

# On the Aerodynamic Efficiency of Insect-Inspired Micro Aircraft Employing Asymmetrical Flapping

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Using a quasi-steady, blade-element analysis, we investigated the role of asymmetrical flapping mechanisms in hovering flight, for insect inspired micro air vehicles. The current analysis was applied to a 30 cm half-span wing, beating not more than 6 Hz. An implementation of asymmetrical flapping exhibited significantly greater lift generation, which can be attributed to the increase in angular velocity squared form for lift that occurs with increasing asymmetry. Significant improvements in the lift-to-power ratio were observed, for a house-fly-like mode of flapping, when the wing-beat frequency was below the natural frequency. At a frequency ratio of 0.3, a 75% increase in performance was observed with the use of asymmetrical flapping. At flapping frequencies above the natural frequency, however, asymmetry was found to be detrimental to performance, due to an increase in inertial forces. In a low inertia, an inclined stroke plane system, characteristic of dragonflies, we see that, in its most efficient flapping condition, asymmetrical flapping is detrimental to performance. However, in compliant systems in which elastic forces are significant, we see that asymmetry can improve the aerodynamic efficiency of the wing-actuation system.

## Nomenclature

$C_D$	=	drag coefficient
$C_L$	=	lift coefficient
$C_{\text{rotation}}$	=	rotational pressure coefficient
$c$	=	wing chord, m
$D$	=	drag, N
$F_{\text{rotation}}$	=	force due to rotational wing movement, N
$f$	=	wing-beat frequency
$H_r$	=	vertical force in the local frame of reference, N
$H_{r,\text{vert}}$	=	vertical force in global frame of reference, N
$I$	=	inertia, $\text{kg} \cdot \text{m}^2$
$k$	=	spring constant, $\text{N} \cdot \text{m}/\text{rad}$
$L$	=	lift, N
$P$	=	power, W
$r$	=	radius from wing root, m
$S$	=	wing area, $\text{m}^2$
$T$	=	flapping period, s
$t$	=	time, s
$t^*$	=	time scaled to produce asymmetrical wing rotation, s
$V$	=	tangential velocity, $\text{m}/\text{s}$
$\hat{x}_0$	=	nondimensional axis of rotation from leading edge
$\alpha$	=	angle of attack, rad
$\alpha_m$	=	mean angle of attack, rad
$\alpha_0$	=	change in angle of attack, rad
$\beta$	=	stroke plane angle, rad
$\Gamma$	=	elastic-to-aerodynamic torque ratio

$\eta$	=	ratio of downstroke time to total period
$\rho$	=	density, $\text{kg}/\text{m}^3$
$\tau$	=	torque, $\text{N} \cdot \text{m}$
$\phi$	=	flapping angle, rad
$\phi_{\text{proj}}$	=	flapping angle projected onto vertical plane, rad
$\phi_0$	=	flapping amplitude, rad
$\Omega$	=	frequency ratio
$\omega$	=	flapping-wing angular velocity, $\text{rad}/\text{s}$
$\omega_n$	=	natural frequency, $\text{rad}/\text{s}$

## I. Introduction

AN ACTIVE topic of unmanned aerial vehicles (UAVs) research considers biologically inspired flapping-wing aircraft. The topic has been facilitated by technological advances in many fields including microelectronics, sensors, microelectromechanical systems, and micro-manufacturing. Biologically inspired flapping-wing aircraft may offer benefits including increased efficiency at low Reynolds numbers [1,2] exhibited by micro air vehicles (MAVs) and small UAVs. In addition, flapping-wing flyers exhibit a range of flight maneuvers not achievable by any single fixed-wing or rotary-wing device.

When modelling the aerodynamics of flapping wings, researchers often use sinusoidal flapping profiles with symmetrical upstroke to downstroke to simplify the modelling of the wing articulation [3–5]. However, studies have shown that insect flapping profiles vary according to aerodynamic demands [6–8] and can be asymmetrical [9,10]. An asymmetrical stroke is one defined by different periods spent in upstroke vs downstroke (see Fig. 1).

This phenomenon is present in many insect species, including Diptera and Odonata. Dudley [9] suggested that an asymmetrical flapping profile could be more apparent in insects with synchronous muscles such as Odonata [9]. These insects tend to rely less on resonance [11] and consequently are not limited to the simple harmonic sinusoidal flapping profile characteristic of a resonant system. Stroke asymmetry is also present in Dipteran insects, although it is less pronounced [10,12]. Dipteran insects appear to rely on nonsinusoidal flapping profiles, or “click” mechanisms [13–15]. These click mechanisms modulate the elasticity of the thorax structure in such a way as to produce a nonsymmetrical downstroke to upstroke. Such mechanisms have been shown in some studies to improve efficiency in lift generation [16–18]. Computational work

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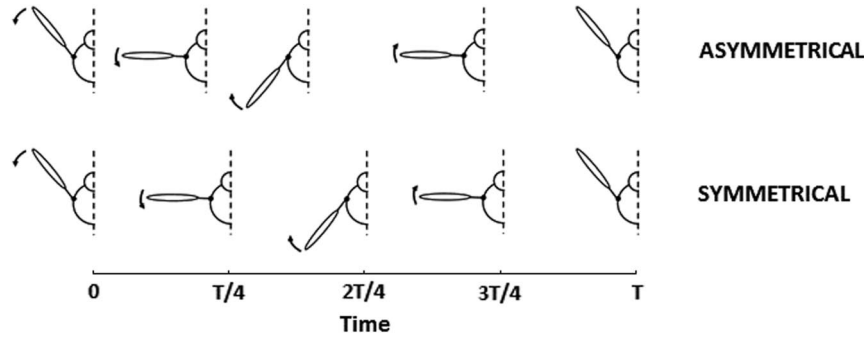


Fig. 1 Illustration of asymmetrical flapping vs symmetrical flapping.

performed by Yu and Tong [8] shows that asymmetrical flapping increases thrust generation in forward flight. Although it is known that stroke asymmetry is present in most insects, the advantages associated with asymmetrical flapping are relatively unknown. This paper aims to simulate and analyze asymmetrical flapping, the extent to which it is beneficial to the aerodynamic efficiency in hover, and how it could be applied in the context of a real world MAV platform. This will be performed for two distinct modes of flapping: Dipteran and Odonate.

## II. Methods

### A. Flapping Modes

This paper will focus on the two most commonly studied modes of insect-inspired flapping, namely, an inertial-dominated, horizontal stroking mode characteristic of Diptera and an aerodynamically dominated, inclined stroking mode characteristic of Odonata.

One of the characteristics of Dipteran flapping is the horizontal stroke plane (see Fig. 2) [3,4]. Additionally, Dipteran flight relies on indirect actuation whereby the muscles attach to an elastic thorax that transfers force to the wing [9]. Inherent in this is the use of resonance to amplify the displacements from the actuator to the wing [11]. As a result, the elasticity of the thorax plays a dominant role in the dynamics of the system, in particular, influencing the natural frequency of the system. The natural frequency is the excitation frequency whereby inertial and elastic forces are in balance and the aerodynamic loads are the only loads that need to be overcome [19,20]. The natural frequency is related to the elasticity and inertia of the system by

$$\omega_n = \sqrt{\frac{k}{I}} \quad (1)$$

Matching the wing-beat frequency to the resonance point helps to reduce the energy otherwise lost to inertial work, which can be significant in Diptera [4,11]. This is one of the methods proposed for

reducing the energy costs associated with flapping flight [21–23]. The elastic thorax in Diptera promotes the theory of resonance. This would suggest that Dipteran flapping follows a simple harmonic motion. While studies by Ennos [10] show that the asymmetry in Diptera is significantly less as compared to other insects, an asymmetry that is contrary to simple harmonic theory still exists. In fact, studies have suggested that the elastic thoracic structure is used as a click mechanism that modulates the forces acting on the wing to produce nonsinusoidal, asymmetric flapping profiles. Such a

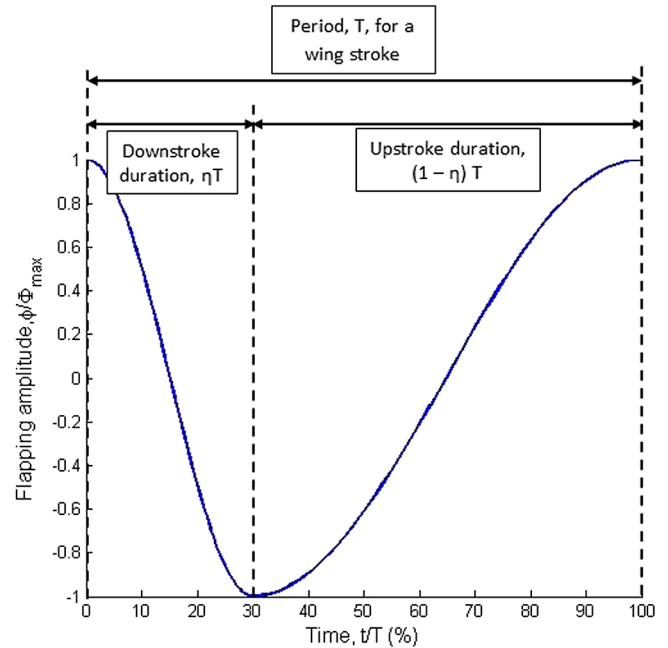


Fig. 3 The asymmetrical flapping profile is represented by two half-cosine waves. The figure illustrates a flapping profile with  $\eta = 0.3$ .

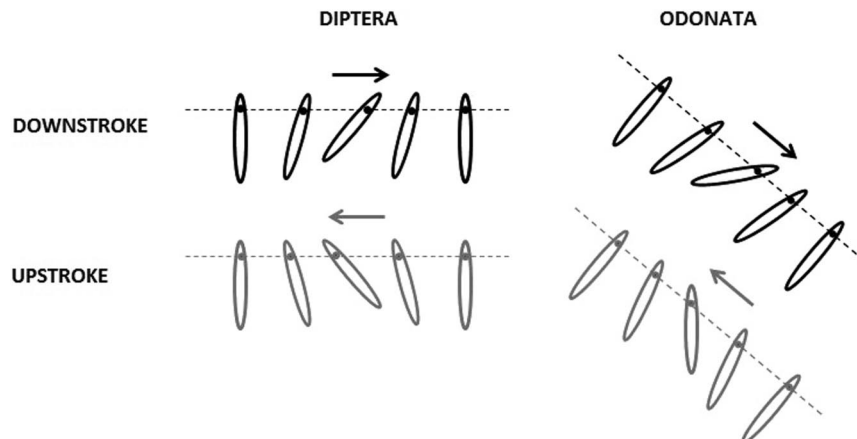


Fig. 2 Illustration of Dipteran vs Odonate flight (adapted from [3,19]).

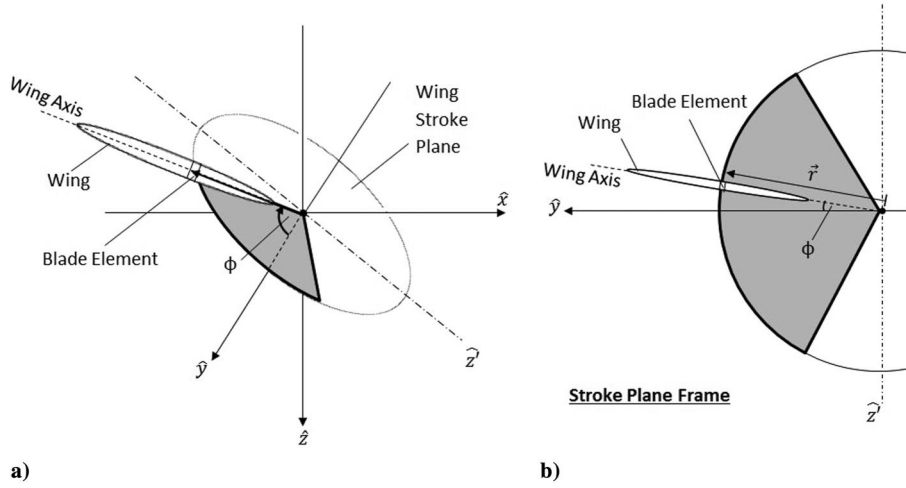


Fig. 4 Illustration of the coordinate system used: a) global coordinate system and b) stroke plane frame.

mechanism produces similar energy savings compared to a resonant system but allows for efficient operation at lower wing-beat frequencies. Through the use of numerical methods and a quadratic damping term, Tang and Brennan [18] showed that the click mechanism improves the aerodynamic efficiency even at a frequency ratio as low as  $\Omega = 0.1$ , where  $\Omega$  is defined as

$$\Omega = \frac{2\pi f}{\omega_n} \quad (2)$$

In dragonflies, it is believed that elastic mechanisms could be present, although much reduced [11]. This is due to several characteristics that define the dragonfly wing-actuation system. Studies [9] have shown that the dragonfly employs a direct-drive mechanism whereby the actuators/muscles of the dragonfly are attached to the wing root and actuate the wing directly. Additionally, dragonflies flap their wing in an inclined stroke plane and use active control of the angle of attack to modulate the aerodynamic forces acting on the wing [3,24]. It is these features that define the dragonfly planform and differentiate it from other insect species (see Fig. 2). Another significant difference between Odonate and Dipteran flight is the proportion of aerodynamic work to inertial work. Dipteran insects have a significantly higher proportion of inertial to aerodynamic work [4,11,25]. It is these differences that make the Odonate mode of flapping distinct from the Dipteran. As such, the assumptions and idealizations used to analyze both these modes are different. This paper will analyze the effects of asymmetrical flapping as it applies to both these unique styles of flapping. It should, however, be noted that we do not attempt to apply comparison between the two modes of flapping. We simply aim to analyze what is a phenomenon observed in both modes of flapping respective to those modes.

### B. Quasi-Steady Model

To quantify the performance of the asymmetrical profile, a numerical quasi-steady blade-element model of the aerodynamic forces acting on the flapping wing was produced. Similar studies have been performed by Weis-Fogh [4] and Norberg [26]. Sane and Dickinson [27] also presented a similar analysis, with additional aerodynamic effects due to wing rotation. The wing was divided into spanwise sections and the instantaneous forces calculated on each section based on the local translational and angular velocity. The forces across all slices were then summed to obtain the forces/torques acting on the wing at any point in time.

The wing kinematics were represented using equations from Wang [24] for a wing flapping in an inclined stroke plane with angle  $\beta$  (see Fig. 2). In Wang's representation of the dragonfly flapping profile, the flapping angle  $\phi$  is represented using a sinusoid. A similar flapping profile is employed in this study. To model the asymmetric

flapping, two half-cosine waves were used with different angular velocities (see Fig. 3).

The downstroke can be expressed as

$$\phi(t) = \phi_0 \cos\left(\frac{\pi t}{\eta T}\right) \quad (3)$$

for  $0 < t < \eta T$ , where  $\eta = T_{\text{Downstroke}}/T$ . The upstroke is represented by

$$\phi(t) = \phi_0 \cos\left[\frac{\pi}{(\eta - 1)T}(t - \eta T) + \pi\right] \quad (4)$$

for  $\eta T < t < T$ . Therefore,  $\eta$  represents the degree of asymmetry in the flapping profile. If  $\eta = 0.5$ , the flapping profile is symmetric. However, if  $\eta < 0.5$ , the downstroke period is shorter than the upstroke. The opposite is true for  $\eta > 0.5$ . The angle of attack  $\alpha$  in the stroke plane is represented using a sinusoid,

$$\alpha = \alpha_m - \alpha_0 \sin(2\pi f t^*) \quad (5)$$

Once the flapping and pitch profile have been determined, the local velocity at each spanwise section, or blade element, of the wing as well as the lift and drag coefficients can be determined. The local velocity is calculated by

$$\vec{V} = |\vec{r}| \dot{\phi} \quad (6)$$

where  $r$  and  $\phi$  are illustrated in Fig. 4.

Equations for the lift and drag coefficients were obtained from three-dimensional experiments performed by Wang et al. [28] and Dickinson et al. [29]. Similar coefficients were obtained by Wang et al. computationally and used as the basis for analyzing dragonfly flight [3,29],

$$C_L = 0.225 + 1.58 \sin(2.13\alpha - 7.2) \quad (7)$$

$$C_D = 1.92 - 1.55 \cos(2.04\alpha - 9.82) \quad (8)$$

Similarly, the rotational pressure forces [27,29] for each section of the wing were modelled as

$$C_{\text{rotation}} = \pi(0.75 - \hat{x}_0) \quad (9)$$

where  $\hat{x}_0$  is the nondimensional axis of rotation from the leading edge. Following that, the forces due to the translational (refer to Fig. 5) and rotational (refer to Fig. 6) motion of the wing could be calculated.

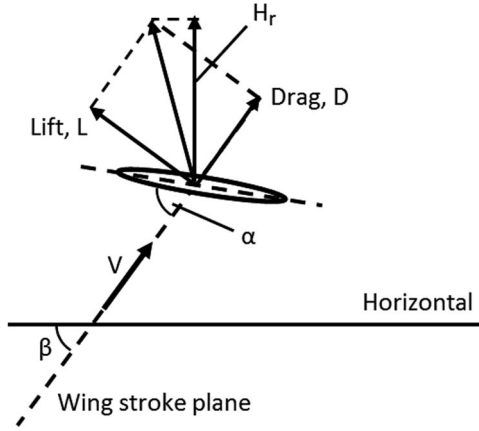


Fig. 5 Illustration of the aerodynamic forces due to the flapping motion acting on a single section of the wing.

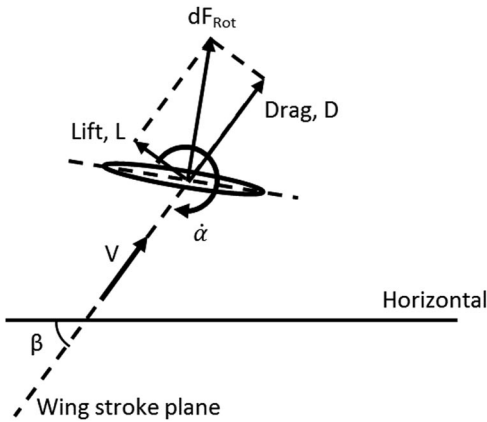


Fig. 6 Illustration of the pressure force due to rotational circulation acting on a single section of the wing.

The aerodynamic forces at each section of the wing are given by

$$dL = 0.5\rho V^2 C_L dS \quad (10)$$

$$dD = 0.5\rho V^2 C_D dS \quad (11)$$

$$dF_{\text{rotation}} = C_{\text{rotation}} \rho c^2 V \dot{\alpha} dr \quad (12)$$

The overall aerodynamic force can then be calculated by summing the force of all wing sections. The wing was divided into 27 wing sections. The stroke was analyzed over a 1000 time steps. It should be noted that the directions of the lift and drag forces are with respect to the velocity of the wing section. To determine the actual lift force generated by the wing in the global reference frame, a coordinate transformation was employed. The vertical force in the frame of reference of the blade element is given by Eq. (13):

$$H_r = L \cos \beta + D \sin \beta + F_{\text{rotation}} \sin(\alpha - \beta) \quad (13)$$

The vertical force with respect to the global frame of reference is

$$H_{r,\text{vert}} = H_r \cos \phi_{\text{Proj}} \quad (14)$$

An equation for  $\cos \phi_{\text{Proj}}$  was calculated using the formulation from Norberg [26]. Therefore, the vertical force with respect to the global frame of reference is

$$H_{r,\text{vert}} = H_r \sqrt{1 - \sin^2 \alpha \sin^2 \beta} \quad (15)$$

To determine the feasibility of asymmetrical flapping, we used the aerodynamic efficiency as the performance metric, which is the ratio of the mean aerodynamic lift to the mean power consumed (i.e.,  $\bar{L}/\bar{P}$ ). This is a common performance metric used in the design of aircraft, as it relates to how much power is required to maintain steady flight [30]. In platforms that are capable of hovering, it also relates to how much power is required to keep the platform in a steady hover mode [16]. To determine the power consumed in the wing stroke due to just the flapping motion, we use the equation

$$P = \tau \omega \quad (16)$$

where

$$\tau = \tau_{\text{Aero}} + \tau_{\text{Inertial}} + \tau_{\text{Elastic}} \quad (17)$$

$$\omega = \dot{\phi} \quad (18)$$

The torques due to inertial and elastic forces were modelled as being linear, i.e.,  $\tau_{\text{Inertial}} = I\dot{\phi}$  and  $\tau_{\text{Elastic}} = k\phi$ . In addition to the power associated with the flapping motion, we also investigated the aerodynamic and inertial power associated with wing rotation, which is one of the main mechanisms of lift generation [29]. The power associated with wing rotation is given as

$$P_{\text{Rotation}} = \tau_{\text{Rotation}} \dot{\alpha} \quad (19)$$

where

$$\tau_{\text{Rotation}} = \tau_{\text{Rotation,Aero}} + \tau_{\text{Rotation,Inertial}} \quad (20)$$

To determine the  $\tau_{\text{Rotation,Aero}}$ , we assumed the aerodynamic forces act through a constant center of pressure at 75% of the chord [27,29]. While this is only an approximation, it should be noted that studies have shown that  $\tau_{\text{Rotation,Aero}}$  is small compared to the aerodynamic torques generated by the flapping mode [25]. As such, any inaccuracies resulting from such an approximation will be minimal. A linear model, i.e.,  $I_{\text{Rotation}} \ddot{\alpha}$ , was used to represent the torque due to the rotational inertia  $\tau_{\text{Rotation,Inertial}}$ .

### C. System Parameters

The parameters used in the simulation were selected to best represent the characteristics of Diptera and Odonata as described by Wang [3,31]. These parameters are highlighted in Table 1. The wing size was selected to be representative of birds or prehistoric dragonflies [32,33], which are in the order of MAVs and small UAVs.

The design of the wing was based off a dragonfly wing profile. A larger 30 cm wing design was selected, operating at lower wing-beat frequencies. This wing size is of the order of that found on prehistoric dragonflies (i.e., the *Meganeuropsis Permiana* had a wingspan of 70 cm [34]). The larger wing allows for larger aerodynamic forces to be generated, while the lower wing-beat frequencies reduce the effect of mechanical loads on the system. For the nominal wing-beat frequency, 5 Hz was selected, as the studies of birds and insects by Greenewalt [23] have shown that similarly sized birds flap at 3–10 Hz. Although the wing-beat frequencies and wing spans differ from that of the Diptera and Odonates, the wing kinematics copy the Diptera and dragonfly, respectively. The inertia of the wing was based on a Mylar and carbon construct described by Lentink et al. [35]. The final design is shown in Fig. 7. The moment of inertia of the wing in the flapping and pitching axis was calculated and shown in Table 2. It should be noted that this wing design was dragonfly inspired and as such has low inertia and is aerodynamically dominated. This is different from the Dipteran system, which is inertially dominated. As such, the Dipteran mode of flapping will use the same wing shape, but

**Table 1** Flapping-wing parameters

Property	Diptera-inspired flapping mode	Dragonfly-inspired flapping mode
$f$ , Hz	1.6, 2.5, 5.0, 6.0	5.0
$\beta$ , deg	0.0	60.0
$\phi_0$ , deg	45.0	45.0
$\alpha_0$ , deg	45.0	45.0
$\alpha_m$ , deg	90.0	75.0, 90.0, 105.0, 120.0

with a different treatment of inertia. This will be presented in Sec. III.A.

### III. Results

#### A. Analysis of Dipteran Flapping Mode

As mentioned previously in Sec. II.A, studies performed by Tang and Brennan [18] showed that the click mechanism improves the aerodynamic efficiency even at a frequency ratio of  $\Omega = 0.1$ . We expand on this analysis by using the quasi-steady, blade-element model listed in Sec. II.B. We compared the effects of implementing varying degrees of asymmetry on a Dipteran flapping profile by changing  $\eta$ . This was performed for a range of frequency ratios centered around the resonant frequency. Rather than using the wing inertias calculated in Table 2, the inertia of the wing was selected to better represent the inertia-dominated characteristics of Dipteran insects. Weis-Fogh [4] showed that the proportion of inertial work for Dipteran insects was on average 0.52 of the combined aerodynamic and inertial work. Using this ratio and knowing the aerodynamic work is calculated by integrating the aerodynamic power over time, the inertial work can be determined. The inertia can thus be calculated using

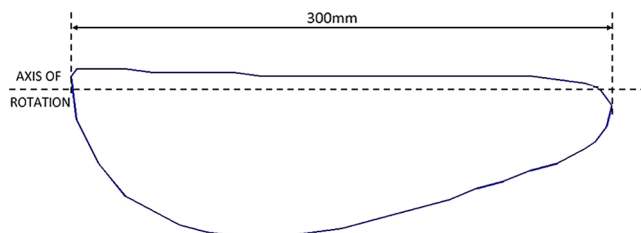
$$I = \frac{\text{Inertial work}}{8\pi^2 f^2 \phi_0^2} \quad (21)$$

Once the inertia is known, the elasticity of the system  $k$  is calculated using Eq. (1), assuming a natural frequency of 5 Hz. We first observed the changes in lift associated with the frequency change and asymmetrical flapping. This was performed for a range of wing-beat frequencies above and below the resonant frequency (i.e.,  $\Omega = 0.3, 0.5, 1.0, 1.2$ ). Lift was calculated for values of  $\eta$  between 0.1 and 0.5. As expected, lift generation decreased with a decreased wing-beat frequency. The mean lift of the system is shown in Fig. 8. However, we notice that across all wing-beat frequencies there is an increase in mean lift with decreasing  $\eta$ .

To explain this phenomenon, we observe the relationship between  $\eta$  and the time-averaged square of the angular velocity  $\omega^2$ . This is significant because lift is approximately proportional to the square of the angular velocity (i.e.,  $L \propto \omega^2$ ). The time-averaged square of the angular velocity can be calculated according to Eq. (22):

$$\overline{\omega^2} = \frac{\int_0^T \omega^2 dt}{T} \quad (22)$$

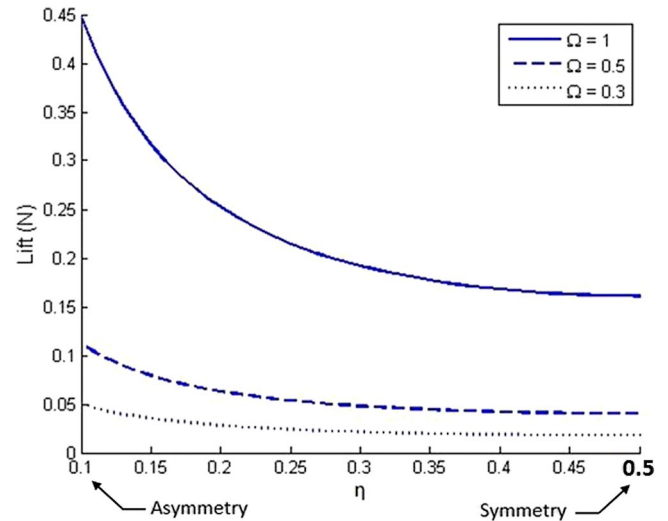
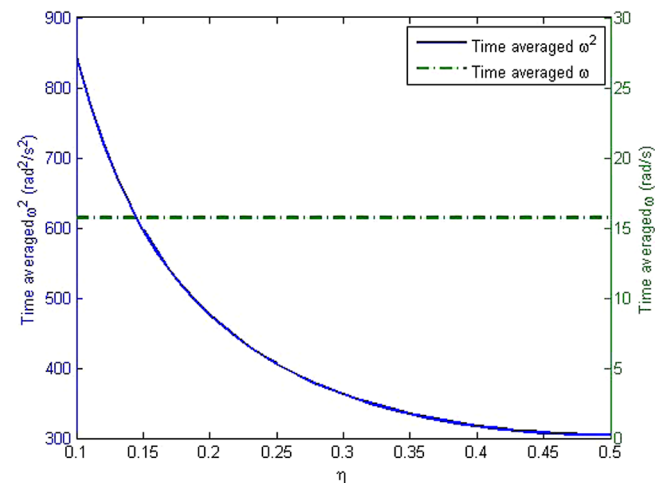
The results are illustrated in Fig. 9 for the case of  $\Omega = 1$ . The effect of decreasing  $\eta$  is to increase  $\omega$  in the downstroke and decrease  $\omega$  in

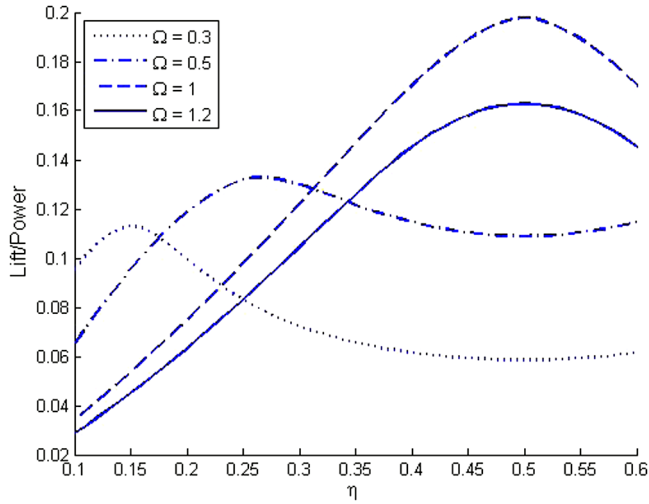
**Fig. 7** Illustration of a dragonfly-inspired wing.**Table 2** Inertial breakdown of the wing

Object	Flapping inertia, $\text{kg} \cdot \text{m}^2$	Rotational inertia, $\text{kg} \cdot \text{m}^2$
Skin	$1.14 \times 10^{-6}$	$6.58 \times 10^{-8}$
Spars + ribs	$1.22 \times 10^{-5}$	$1.37 \times 10^{-7}$
Total	$1.33 \times 10^{-5}$	$2.02 \times 10^{-7}$

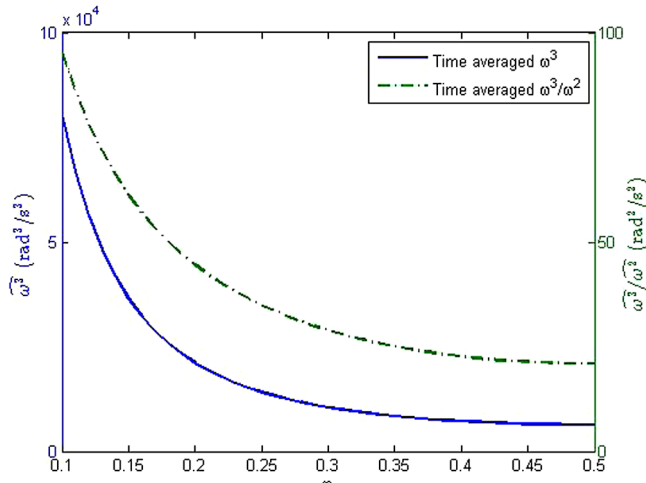
the upstroke, leading to no significant change in time-averaged  $\omega$ . However, we see that, when taken across the entire period of the wing-stroke,  $\omega^2$  is higher. Correspondingly, as lift is  $\propto \omega^2$ , there is an increase in lift with decreasing  $\eta$ .

Following the analysis of lift, the lift-to-power ratios were calculated across a range of  $\eta$  values. This is illustrated in Fig. 10. At the resonant frequency, it can be seen that the use of asymmetrical flapping has detrimental effects on the aerodynamic efficiency of the system. However, at wing-beat frequencies below the natural frequency, the benefits of asymmetrical flapping become more pronounced. We see that as the wing-beat frequency decreases the value of  $\eta$  that produces the optimal  $L/P$  decreases. This would suggest that at progressively lower wing-beat frequencies the wing stroke that produces maximum  $L/P$  is more asymmetrical. In

**Fig. 8** Mean lift generated for a Dipteran-inspired flapping mode across a range of  $\eta$  for different wing-beat frequencies.**Fig. 9** Time-averaged angular velocity and time-averaged square of the angular velocity vs  $\eta$  performed for a Dipteran-inspired flapping mode ( $\Omega = 1$ ).



**Fig. 10** Aerodynamic efficiency for a Dipteran-inspired flapping mode,  $\bar{L}/P$ , across a range of  $\eta$  and different wing-beat frequencies.



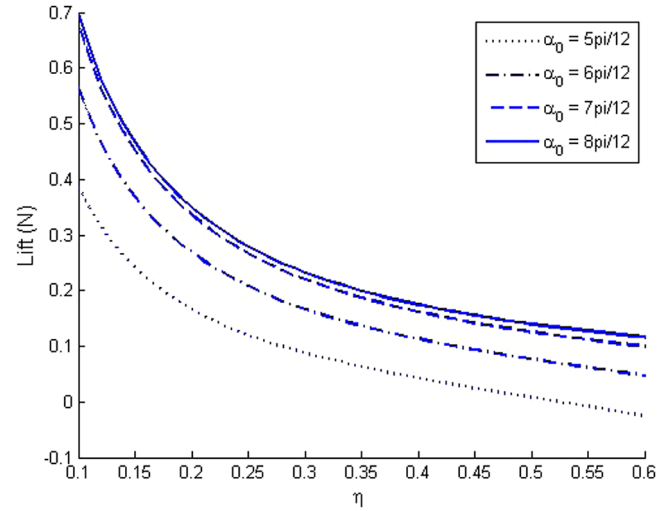
**Fig. 11** Time-averaged  $\omega^3$  and  $\omega^3/\omega^2$  vs  $\eta$  performed for a Dipteran-inspired flapping mode ( $\Omega = 1$ ).

addition, the improvements in efficiency achieved between the optimal asymmetrical flapping profile and a symmetrical profile (i.e.,  $\eta = 0.5$ ) become more significant. At  $\Omega = 0.3$ , the value of  $\eta$  that produces the ideal  $L/P$  is 0.18, and there is a 75% increase in performance with the use of asymmetry.

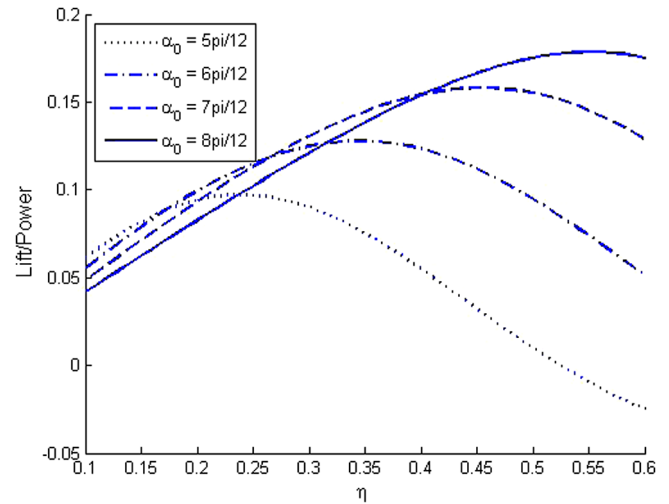
We also observe the change in time-averaged  $\omega^3$  (i.e.,  $\bar{\omega}^3$ ) with  $\eta$  (see Fig. 11). This value is important for analyzing the inertial power of the system. The significance of this will be discussed in Sec. IV.

### B. Analysis of Dragonfly Mode of Flapping

As discussed in Sec. I, the dragonfly mode of flapping differs from the Dipteran mode in a few ways. The dragonfly mode of flapping is characterized by motion in an inclined stroke plane and active modulation of the angle of attack of the wing to manipulate aerodynamic forces. Another characteristic of the dragonfly flapping profile is reduced reliance on resonant mechanisms as demonstrated by the energetic and functional requirements of a dragonfly wing-actuation system [11]. This is further supported by their use of a direct-drive mechanism that largely bypasses the elastic thorax. To simulate this effect, we initially modeled the system as a direct-drive mechanism with no elasticity (i.e.,  $k = 0$ ). Another characteristic of the dragonfly wing-actuation system is the significantly larger proportion of aerodynamic work vs inertial work [11,25,36,37]. Initial quasi-steady analyses performed on the dragonfly flapping



**Fig. 12** Lift generated for a Odonate-inspired flapping mode by the system for different values of  $\eta$  and different  $\alpha_0$ .

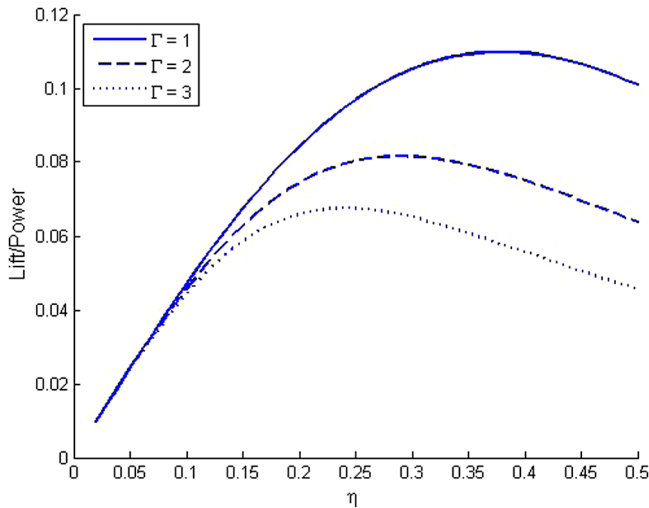


**Fig. 13** Aerodynamic efficiency for a Odonate-inspired flapping mode,  $\bar{L}/P$ , for different values of  $\eta$  and different  $\alpha_0$ .

profile without asymmetrical flapping showed the ratio of aerodynamic work to inertial work was 12.0 for the system described in Sec. II.C. This shows that the simulated system is dominated by aerodynamic forces and is congruent with the dragonfly mode of flapping.

We compared the effects of implementing varying degrees of asymmetry on the dragonfly flapping profile by changing  $\eta$ . This was performed for a range of  $\alpha_0$ . The flapping amplitude  $\phi_0$  was kept at 45 deg to remain consistent with the flapping amplitude used in Sec. III.A. The elasticity of the system was initially assumed to be zero to better represent the direct-drive wing-actuation system design. The mean lift of the system is shown in Fig. 12. Across all  $\alpha_0$ , there is an increase in the mean lift generation with decreasing values of  $\eta$ . The increases in lift due to asymmetrical flapping are substantially more pronounced in the dragonfly mode of flapping. This can be explained by the fact that a dragonfly generates most of its lift on the downstroke [25,31]. By decreasing  $\eta$ , the velocity in the downstroke is increased, resulting in more lift being generated across a shorter time. This is in contrast to the Dipteran mode of flapping, which generates substantial lift in both the upstroke and downstroke. During Dipteran flapping, increases in lift due to a quicker downstroke are largely countered by a reduction in lift in the upstroke.





**Fig. 14** Aerodynamic efficiency for a Odonate-inspired flapping mode,  $L/\bar{P}$ , for different values of  $\eta$  and different  $\Gamma$  ( $\alpha_0 = 7\pi/12$ ).

We then observed the aerodynamic efficiency of the lift generation mechanism. The metric used to quantify this efficiency is the mean-lift-to-mean-power ratio, i.e.,  $L/\bar{P}$ . As shown in Fig. 13, the aerodynamic efficiency of the system is significantly affected by varying the asymmetry in the flapping profile. For lower values of  $\alpha_0$ , improvements in aerodynamic efficiency of up to 9.8 times are observed with the implementation of asymmetrical flapping vs symmetrical flapping. However, the highest values of aerodynamic efficiency are observed where  $\alpha_0 = 8\pi/12$  and no asymmetry is used.

The previous cases assumed an idealized Odonate mode of flapping with no elasticity present in the system. However, elasticity is a characteristic that is inherent in all natural systems and compliant designs [38–40]. To model the effects of elasticity in the dragonfly wing system, the elasticity was varied such that the ratio of elastic to aerodynamic torques, i.e.,  $\Gamma = \tau_{\text{Elastic,max}}/\tau_{\text{Aero,max}}$ , were 1.0, 2.0, and 3.0, respectively. The aerodynamic efficiencies are illustrated in Fig. 14. With increasing elasticity of the system, improvements to aerodynamic efficiency are apparent. At  $\Gamma = 3$ , there is a 50% increase in aerodynamic efficiency with the implementation of asymmetry in the flapping profile ( $\eta = 0.23$ ).

#### IV. Discussion

In this paper, we investigated the effect of an asymmetric flapping profile for hover in the context of a real-world insect-inspired wing-actuation system.

To model the aerodynamics of the system, we used a quasi-steady blade-element model. This is a modification of the work performed by Sane and Dickinson [27]. The quasi-steady blade-element model detailed in Sec. II.B allows us to capture representative lift and drag forces in three-dimensional space, while also allowing us to investigate the combined effects of flapping and pitching wing movements. Sinusoidal wing kinematics that included a factor of asymmetry  $\eta$ , which changed the time spent in the upstroke to that in the downstroke were prescribed. Elastic and inertial models that were proportional to the prescribed angular position  $\phi$  and angular acceleration  $\ddot{\phi}$ , respectively, were developed. This was performed for two distinct modes of flapping, namely, Dipteran and Odonate. Because of the differences between these two flapping modes, the treatment of inertia and elasticity differed. Analysis of the Dipteran mode of flapping used inertial values derived from the inertial work, which studies have shown to be approximately 0.52 of the combined aerodynamic and inertial work [4]. Assuming resonant flapping, which is characteristic of Diptera, the elastic constant was calculated using Eq. (1). For Odonates, the assumption of resonant flapping is not applicable. As such, we opted to perform the quasi-steady

analysis over a range of elastic values from an idealized, zero-elasticity case to a case with dominant elastic forces. This allowed us to analyze the effects of asymmetrical flapping for a range of elasticity values. Inertial values were based on a dragonfly-inspired wing design used in a representative MAV system [41–43]. The same wing inertia is too low for use in a Dipteran system as Diptera are characterized by dominant inertial forces. Another fundamental difference between the Odonate and Dipteran modes is the inclined stroke plane used by Odonata. This has also been reflected in the analysis.

The quasi-steady model does have its limitations in analyzing the aerodynamic forces being applied to the system. However, it should also be noted that effects like unsteady wake and added mass forces, while important, are not the primary means of lift generation. This paper aims to investigate the effects of asymmetrical flapping on the primary mode of lift generation to determine system level suitability. Another assumption was the use of lift and drag coefficients presented by Wang et al. [28] and Dickinson et al. [29]. Similar coefficients were obtained by Wang computationally and used as the basis for analyzing dragonfly flight [3,28]. To determine the effect of lift and drag coefficients on the results of the quasi-steady analysis, a sensitivity analysis was performed in which lift and drag coefficients were simultaneously increased by  $\pm 50\%$  and the analysis redone. The results are shown in the Appendix. These results demonstrate that changing the lift and drag coefficients has no significant impact on the overall trends. Although the accuracy of the quasi-steady analysis might not be absolute in nature, it is a fair investigation of the relationship between the aerodynamic modes, inertial effects, and elasticity. Without simplifying assumptions, any sort of coherent design considerations become infeasible. This paper will set the basis for further, more accurate analysis that includes nonlinear elasticity and nonsteady aerodynamic effects.

In Sec. III.A, we analyzed the effects of asymmetrical flapping on the aerodynamics of the Dipteran flapping mode. Previous studies by Tang and Brennan [18] showed that the improvements in aerodynamic efficiency with the use of a click mechanism could be attributed to the phase shifting effect and the generation of higher wing velocities, both of which are also present in asymmetrical flapping. Initially, the mean lift over the wing stroke period was considered. We see an increase in lift generated with increased wing-beat frequency. As lift is proportional to  $\omega^2$ , a quicker wing-beat frequency would result in greater lift forces being generated. We then observed the changes in lift associated with changing asymmetry. Increasing asymmetry (i.e., reducing  $\eta$ ) increased the lift that was being generated. To understand why, we observed the change in the time-averaged  $\omega$  and  $\omega^2$  across the wing-beat with changing  $\eta$  (see Fig. 9). We observed that asymmetry had no effect on  $\hat{\omega}$ , as any gains in angular velocity from a quicker downstroke were balanced by the losses in angular velocity in the upstroke. However, when we considered the change in  $\hat{\omega}^2$  with  $\eta$ , we saw that  $\hat{\omega}^2$  increased with increasing asymmetry, which explained the observed increases in lift. Following the lift force analysis, the aerodynamic efficiency was considered. The mean lift over mean power was used as the measure of aerodynamic efficiency. The analysis was performed for a range of wing-beat frequencies from high-frequency, inertially dominated systems to low-frequency, elastically dominated systems. It was observed that with inertially dominated systems there was no advantage to using asymmetrical flapping (see Fig. 10). However, in systems dominated by elasticity, this nonresonant mode of asymmetrical flapping has the potential to improve the aerodynamic efficiency. At a frequency ratio,  $\Omega = 0.3$ , a 75% increase in performance was observed with the use of asymmetrical flapping (see Fig. 10). An explanation for this can be derived from observations of the forces applied to a compliant flapping-wing mechanism. In a harmonic system with both elastic and inertial forces, the ratio of elastic to inertial forces is given by  $k/I\omega^2$ . At lower wing-beat frequencies, elastic forces are dominant, and a portion of work is used to overcome excess elasticity. Elastic power is proportionate to  $\omega$ , while lift generation is proportionate to  $\omega^2$ . Therefore, when  $\eta$  is decreased,  $\hat{\omega}^2$  increases, while  $\hat{\omega}$  stays approximately the same (see

Fig. 9), leading to higher aerodynamic efficiency. The opposite is true at higher wing-beat frequencies. Inertial forces dominate, however, inertial power is proportional to  $\omega^3$ . As shown in Fig. 11, with decreasing  $\eta$ , both  $\widehat{\omega}^2$  and  $\widehat{\omega}^3$  increase; however,  $\widehat{\omega}^3$  increases faster than  $\widehat{\omega}^2$ . An indication of the amount of inertial power that is consumed during the wing beat as inertial power is proportional to  $\omega^3$  is  $\widehat{\omega}^3$ . This indicates that as  $\eta$  reduces the amount of inertial power required increases faster than the lift generated, which explains the reduction in aerodynamic efficiency with decreasing  $\eta$ . This suggests that in a nonresonant flight mode below the natural frequency asymmetrical flapping can be used to tune the system to obtain increased aerodynamic efficiency in hover. In fixed-frequency systems, it can also serve as a secondary or alternate means to modulate the lift of the system. Brennan et al. [44] and Chin and Lau [16,17] observed improved aerodynamic efficiency at low frequencies with the use of a click-induced asymmetrical flapping profile. Caution must be used when testing theory about insect flight against empirical results from synthetic artifacts due to the gap that appears to exist between the level of optimization of insects and current mechanical systems [45]. Differences also exist between the wings used by insects, which appear to have complex and optimized aerostructural properties [46], and those used in current mechanical systems. With these considerations in mind, it seems that, unlike our linear-elastic system, Brennan [44] and Chin and Lau [16,17] have observed improved aerodynamic efficiency even at higher, inertial-dominated wing-beat frequencies. This would suggest that the energy-saving mechanism is in part due to the nonlinear elastic nature of the click mechanism. Results from Chin and Lau [16,17] suggest that the click mechanism is at least a third-order polynomial that has the potential to offer finer optimization of the force profile, and hence aerodynamic efficiency, than just a linear elastic storage mechanism. Another variable that needs to be considered is the effect asymmetrical flapping has on wing flexure. Insect wings have finite stiffness, and as such will experience chordwise and spanwise bending when aerodynamically or inertially loaded [35,47,48]. Asymmetrical flapping serves to modulate velocity in the up and down strokes, which in turn influences the degree of passive wing rotation and hence the angle of attack of the wing.

In Sec. III.B, we modified the flapping model to reflect the key characteristics of dragonfly flight, which include an inclined stroke plane and a lower reliance on elasticity. This allows us to determine if the asymmetry observed in dragonfly flight is beneficial to hover performance. Varying degrees of asymmetrical flapping were applied to the model and tested for different mean angles of attack. It was found that for lower values of  $\alpha_0$  varying the downstroke-to-upstroke ratio resulted in a significant improvement in performance. The best aerodynamic efficiencies were observed when  $\alpha_0$  was high, and no asymmetry was used. This can be attributed to the fact that, like the previous case, which was inertially dominated (Sec. III.A,  $\Omega > 1.0$ ), the increases in lift were proportional to  $\omega^2$ , while the required power was proportional to  $\omega^3$ . As such, the increase in flapping velocity in the downstroke was not beneficial to the aerodynamic efficiency of the system. The improvements in performance using asymmetrical flapping at lower  $\alpha_0$  values were associated with the phase shifting effect. This led to the suggestion that the use of asymmetrical flapping in hover is secondary to modulating the angle of attack in the dragonfly flapping profile. This is congruent with findings by Wakeling and Ellington [12], who showed significant but small variations in the downstroke-to-upstroke ratio in the Odonate mode of flight. Another explanation for these variations could be the elasticity in the dragonfly. In many flapping-wing systems, including the dragonfly, some elasticity will be present. Although the elasticity might not be introduced to tune a resonant system, it is a design parameter in many compliant flapping-wing systems [16,17,49]. Therefore, it is necessary to observe the dragonfly wing-actuation system with elasticity introduced. Results showed that asymmetrical flapping improved performance significantly with an increase in elasticity. The increases in performance can be attributed to the fact that the power required to overcome elastic forces was proportional to  $\omega$ . Therefore, by

increasing the speed of the downstroke, the aerodynamic forces increased faster than the power required to overcome elastic forces. So in systems in which elastic forces are significant the benefits of having asymmetrical flapping can be realized.

## V. Conclusions

This paper has presented the use of a quasi-steady blade-element model that was used to investigate the benefits of asymmetrical flapping in both an inertia-dominated, horizontal stroking system and an aerodynamically dominated, inclined stroke plane system, characteristic of Dipteran and Odonate insects, respectively. The results show that in a Diptera-inspired system stroke asymmetry increases the lift generation, which can be attributed to higher  $\omega^2$  across the wing beat. Additionally, at wing-beat frequencies below the natural frequency, asymmetrical flapping improves aerodynamic efficiency. At and above the natural frequency, asymmetrical flapping is detrimental to performance, which was explained by the higher proportion of inertial power above the natural frequency, which is sensitive to the time-averaged  $\omega^3$ . One should also note that this finding is contradictory to results for a click mechanism that shows higher aerodynamic efficiencies up to and at the resonant frequency. This suggests that the energy savings of a click mechanism are not just an aerodynamic effect but rather a nonlinear elastic phenomenon. Asymmetrical flapping could be a means to optimize the aerodynamic efficiency of nonresonant systems below the natural frequency. It can also be used as an alternate means of lift modulation. The use of asymmetrical flapping was then investigated in an idealized dragonfly mode of flight with no elasticity initially. Asymmetrical flapping was found to improve aerodynamic efficiency for some mean angles of attack. However, the most efficient flight was still achieved via tuning the angle of attack of the wing. With the introduction of elasticity into the system, it was observed that asymmetrical flapping produced significant improvements to aerodynamic efficiency. This suggests that the benefits of asymmetrical flapping in hover can be better realized in systems suffering from or exploiting elasticity.

## Appendix: Sensitivity Analysis

### Appendix A: Dipteran Flapping Mode, $C_L, C_D$ Decreased by 50%

See Figs. A1 and A2.

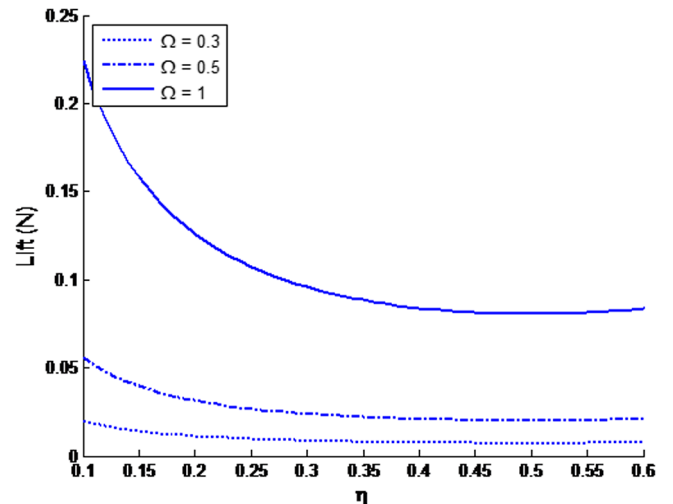


Fig. A1 Lift for a Dipteran-inspired flapping mode.



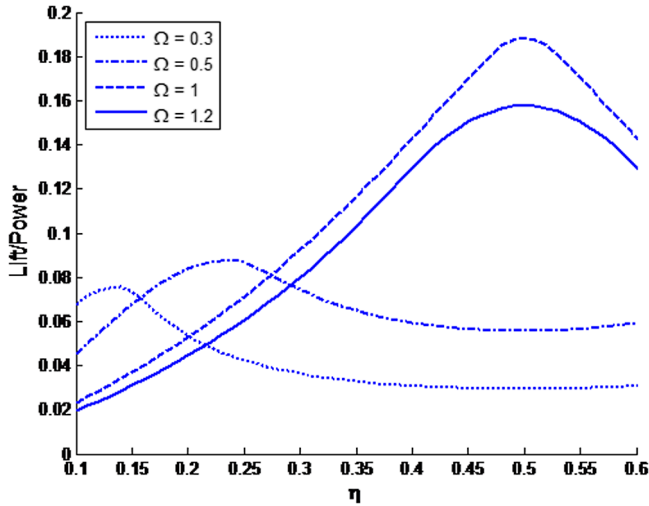


Fig. A2 Aerodynamic efficiency for a Dipteran-inspired flapping mode.

### Appendix B: Dipteran Flapping Mode, $C_L, C_D$ Increased by 50%

See Figs. B1 and B2.

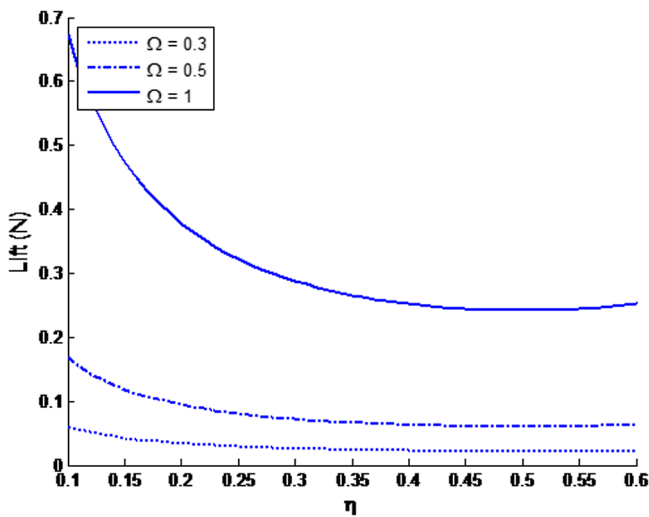


Fig. B1 Lift for a Dipteran-inspired flapping mode.

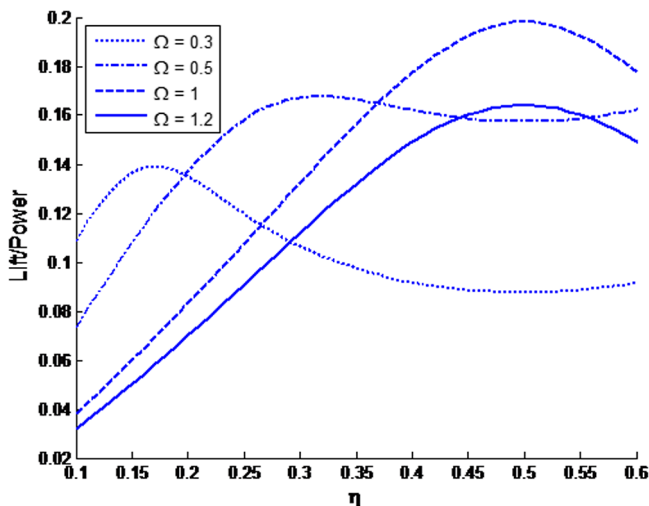


Fig. B2 Aerodynamic efficiency for a Dipteran-inspired flapping mode.

### Appendix C: Odonate Flapping Mode, $C_L, C_D$ Decreased by 50%

See Figs. C1–C3.

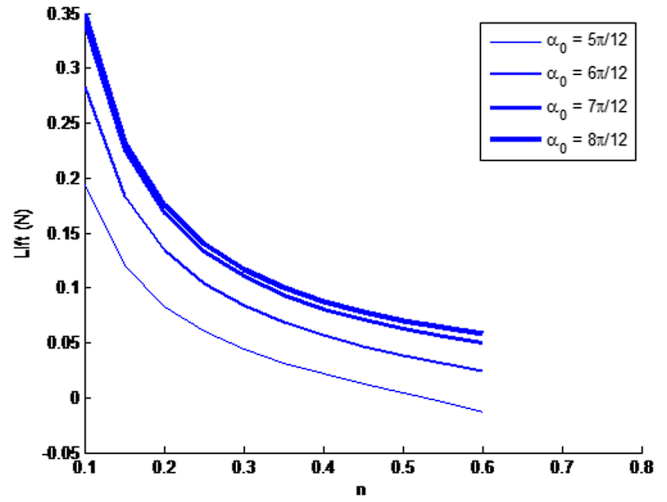


Fig. C1 Lift for an Odonate-inspired flapping mode.

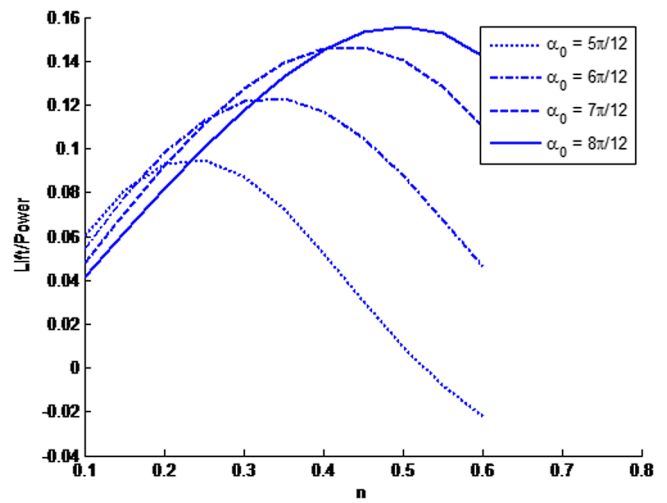


Fig. C2 Aerodynamic efficiency for an Odonate-inspired flapping mode.

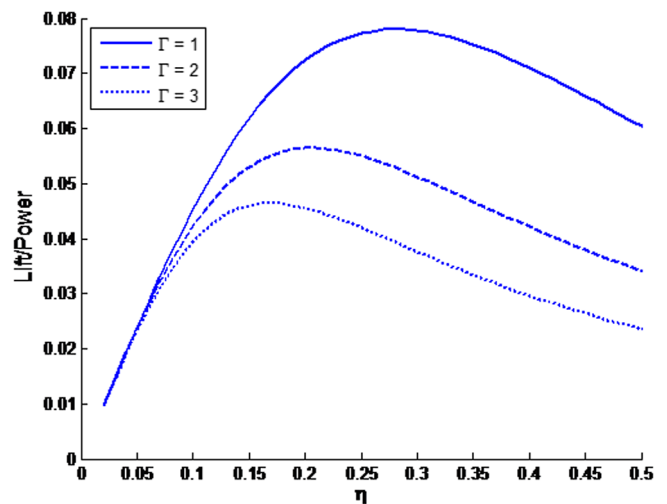


Fig. C3 Aerodynamic efficiency for an Odonate-inspired flapping mode with elasticity.

## Appendix D: Odonate Flapping Mode, $C_L, C_D$ Increased by 50%

See Figs. D1–D3.

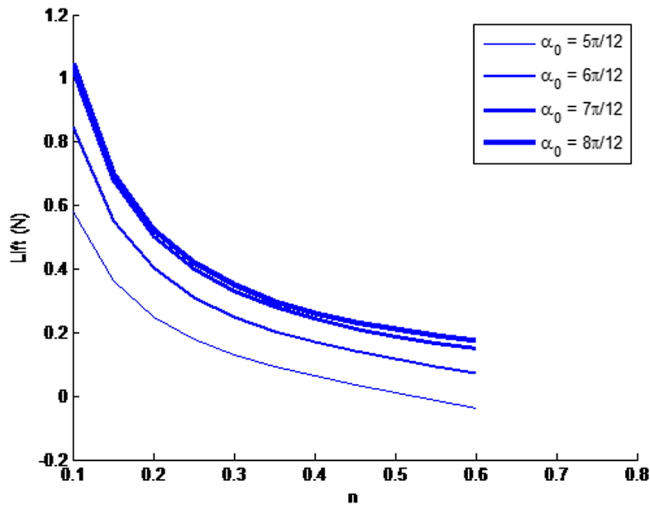


Fig. D1 Lift for an Odonate-inspired flapping mode.

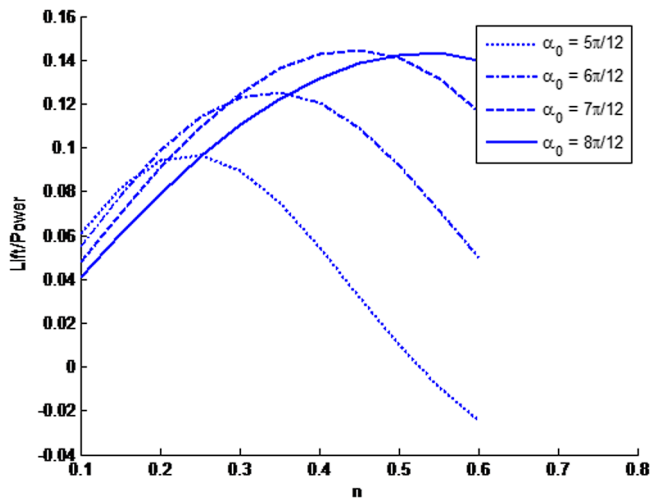


Fig. D2 Aerodynamic efficiency for an Odonate-inspired flapping mode.

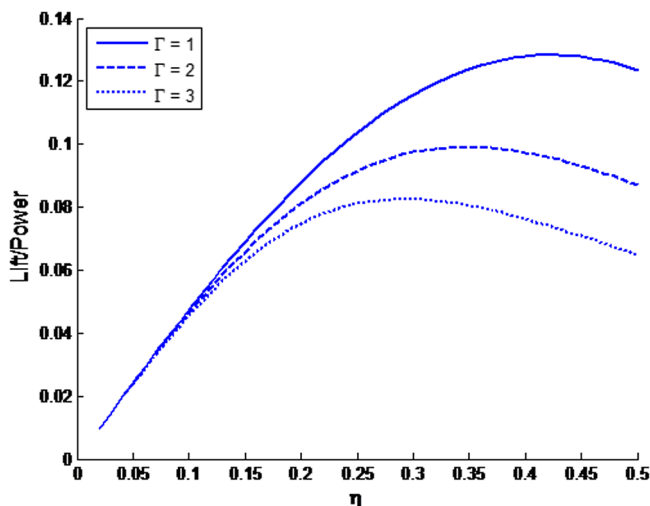


Fig. D3 Aerodynamic efficiency for an Odonate-inspired flapping mode with elasticity.

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