CS 405/6001: Game Theory and Algorithmic Mechanism Design

Problem Set 1
Course Homepage

[Week 1]

1. "The outcome of every play of the game of chess is either a victory for White, a victory for Black, or a draw." Is this statement equivalent to the result of von Neumann? Justify your answer.
   Note: the result of von Neumann states that,
   "In chess, one and only one of the following statements is true
   - W has a winning strategy
   - B has a winning strategy
   - Each player has a draw guaranteeing strategy"

2. Consider a simplified version of chess, where the game starts with only two kings on the board: a white king (W) and a black king (B). The rules of the game are as follows:
   - Both kings can move one square in any direction (standard chess king moves).
   - If one king can move to the square occupied by the other king, the game ends, and the player whose king made the move wins.
   - If neither player can move their king without being captured in the next move, the game ends in a draw.

   Now in this game, answer the following:
   (a) Can White guarantee a draw or a win in this game regardless of Black's moves? If so, what is the strategy?
   (b) Can Black guarantee a draw or a win in this game regardless of White's moves? If so, what is the strategy?

3. In a game of chess, two players (White and Black) are rational and intelligent. White always moves first. Let's assume the game is in a specific situation where both players know that a sequence of exactly 4 moves will lead to checkmate by White. However, this sequence is not obvious, and Black is still trying to defend.

   White can either:
   - Make a safe move that preserves the game state but delays checkmate, or
   - Attempt the checkmate sequence, which requires White to predict all of Black's responses perfectly.

   Black can either:
   - Try to prevent checkmate by playing the best defensive moves, or

- Resign, acknowledging that checkmate is inevitable.

Based on this game situation, answer the following:

(a) If both players are rational and intelligent, what should White do?

(b) What should Black do?

(c) What is the likely outcome of the game?

4. Five individuals (A, B, C, D, and E) are deciding whether to adopt a new policy. The policy will be adopted if and only if all five individuals vote in favor. Each person knows their own preference but does not know the preferences of others. They all know, however, that at least three people want the policy to be adopted. This information is common knowledge.

   Each individual gains 3 points if the policy is adopted and they voted for it. They gain 0 points if the policy is adopted and they voted against it. If the policy is not adopted, they gain 1 point regardless of how they voted.

   For this voting, answer the following:

   (a) What will each rational and intelligent individual do?

   (b) Will the policy be adopted?

5. Five pirates of varying ages have a treasure of 100 gold coins. On their ship, they decide to divide the coins according to this method:

   The oldest pirate suggests a way to distribute the coins, and all pirates (including the oldest) cast a vote either in favor or against the proposal. If 50% or more of the pirates vote in favor, the coins are divided as proposed. If not, the proposing pirate is thrown overboard, and the process begins again with the remaining pirates.

   Given that pirates are a ruthless bunch, if a pirate would receive the same number of coins whether they voted for or against a proposal, they would vote against it to ensure the proposing pirate is thrown overboard.

   Assuming all five pirates are intelligent, rational, greedy, and want to stay alive (and surprisingly good at math for pirates), what will the outcome be?

6. Cannibals ambush a safari in the jungle and capture three men, giving them a single chance to escape without being eaten.

   The captives are lined up in order of height and tied to stakes. The man at the back can see the hats on the heads of the two men in front of him, the middle man can only see the hat of the man in front, and the man at the front cannot see anyone's hat. The cannibals show the men five hats—three black and two white.

   Afterward, blindfolds are placed over each man's eyes, and a hat is put on each of their heads, with the two remaining hats hidden. The blindfolds are then removed, and the cannibals tell the men that if one of them can correctly guess the color of the hat he is wearing, they will all be set free.

The man at the rear, who can see both of his companions' hats but not his own, says, "I don't know." The man in the middle, who can see only the hat of the man in front, also says, "I don't know." But the man at the front, who cannot see anyone's hat, confidently says, "I know!"

How did he figure out the color of his hat, and what color was it? Your response should explain the logical conclusions that led each cannibal to react the way they did.

[Week 2]

7. Find a game that has at least one equilibrium, but in which iterative elimination of dominated strategies yields a game with no equilibria.

8. In a first-price auction, participants submit their bids in sealed envelopes. The bidder with the highest bid wins the auction and is required to pay the amount they bid. If there are ties, meaning multiple bidders have the highest bid, a random draw determines which of these top bidders wins, and the winning bidder pays their bid amount.

   (a) In this situation, does the strategy β_i^* of buyer i, in which he bids his private value for the item, weakly dominate all his other strategies?

   (b) Find a strategy of buyer i that weakly dominates strategy β_i^*.

   Does the strategy of bidding one's private value weakly dominate all other strategies? Provide an explanation for your answer.

9. In the three-player game described, where Player I selects a row (A or B), Player II chooses a column (a or b), and Player III picks a matrix (α, β, or γ), determine all the equilibria of the game.

	a	b
A	0, 0, 5	0, 0, 0
B	2, 0, 0	0, 0, 0

α

	a	b
A	1, 2, 3	0, 0, 0
B	0, 0, 0	1, 2, 3

β

	a	b
A	0, 0, 0	0, 0, 0
B	0, 5, 0	0, 0, 4

γ

10. Provide an example of a game $G = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ in strategic form where the game $\hat{G}$, which is obtained by eliminating one strategy from one player's strategy set, has an equilibrium that does not exist in the original game G.

11. Let A and B be two finite sets, and let $u : A \times B \to \mathbb{R}$ be an arbitrary function. Prove that

$$\max_{a\in A} \min_{b\in B} u(a,b) \leq \min_{b\in B} \max_{a\in A} u(a,b)$$

12. In a game involving fifty players, each participant writes down an integer from the set $\{0, 1, \ldots, 100\}$ on a separate slip of paper and submits it. The game master then calculates

the average x of all the numbers submitted. The winner is the player (or players) whose number is closest to $\frac{2}{3}x$. The prize of \$1,000 is shared equally among the winners.

Describe this as a strategic-form game and determine all the Nash equilibria. What would your strategy be in this game, and why?

[Week 3]

13. Show that in any n-person game, the payoff for each player at a Nash equilibrium is always at least as high as their maxmin value.

14. For each of the following games, where Player I is the row player and Player II is the column player:

    (a) Find all the equilibria in mixed strategies, and all the equilibrium payoffs.

    (b) Find each player's maxmin strategy.

    (c) What strategy would you advise each player to use in the game?

	L	R
T	$5,5$	$0,8$
B	$8,0$	$1,1$

Game A

	L	R
T	$9,5$	$10,4$
B	$8,4$	$15,6$

Game B

	L	R
T	$5,16$	$15,8$
B	$16,7$	$8,15$

Game C

	L	R
T	$8,3$	$10,1$
B	$6,-6$	$3,5$

Game D

	L	R
T	$4,12$	$5,10$
B	$3,16$	$6,22$

Game E

	L	R
T	$2,2$	$3,3$
B	$4,0$	$2,-2$

Game F

	L	R
T	$15,3$	$15,10$
B	$15,4$	$15,7$

Game G

15. Assume Country A builds facilities for nuclear weapons development. Country B sends a spy ring with quality α to determine whether Country A is indeed developing nuclear weapons and is considering bombing the facilities. The spy ring from Country B will accurately report nuclear development with probability α if it is occurring, and will incorrectly report it with probability $1-\alpha$. Conversely, if no nuclear development is happening, the spy ring will correctly report this with probability α and falsely indicate development with probability $1-\alpha$. Country A must decide on nuclear development, while Country B, based on the spy report, must choose whether to bomb Country A's facilities. The payoffs for both countries are detailed in the following table.

		Country B	
		Bomb	Don't Bomb
Country A	Don't Develop	$\frac{1}{2},\frac{1}{2}$	$\frac{3}{4},1$
	Develop	$0,\frac{3}{4}$	$1,0$

(a) Depict this situation as a strategic-form game. Are there any dominating strate- gies in the game?

(b) Describe what it means to say that the quality of Country B's spy ring is $\alpha = \frac{1}{2}$. What if $\alpha = 1$?

(c) For each $\alpha \in [\frac{1}{2}, 1]$, find the game's set of equilibria.

(d) What is the set of equilibrium payoffs as a function of α? What is the α at which Country A's maximal equilibrium payoff is obtained? What is the α at which Country B's maximal equilibrium payoff is obtained?

(e) Assuming both countries play their equilibrium strategy, what is the probability that Country A will manage to develop nuclear weapons without being bombed?

16. Ten individuals are arrested for a crime, but the police do not have enough resources to conduct a thorough investigation. Consequently, the chief investigator offers the following deal: if at least one suspect confesses, each confessing suspect will receive a one-year jail term, while those who do not confess will be released. If no one confesses, the police will extend their investigation, and if they still fail to identify the culprit, all suspects will face a ten-year prison sentence.

    (a) Represent this scenario as a strategic-form game where the players are the arrested individuals. Each player's utility is calculated as 10 minus the number of years they spend in jail.

    (b) Identify all the equilibrium points in pure strategies. What does such an equilibrium signify intuitively, and under what conditions is it plausible for this equilibrium to be achieved?

    (c) Determine a symmetric equilibrium in mixed strategies. What is the probability that, at this equilibrium, no one chooses to confess?

    (d) Assume there are n suspects instead of 10. Find a symmetric equilibrium in mixed strategies. As n approaches infinity, what is the limit of the probability that no one volunteers to confess in this symmetric equilibrium? What implications can we draw from this analysis regarding volunteering in large groups?

    Note : Symmetric equilibrium in mixed strategies: an equilibrium $\sigma = (\sigma_i)_{i \in N}$ satisfying $\sigma_i = \sigma_j$ for each $i, j \in N$

17. Consider the following two-player game composed of two stages. In the first stage, one of the two following matrices is chosen by a coin toss (with each matrix chosen with probability $\frac{1}{2}$). In the second stage, the two players play the strategic-form game whose payoff matrix is given by the matrix that has been chosen.

		Karan		
		L	C	R
Isha	T	$0, 0$	$1, -1$	$-1, 10$
	B	$-2, -2$	$-2, -2$	$-3, -12$

		Karan		
		L	C	R
Isha	T	$-1, 1$	$-2, -1$	$-2, -11$
	B	$1, \frac{1}{2}$	$-1, 0$	$-1, 10$

For each of the following cases, find the unique equilibrium:

(a) No player knows which matrix was chosen.

(b) Isha knows which matrix was chosen, but Karan does not know which matrix was chosen.

What effect does adding information to Isha have on the payoffs to the players at equilibrium?

18. Consider a lottery game with n participants competing for a prize of $\$M$ (where $M > 1$). Each player can buy as many numbers as they want from the set $1, 2, \ldots, K$, at a cost of \$1 per number. After all numbers are purchased, the game identifies the numbers that were bought by exactly one player. The winning number is the smallest among these unique numbers. The player who purchased this winning number claims the entire prize. If no number is bought by only one player, no prize is awarded.

    (a) Write down every player's set of pure strategies and payoff function.

    (b) Prove that a symmetric equilibrium is present, meaning there is an equilibrium where all players employ the same mixed strategy.

    (c) Given $p_1 \in (0, 1)$, examine the mixed strategy $\sigma_i(p_1)$ for player i: with probability p_1, the player selects only the number 1, while with probability $1 - p_1$, the player abstains from purchasing any numbers. What specific conditions must be met by M, n, and p_1 to ensure that the strategy profile where each player adopts $\sigma_i(p_1)$ constitutes a symmetric equilibrium?

    (d) Show that if at equilibrium there is a positive probability that player i will not purchase any number, then his expected payoff is 0.

    (e) Demonstrate that if $M < n$, indicating that the number of participants exceeds the prize value at equilibrium, there is a non-zero probability that no player buys a number. From this, conclude that in every symmetric equilibrium, the expected payoff for each player is zero. (Hint: Prove that if every player buys at least one number with probability 1, the expected total number of numbers purchased by all players exceeds the prize value M, leading to at least one player having a negative expected payoff.)

[Week 4]

19. Prove that in every correlated equilibrium, the payoff to each player i is at least his maxmin value in mixed strategies.

$$\underline{v}_i = \max_{\sigma_i \in \Delta S_i} \min_{\sigma_{-i} \in \Delta S_{-i}} U_i(\sigma_i, \sigma_{-i}).$$

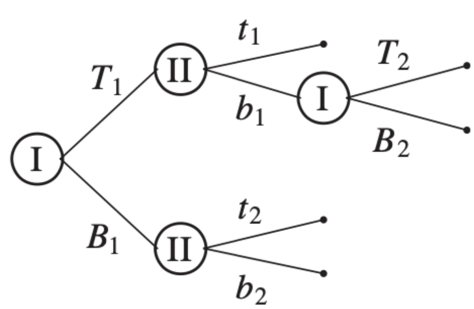
20. Show that there exists a unique correlated equilibrium in the following game, in which $a, b, c, d \in (-\frac{1}{4}, \frac{1}{4})$. Find this correlated equilibrium. What is the limit of the correlated equilibrium payoff as a, b, c, and d approach 0?

		Player II	
		L	R
Player I	T	$1, 0$	$c, 1+d$
	B	$0, 1$	$1+a, b$

21. Represent the following scenario as a game in extensive form. Ehan, Lalit, and Sushant are senior partners at a law firm evaluating candidates to join their team. The candidates under consideration are Krish, Roshni, and Jamal. The decision-making process, reflecting the hierarchy among the three partners, is as follows:

    - Ehan makes the initial proposal of one of the candidates.
    - Lalit follows by proposing a candidate of his own (who may be the same candidate that Ehan proposed).
    - Sushant then proposes a candidate (who may be one of the previously proposed candidates).
    - A candidate who receives the support of two of the partners is accepted into the firm. If no candidate has the support of two partners, all three candidates are rejected.

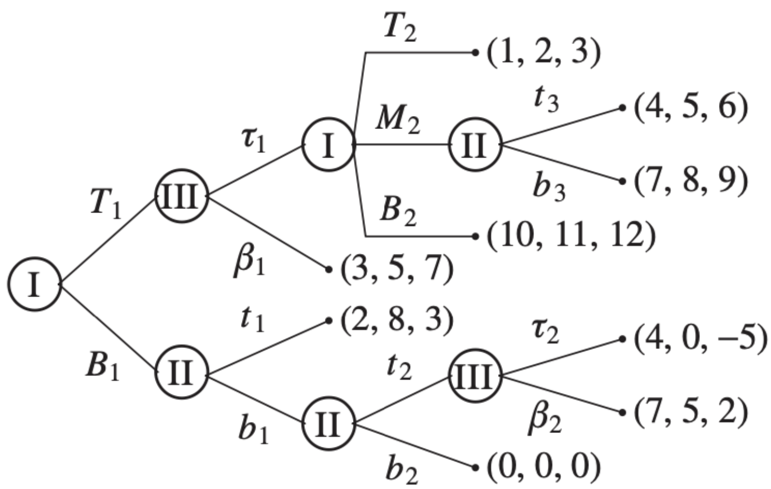
22. By definition, a player's strategy prescribes his selected action at each vertex in the game tree. Consider the following game. Player I has four strategies, $T_1T_2, T_1B_2, B_1T_2, B_1B_2$. Two of these strategies, B_1T_2 and B_1B_2, regardless of the strategy used by Player II, yield the same play of the game, because if Player I has selected action B_1 at the root vertex, he will never get to his second decision vertex. We can therefore eliminate one of these two strategies and define a *reduced strategy* B_1, which only stipulates that Player I chooses B_1 at the root of the game.



In the game appearing in the above figure, the reduced strategies of Player I are T_1T_2, T_1B_2, and B_1. The reduced strategies of Player II are the same as his regular strategies, t_1t_2, t_1b_2, b_1t_2, and b_1b_2, because Player II does not know to which vertex Player I's choice will lead. Formally, a *reduced strategy* τ of player i is a function from a subcollection $\hat{U}_i$ of player i's collection of information sets to actions, satisfying the following two conditions:

(i) For any strategy vector of the remaining players σ_{-i}, given the vector (τ_i, σ_{-i}), the game will definitely not get to an information set of player i that is not in the collection $\hat{U}_i$.

(ii) There is no strict subcollection of $\hat{U}_i$ satisfying condition (i).



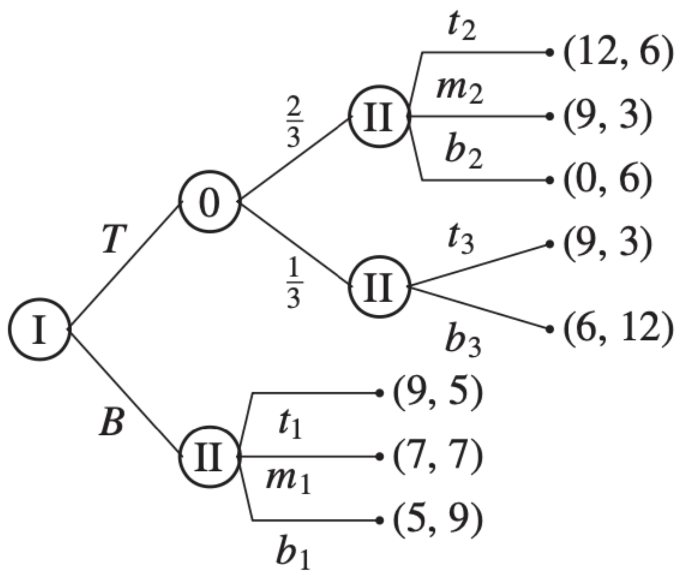
(a) List the reduced strategies of each of the players in the game depicted in the above figure.

(b) What outcome of the game will obtain if the three players make use of the reduced strategies $(B_1), (t_1, t_3), (\beta_1, \tau_2)$?

(c) Can any player increase his payoff by unilaterally making use of a different strategy (assuming that the other two players continue to play according to the strategies of part (b))?

23. Consider the following game: Two players take turns placing quarters on a round table. The coins must not be stacked on top of each other, although they can touch. Each quarter must be fully placed on the table. The player who cannot place a quarter on the table on their turn, without stacking it on an already placed coin, loses the game, and the other player wins. Demonstrate that the first player has a winning strategy.

24. In the game depicted below , if Player I chooses T , there is an ensuing chance move, after which Player II has a turn, but if Player I chooses B, there is no chance move, and Player II has an immediately ensuing turn (without a chance move). The outcome of the game is a pair of numbers (x, y) in which x is the payoff for Player I and y is the payoff for Player II.



(a) What are all the strategies available to Player I?

(b) How many strategies has Player II got? List all of them.

(c) What is the expected payoff to each player if Player I plays B and Player II plays (t_1, b_2, t_3)?

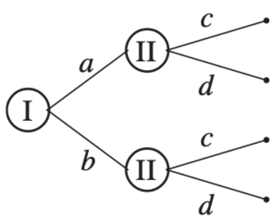
(d) What is the expected payoff to each player if Player I plays T and Player II plays (t_1, b_2, t_3)?

CS 405/6001: Game Theory and Algorithmic Mechanism Design

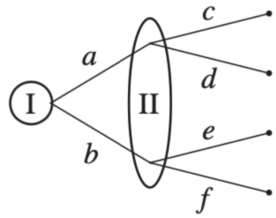
Problem Set 2
[Course Homepage]

[Week 5]

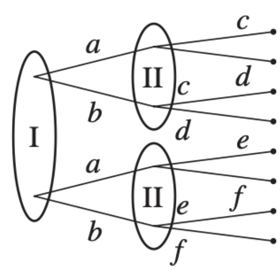
1. Each one of the following figures cannot depict a game in extensive form. For each one, explain why.



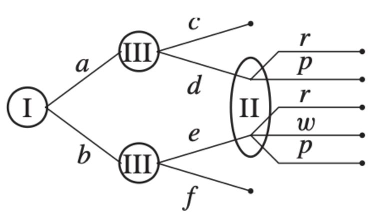
Part A



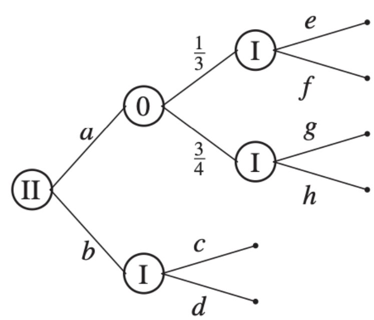
Part B



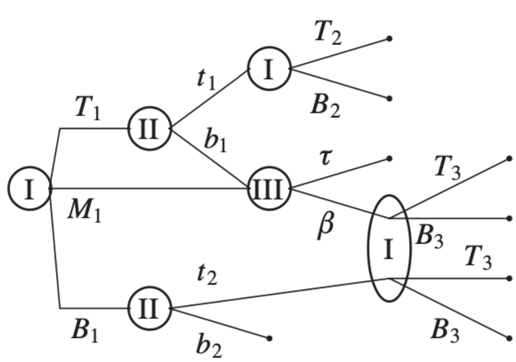
Part C



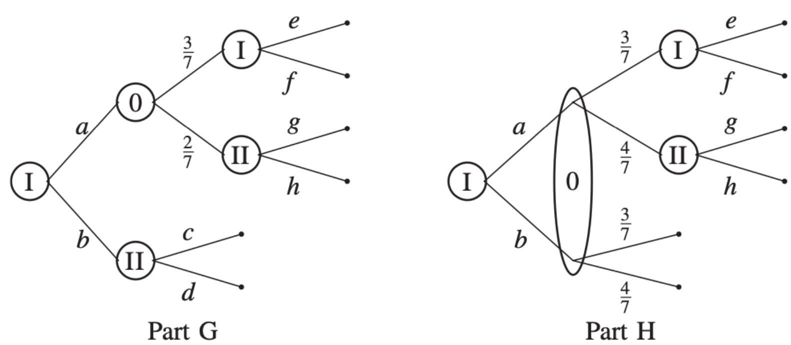
Part D



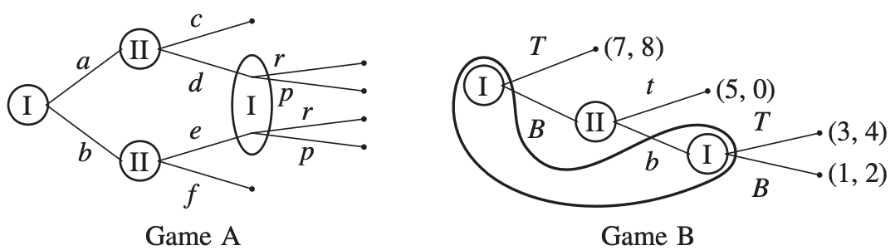
Part E

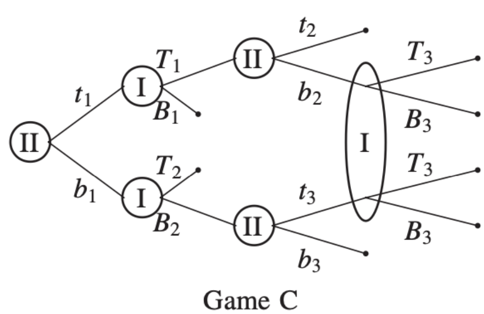


Part F

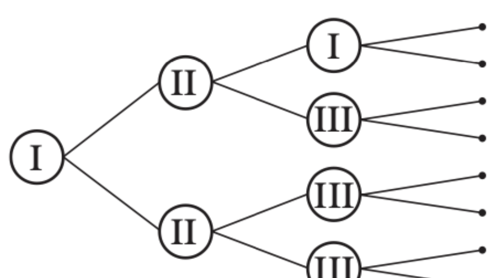


2. In each of the following games, Player I has an information set containing more than one vertex. What exactly has Player I "forgotten" (or could "forget") during the play of each game?





3. Sketch the information sets in the following game tree in each of the situations described in this exercise.

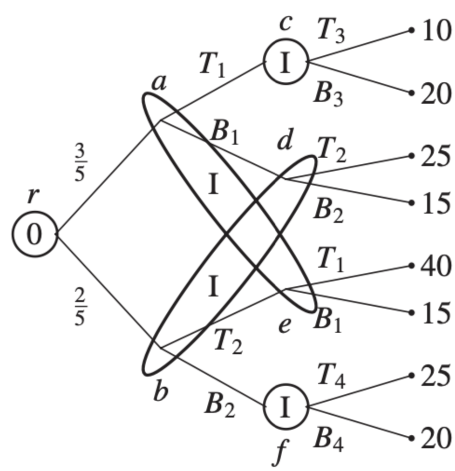


(a) Player II does not know what Player I selected, while Player III knows what Player I selected, but if Player I moved down, Player III does not know what Player II selected.

(b) Player II does not know what Player I selected, and Player III does not know the selections of either Player I or Player II.

(c) At every one of his decision points, Player I cannot remember whether or not he has previously made any moves.

4. (a) What does Player I know, and what does he not know, at each information set in the following game:



(b) How many strategies has Player I got?

(c) The outcome of the game is the payment to Player I. What do you recommend Player I should play in this game?

5. Consider the following game. Player I has the opening move, in which he chooses an action in the set $\{L, R\}$. A lottery is then conducted, with either λ or ρ selected, both with probability $\frac{1}{2}$. Finally, Player II chooses either l or r. The outcomes of the game are not specified. Depict the game tree associated with the extensive-form game in each of the following situations:

(a) Player II, at his turn, knows Player I's choice, but does not know the outcome of the lottery.

(b) Player II, at his turn, knows the outcome of the lottery, but does not know Player I's choice.

(c) Player II, at his turn, knows the outcome of the lottery only if Player I has selected L.

(d) Player II, at his turn, knows Player I's choice if the outcome of the lottery is λ, but does not know Player I's choice if the outcome of the lottery is ρ.

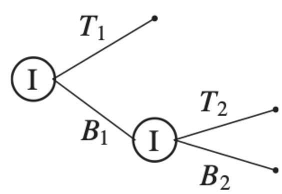
(e) Player II, at his turn, does not know Player I's choice, and also does not know the outcome of the lottery.

6. (a) Let i be a player with perfect recall in an extensive-form game and let σ_i be a mixed strategy of player i. Suppose that there is a strategy vector σ_{-i} of the other players such that $\rho(x;\sigma_i,\sigma_{-i}) > 0$ for each leaf x in the game tree. Prove that there exists a unique behavior strategy b_i equivalent to σ_i.

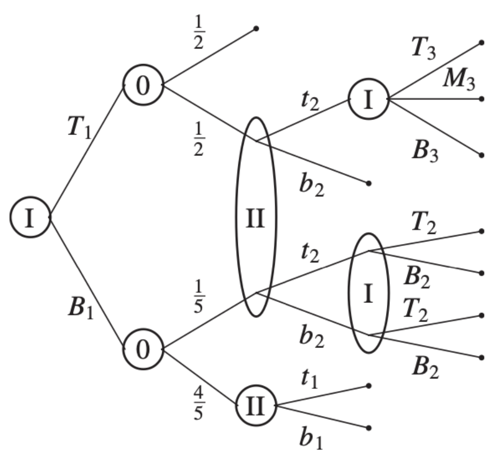
   (b) Give an example of an extensive-form game in which player i has perfect recall and there is a mixed strategy σ_i with more than one behavior strategy equivalent to it.

7. Find a behavior strategy equivalent to the given mixed strategies in each of the following games.

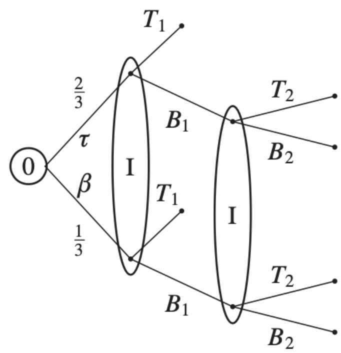
   (a) $s_I = [\frac{1}{2}(B_1, B_2), \frac{1}{2}(T_1, T_2)]$, in the game



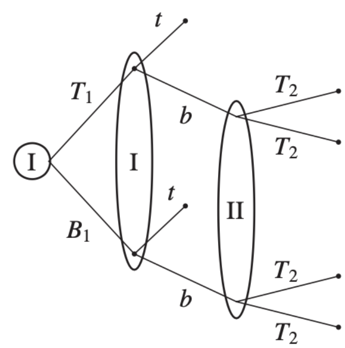
   (b) $s_I = [\frac{3}{7}(B_1B_2M_3), \frac{1}{7}(B_1T_2B_3), \frac{2}{7}(T_1B_2M_3), \frac{1}{7}(T_1T_2T_3)]$ and $s_{II} = [\frac{3}{7}(b_1b_2), \frac{1}{7}(b_1t_2), \frac{1}{7}(t_1b_2), \frac{2}{7}(t_1t_2)]$, in the game



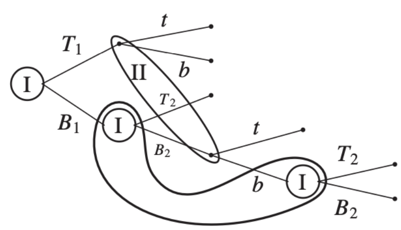
8. In each of the games in the following diagrams, identify which players have perfect recall. In each case in which there is a player with imperfect recall, indicate what the player may forget during a play of the game, and in what way the condition in definition of perfect recall fails to obtain.



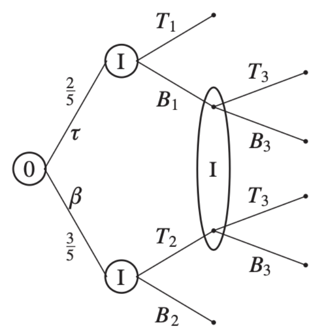
Game A



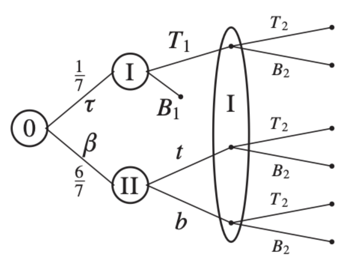
Game B



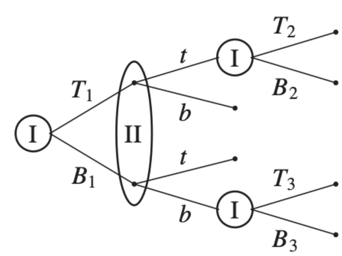
Game C



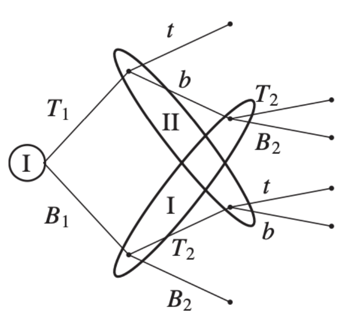
Game D



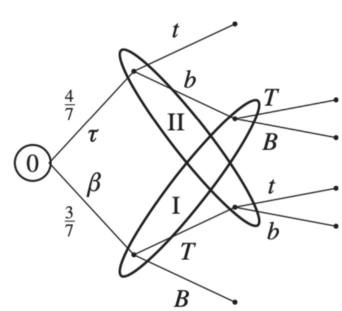
Game E



Game F



Game G

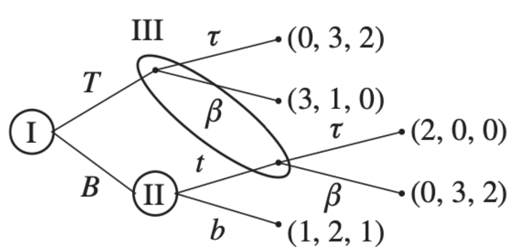


Game H

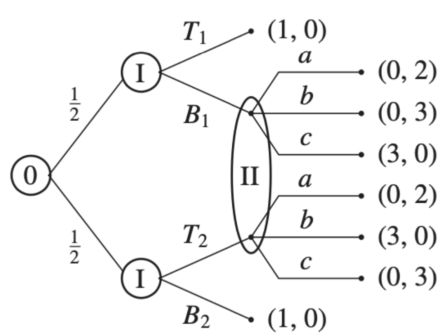
[Week 6]

9. Caesar is at a cafe, trying to choose what to drink with breakfast: beer or orange juice. Brutus, sitting at a nearby table, is pondering whether or not to challenge Caesar to a duel after breakfast. Brutus does not know whether Caesar is brave or cowardly, and he will only dare to challenge Caesar if Caesar is cowardly. If he fights a cowardly opponent, he receives one unit of utility, and he receives the same single unit of utility if he avoids fighting a brave opponent. In contrast, he loses one unit of utility if he fights a brave opponent, and similarly loses one unit of utility if he dishonors himself by failing to fight a cowardly opponent. Brutus ascribes probability 0.9 to Caesar being brave, and probability 0.1 to Caesar being a coward. Caesar has no interest in fighting Brutus: he loses 2 units of utility if he fights Brutus, but loses nothing if there is no fight. Caesar knows whether he is brave or cowardly. He can use the drink he orders for breakfast to signal his type, because it is commonly known that brave types receive one unit of utility if they drink beer (and receive nothing if they drink orange juice), while cowards receive one unit of utility if they drink orange juice (and receive nothing if they drink beer). Assume that Caesar's utility is additive; for example, he receives three units of utility if he is brave, drinks beer, and avoids fighting Brutus. Answer the following questions:
   (a) Describe this situation as an extensive-form game, where the root of the game tree is a chance move that determines whether Caesar is brave (with probability 0.9) or cowardly (with probability 0.1).
   (b) Find all the Nash equilibria of the game.
   (c) Find all the sequential equilibria of the game.

10. Find all the sequential equilibria of the following game.



11. Consider the following extensive-form game.



(a) Prove that in this game at every Nash equilibrium Player I plays (T_1, B_2).

(b) List all the Nash equilibria of the game.

(c) Which of these Nash equilibria can be completed to a sequential equilibrium, and for each such sequential equilibrium, what is the corresponding belief of Player II at his information sets? Justify your answer.

12. Two or three players are about to play a game: with probability $\frac{1}{2}$ the game involves Players 1 and 2 and with probability $\frac{1}{2}$ the game involves Players 1, 2, and 3. Players 2 and 3 know which game is being played. In contrast, Player 1, who participates in the game under all conditions, does not know whether he is playing against Player 2 alone, or against both Players 2 and 3. If the game involves Players 1 and 2 the game is given by the following matrix, where Player 1 chooses the row, and Player 2 chooses the column:

	L	R
T	$0,0$	$2,1$
B	$2,1$	$0,0$

with Player 3 receiving no payoff. If the game involves all three players, the game is given by the following two matrices, where Player 1 chooses the row, Player 2 chooses the column, and Player 3 chooses the matrix:

W

	L	R
T	$1,2,4$	$0,0,0$
B	$0,0,0$	$2,1,3$

E

	L	R
T	$2,1,3$	$0,0,0$
B	$0,0,0$	$1,2,4$

(a) What are the states of nature in this game?

(b) How many pure strategies does each player have in this game?

(c) Depict this game as a game with incomplete information.

(d) Describe the game in extensive form.

(e) Find two Bayesian equilibria in pure strategies.

(f) Find an additional Bayesian equilibrium by identifying a strategy vector in which all the players of all types are indifferent between their two possible actions.

13. Let $L > M > 0$ be two positive real numbers. Two players play a game in which the payoff function is one of the following two, depending on the value of the state of nature s, which may be 1 or 2:

Player II (columns), Player I (rows):

	A	B
A	M, M	$1, -L$
B	$-L, 0$	$0, 0$

The state game for $s = 1$

Player II (columns), Player I (rows):

	A	B
A	$0, 0$	$0, -L$
B	$-L, 1$	M, M

The state game for $s = 2$

The probability that the state of nature is $s = 2$ is $p < \frac{1}{2}$. Player I knows the true state of nature, and Player II does not know it. The players would clearly prefer to coordinate their actions and play (A, A) if the state of nature is $s = 1$ and (B, B) if the state is $s = 2$, which requires that both of them know what the true state is. Suppose the players are on opposite sides of the globe, and the sole method of communication available to them is e-mail. Due to possible technical communications disruptions, there is a probability of $\epsilon > 0$ that any e-mail message will fail to arrive at its destination. In order to transfer information regarding the state of nature from Player I to Player II, the two players have constructed an automated system that sends e-mail from Player I to Player II if the state of nature is $s = 2$, and does not send any e-mail if the state is $s = 1$. To ensure that Player I knows that Player II received the message, the system also sends an automated confirmation of receipt of the message (by e-mail, of course) from Player II to Player I the instant Player I's message arrives at Player II's e-mail inbox. To ensure that Player II knows that Player I received the confirmation message, the system also sends an automated confirmation of receipt of the confirmation message from Player I to Player II the instant Player II's confirmation arrives at Player I's e-mail inbox. The system then proceeds to send an automated confirmation of the receipt of the confirmation of the receipt of the confirmation, and so forth. If any of these e-mail messages fail to arrive at their destinations, the automated system stops sending new messages. After communication between the players is completed, each player is called upon to choose an action, A or B.

Answer the following questions:

(a) Depict the situation as a game with incomplete information, in which each type of each player is indexed by the number of e-mail messages he has received.

(b) Prove that the unique Bayesian equilibrium where Player I plays A when $s = 1$ is for both players to play A under all conditions.

(c) How would you play if you received 100 e-mail confirmation messages? Explain your answer.

14. This exercise illustrates that in situations in which a seller has more information than a buyer, transactions might not be possible. Consider a used car market in which a fraction q of the cars ($0 \leq q \leq 1$) are in good condition and $1 - q$ are in bad condition (lemons). The seller (Player 2) knows the quality of the car he is offering to sell while the buyer (Player 1) does not know the quality of the car that he is being offered to buy. Each used car is offered for sale at the price of \$$p$ (in units of thousands of dollars). The payoffs to the seller and the buyer, depending on whether or not the transaction is completed, are described in the following tables:

	Sell	Don't Sell
Buy	$6-p, p$	$0, 5$
Don't Buy	$0, 5$	$0, 5$

State game if car in good condition

	Sell	Don't Sell
Buy	$4-p, p$	$0, 0$
Don't Buy	$0, 0$	$0, 0$

State game if car in bad condition

Depict this situation as a Harsanyi game with incomplete information, and for each pair of parameters p and q, find all the Bayesian equilibria.