The Logic of Compound Statements

Logic is the study of what counts as a good reason for what and why.

A deductively valid inference is one for which there is no situation in which all premises are true but the conclusion is not.

An argument is a sequence of statements aimed at demonstrating the truth of an assertion. The assertion at the end of the sequence is called the conclusion and the preceding statements are called premises.

Statement:

A statement (or proposition) is a sentence that is true or false but not both.

p but q means p and q

neither p nor q means $\sim p$ and $\sim q$

Logically Equivalent: ($\equiv$)

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$p \wedge q$ and $q \wedge p$ always have the same truth values, so they are logically equivalent.

Types of Proposition:

- Tautology - Always true
- Contradiction - Always false
- Contingent - Sometimes true, sometimes false

Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1.	*Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2.	*Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3.	*Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4.	*Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5.	*Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6.	*Double negative law:*	$\sim(\sim p) \equiv p$	
7.	*Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8.	*Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9.	*De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10.	*Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11.	*Negations of* **t** *and* **c**:	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Maths builds certainty by using logical rules like Modus ponens. When arguments are shaped to fit these rules, deduction becomes mechanical not mysterious.

Logic isn't about truth - it's about structure.

In practice -

- Use logic for structure
- Use truth for substance
- Use relevance for usefulness