Conditional Statements

Consider conditional statements as a promise. In logic, an implication $p \to q$ is considered false only when p is true and q is false. That's the only scenario where a promise is broken.

$$p \to q \equiv \sim p \vee q$$

$$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim Q \vee P)$$
$$\equiv (p \to q) \wedge (q \to p)$$

Conditional

$p \to q$ Hypothesis $\to$ Conclusion
(or antecedent) (or consequent)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi Conditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Use this	When you mean	Example
IF	One way condition	If $x > 0$, then $x^2 > 0$
IFF	Two way condition	x is even iff $x \% 2 = 0$

1) r is a sufficient condition for s : $r \to s$

2) r is a necessary condition for s ; $\neg r \to \neg s$

$(r \vee \neg s) \equiv s \to r$

3) r is a necessary and sufficient condition : $r \leftrightarrow s$

Converse, Inverse and Contrapositive:

- Contrapositive of $p \to q \equiv \sim q \to \sim p$

p	q	$p \to q$	$\sim q \to \sim p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

An implication and its contrapositive is equivalent.

- Converse of $p \to q = q \to p$
- Inverse of $p \to q = \sim p \to \sim q$

Caution:

If we mechanically accept logical rules without context or understanding, we risk making conclusion that are technically valid but meaningless or even misleading.

For ex:- If the moon is made of green cheese, then $2+2=4$.

This is an $F \to T \equiv T$ statement, but the conclusion has nothing to do with the premise. Yet, logic says the implication is true, just because the premise is false. This is logically correct, but completely useless in the real world reasoning.