**Practice on MLE (sec 7.2), Evaluating Point Estimators (sec 7.7), and Bayesian Estimator (sec 7.8)**

# 1 Questions

1. Consider the task of performing line fitting to a set of points $(x_i, y_i)$. For this question, model $y_i$ as having the equation $mx_i + c$ (we do NOT know $m$ and $c$) but with Gaussian Noise added to it in the form $\mathcal{N}(0, \sigma^2)$. Use the concept of MLE to find the estimate for $m$, $c$ and $\sigma$.

> We model our data as
> $$y_i = mx_i + c + \epsilon$$
> where $\epsilon$ is the noise sampled from a Gaussian.
> Thus, $y_i \sim \mathcal{N}(mx_i + c, \sigma^2)$
> $$P = \sum p(y_i|x_i, m, c) = \sum_{i=1}^{n} \frac{\exp\left(\frac{-(y_i - mx_i - c)^2}{2\sigma^2}\right)}{\sigma\sqrt{2\pi}}$$
> Set partial derivative of $logP$ to 0 with respect to both $m$ and $c$ to obtain,
> $$\begin{aligned} c' &= \sum_{i=1}^{n} \frac{y_i - m'x_i}{n} \\ m' &= \frac{\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y)}{\sum_{i=1}^{n}(x_i - \mu_x)^2} \end{aligned}$$
> where $\mu_x = (\sum x_i)/n$ and $\mu_y = (\sum y_i)/n$. use the MLE Gaussian estimation on $y_i - m'x_i - c'$ to get the variance,
> $$\sigma'^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - m'x_i - c')^2$$

2. Find a MLE estimate for the Geometric Distribution probability $p$, where $P(x, p) = (1-p)^{x-1}p$.

> Solution - $p' = \frac{n}{\sum x_i} = \frac{1}{\mu_x}$

3. Find a MLE estimator for $\theta$ for a sample size of $n$ in the two sided exponential family with the pdf

$$f(x) = \frac{1}{2}e^{-|x-\theta|} \quad \forall\ x \in \mathcal{R}$$

Is this unbiased?

Solution- Use basic MLE derivation to obtain that $x'$, the Median of the sample, is a MLE estimator of $\theta$.
Reason using symmetry of the distribution and that we can take another sample mirrored around the true $\theta$ that the final bias will be 0.

4. Use MLE for normal distribution to estimate $\sigma^2$ while $\mu$ is known. What is the expected value of estimator?

Derive a) $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$

## 2 Textbook Problems

Chapter 7, Problems 62, 63, 65 and Examples 7.8b, 7.8c, 7.8d