Math 54: Homework 3

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Assignment

1.5: 1, 5, 9, 23, 24, 25, 38, 39.
1.7: 1, 7, 9, 11, 21, 22, 31, 32, 33, 34, 37, 38.

Section 1.5

Problem 1

Determine if the system has a nontrivial solution:

$$\begin{aligned} 2x_1 - 5x_2 + 8x_3 &= 0 \\ 2x_1 - 7x_2 + x_3 &= 0 \\ 4x_1 + 2x_2 + 7x_3 &= 0 \end{aligned}$$

Solution:

$$\begin{pmatrix} 2 & -5 & 8 \\ 2 & -7 & 1 \\ 4 & 2 & 7 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & -5 & 8 \\ 0 & -2 & 7 \\ 0 & -8 & -9 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & -5 & 8 \\ 0 & -2 & 7 \\ 0 & 0 & -37 \end{pmatrix}$$

There are infinitely many solutions.

Problem 5

Write the solution set of the given homogeneous system in parametric vector form.

Solution:

$$\begin{pmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ & -3 & -6 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 3 & 1 \\ 0 & 3 & 6 \\ & -3 & -6 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 &= -5x_3 \\ x_2 &= -2x_3 \end{aligned}$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -5x_3 \\ -2x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -5 \\ -2 \\ 1 \end{pmatrix} \text{ (with } x_3 \text{ free)}$$

Problem 9

Describe parametric solution set.

Solution:

$$\begin{pmatrix} 3 & -9 & 6 \\ -1 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 1 & -3 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \text{ (with } x_2, x_3 \text{ free)}$$

Problem 23

True or False.

Solution:

a.) True, homogenous solutions always at least have the trivial solution.

b.) False, it provides an implicit description of solution set.

c.) False, it always has the trivial solution.

d.) False, it is a line through p parallel to v.

e.) True.

Problem 24

True or False.

Solution:

a.) False, at least one entry is nonzero

b.) True.

c.) True.

d.) False, being parallel to a point doesn't have any meaning. It is move parallel to the line through p and the origin.

e.) True.

Problem 25??

Let w be any solution of Ax=b.

Show v_h = w−p is a solution of Ax=0.

Solution:

$$A \cdot \mathbf{w} = \mathbf{b}$$

$$A \cdot \begin{bmatrix} x_1 \\ ... \\ x_n \end{bmatrix} = \mathbf{p} + x_1 \begin{bmatrix} a_1 \\ ... \\ a_n \end{bmatrix}$$

$$\begin{aligned} A \cdot \mathbf{x} &= \mathbf{0} \\ &= \sum_i x_i \mathbf{v}_i \end{aligned}$$

Problem 38

Solution: Yes, there exists a $\mathbf{z} \in \mathbb{R}^3$, because for all Ax, $\mathbf{z}$= $\vec{0}$ will always allow for a solution.

Problem 39

Solution:

$$\begin{aligned} Ax &= 0 \\ Au &= \sum_i^n u_i \mathbf{v}_i = 0 \end{aligned}$$

$$\begin{aligned} A(cu) &= c \sum_i^n u_i \mathbf{v}_i \\ &= c \cdot 0 \\ &= 0 \end{aligned}$$

Section 1.7

Problem 1

Determine linear depedence

Solution: Linearly independent. It can be reduced to REF with no free variables.

Problem 7

Determine if columns are linearly independent.

Solution: Linearly dependent. There are 4 columns and 3 entries. By Theorem 8, this is a linearly dependent set.

Problem 9??

Solution:

a.) v1 and v2 are multiples... so all?

b.) v1 and v2 are multiples... so all?

Problem 11

Solution:

$$\begin{pmatrix} 1 & -1 & 4 \\ 3 & -5 & 7 \\ -1 & 5 & h \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -1 & 4 \\ 0 & -2 & -5 \\ 0 & 4 & h+4 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -1 & 4 \\ 0 & -2 & -5 \\ 0 & 0 & h-6 \end{pmatrix}$$

Linearly dependent when $h = 6$.

Problem 21

True or False.

Solution:

a.) False, it's linearly independent when it *only* has the trivial solution.

b.) False, at least one vector is a linear combination of the others.

c.) True.

d.) True.

Problem 22?

True or False.

Solution:

a.) True.

b.) False, each vector could be the zero vector.

c.) True.

d.) True (99% sure).

Problem 31

Find nontrivial solution of Ax=0.

Solution:

$$x = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Problem 32

Find nontrivial solution of Ax=0.

Solution:

$$x = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Problem 33

Solution: True, at least one vector (v_3) is a linear combination of the other vectors.

Problem 34

Solution: True, any set with the zero vector is linearly dependent.

Problem 37

Solution: True, the linear combinations that would lead $\{v_1, v_2, v_3\}$ to be dependent would also hold for $\{v_1, v_2, v_3, v_4\}$ if you scale v_4 by 0.

Problem 38

Solution: True. Any linear combinations that would result in $\{v_1, v_2, v_3\}$ being dependent would also hold for $\{v_1, v_2, v_3, v_4\}$ if you scale v_4 by 0.