CS 228 : Logic in Computer Science

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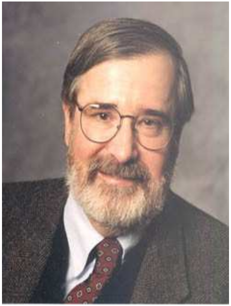
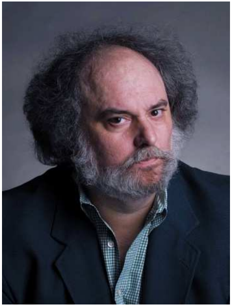
Linear Temporal Logic

Model Checking

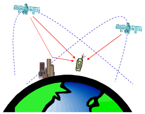



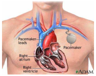

- Year 2007 : ACM confers the Turing Award to the pioneers of Model Checking: Ed Clarke, Allen Emerson, and Joseph Sifakis
- `https://amturing.acm.org/award_winners/clarke_1167964`

Model checking

- Model checking has evolved in last 25 years into a widely used verification and debugging technique for software and hardware.
- Model checking used (and further developed) by companies/institutes such as IBM, Intel, NASA, Cadence, Microsoft, and Siemens, and has culminated in many freely downloadable software tools that allow automated verification.

What is Model Checking?



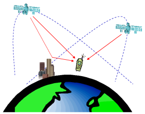


System → specification

satisfy?

good/bad properties

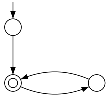
What is Model Checking?



System $\xrightarrow{\text{satisfy?}}$ specification

↓ ↓

System Model $\xrightarrow[\models ?]{\text{model-checking}}$ Spec Model



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Model Checker as a Black Box

- Inputs to Model checker : A finite state system M, and a property P to be checked.
- Question : Does M satisfy P?
- Possible Outputs
  - Yes, M satisfies P
  - No, here is a counter example!.

What are Models?

Transition Systems

- States labeled with propositions
- Transition relation between states
- Action-labeled transitions to facilitate composition

What are Properties?

Example properties

- Can the system reach a deadlock?
- Can two processes ever be together in a critical section?
- On termination, does a program provide correct output?

Notations for Infinite Words

- Σ is a finite alphabet
- Σ^* set of finite words over Σ
- An infinite word is written as $\alpha = \alpha(0)\alpha(1)\alpha(2)\ldots$, where $\alpha(i) \in \Sigma$
- Such words are called ω-words
- a^ω, $a^7.b^\omega$

Transition Systems

A Transition System is a tuple $(S, Act, \rightarrow, I, AP, L)$ where

- S is a set of states
- Act is a set of actions
- $s \xrightarrow{\alpha} s'$ in $S \times Act \times S$ is the transition relation
- $I \subseteq S$ is the set of initial states
- AP is the set of atomic propositions
- $L : S \rightarrow 2^{AP}$ is the labeling function

Traces of Transition Systems

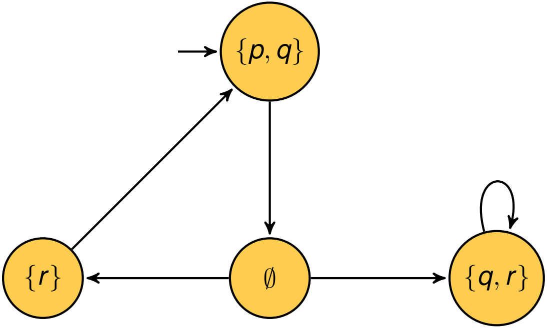
- Labels of the locations represent values of all observable propositions $\in AP$
- Captures system state
- Focus on sequences $L(s_0)L(s_1)\ldots$ of labels of locations
- Such sequences are called traces
- Assuming transition systems have no terminal states,
  - Traces are infinite words over 2^{AP}
  - Traces $\in (2^{AP})^{\omega}$
  - Go to the example slide and define traces

Traces of Transition Systems

Given a transition system $TS = (S, Act, \rightarrow, I, AP, L)$ without terminal states,

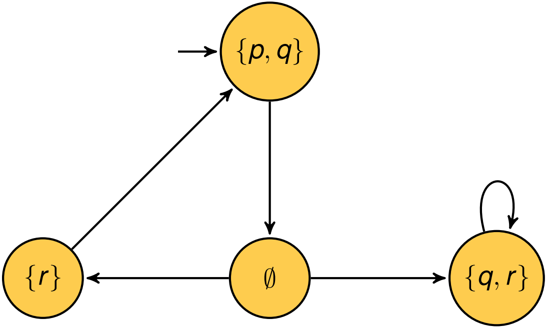
- All maximal executions/paths are infinite
- Path $\pi = s_0 s_1 s_2 \ldots$, $trace(\pi) = L(s_0)L(s_1)\ldots$
- For a set Π of paths, $Trace(\Pi) = \{trace(\pi) \mid \pi \in \Pi\}$
- For a location s, $Traces(s) = Trace(Paths(s))$
- $Traces(TS) = \bigcup_{s \in I} Traces(s)$

Example Traces



$AP = \{p, q, r, t\}$

Example Traces



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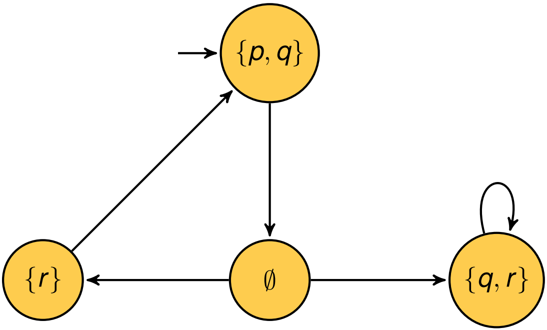
- $\{p, q\}\emptyset\ \{q, r\}^\omega$

Example Traces



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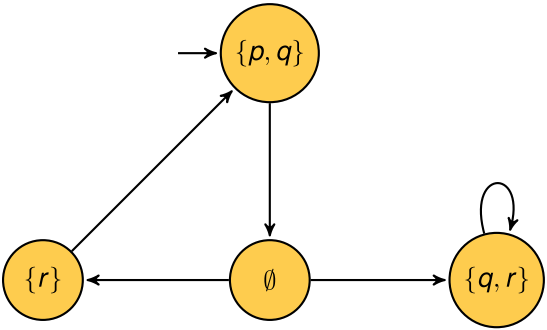
- $(\{p, q\}\emptyset\{r\})^\omega$

Example Traces



$AP = \{p, q, r, t\}$

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- $(\{p, q\}\emptyset\{r\})^\omega$
- $(\{p, q\}\emptyset\{r\})^*\ \{p, q\}\emptyset\ \{q, r\}^\omega$

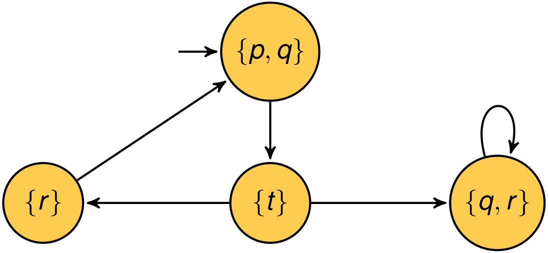
Linear Time Properties

- Linear-time properties specify traces that a TS must have
- A LT property P over AP is a subset of $(2^{AP})^\omega$
- TS over AP satisfies a LT property P over AP

$$TS \models P \text{ iff } Traces(TS) \subseteq P$$

- $s \in S$ satisfies LT property P (denoted $s \models P$) iff $Traces(s) \subseteq P$

Specifying Traces



- Whenever p is true, r will eventually become true
  - $\{A_0 A_1 A_2 \cdots \mid \forall i \geqslant 0, p \in A_i \to \exists j \geqslant i, r \in A_j\}$
- q is true infinitely often
  - $\{A_0 A_1 A_2 \cdots \mid \forall i \geqslant 0, \exists j \geqslant i, q \in A_j\}$
- Whenever r is true, so is q
  - $\{A_0 A_1 \cdots \mid \forall i \geqslant 0, r \in A_i \to q \in A_i\}$

Syntax of Linear Temporal Logic

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- Propositional logic formulae over *AP*
  - $a \in AP$ (atomic propositions)
  - $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$

Syntax of Linear Temporal Logic

Given *AP*, a set of propositions,

- Propositional logic formulae over *AP*
  - $a \in AP$ (atomic propositions)
  - $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$
- Temporal Operators
  - $\bigcirc\varphi$ (Next φ)
  - $\varphi \, \mathsf{U} \psi$ (φ holds until a ψ-state is reached)
- LTL : Logic for describing LT properties

Semantics (On the board)

LTL formulae φ over AP interpreted over words $w \in \Sigma^\omega$, $\Sigma = 2^{AP}$,
$w \models \varphi$