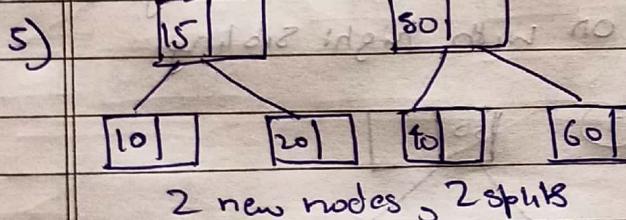
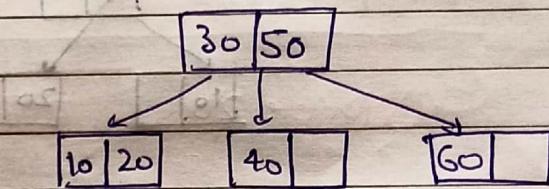
Split node



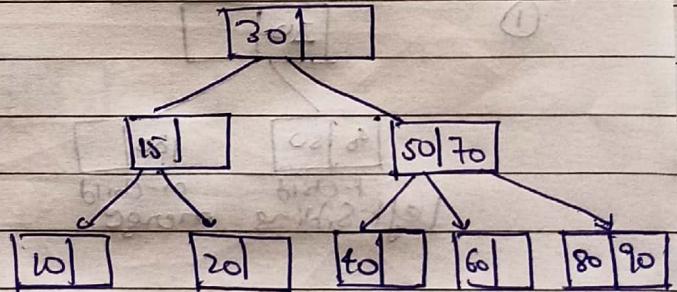
3) Insert 50, 60



4) Insert 10



5) Insert 70, 80, 90



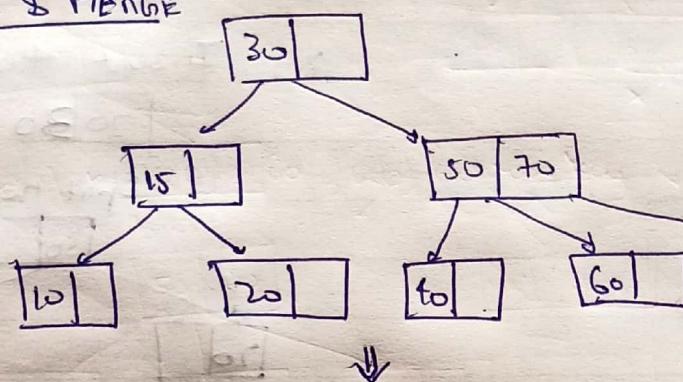
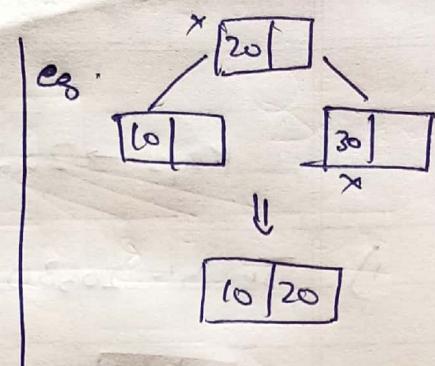
i) Case I → Delete 90 [if key == leaf node]

[leaf node → directly delete]

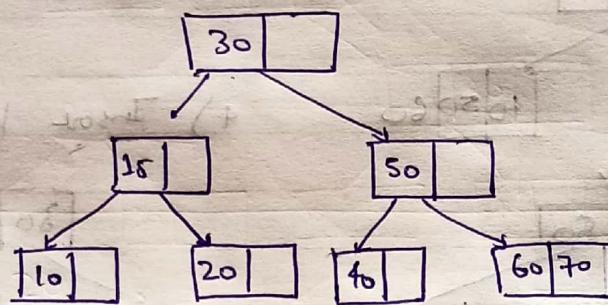
SIMPLE DELETE

ii) Case II → Delete 80

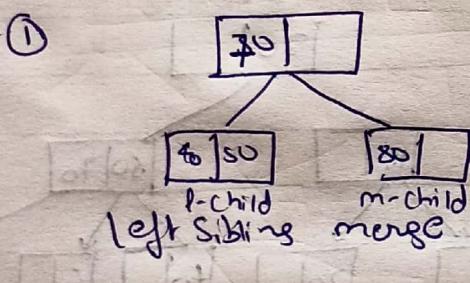
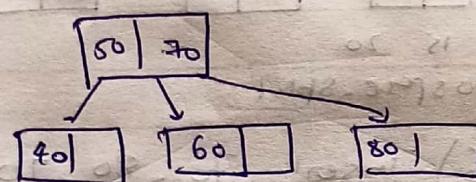
DELETE & MERGE



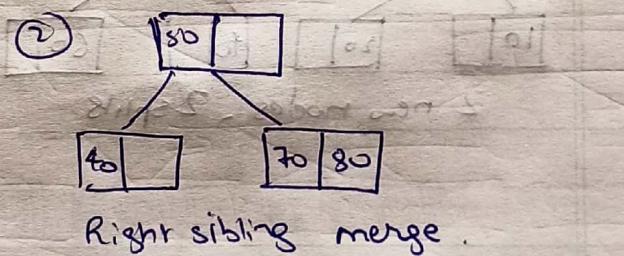
If 80 gone, vacant node.



If we want to delete 60.
We either merge with left sibling
or with right sibling.

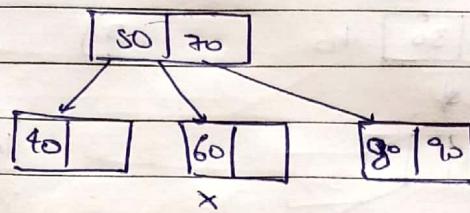


left sibling merge.



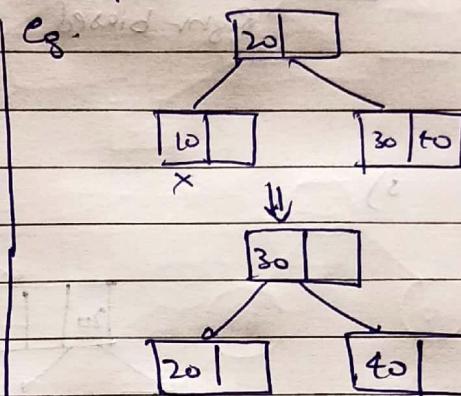
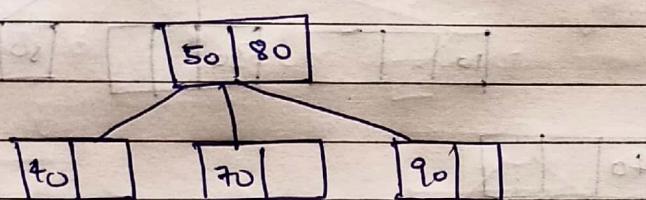
Right sibling merge.

Case 3 : Borrow

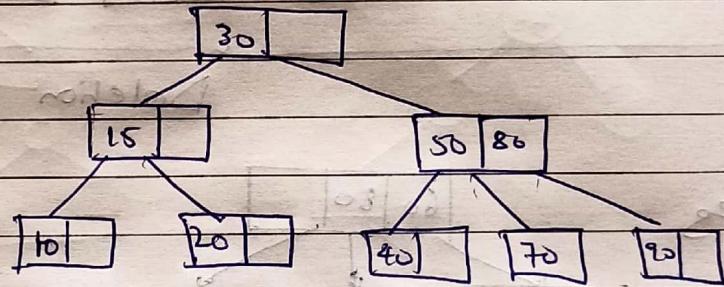


Delete 60

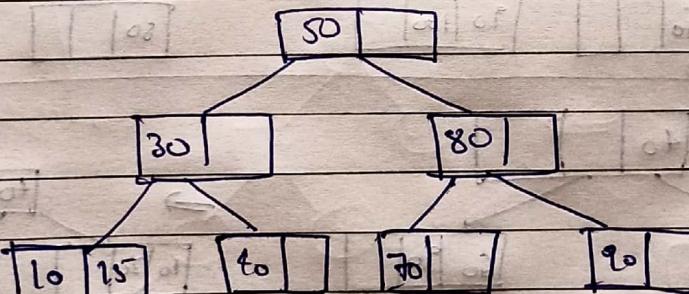
→ Now if we delete 60, we borrow key via parent



Q) Delete 20

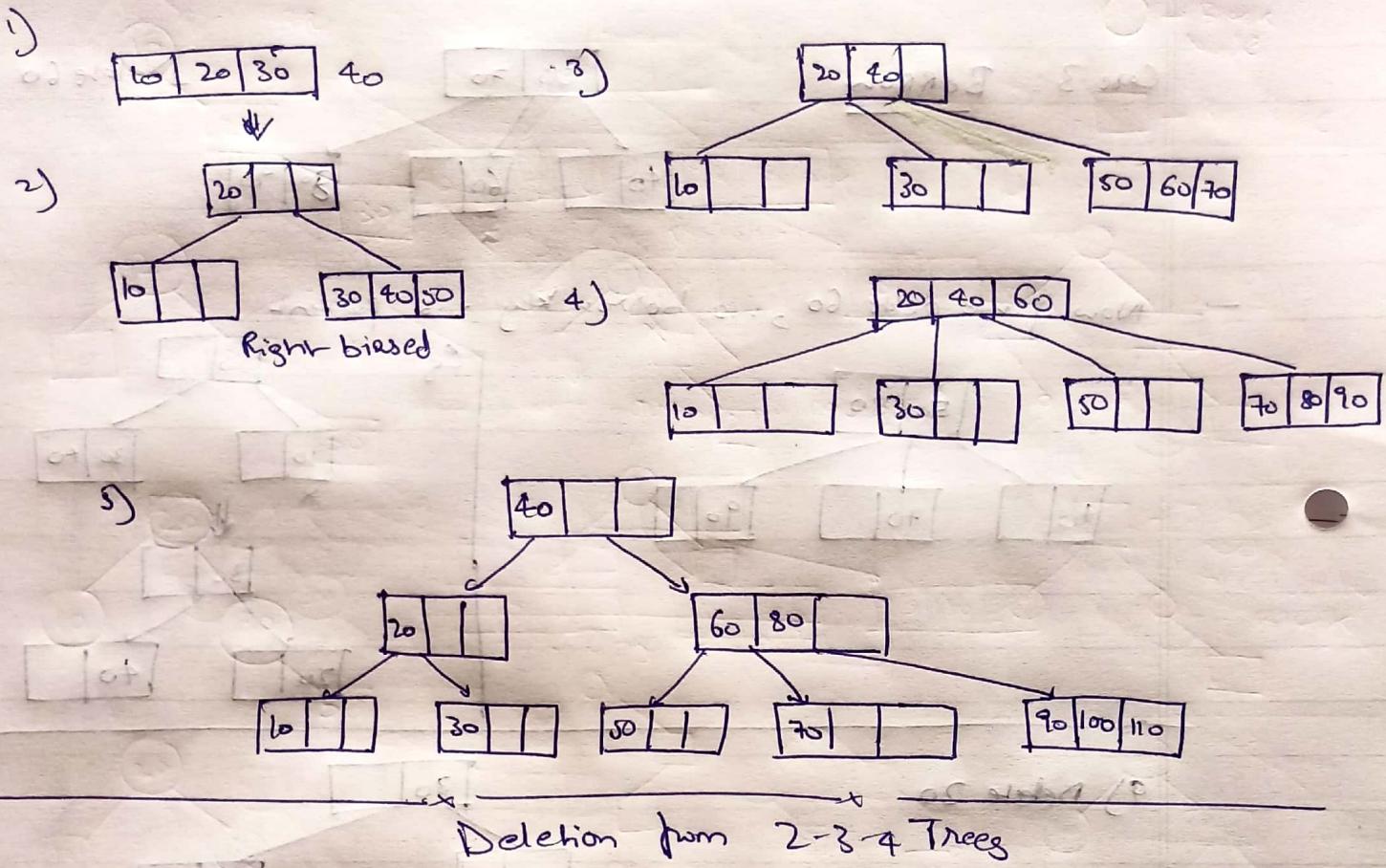


Answer:

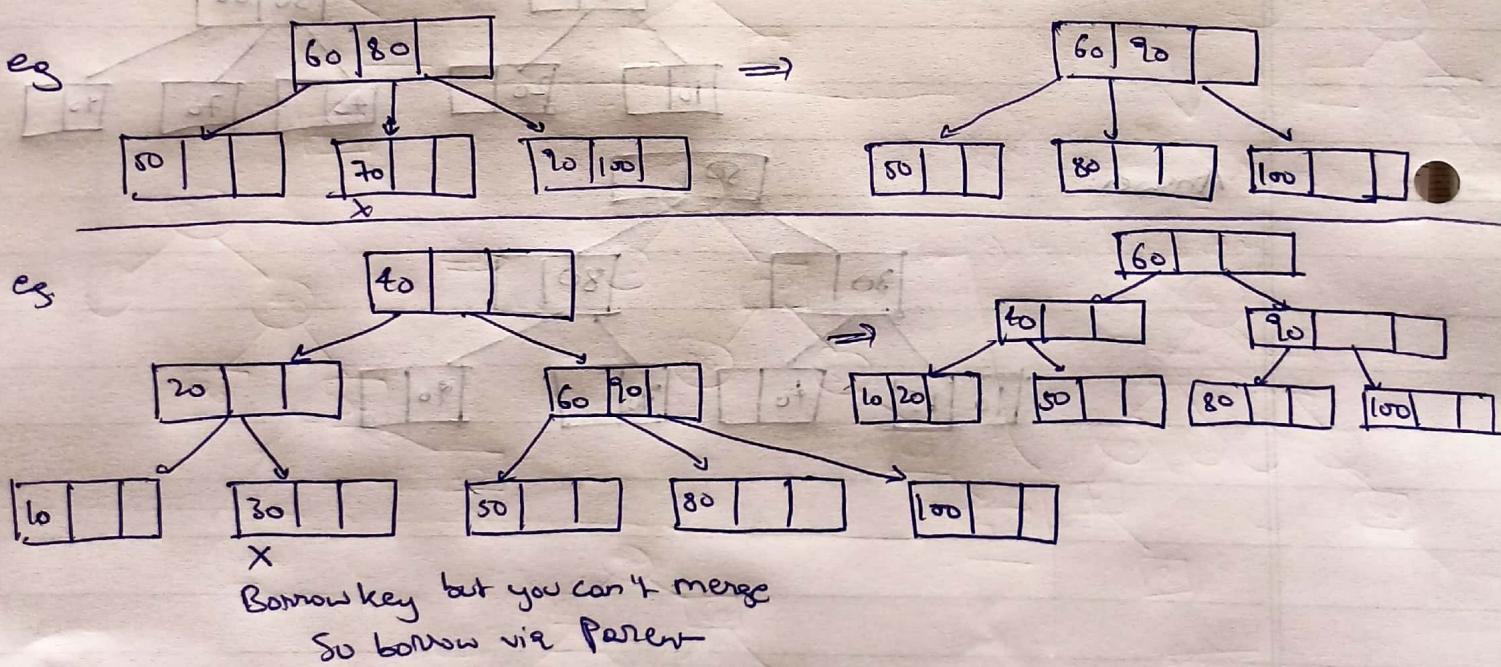


2-3-4 Trees

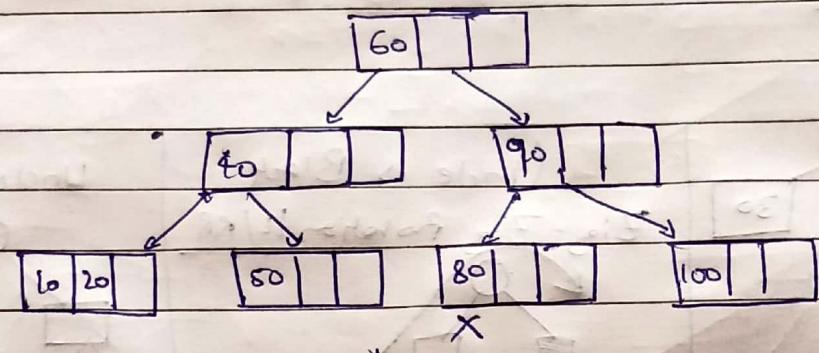
Keys → 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110



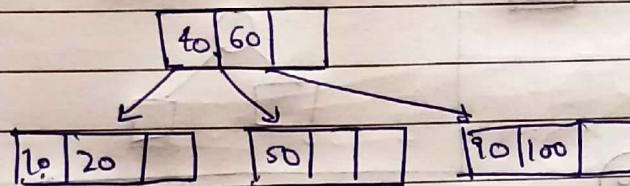
Deletion from 2-3-4 Trees



eg-



i) Can't borrow, so merge.



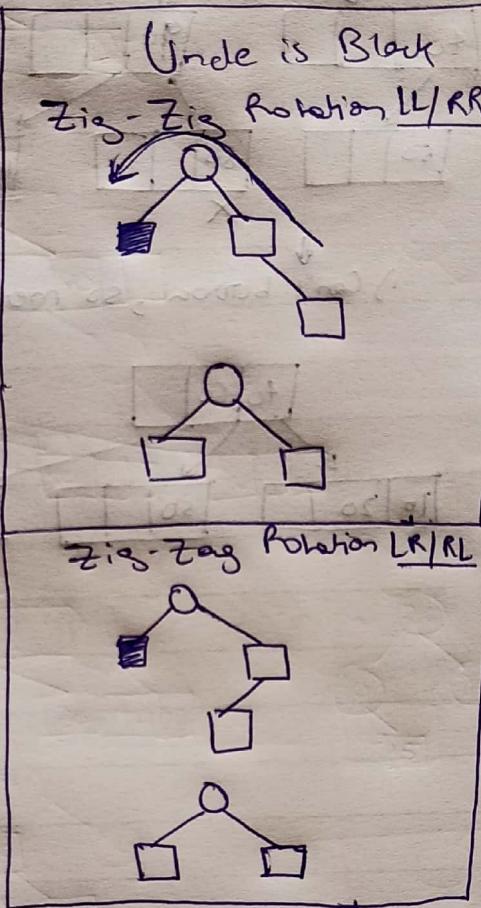
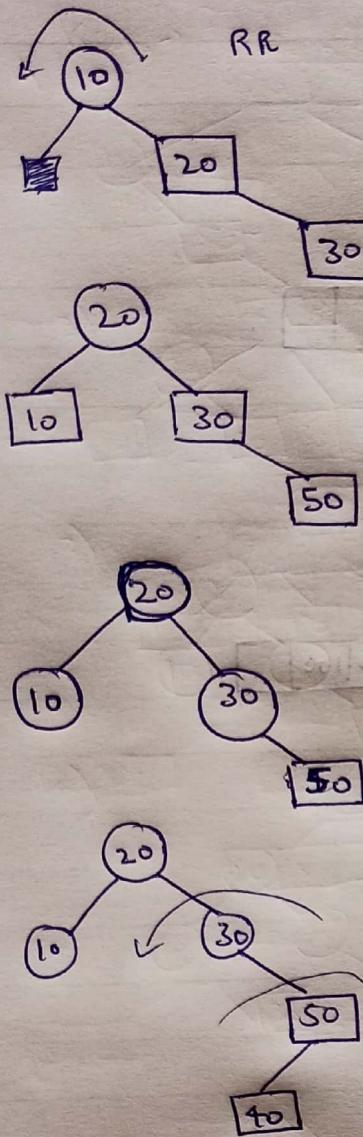
RED-BLACK TREE

keys - 10, 20, 30, 50, 40, 60, 70, 80, 45, 8

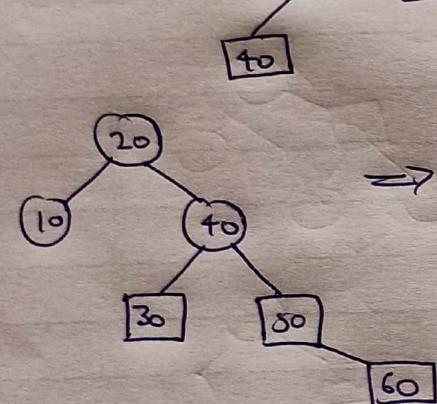
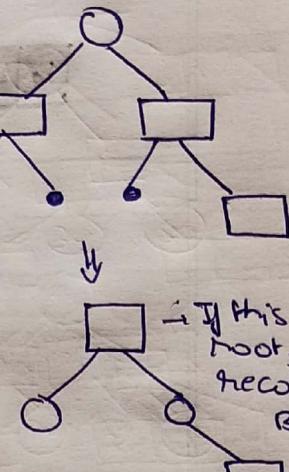
■ → NULL

○ → Black Node

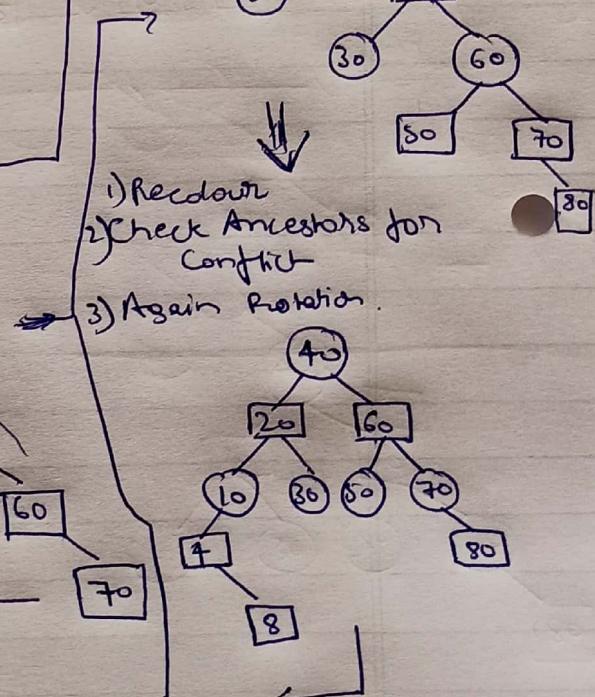
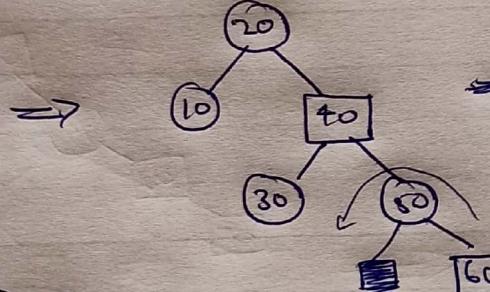
□ → Red Node



Uncle is Red

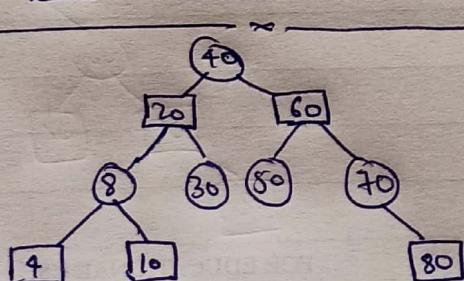


⇒



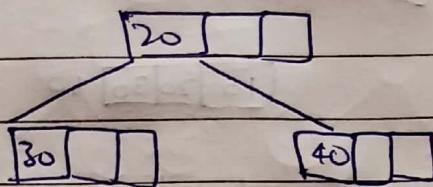
Zig-Zag Rotation

Final Tree:



RED-BLACK vs 2-3-4 TREE

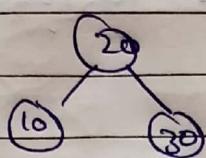
2-3-4



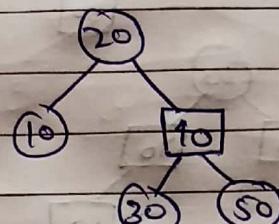
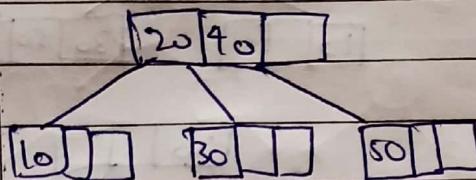
$\circ \rightarrow$ Black

$\square \rightarrow$ Red

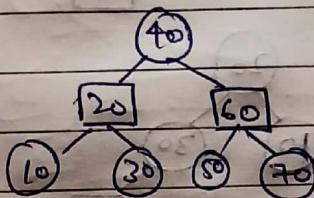
RED-BLACK



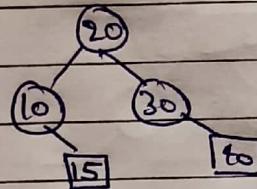
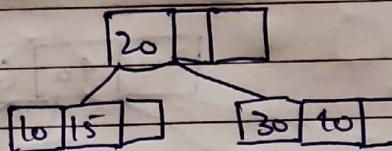
2)



3)



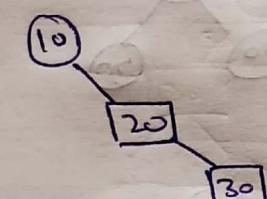
4)



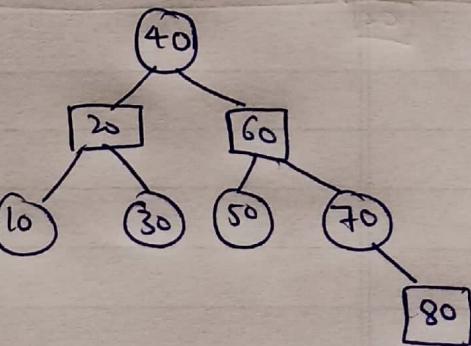
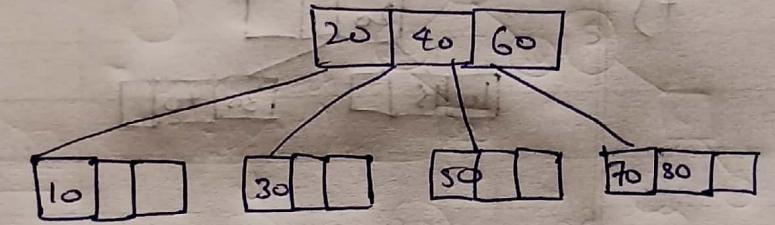
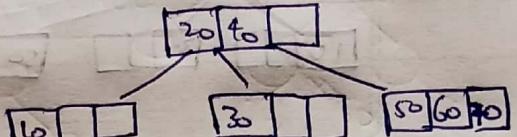
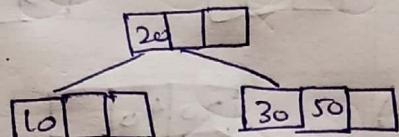
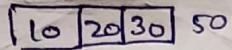
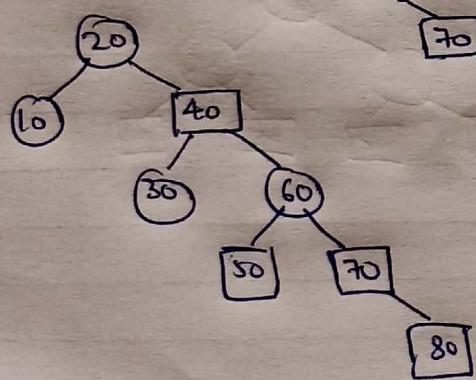
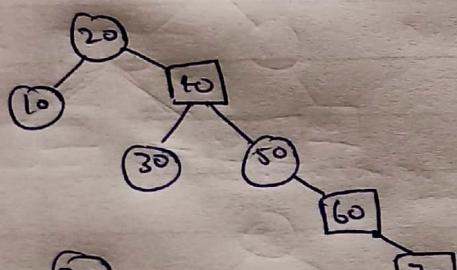
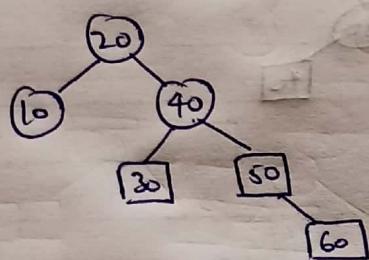
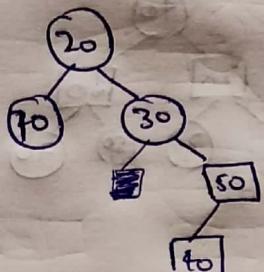
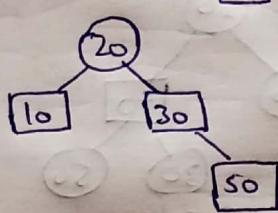
RED-BLACK
TREE

2-3-4
TREE

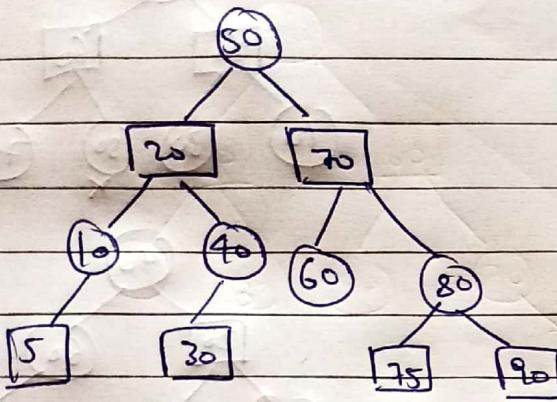
Keys → 10, 20, 30, 50, 40, 60, 70, 80



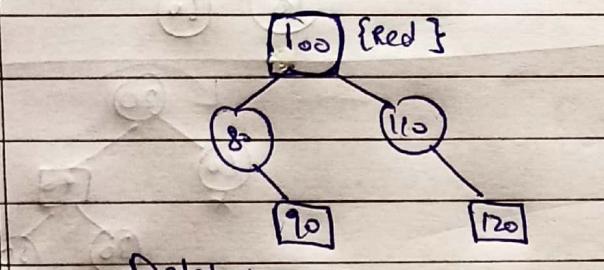
○ → Black
□ → Red



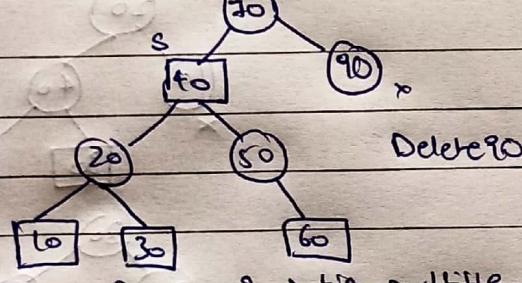
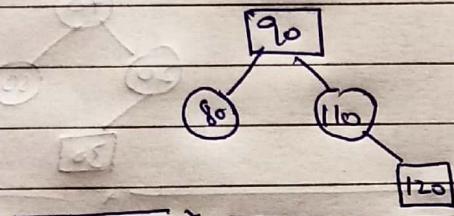
Deletion in Red-Black Tree



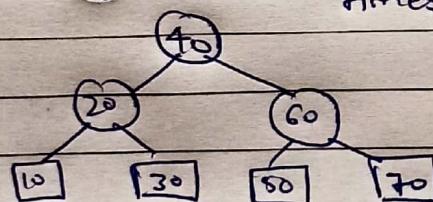
e.g.



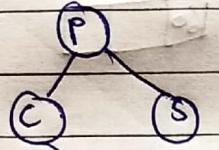
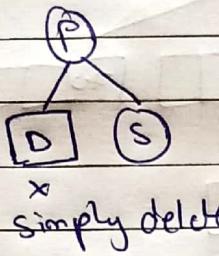
Delete 100



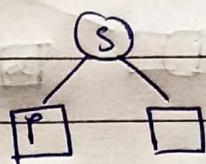
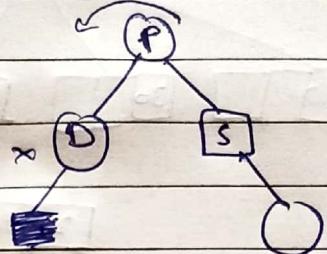
Perform rotation multiple times.



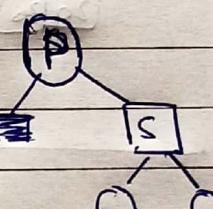
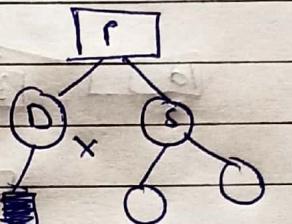
Case 1:



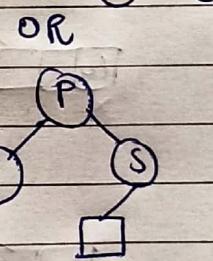
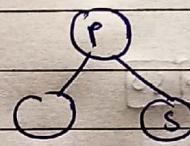
Case 2:



Case 3:



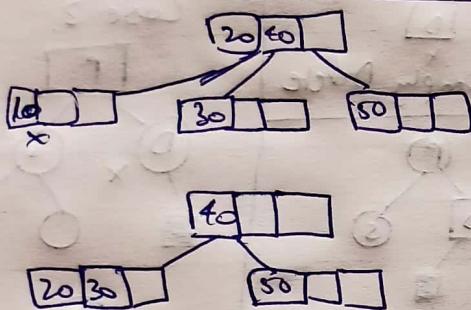
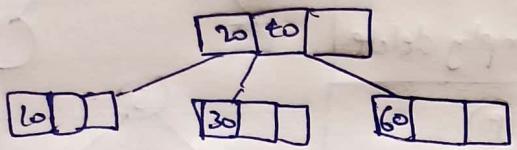
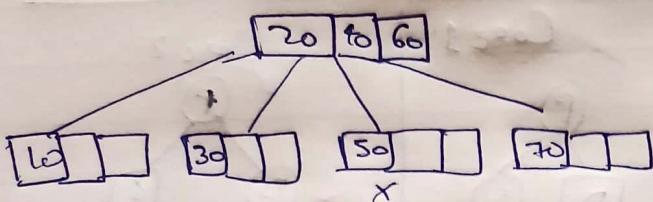
Then the black node (child) will take its place
↓



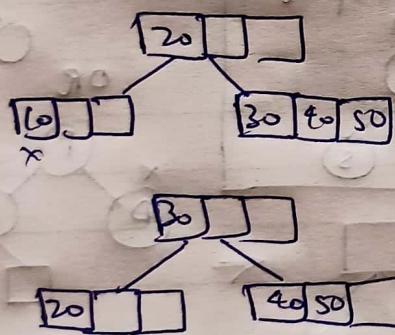
OR



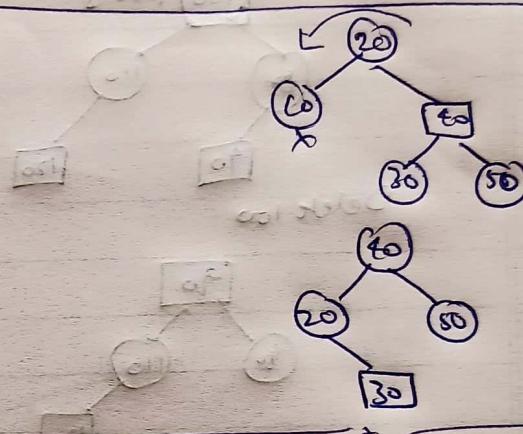
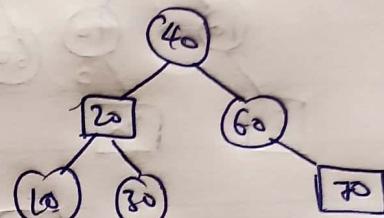
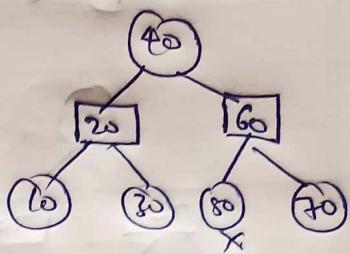
Red - Black Tree Deletion



red-left child
(black) child



2-3-4 Trees



Zig-Zag

