# Direct Proof and CounterExample 6 : Floor and Ceiling

## Floor of x :

$$\lfloor x \rfloor = n \iff n \leq x < n+1$$

$x \in R$, $n \in Z$

## Ceiling of x:

$$\lceil x \rceil = n \iff n-1 < x \leq n$$

For ex:-

$$\left\lfloor \frac{25}{4} \right\rfloor = \lfloor 6.25 \rfloor = 6$$

$$\lceil 6.25 \rceil = 7$$

## Theorems:

1) For every real number $x$ and every integer $m$,

$$\lfloor x+m \rfloor = \lfloor x \rfloor + m$$

2) For any integer $n$,

$$\left\lfloor \frac{n}{2} \right\rfloor = \begin{cases} n/2 & \text{if } n \text{ is even} \\ \frac{n-1}{2} & \text{if } n \text{ is odd} \end{cases}$$

3) If $n$ is any integer and $d$ is positive integer, and if $q = \lfloor n/d \rfloor$ and $r = n - d \cdot \left\lfloor \frac{n}{d} \right\rfloor$ then,

$$n = dq + r \text{ and } 0 \leq r < d$$

## Problems :

1) To prove,

$$\forall x \in R, \ \lfloor x-1 \rfloor = \lfloor x \rfloor - 1$$

Let $\lfloor x \rfloor = n$

$$n \leq x < n+1$$

$$n-1 \leq x-1 < n$$

$$\lfloor x-1 \rfloor = n-1$$

Since, $n = \lfloor x \rfloor$

$$\lfloor x-1 \rfloor = \lfloor x \rfloor - 1$$

Hence, proved.