# Valid and Invalid Arguments

The logical form of an argument can be abstracted from its content. For example,

If Socrates is a man, then Socrates is mortal.
Socrates is a man
$\therefore$ Socrates is mortal → ARGUMENT-FORM
If $p$ then $q$
$p$ → PREMISES
$\therefore q$ → CONCLUSION

ARGUMENT

## Testing and Argument form of validity:

- Identify the premises and conclusion of the argument form.
- Construct a truth table showing truth values of all premises and the conclusion.
- A row of the truth table in which all premises are true is called a critical row. If the conclusion is every critical row is true, then the argument is valid else invalid.

$$p \to q \vee \sim r$$
$$q \to p \wedge r$$
$$\therefore p \to r$$

**Caution!** If at least one premise of an argument is false, then we have no information about the conclusion: It might be true or it might be false.

**Solution** The truth table shows that even though there are several situations in which the premises and the conclusion are all true (rows 1, 7, and 8), there is one situation (row 4) where the premises are true and the conclusion is false.

| | | | | | | premises | | conclusion | |
|---|---|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $r$ | $\sim r$ | $q \vee \sim r$ | $p \wedge r$ | $p \to q \vee \sim r$ | $q \to p \wedge r$ | $p \to r$ | |
| T | T | T | F | T | T | T | T | T | cR |
| T | T | F | T | T | F | T | F | | |
| T | F | T | F | F | T | F | T | | |
| T | F | F | T | T | F | T | T | F | cR |
| F | T | T | F | T | F | T | F | | |
| F | T | F | T | T | F | T | F | | |
| F | F | T | F | F | F | T | T | T | cR |
| F | F | F | T | T | F | T | T | T | cR |

This row shows that an argument of this form can have true premises and a false conclusion. Hence this form of argument is invalid.

**TABLE 2.3.1** Valid Argument Forms

| | | | |
|---|---|---|---|
| **Modus Ponens** | $p \to q$<br>$p$<br>$\therefore q$ | **Elimination** | **a.** $p \vee q$, $\sim q$, $\therefore p$ **b.** $p \vee q$, $\sim p$, $\therefore q$ |
| **Modus Tollens** | $p \to q$<br>$\sim q$<br>$\therefore \sim p$ | **Transitivity** | $p \to q$<br>$q \to r$<br>$\therefore p \to r$ |
| **Generalization** | **a.** $p$, $\therefore p \vee q$ **b.** $q$, $\therefore p \vee q$ | **Proof by Division into Cases** | $p \vee q$<br>$p \to r$<br>$q \to r$<br>$\therefore r$ |
| **Specialization** | **a.** $p \wedge q$, $\therefore p$ **b.** $p \wedge q$, $\therefore q$ | | |
| **Conjunction** | $p$<br>$q$<br>$\therefore p \wedge q$ | **Contradiction Rule** | $\sim p \to \mathbf{c}$<br>$\therefore p$ |

Fallacies:

- Ambiguous premises: Using ambiguous premises.
- Circular Reasoning: Assuming what is to be proved without having derived it from premises.
- Jumping to a conclusion: Jumping to a conclusion without adequate grounds.
- Converse Error: This fallacy is known as affirming the consequent.

$P \to Q$
$Q$
$\therefore P$ (incorrect inference)

- Inverse Error: This fallacy is known as denying the antecedent.

$P \to Q$
$\neg P$
$\therefore \neg Q$ (incorrect inference)