MATHEMATICAL INDUCTION

Principle of Mathematical Induction:

Let $P(n)$ be a property that is defined for integers n, and let a be a fixed integer. Suppose the following statements are true.

1) $P(a)$ is true

2) For every integer $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true.

Then the statement for every integer $n \geq a$, $P(n)$ is true.

A Beautiful Proof of Mathematical induction,

Proposition: For every integer $n \geq 8$, n¢ can be obtained using 3¢ and 5¢ coins.

$8 \rightarrow 3+5$

$9 \rightarrow 3+3+3$

$10 \rightarrow 5+5$

$11 \rightarrow 3+3+5$

$12 \rightarrow 3+3+3+3$

$13 \rightarrow 3+5+5$

If we follow this pattern we will always get the next integer.

If we see there is a pattern:

Case I: When one of the coins has 5. To increment by 1 we replace the 5 by $3+3$

Case II: When there are no 5, we replace $3+3+3$ by $5+5$

There will never be a case of only $3+3$ because $n \geq 8$

STRONG MATHEMATICAL INDUCTION:

Situation	Use
When the next case depends on only previous case	Weak Induction is enough
When next case depends on several earlier cases For ex:- Fibonacci Sequence	Strong Induction is needed.

Principle of Strong Mathematical Induction:

Let $P(n)$ be a property that is defined for integers n, and let a and b be fixed integers with $a \le b$. Suppose the following two statements are true:

1. $P(a), P(a+1), \ldots,$ and $P(b)$ are all true (basis)
2. For every integer $k \ge b$, if $P(i)$ is true for each integer i from a through k, then $P(k+1)$ is true. (inductive step)

Then the statement for every integer $n \ge a$, $P(n)$ is true.

Ex 5.4.2) Sequence: $a_0 = 0$, $a_1 = 4$, $a_k = 6a_{k-1} - 5a_{k-2}$ for $k \ge 2$

a) First 4 terms:

$a_0 = 0$

$a_1 = 4$

$a_2 = 6a_1 - 5a_0 = 6 \cdot 4 - 5 \cdot 0 = 24$

$a_3 = 6a_2 - 5a_1 = 6 \cdot 24 - 5 \cdot 4$

$= 144 - 20 = 124$

b) Hypothesis: $a_n = 5^n - 1$

Basis;

$a_0 = 5^0 - 1 = 1 - 1 = 0$ (True)

$a_1 = 5^1 - 1 = 4$ (True)

$a_2 = 5^2 - 1 = 24$ (True)

$a_3 = 5^3 - 1 = 125 - 1 = 124$ (True)

Induction Step: Let $a_i = 5^i - 1$ for $0 \le i \le k$ be true.

We need to prove,

$$a_{k+1} = 5^{k+1} - 1$$

Solving LHS,

$$a_{k+1} = 6a_k - 5a_{k-1}$$
$$6(5^k - 1) - 5(5^{k-1} - 1)$$
$$6 \cdot 5^k - 6 - 5^k + 5$$
$$5^k(6-1) - 1 = 5^{k+1} - 1$$

LHS = RHS

Hence, proved

WELL ORDERING PRINCIPLE:

Every non empty set of non-negative integers has a least (smallest) element.