PHYSICS 5A, FALL 2018, HW 2: DUE 9/11, 9:30AM

The maximum points you can achieve is 100.

Problem 1. 10 pts

K.K. 1.25

Problem 2. 15 pts

K.K. 1.26

Problem 3. 15 pts

K.K. 2.8

Problem 4. 15 pts

K.K. 2.9

Problem 5. 15 pts

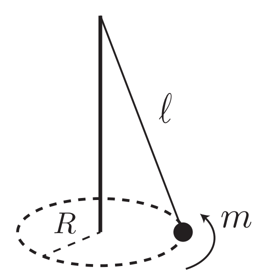
K.K. 2.16

Problem 6. 15 pts

A small ball of mass m tied to a massless string of length ℓ is moving around a circle of radius R, as shown in the Fig. Determine (in terms of m, ℓ, R, g):

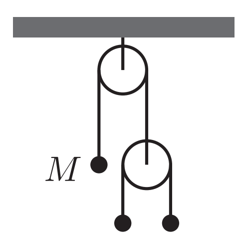
a) The rotation frequency f

b) The string tension T.



Problem 7. 20 pts

An Atwood machine composed of three masses M, m_1, m_2, with massless strings and pulleys, hangs as shown in the Fig. What should M be, in terms of m_1 and m_2, so that the mass M doesn't move?

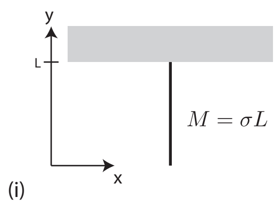


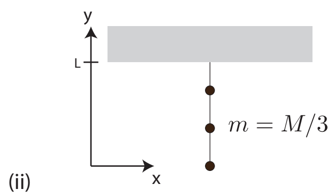
Problem 8. 20 pts *Bouncing turtle*. A spherical turtle is thrown straight up and reaches a height h. It then falls, hits the ground, and bounces straight back up, reaching height fh. This repeats: after each bounce it reaches a fraction f of its previous height. (Between bounces, ignore air friction: the turtle is acted on only by gravity). Find the total distance traveled, and also the total time, before it comes to rest. What is its average speed during this period?

Hint: the Zeno-like sum $\sum_{n=0}^{\infty} f^n = \frac{1}{1-f}$ may prove useful.

Massive string preamble. In our discussion of strings and tension thus far, we have assumed that the mass of the string is negligible (i.e., zero). This implies the total force on any segment of the string is zero, since $\mathbf{F}_s = 0 \cdot \mathbf{a}_s$ and $\mathbf{a}_s$ should be finite. Consequently, the tension forces exerted at the two ends of any string segment must be equal and opposite, e.g., the tension is constant along the length of the string. In the following two problems we'll instead consider a *massive* string characterized by its mass per unit length σ, i.e., a string of length L has mass $M = \sigma L$.

Problem 9. 15 pts





A massive string of length L and linear density σ hangs motionless from a ceiling, as shown in Fig (i). Gravity pulls down on the string (let g denote the acceleration due to gravity). Letting $y = 0$ denote the bottom of the string, how does the string tension $T(y)$ vary with the height y? To understand how to answer this problem, we'll break it into three steps.

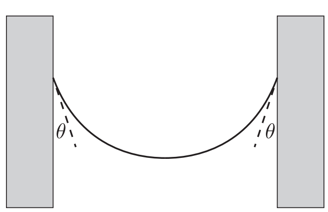
a) Let's start with something simpler: suppose three masses, each of mass $m = M/3$, are tied together with three segments of *massless* string each of length $L/3$ and hung from the ceiling, as shown in Fig (ii). (Note: the total mass of the whole hanging apparatus is thus M, as before). Each mass is pulled down by gravity. Labeling the string segments $i = 0, 1, 2$ from bottom to top, what is the string tension T_i in each segment?

b) Now generalize (a): suppose there are N masses separated by massless strings of length $\Delta\ell = L/N$, each of weight $\Delta m = M/N$. Now what is the tension T_i in the i-th piece of string? Letting y_i denote the top height of each piece of string, re-express T_i in terms of $y_i, \sigma = M/L, g$.

c) Using your solution from (b), you can now take the limit of $N \to \infty$: what is the tension in the string $T(y)$ at height y? Give your answer in terms of y, σ, g.

Comment: now you've solved the problem of tiny massive beads tied to a sequence of short massless strings. But this is exactly how a *massive* string behaves! Each infinitesimal segment of the string has mass $\sigma d\ell$, and is acted on by the tension $T(\ell)$ on either side of the segment,

Problem 10. 25 pts



Another massive string, with total mass M, is attached to two walls and hangs motionless, as shown in the Fig. Gravity pulls down on the string. I could ask you to find the shape of the string, but will spare you! Instead, suppose you measure that the string makes an angle θ with respect to the right and left wall, as indicated. Because the string is massive, the tension varies along the length of the string. What is the tension at the *midpoint* of the string?

Hint: You will *not* need to find the tension along the length of the entire string, or find its shape, in order to solve this problem. Mentally divide the string into its left and right halves, which are mirror images. What is the total force on the left half?