Logic in CS Autumn 2024

Problem Sheet 9

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1. Consider the following set of sentences $\Gamma = \{F_1, F_2, F_3, F_4\}$ such that

$$\begin{array}{rcl} F_1 & = & \forall x\,(\forall y\,(C(x,y) \to R(y)) \to H(x)) \\ F_2 & = & \forall x\,(G(x) \to R(x)) \\ F_3 & = & \forall x\,(\exists y\,(C(y,x) \wedge G(y)) \to G(x)) \\ F_4 & = & \forall x\,(G(x) \to H(x)). \end{array}$$

   (a) What is the signature τ of Γ?

   (b) Skolemize $F_1, \ldots, F_4$ and obtain $G_1, \ldots, G_4$. What is the signature of $G_1, \ldots, G_4$?

   (c) Show that propositional resolution gives $\emptyset$ by resolution applied on ground instances of $G_1, \ldots, G_4$.

2. Give an example of a finite set of clauses F in first-order logic such that $Res^*(F)$ is infinite.

3. Give an example of a signature τ that has at least one constant symbol and a τ-formula F (that does not mention equality) such that F is satisfiable but does not have a Herbrand model.

4. A closed formula is in the class $\exists^*\forall^*$ if it has the form $\exists x_1 \ldots \exists x_m \forall y_1 \ldots \forall y_n F$, where F is quantifier-free and $m, n \geq 0$.

   (a) Prove that if an $\exists^*\forall^*$-formula over a signature with no function symbols has a model then it has a finite model.

   (b) Suggest an algorithm for deciding whether a given $\exists^*\forall^*$-formula over a signature with no function symbols has a model.

   (c) Argue that the satisfiability problem for the class of $\forall^*$-formulas that may mention function symbols is undecidable.

5. Execute ground resolution to show that the following formula is unsatisfiable:

$$\forall x \forall y((P(x) \wedge \neg Q(y,y)) \to Q(x,y)) \wedge \neg\exists x(P(x) \wedge \exists y(Q(y,y) \wedge Q(x,y))) \wedge \exists y(P(y))$$