CS 228 : Logic in Computer Science

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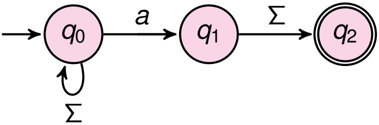
Recap

- FOL over words : Satisfiability
- Translation from formulae φ to equivalent DFA A, $L(\varphi) = L(A)$
- Proof by structural induction, with $\neg, \wedge, \vee$ mapping to complementation, intersection and union
  - Union in DFA–> disjunction in logic
  - Intersection in DFA–> conjunction in logic
  - Complementation in DFA –> Negation in logic
- How to handle quantifiers?

Non-determinism



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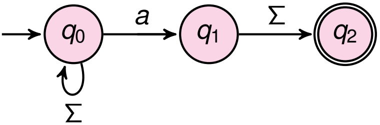

- Assume we relax the condition on transitions, and allow
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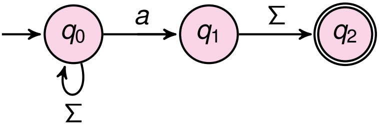

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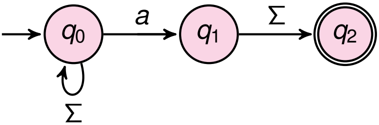

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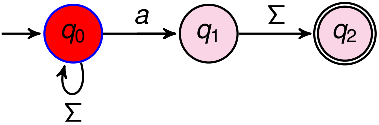

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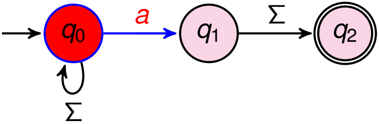

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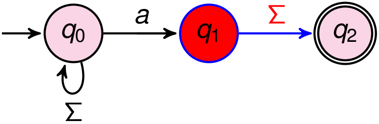

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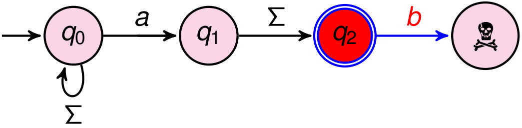

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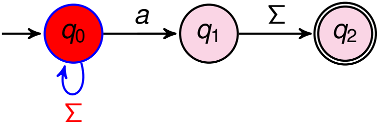

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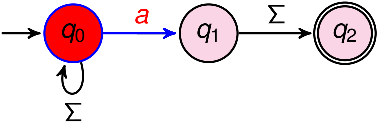

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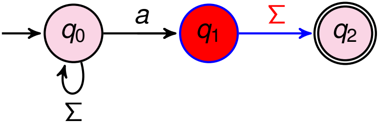

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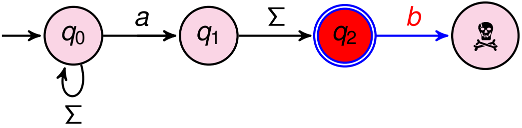

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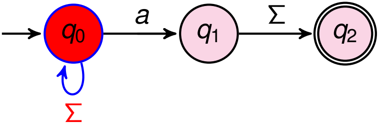

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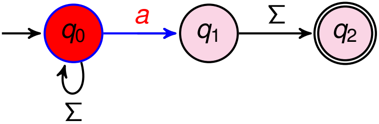

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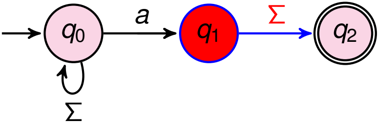

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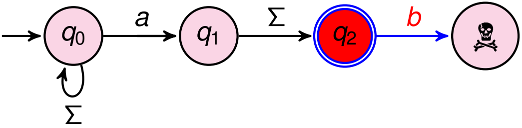

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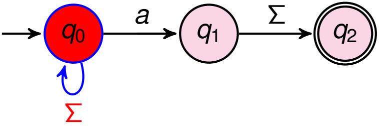

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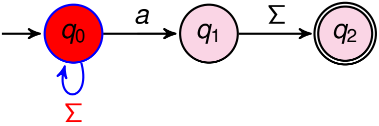

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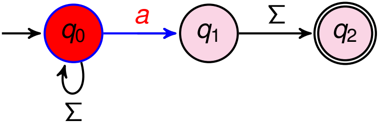

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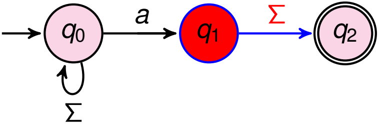

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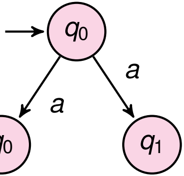
Nondeterministic Finite Automata(NFA)

- $N = (Q, \Sigma, \delta, Q_0, F)$
  - Q is a finite set of states
  - $Q_0 \subseteq Q$ is the set of initial states
  - $\delta : Q \times \Sigma \rightarrow 2^Q$ is the transition function
  - $F \subseteq Q$ is the set of final states
- Acceptance condition : A word w is accepted iff it has atleast one accepting path

Run Tree of *aaab*



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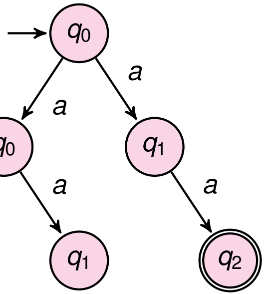

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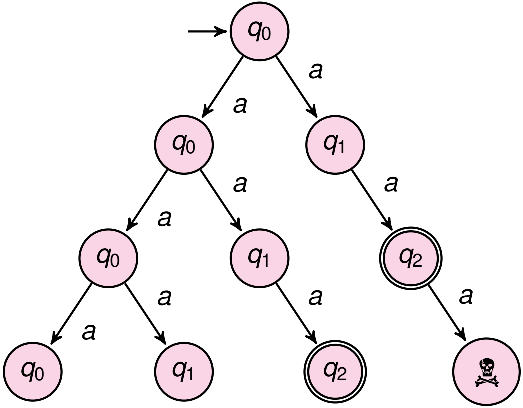

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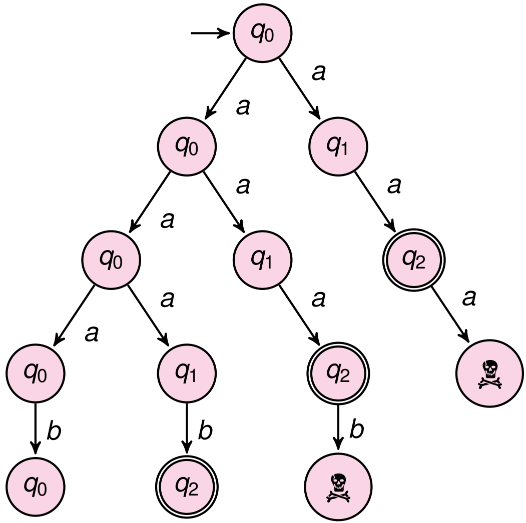

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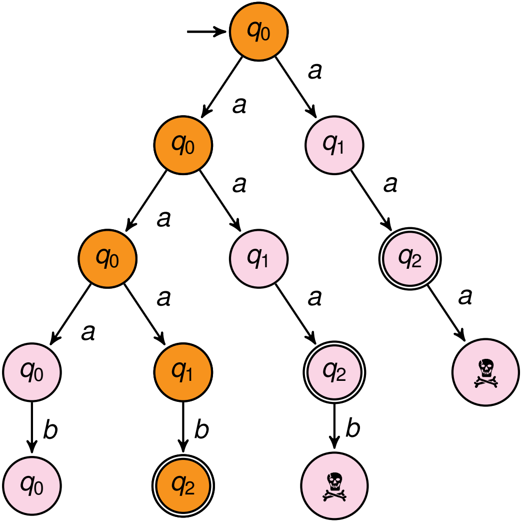

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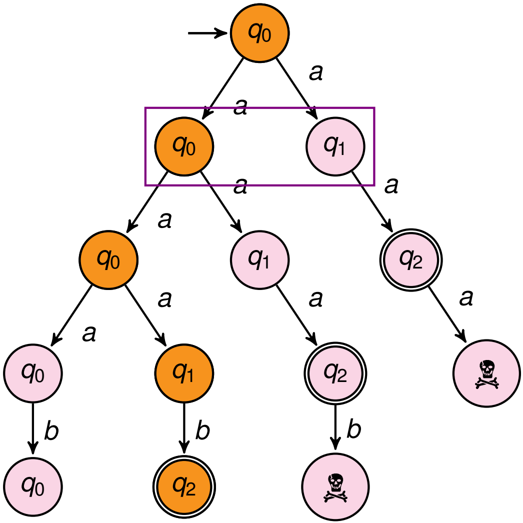

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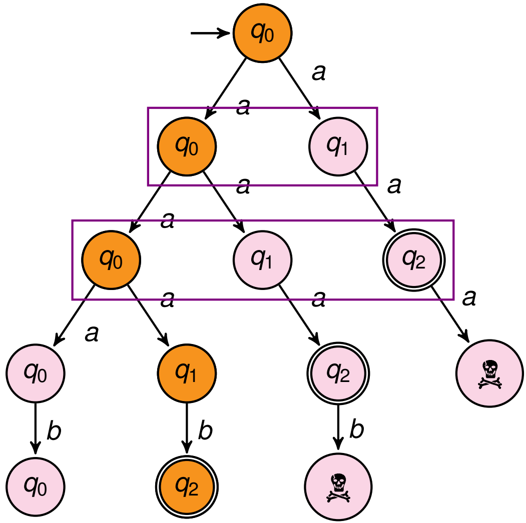

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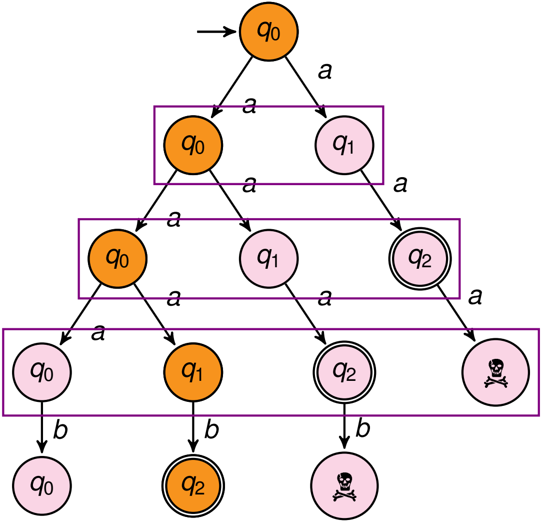

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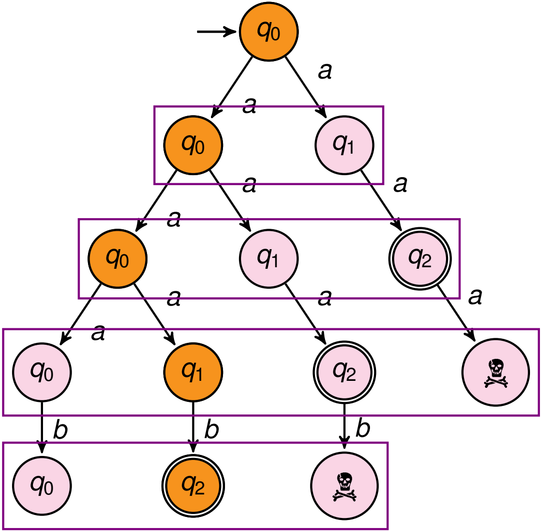
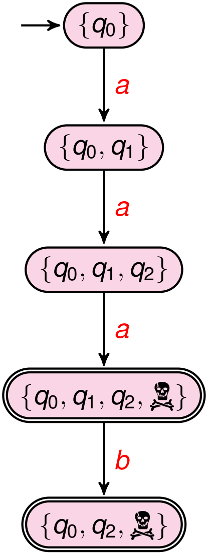


The Single Run



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  - Accept if the obtained set of states contains a final state

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NFA = DFA

$$x \in L(D) \leftrightarrow \hat{\Delta}(Q_0, x) \in F'$$

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$$\hat{\delta}(Q_0, x) \cap F \neq \emptyset$$

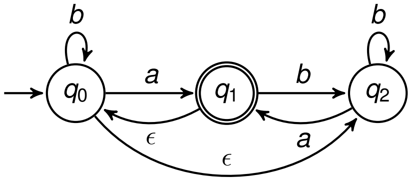
$$\leftrightarrow$$

$$x \in L(N)$$

Regularity

A language L is regular iff there exists an NFA A such that $L = L(A)$

ϵ-NFA



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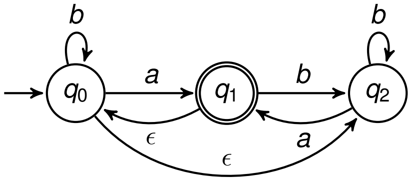

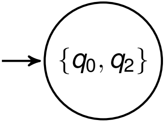

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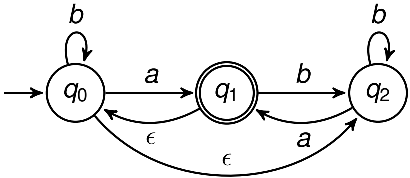

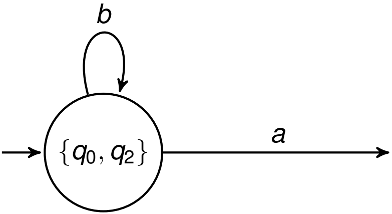

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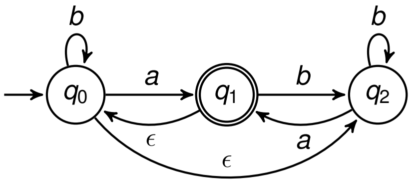

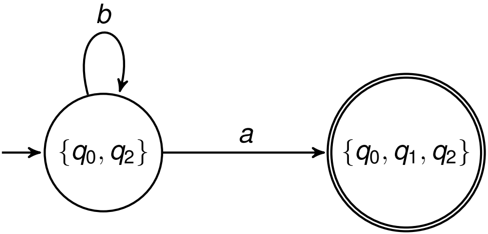

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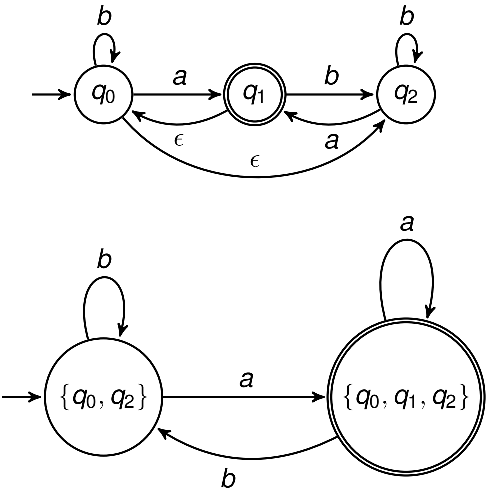
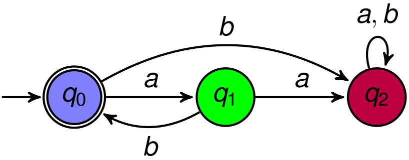


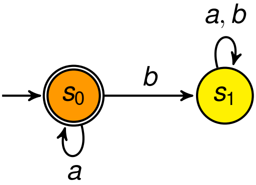
ϵ-NFA and DFA

- ϵ-close the initial states of the ϵ-NFA to obtain initial state of DFA
- From a state S, compute $\Delta(S, a)$ and ϵ-close it
- All states in the DFA are ϵ-closed
- Final states are those which contain a final state of the ϵ-NFA

Closure under Concatenation

- Given regular languages L_1, L_2, is $L_1.L_2$ regular





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