SEQUENCES

Summation Notation :

$\sum_{k=m}^{n} a_k$ read as summation from k equals m to n of a sub k.

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + \cdots + a_n$$

n ← upper limit of summation

$k=m$ ← lower limit of summation

k → index of summation

Ex 5.1-4) $a_1 = -2$ $a_4 = 1$

$a_2 = -1$ $a_5 = 2$

$a_3 = 0$

a) $\sum_{k=1}^{5} a_k = -2-1+0+1+2 = 0$

c) $\sum_{k=1}^{2} a_{2k} = a_{2\cdot 1} + a_{2\cdot 2} = a_2 + a_4 = -1+1 = 0$

Telescoping Sum :

Successive terms cancel out. Like a collapsing telescope, which on collapsing leaves only the first and last parts.

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

Product Notation :

$\prod_{k=m}^{n} a_k$ read as product from k equals m to n of a-sub-k

$$\prod_{k=m}^{n} a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdots a_n$$

Properties of Summation and Products :

1) $\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$

2) $c \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} c \cdot a_k$

3) $\left(\prod_{k=m}^{n} a_k\right) \cdot \left(\prod_{k=m}^{n} b_k\right) = \prod_{k=m}^{n} (a_k \cdot b_k)$

Change of variable :

$$\sum_{j=2}^{4} (j-1)^2 = (2-1)^2 + (3-1)^2 + (4-1)$$
$$= 1^2 + 2^2 + 3^2$$

Suppose, $k = j-1$ (More generally)

Limits : $k = 2-1 = 1$; $k = 4-1 = 3$

$$\sum_{k=1}^{3} k^2$$

Factorial and n choose r notation :

$${}^{n}C_{r+1} = \frac{n-r}{r+1}\,{}^{n}C_r$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n(n-1)! & \text{if } n \geq 1 \end{cases}$$

$${}^{n}C_r = \frac{n!}{r!\,(n-r)!} \quad \text{(n choose r)}$$

n chose r and represents the number of size r that can be chosen from a set with n elements.

Basic Summation Formulas :

1) $$\sum_{k=1}^{n} 1 = n$$

2) $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

3) $$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

4) $$\sum_{k=1}^{n} k^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

5) $$\sum_{k=1}^{n} k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

Arithmetic Series:

$$\sum_{k=0}^{n-1} (a+kd) = \frac{n}{2}\left[2a+(n-1)d\right]$$

Geometric Series :

$$\sum_{k=0}^{n-1} ar^k = a \cdot \frac{1-r^n}{1-r} \quad (\text{for } r \neq 1)$$

Infinite geometric Series: $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ (if $|r| < 1$)

Alternating Series:

$\sum_{k=1}^{n} (-1)^{k+1}$ 1 if n odd, 0 if n even

$\sum_{k=1}^{n} (-1)^{k+1} k$ $\left\lceil \frac{n}{2} \right\rceil$