

## SEQUENCES

### Summation Notation :

$\sum_{k=m}^n a_k$  read as summation from k equals m to n of a sub k.

$n \leftarrow$  upper limit of summation

$$\sum_{k=m}^n a_k = a_m + a_{m+1} + \dots + a_n$$

$m \leftarrow$  lower limit of summation

$\downarrow$   
index of summation

$$\text{Ex 5.1-4)} \quad a_1 = -2 \quad a_4 = 1$$

$$a_2 = -1 \quad a_5 = 2$$

$$a_3 = 0$$

$$\text{a) } \sum_{k=1}^5 a_k = -2 - 1 + 0 + 1 + 2 = 0$$

$$\text{c) } \sum_{k=1}^2 a_{2k} = a_{2 \cdot 1} + a_{2 \cdot 2} = a_2 + a_4 = -1 + 1 = 0$$

Telescoping Sum: Successive terms cancel out.

Like a collapsing telescope, which on collapsing leaves only the first and last part.

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

### Product Notation :

$\prod_{k=m}^n a_k$  read as product from k equals m to n of a sub k

$$\prod_{k=m}^n a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdots \cdots a_n$$

## Properties of Summation and Products:

$$1) \sum_{k=m}^n a_k + \sum_{k=m}^n b_k = \sum_{k=m}^n (a_k + b_k)$$

$$2) c \sum_{k=m}^n a_k = \sum_{k=m}^n c \cdot a_k$$

$$3) \left( \prod_{k=m}^n a_k \right) \cdot \left( \prod_{k=m}^n b_k \right) = \prod_{k=m}^n (a_k \cdot b_k)$$

Change of variable:

$$\sum_{j=2}^4 (j-1)^2 = (2-1)^2 + (3-1)^2 + (4-1)^2 \\ = 1^2 + 2^2 + 3^2$$

Suppose,  $k = j-1$  (More generally)

Limits:  $k = 2-1 = 1$ ;  $k = 4-1 = 3$

$$\sum_{k=1}^3 k^2$$

Factorial and n choose r notation:  ${}^n C_{r+1} = \frac{n-r}{r+1} {}^n C_r$

$$n! = \begin{cases} 1 & \text{if } n=0 \\ n(n-1)! & \text{if } n \geq 1 \end{cases}$$

$${}^n C_r = \frac{n!}{r!(n-r)!} \quad (\text{n choose r})$$

n choose r and represents the number of size r that can be chosen from a set with n elements.

## Basic Summation Formulas :

$$1) \sum_{k=1}^n 1 = n$$

$$2) \sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$3) \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$4) \sum_{k=1}^n k^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$$

$$5) \sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

## Arithmetic Series:

$$\sum_{k=0}^{n-1} (a + kd) = \frac{n}{2} [2a + (n-1)d]$$

## Geometric Series:

$$\sum_{k=1}^n ar^k = a \cdot \frac{1-r^n}{1-r} \quad (\text{for } r \neq 1)$$

$$\text{Infinite geometric series: } \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} \quad (\text{if } |r| < 1)$$

## Alternating Series:

$$\sum_{k=1}^n (-1)^{k+1} \quad 1 \text{ if } n \text{ odd}, 0 \text{ if } n \text{ even}$$

$$\sum_{k=1}^n (-1)^{k+1} k \quad \left[ \frac{n}{2} \right]$$