Physics 5a: Homework 1

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Assignment

1.2: 1, 5, 7, 11, 15, 23, 26, 30.

Problem 1.2.1

Determine which matrices are in RREF and which are only in REF.

Solution:

RREF: a, b

REF: d

Problem 1.2.5

Determine the possible echelon forms of a nonzero 2x2 matrix using Example 1 notation.

Solution:

$$REF: \begin{pmatrix} \blacksquare & * \\ 0 & \blacksquare \end{pmatrix},$$

$$RREF: \begin{pmatrix} \blacksquare & 0 \\ 0 & \blacksquare \end{pmatrix}, \begin{pmatrix} \blacksquare & * \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \blacksquare \\ 0 & 0 \end{pmatrix}$$

Problem 1.2.7

Find general solutions for matrix:

$$M = \begin{pmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{pmatrix} \xrightarrow{R_2=R_2-3R_1} \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{pmatrix} \xrightarrow{R_2=-1/5R_2} \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$Sol_M = \begin{cases} x_1 = -3x_2 - 5 \\ x_2 = \text{free variable} \\ x_3 = 3 \end{cases}$$

Problem 1.2.11

Find general solutions for matrix:

$$M = \begin{pmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{pmatrix} \xrightarrow[R_3=R_3+2R_1]{R_2=R_2+3R_1} \begin{pmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1=1/3R_1} \begin{pmatrix} 1 & -4/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Sol_M = \begin{cases} x_1 = \frac{4}{3}x_2 - \frac{2}{3}x_3 \\ x_2 = \text{free variable} \\ x_3 = \text{free variable} \end{cases}$$

Problem 1.2.15

Determine if systems are consistent and if so, are unique.

Solution:

a) System is consistent and unique.

b) System is inconsistent.

Problem 1.2.23

Is a 3x5 coefficient matrix with 3 pivots consistent?

Solution: Yes, the augmented matrix will have pivots in all 3 rows *before* the final column. It will just have 2 free variables, but that doesn't affect consistency.

Problem 1.2.26

Explain why a system of three equations with a corresponding coefficient matrix with three pivots is unique.

Solution: If there are three pivots in the coefficient matrix, that means all three variables have a solution. If three variables in a system of three equations all have solutions, then there are no free variables, implying that is the unique solution.

Problem 1.2.30

Give an example of an inconsistent underdetermined system of two equations in three unknowns.

Solution:

$$\begin{cases} x_1 + x_2 + x_3 = 3 \\ x_1 + x_2 + x_3 = 4 \end{cases}$$