Logic in CS Autumn 2024

Problem Sheet 8

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1. **Theorem 1 (Propositional Compactness Theorem)** *Let S be a set of propositional formulae. S is satisfiable iff every finite subset of S is satisfiable.*

   Prove the above theorem using soundness and completeness of natural deduction.

2. **Theorem 2 (Compactness theorem for FOL)** *Let S be a set of formulae in first-order logic. S is satisfiable iff every finite subset of S is satisfiable.*

   Using the fact that FOL is semi-decidable, prove the compactness theorem for FOL.

3. In Problem Sheet 7, we saw how to write the following in MSO: "There is a path from node s to node t in the graph" using the signature $\tau = \{E\}$.

   Show, using compactness theorem, that you cannot capture this using FOL with the same signature.