Direct Proof: Quotient Remainder Theorem

The Quotient Remainder Theorem:

Given any integer n and a positive integer d, there exist unique integer q and r such that

$$n = dq + r \text{ and } 0 \le r < d$$

n div d = Integer quotient (q)

n mod d = Non-negative integer remainder (r)

$$0 \le r < d$$

Parity:

The fact that any integer is either odd or even is called the parity property.

Theorem: Any two consecutive integers have opposite parity.

The Triangle Inequality: (Also an example of proof by cases)

$$|x| + |y| \ge |x+y|$$

similar In a triangle sum of two sides is greater than 3rd side.

Case I: $x > 0$, $y > 0$

$|x| + |y| = x + y$ $\quad |x| + |y| \ge |x+y|$

$|x+y| = x+y$

Case II: $x > 0$, $y < 0$ $\quad |x| + |y| \ge |x+y|$

$|x| + |-y| = x+y$

$|x-y|$, Here $x-y$ or $y-x$

Case III: $x < 0$, $y > 0$ $\quad$ Similar to 3 $\quad |x| + |y| \ge |x+y|$

Case IV: $x < 0$, $y < 0$ $\quad |-x| + |-y| = x+y$

$$|-x-y| = |-(x+y)| = x+y$$

$\therefore$ Combining all 4 cases, $|x|+|y| \geq |x+y|$

Good Problems:

1.) Find integers q and r such that $n = dq + r$ and $0 \leq r < d$.

$n = -45$, $d = 11$

$-45 = 11 \times -5 + 10$