Math 54: Week 2 Discussion

Abhijay Bhatnagar

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Inconsistent Matrix

True or false? A system with fewer equations than unknowns is always consistent? Answer: False

Ex:

$$x + y + z = 2 \tag{1}$$
$$x + y + z = 3 \tag{2}$$

Theorem: A system of linear equations has no solutions, exactly 1 solution, or infinity solutions.

Proof. If any matrix reduces to:

$$\begin{matrix} x_1 & * & * & ... & a \\ 0 & x_2 & & ... & ... \\ 0 & 0 & 0 & ... & c \end{matrix}$$

Then the last row is the equation, 0 = c, which is nonzero. Contradiction, no solution.

If any matrix reduces to:

$$\begin{matrix} x_1 & * & & ... & a \\ 0 & x_2 & & ... & b \\ 0 & 0 & x_3 & ... & c \\ ... & & & & \end{matrix}$$

Then the rows reduce to $\{x_1, x_2, x_3\} = \{a, b, c\}$ which is nonzero. There is no contradiction.

If any matrix reduces to:

$$\begin{matrix} x_1 & & & & a \\ 0 & 0 & x_3 & & b \\ 0 & 0 & 0 & x_4 & c \\ ... \end{matrix}$$

Then the rows reduce to $\{x_1, x_2, x_3, x_4\} = \{a, free, b, c\}$. There is no contradiction, but infinitely many solutions for x_2.

$\square$

Definition: The non-pivot columns are the free variables, the pivot columns are the basic variables

Definition: A matrix is nxm if it has n rows and m columns (so height by width). E.g.

$$\begin{matrix} 1 & 4 \\ 3 & 5 \\ 5 & 7 \end{matrix}$$

Is 3x2.

Homework Review

Problem 20. Determine all h s.t. augmented matrix is consistent:

$$\begin{matrix} 1 & h & -3 \\ -2 & 4 & 6 \end{matrix}$$

Solution:

We know it is consistent if and only if there is no pivot in last column.

$$\begin{matrix} 1 & h & -3 \\ 0 & 4+2h & 0 \end{matrix}$$

There is never a pivot in last column.

Problem 28. Suppose a,b,c,d sonstants where a $\neq 0$, and below system is consistent for all f, g. What em say about a,b,c,d?

$$ax_1 + bx_2 = f \tag{3}$$
$$cx_1 + dx_2 = g \tag{4}$$

Solution:

$$\begin{matrix} a & b & f \\ c & d & g \end{matrix}$$

Substract c/a, Row 1 from Row 2, yielding:

$$\begin{matrix} a & b & f \\ 0 & d - bc/a & g - fc/a \end{matrix}$$

We want this to be consistent for all f, g , but as f, g can be anything , so can $g - fc/a$. So for some f, g, $g - fc/a \neq 0$ (let $f = 0$, $g = 1$). So to be consistent for all f, g, we need:

$$\begin{matrix} [a] & b & f \\ 0 & [d - bc/a] & g - f/ca \end{matrix}$$

pivots.

$\therefore$d-bc/a $\neq 0 \implies d \neq cb/a$

New Material

Definition: A vector in $\mathbb{R}^n$ is a nx1 matrix. For instance:

$$\begin{matrix} 2 \\ 5 \\ 7 \end{matrix}$$

$\in \mathbb{R}^3$

Definition: Vector Definition

$$x_1$$
$$...$$
$$x_n$$

$+$

$$y_1$$
$$...$$
$$y_n$$

$=$

$$x_1 + y_1$$
$$...$$
$$x_n + y_n$$

Definition: Scaling. Given scalar $c \in \mathbb{R}$, and vector $\{x_1, ..., x_n\} \in \mathbb{R}^n$

c

$$x_1$$
$$...$$
$$x_n$$

$=$

$$cx_1$$
$$...$$
$$cx_n$$

Theorem: Vectors $u, v, w \in R^n$ and scalars $c, d \in \mathbb{R}$ satisfy the following properties (which we will later call axioms defining a vector space:

1. Associative: $u + (v + w) = (u + v) + w$
2. Communtative: $u + v = v + u$
3. Distributive: c(u+v) = cu + cv
   (c+d)u = cu + du
4. Scalar Associative: c(du) = (cd)u
5. Zero vector, there is a zero vector $0^hat, s.t. o^hat + v = v\forall$ v.
6. Multiplication identity: 1*v = v