

# Physics 5a: Homework 1

Abhijay Bhatnagar

August 29, 2018

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## Assignment

1.1: 1, 3, 5, 7, 11, 15, 20, 23, 24, 28.

## Problem 1.

Solve system:

$$x_1 + 5x_2 = 7 \quad (1)$$

$$-2x_1 - 7x_2 = -5 \quad (2)$$

**Solution:** Beginning the process to convert to

Converted to an augmented matrix:

$$\begin{pmatrix} 1 & 5 & 7 \\ -2 & -7 & -5 \end{pmatrix}$$

Proceeding to convert to RREF

$$R_2 = R_2 + 2R_1:$$

$$\begin{pmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{pmatrix}$$

$$R_2 = R_2/3 \text{ (REF):}$$

$$\begin{pmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{pmatrix}$$

$$R_1 = R_1 - 5R_2:$$

$$\begin{pmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\therefore (x_1, x_2) = (-8, 3)$$

### Problem 3.

Find point of intersection of the lines  $x_1 - 5x_2 = 1$  and  $3x_1 - 7x_2 = 5$ .

**Solution:** Converted to an augmented matrix:

$$\begin{pmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{pmatrix} \xrightarrow{R_2=R_2-R_1} \begin{pmatrix} 1 & 5 & 7 \\ 0 & -7 & -9 \end{pmatrix} \xrightarrow{R_2=-1/7R_2} \begin{pmatrix} 1 & 5 & 7 \\ 0 & 1 & 9/7 \end{pmatrix} \xrightarrow{R_1=R_1-5R_2} \begin{pmatrix} 1 & 0 & 4/7 \\ 0 & 1 & 9/7 \end{pmatrix}$$

$$\therefore (x_1, x_2) = (4/7, 9/7)$$

### Problem 5.

State in words the next two elementary row operations involved in solving a particular matrix.

1. Do a row addition operation of Row 3 onto Row 2, specifically adding Row 2 three times.
2. Do a row subtraction operation of Row 3 onto Row 1, specifically subtract it five times.

### Problem 7.

Row operations on: 
$$\begin{pmatrix} 1 & 7 & 3 & -4 \\ & 1 & -1 & 3 \\ & & & 1 \\ & & 1 & -2 \end{pmatrix}$$

1.  $R_2 = R_2 + R_4$

$$\begin{pmatrix} 1 & 7 & 3 & -4 \\ & 1 & 0 & 1 \\ & & & 1 \\ & & 1 & -2 \end{pmatrix}$$

2.  $R_1 = R_1 - 3R_4$

$$\begin{pmatrix} 1 & 7 & 0 & 2 \\ & 1 & 0 & 1 \\ & & & 1 \\ & & 1 & -2 \end{pmatrix}$$

3.  $R_1 = R_1 - 7R_2$

$$\begin{pmatrix} 1 & 0 & 0 & 16 \\ & 1 & 0 & 1 \\ & & & 1 \\ & & 1 & -2 \end{pmatrix}$$

$R_3 \implies 0 = 1 \therefore$  there are no solutions.

### Problem 11.

Solve a linear system with the following augmented matrix:

$$\begin{pmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{pmatrix}$$

**Solution:**

$$\begin{pmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{pmatrix} \xrightarrow[R_3=R_3-3R_2]{R_1, R_2=R_2, R_1} \begin{pmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & -2 & -8 & 12 \end{pmatrix} \xrightarrow{R_3=R_3+2R_2} \begin{pmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$R_3 \implies 0 = 2 \therefore$  there are no solutions.

### Problem 15.

Determine if the following linear system is consistent.

$$\begin{pmatrix} 1 & & 3 & & 2 \\ & 1 & & -3 & 3 \\ & -2 & 3 & 2 & 1 \\ 3 & & & 7 & -5 \end{pmatrix}$$

**Solution:**  $R_3 = R_3 + 2R_2, R_4 = R_4 - 3R_3 :$

$$\begin{pmatrix} 1 & & 3 & & 2 \\ & 1 & & -3 & 3 \\ & & 3 & -4 & 7 \\ & & & 7 & -11 \end{pmatrix}$$

Each of these rows have a pivot left of the solution column, therefore the system is consistent.

## Problem 20.

Determine value of  $h$  s.t. the following matrix is the augmented matrix of a consistent linear system:

$$\begin{pmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{pmatrix}$$

**Solution:** Reducing to REF:

$$\begin{pmatrix} 1 & h & -3 \\ 0 & 4 + 2h & 0 \end{pmatrix}$$

The system is consistent  $\iff 4 + 2h \neq 0 \iff h \neq -2$

## Problem 23.

Identify statements as *true* or *false* and justify answer. If true, refer to definition/theorem. If false, give location of misquoted statement or counter example.

- a.) *Every elementary row operation is reversible.*

True. Explained on bottom of page 6.

- b.) *A 5 x 6 matrix has six rows.*

False. Matrices are row x col, definition given on page 4.

- c.) *The solution set of a linear system involving variables  $x_1, \dots, x_n$  is a list of numbers  $s_1, \dots, s_n$  that makes each equation in the system a true statement when the values  $s_1, \dots, s_n$  are substituted for  $x_1, \dots, x_n$ , respectively.*

False. The solution set is the set of all possible solutions to a linear system. When there is only one solution, it can be considered a list of numbers in which that previous statement can be applied; however, when there are no solutions or infinitely many solutions, it is more accurately described as either no numbers, or a set of infinitely many numbers under specific conditions. Refer to 3.

- d.) *Two fundamental questions about a linear system involve existence and uniqueness.*

True. Refer to Page 7.

## Problem 24.

- (a) *Elementary row operations on an augmented matrix never change the solution set of the associated linear system.*

True. Refer to page 5.

- (b) *Two matrices are row equivalent if they have the same number of rows.*

False. Row equivalence refers to interchangeability through elementary row operation, see page 6.

- (c) *An inconsistent system has more than one solution.*

False. An inconsistent system has no solutions, see page 4.

- (d) *Two linear systems are equivalent if they have the same solution set.*

True. Refer to page 4.

## Problem 28.

Suppose  $a, b, c$ , and  $d$  are constants such that  $a$  is not zero and the system below is consistent for all possible values of  $f$  and  $g$ . What can you say about the numbers  $a, b, c$ , and  $d$ ? Justify your answer.

$$ax_1 + bx_2 = f$$

$$cx_1 + dx_2 = g$$

### Solution:

Augmented matrix:

$$\begin{pmatrix} a & b & f \\ c & d & g \end{pmatrix}$$

Since  $a \neq 0$ , we can divide  $R_1$  by  $a$ :

$$\begin{pmatrix} 1 & b/a & f/a \\ c & d & g \end{pmatrix}$$

Covert to REF  $R_2 = R_2 - cR_1$

$$\begin{pmatrix} 1 & b/a & f/a \\ c & d - bc/a & g - cf/a \end{pmatrix}$$

Since the system is consistent:  $d - \frac{bc}{a} \neq 0$