Earthquake location determination methods.

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Nonlinear Earthquake Location Determination Method

The **arrival-time of wave at a location x = (*x*, *y*, *z*)** for a seismic source located at **x**0 = (*x*0, *y*0, *z*0) is given by:

$$t_{arr} = t_0 + \frac{1}{v}\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]^{\frac{1}{2}}$$

Actually it's a nonlinear inverse problem with 4 unknowns.

Goal : Solve for the earthquake's hypocenter and origin time by **minimizing the difference between the observed arrival times at various stations and the predicted arrival times (**r_i**)**, which are based on the location and velocity model.

Misfit (E)

t^i_{pre} - Predicted arrival time

t^i_{obs} - Observed arrival time

$$t^i_{\text{pre}} = T^0 + t^i_{\text{model}} + t^i_{\text{corr}},$$

$$r_i = t^i_{\text{obs}} - t^i_{\text{pre}}$$

To Sum the residuals -

L2 Norm

$$E = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} (t^i_{\text{obs}} - t^i_{\text{pre}})^2.$$

The least-square solutions work well if the residuals are obtained by **uncorrelated Gaussian noise.**

However, the minimization of *L2* norm is **not robust for data with large outliers**.

L1 Norm

$$E = \sum_{i=1}^{N} | t_{\text{obs}}^{i} - t_{\text{pre}}^{i} |$$

However, it is a **non-smooth optimization problem** and is not much preferred in the standard location programs due to the **high computational cost**.

So,

Likelihood Function Lomax et al. (2011)

$$L(\mathbf{x}) = \exp\left[-\frac{1}{2}\sum_{i=1}^{N}\frac{(t_{\text{obs}}^{i} - t_{\text{pre}}^{i})^2}{\sigma_i^2}\right]$$

Maximizing the ***L*(x)** will give the maximum likelihood of x, y, z, t.

Grid Search

- Possible range of position is defined. - Grid

```
x_range = np.linspace(0, 100, 20)
y_range = np.linspace(0, 100, 20)
z_range = np.linspace(0, 50, 10)
t_range = np.linspace(0, 50, 100)
```

- For each point on the grid, r_i is calculated using L1, L2 or L methods.

```
best_misfit = float('inf')
best_hypocenter = None
best_origin_time = None

for x in x_range:
    for y in y_range:
        for z in z_range:
            for t0 in t_range:
                predicted_times = compute_travel_times(np.array([x, y, z]), t0)
                misfit = np.sum((observed_arrival_times - predicted_times) ** 2)  # L2 norm
```

- Smallest misfit is chosen as the best estimate.

```
                if misfit < best_misfit:
                    best_misfit = misfit
                    best_hypocenter = np.array([x, y, z])         # L2 norm minimisation
                    best_origin_time = t0
```

Challenges of Nonlinear Location Methods

1. **Computational Cost**: Grid search methods can be computationally expensive

2. **Non-Smooth Optimization**: Minimizing the L1 norm or other non-smooth objective functions is challenging because traditional gradient-based optimization methods are not applicable. This can slow down the optimization process.

3. **Complex Earth Models**: In regions with complex Earth structures (e.g., varying velocities), the nonlinearities in the travel-time calculations can become more pronounced, making it harder to find the true global minimum.

References

- Earthquake Location Methods - Ezgi Karasözen, Bülent Karasözen
- Linear and nonlinear earthquake location approaches in a case study overview – Amin Abbasi
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