Practice on Density Estimation

1 Questions

1. (Textbook Problem) Solve question 1 in Chapter 20 of the All of Statistics book (https://egrcc.github.io/docs/math/all-of-statistics.pdf)

2. If the true density $f(x) \sim U(0,1)$, estimate the risk of the histogram and kernel density estimator both with width h. Assume KDE uses a uniform kernel. Compare the risk of histogram and KDE.

3. If your goal is to reduce the risk of estimating the cumulative distribution function CDF $F(x)$, would you rather use the empirical CDF $\hat{F}(x) = \frac{1}{n}\sum_i I(X_i \leq x)$, or first use KDE to estimate the density $\hat{f}(x) = \frac{1}{nh}\sum_i K(\frac{x-X_i}{h})$, and then estimate the CDF via integration as $\hat{F_K}(x) = \int_\infty^x \hat{f}(u)du$? Assume a uniform Kernel, and first give an expression for $\hat{F_K}(x)$, and then contrast with $\hat{F}(x)$.