Indirect Argument : Contradiction and Contraposition

Proof by Contradiction or reductio ad absurdum
or Reduce an argument to an impossibility or absurdity

Proof by Contraposition:

To prove, $P \to Q$
we prove its contraposition $\neg Q \to \neg P$

Any statement that can be proved by method of contra-position can be proved by contradiction. But the converse is not true.
$(Q \to P)$

Proof by Contradiction
$\neg(p \to q) \equiv \neg(\neg p \vee q) \equiv p \wedge \neg q$
We contradict the assumption and here is the proof.

Good Problems:

i) For all positive real numbers r, s,

$$\sqrt{r+s} \neq \sqrt{r} + \sqrt{s}$$

Proof by Contradiction:

Assume, $\exists$ positive real numbers r, s

$$\sqrt{r+s} = \sqrt{r} + \sqrt{s}$$

$$(\sqrt{r+s})^2 = (\sqrt{r} + \sqrt{s})^2$$

$$\Rightarrow r+s = r+s+2\sqrt{r}\sqrt{s}$$

$$2\sqrt{r}\sqrt{s} = 0$$

$$\sqrt{r} = 0 \text{ or } \sqrt{s} = 0$$

Not possible since r, s are +ve real numbers and $r \neq 0$ and $s \neq 0$

Hence, by reductio ad absurdum, our assumption is absurd. Hence $\forall r, s \in R^+$ $\sqrt{r+s} \neq \sqrt{r} + \sqrt{s}$

2) Sieve of Eratosthenes:

Find all prime numbers $\leq n$

a) Write all integers from 2 to n

b) Cross out multiples of 2 except 2, multiples of 3 except 3, repeat till $\lfloor \sqrt{n} \rfloor$

c) The number left are the prime numbers:

For ex:- Find prime numbers till 10

$2 \quad 3 \quad \cancel{4} \quad 5 \quad \cancel{6} \quad 7 \quad \cancel{8} \quad \cancel{9} \quad \cancel{10}$

$$\lfloor \sqrt{10} \rfloor = 3$$

Prime list = [2, 3, 5, 7]