

Hit or Miss?

State which of the proofs below is correct or incorrect. For the incorrect ones, please explain clearly where the logical error in the proof lies. Simply saying that the claim or the induction hypothesis is false is not a valid explanation of what is wrong with the proof. You do not need to elaborate if you think the proof is correct.

Parts Claim: For all positive numbers $n \in \mathbb{R}$, $n^2 \geq n$. proof The proof will be by induction on n .

A Coin Game

Your "friend" Stanley Ford suggests you play the following game with him. You each start with a single stack of n coins. On each of your turns, you select one of your stacks of coins (that has at least two coins) and split it into two stacks, each with at least one coin. Your score for that turn is the product of the sizes of the two resulting stacks (for example, if you split a stack of 5 coins into a stack of 3 coins and a stack of 2 coins, your score would be $3 \cdot 2 = 6$). You continue taking turns until all your stacks have only one coin in them. Stan then plays the same game with his stack of n coins, and whoever ends up with the largest total score over all their turns wins.

Prove that no matter how you choose to split the stacks, your total score will always be $n(n-1)2$. (This means that you and Stan will end up with the same score no matter what happens, so the game is rather pointless.)

Grid Induction

Pacman is walking on an infinite 2D grid. He starts at some location $(i, j) \in \mathbb{N}^2$ in the first quadrant, and is constrained to stay in the first quadrant (say, by walls along the x and y axes). Every second he does one of the following (if possible):

Walk one step down, to $(i, j-1)$.

Walk one step left, to $(i-1, j)$.