INDIRECT ARGUMENT : TWO FAMOUS THEOREM

Well Ordering Principle : Every non empty set of integers has a least smallest element.

1) Irrationality of $\sqrt{2}$:

Suppose that $\sqrt{2}$ is rational.

So, $\sqrt{2} = \frac{m}{n}$ where m, n are integers with no common factors.

$$2 = \frac{m^2}{n^2}$$

$$m^2 = 2n^2$$

If m^2 is even then m is even.

$$\therefore m^2 = (2k)^2 = 2n^2$$

$$\text{or, } n^2 = 2k^2$$

If n^2 is even, then n is even

Thus m, n have a common factor 2 which contradicts our supposition. Hence, $\sqrt{2}$ is irrational.

Proposition : For any integer a and any prime number p, if $p \mid a$ then $p \nmid (a+1)$

2) There are infinitely many primes.

Suppose there are finite primes.

$p_1, p_2, p_3 \ldots p_n$ (Only these prime exist)

Let $Q = (p_1 \times p_2 \times p_3 \cdots p_n) + 1$

Case I: Q is prime : If this is true then Q was not in our original list of primes, which is contradictory.

Case II: Q is not prime: If Q is not prime then it must be divisible by some prime in the original list. But that is not possible as it will leave remainder 1.

So, Q will divisible by some prime which is not in the original list.

Therefore, our list is not complete and our assumption is false.

Therefore, there are infinitely many primes.

Good Problems:

1) Is 40.272727.... rational?

Let $x = 40.272727\ldots$

$100x = 4027.272727\ldots$

$99x = 4027 - 40$

$x = \frac{443}{11}$

Type of Number	Decimal Form
Rational	Either terminates or repeats
Irrational	Never terminates, never repeats.

) T --$\sqrt{2}$ is

uppose 6 - rational

$6 - 7\sqrt{2} = \frac{p}{q}$; $p, q \in \mathbb{Z}$

$6 - \frac{p}{q} = 7\sqrt{2}$

$\sqrt{2} = \frac{6q - p}{7 \cdot q}$

Integer

Integer