Math 54: Homework 3

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Assignment

1.5: 1, 5, 9, 23, 24, 25, 38, 39.
1.7: 1, 7, 9, 11, 21, 22, 31, 32, 33, 34, 37, 38.

Section 1.5

Problem 1

Determine if the system has a nontrivial solution:

$$\begin{aligned} 2x_1 - 5x_2 + 8x_3 &= 0 \\ 2x_1 - 7x_2 + x_3 &= 0 \\ 4x_1 + 2x_2 + 7x_3 &= 0 \end{aligned}$$

Solution:

$$\begin{pmatrix} 2 & -5 & 8 \\ 2 & -7 & 1 \\ 4 & 2 & 7 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & -5 & 8 \\ 0 & -2 & 7 \\ 0 & -8 & -9 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & -5 & 8 \\ 0 & -2 & 7 \\ 0 & 0 & -37 \end{pmatrix}$$

There is a nontrivial solution.

Problem 5

Write the solution set of the given homogeneous system in parametric vector form.

Solution:

$$\begin{pmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ & -3 & -6 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 3 & 1 \\ 0 & 3 & 6 \\ & -3 & -6 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 &= 5x_3 \\ x_2 &= -2x_3 \end{aligned}$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} \text{ (with } x_3 \text{ free)}$$

Problem 9

Describe parametric solution set.

Solution:

$$\begin{pmatrix} 3 & -9 & 6 \\ -1 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 1 & -3 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \text{ (with } x_2, x_3 \text{ free)}$$

Problem 23

True or False.

Solution:

a.) True, homogenous solutions always at least have the trivial solution.

b.) False, it provides an implicit description of solution set.

c.) False, it always has the trivial solution.

d.) False, it is a line through p parallel to v.

e.) True.

Problem 24

True or False.

Solution:

a.) False, at least one entry is nonzero

b.) True, definition of plane.

c.) True, b has to be zero.

d.) True, p is translation vector.

e.) False, only if there is a vector solution to $Ax = b$.

Problem 25

Let w be any solution of Ax=b.

Show v_h = w−p is a solution of Ax=0.

Solution:

$$\begin{aligned}
A \cdot \mathbf{w} &= \mathbf{b} \\
A \cdot \mathbf{p} &= \mathbf{b} \\
A \cdot \mathbf{v}_h &= 0 \\
A \cdot (p + v_h) &= Ap + Av_h \\
&= b + 0 \\
&= b
\end{aligned}$$

Problem 38

Solution: No, an inconsistent solution implies there is a zero row. If $Ax = z$ has a solution, it has a free variable.

Problem 39

Solution:

$$\begin{aligned}
Ax &= 0 \\
Au &= \sum_i^n u_i \mathbf{v}_i = 0
\end{aligned}$$

$$\begin{aligned}
A(cu) &= c \sum_i^n u_i \mathbf{v}_i \\
&= c \cdot 0 \\
&= 0
\end{aligned}$$

Section 1.7

Problem 1

Determine linear dependence

Solution: Linearly independent. It can be reduced to REF with no free variables.

Problem 7

Determine if columns are linearly independent.

Solution: Linearly dependent. There are 4 columns and 3 entries. By Theorem 8, this is a linearly dependent set.

Problem 9

Solution:

a.) None. $\text{span}\{v_1, v_2\}$ is a line that v_3 isn't on.

b.) All v_1 and v_2 are linearly dependent.

Problem 11

Solution:

$$\begin{pmatrix} 1 & -1 & 4 \\ 3 & -5 & 7 \\ -1 & 5 & h \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -1 & 4 \\ 0 & -2 & -5 \\ 0 & 4 & h+4 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -1 & 4 \\ 0 & -2 & -5 \\ 0 & 0 & h-6 \end{pmatrix}$$

Linearly dependent when $h = 6$.

Problem 21

True or False.

Solution:

a.) False, it's linearly independent when it *only* has the trivial solution.

b.) False, at least one vector is a linear combination of the others.

c.) True, columns ¿ rows.

d.) True, definition of dependence.

Problem 22

True or False.

Solution:

a.) True.

b.) False, each vector could be the zero vector.

c.) True, definition of dependence.

d.) False, not necessarily the reason for dependence.

Problem 31

Find nontrivial solution of Ax=0.

Solution:

$$x = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Problem 32

Find nontrivial solution of Ax=0.

Solution:

$$x = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Problem 33

Solution: True, at least one vector (v_3) is a linear combination of the other vectors.

Problem 34

Solution: True, any set with the zero vector is linearly dependent.

Problem 37

Solution: True, the linear combinations that would lead $\{v_1, v_2, v_3\}$ to be dependent would also hold for $\{v_1, v_2, v_3, v_4\}$ if you scale v_4 by 0.

Problem 38

Solution: True. Any linear combinations that would result in $\{v_1, v_2, v_3\}$ being dependent would also hold for $\{v_1, v_2, v_3, v_4\}$ if you scale v_4 by 0.