Physics 5a: Homework 1

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Assignment

1.1: 1, 3, 5, 7, 11, 15, 20, 23, 24, 28.

Problem 1.

Solve system:

$$x_1 + 5x_2 = 7 \tag{1}$$
$$-2x_1 - 7x_2 = -5 \tag{2}$$

Solution: Beginning the process to convert to

Converted to an augmented matrix:

$$\begin{pmatrix} 1 & 5 & 7 \\ -2 & -7 & -5 \end{pmatrix}$$

Proceeding to convert to RREF

$R_2 = R_2 + 2R_1$:

$$\begin{pmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{pmatrix}$$

$R_2 = R_2/3$ (REF):

$$\begin{pmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{pmatrix}$$

$R_1 = R_1 - 5R_2$:

$$\begin{pmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{pmatrix}$$

$\therefore (x_1, x_2) = (-8, 3)$

Problem 3.

Find point of intersection of the lines $x_1 - 5x_2 = 1$ and $3x_1 - 7x_2 = 5$.

Solution: Converted to an augmented matrix:

$$\begin{pmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{pmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{pmatrix} 1 & 5 & 7 \\ 0 & -7 & -9 \end{pmatrix} \xrightarrow{R_2 = -1/7 R_2} \begin{pmatrix} 1 & 5 & 7 \\ 0 & 1 & 9/7 \end{pmatrix} \xrightarrow{R_1 = R_1 - 5R_2} \begin{pmatrix} 1 & 0 & 4/7 \\ 0 & 1 & 9/7 \end{pmatrix}$$

$\therefore (x_1, x_2) = (4/7, 9/7)$

Problem 5.

State in words the next two elementary row operations involved in solving a particular matrix.

1. Do a row addition operation of Row 3 onto Row 2, specifically adding Row 2 three times.
2. Do a row subtraction operation of Row 3 onto Row 1, specifically subtract it five times.

Problem 7.

$$\text{Row operations on: } \begin{pmatrix} 1 & 7 & 3 & -4 \\ & 1 & -1 & 3 \\ & & & 1 \\ & & 1 & -2 \end{pmatrix}$$

1. $R_2 = R_2 + R_4$

$$\begin{pmatrix} 1 & 7 & 3 & -4 \\ & 1 & 0 & 1 \\ & & & 1 \\ & & 1 & -2 \end{pmatrix}$$

2. $R_1 = R_1 - 3R_4$

$$\begin{pmatrix} 1 & 7 & 0 & 2 \\ & 1 & 0 & 1 \\ & & & 1 \\ & & 1 & -2 \end{pmatrix}$$

3. $R_1 = R_1 - 7R_2$

$$\begin{pmatrix} 1 & 0 & 0 & 16 \\ & 1 & 0 & 1 \\ & & & 1 \\ & & 1 & -2 \end{pmatrix}$$

$R_3 \implies 0 = 1 \therefore$ there are no solutions.

Problem 11.

Solve a linear system with the following augmented matrix:

$$\begin{pmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{pmatrix} \xrightarrow[R_3=R_3-3R2]{R_1,R_2=R_2,R_1} \begin{pmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & -2 & -8 & 12 \end{pmatrix} \xrightarrow{R_3=R_3+2R_2} \begin{pmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$R_3 \implies 0 = 2 \therefore$ there are no solutions.

Problem 15.

Determine if the following linear system is consistent.

$$\begin{pmatrix} 1 & & 3 & & 2 \\ & 1 & & -3 & 3 \\ & -2 & 3 & 2 & 1 \\ 3 & & & 7 & -5 \end{pmatrix}$$

Solution: $R_3 = R_3 + 2R_2, R_4 = R_4 - 3R_3:$

$$\begin{pmatrix} 1 & & 3 & & 2 \\ & 1 & & -3 & 3 \\ & & 3 & -4 & 7 \\ & & & 7 & -11 \end{pmatrix}$$

Each of these rows have a pivot left of the solution column, therefore the system is consistent.

Problem 20.

Determine value of h s.t. the following matrix is the augmented matrix of a consistent linear system:

$$\begin{pmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{pmatrix}$$

Solution: Reducing to REF:

$$\begin{pmatrix} 1 & h & -3 \\ 0 & 4+2h & 0 \end{pmatrix}$$

The system is consistent $\iff 4 + 2h \neq 0 \iff h \neq -2$

Problem 23.

Identify statements as *true* or *false* and justify answer. If true, refer to definition/theorem. If false, give location of misquoted statement or counter example.

a.) *Every elementary row operation is reversible.*
True. Explained on bottom of page 6.

b.) *A 5 x 6 matrix has six rows.*
False. Matrices are row x col, definition given on page 4.

c.) *The solution set of a linear system involving variables $x_1, ..., x_n$ is a list of numbers $s_1, ..., s_n$ that makes each equation in the system a true statement when the values $s_1, ..., s_n$ are substituted for $x_1, ..., x_n$, respectively.*
False. The solution set is the set of all possible solutions to a linear system. When there is only one solution, it can be considered a list of numbers in which that previous statement can be applied; however, when there are no solutions or infinitely many solutions, it is more accurately described as either no numbers, or a set of infinitely many numbers under specific conditions. Refer to 3.

d.) *Two fundamental questions about a linear system involve existence and uniqueness.*
True. Refer to Page 7.

Problem 24.

(a) *Elementary row operations on an augmented matrix never change the solution set of the associated linear system.*
True. Refer to page 5.

(b) *Two matrices are row equivalent if they have the same number of rows.*
False. Row equivalence refers to interchangeability through elementary row operation, see page 6.

(c) *An inconsistent system has more than one solution.*
False. An inconsistent system has no solutions, see page 4.

(d) *Two linear systems are equivalent if they have the same solution set.*
True. Refer to page 4.

Problem 28.

Suppose $a, b, c,$ and d are constants such that a is not zero and the system below is consistent for all possible values of f and g. What can you say about the numbers $a, b, c,$ and d? Justify your answer.

$ax_1 + bx_2 = f$
$cx_1 + dx_2 = g$

Solution:

Augmented matrix:

$$\begin{pmatrix} a & b & f \\ c & d & g \end{pmatrix}$$

Since $a \neq 0$, we can divide R_1 by a:

$$\begin{pmatrix} 1 & b/a & f/a \\ c & d & g \end{pmatrix}$$

Covert to REF $R_2 = R_2 - cR_1$

$$\begin{pmatrix} 1 & b/a & f/a \\ c & d - bc/a & g - cf/a \end{pmatrix}$$

Since the system is consistent: $d - \frac{bc}{a} \neq 0$