Math 54: Week 2 Discussion

Abhijay Bhatnagar

August 28, 2018

Review From Last Week

Configuration $\{q_i\} \implies$ picture of system (no v.)
Trajectory $\{q_i(t)\} \implies$ movie of system: $v = \frac{dq}{dt}, a = \frac{d^2q}{dt^2}$
Newton $\{q_i(t)\, v_i(0) \implies \{q_i(t)\}$ (Phase space, config not enough)

Kinematics

Taylor series expansion for velocity..?

$$v(t) = -\frac{g}{a}(1 - e^(\alpha t)) \quad (1)$$
$$v(0) = 0 \quad (2)$$

T.E. $= e^(- \alpha t)$

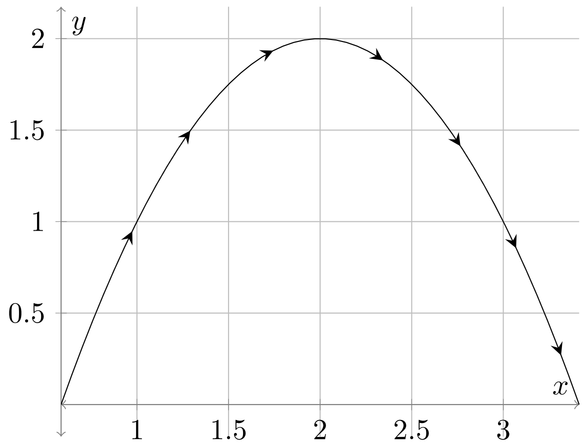
Use Taylor series approximation for T.E.

$$v(t) = -\frac{g}{a}(1 - (1 - \alpha t + 1/2\alpha^2 t^2 + ...)) \quad (3)$$
$$= -gt + \frac{g}{2}\alpha t \quad (4)$$

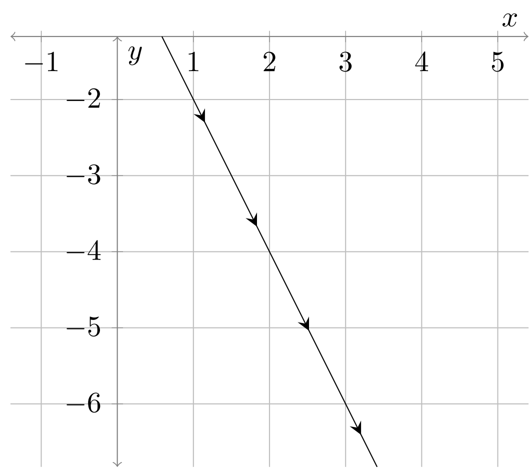
The taylor expansion is good when x is small $\implies \alpha t << 1 \implies t << \frac{1}{\alpha}$.

Visualizations of positional derivatives

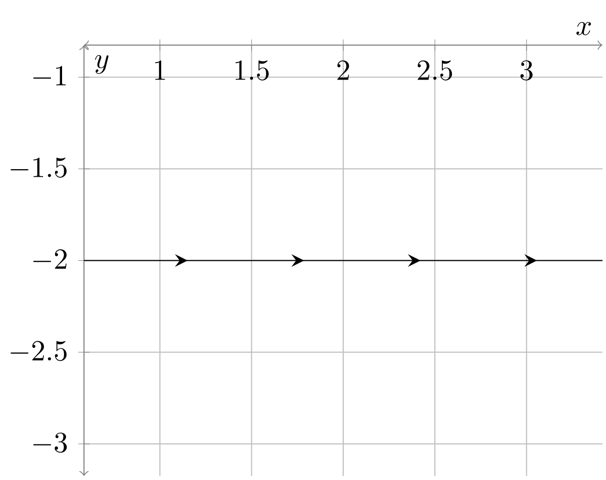
$x(t) =$



$v(t) =$



$a(t) =$



Derivative exercises

1. Exercise in finding 1st and 2nd derivatives of $x(t) = cost(\alpha t)$
2. Exercise in finding 1st and 2nd derivatives of $x(t) = \frac{1}{2}at^2$
3. Exercise in finding 1st and 2nd derivatives of $x(t) = \frac{1}{2}at^2 + v_0t + x_0$

Integral exercise

1. Exercise in finding 1st and 2nd antiderivatives of $x(t) = x_0$

Considering Dimensions

$$\begin{aligned} \overrightarrow{r}(t) &= (x(t), y(t)) && (5) \\ &= x(t)\hat{i} + y(t)\hat{j} && (6) \end{aligned}$$

When converting from different coordinate perspectives:

$$\begin{aligned} \overrightarrow{r}(t) = x(t)\hat{i} + y(t)\hat{j} &= \tilde{x}\tilde{\hat{i}} + \tilde{y}\tilde{\hat{j}} && (7) \\ x &\neq \tilde{x} && (8) \\ y &\neq \tilde{y} && (9) \\ \text{... get from notes} & && (10) \end{aligned}$$

Vectors

1. Adding $\overrightarrow{v} + \overrightarrow{u}$
2. Length $\|\overrightarrow{v}\|$
3. Dot $\overrightarrow{v} \cdot \overrightarrow{u}$
4. Cross $\overrightarrow{v}\text{x}\overrightarrow{u}$

Ball-in-cup-esque exercise

Exercise in finding when a ball shot