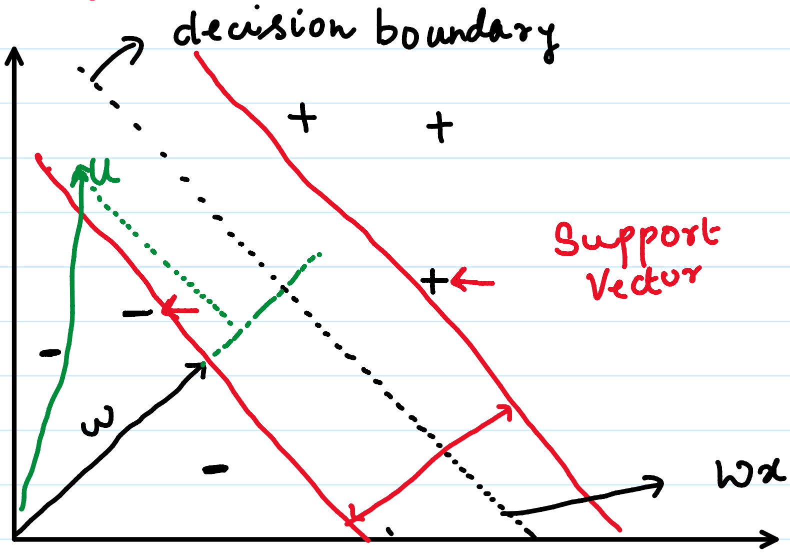
SVM-PART-(A)

11 October 2020 17:37

SUPPORT VECTOR MACHINES



Valdmir Vapnik

+ : SPAM

- : Leg. email

- LINE
- Many lines
- which is best?

$\bar{w} \cdot \bar{u} \geq c$ +ve / -ve

x_+ : (+ve class)

$$w \cdot x_+ + b \geq 1 \qquad (1)$$

x_- : (- ve class)

$$w \cdot x_- + b \leq -1 \qquad (2)$$

Maltonatical y_i : 1 if + ve class

$\qquad\qquad\qquad\qquad -1$ if - ve class

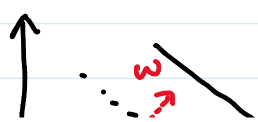
$y_i\,(w \cdot x_i + b) \geq 1$ [+ve class], $y_i \rightarrow 1$

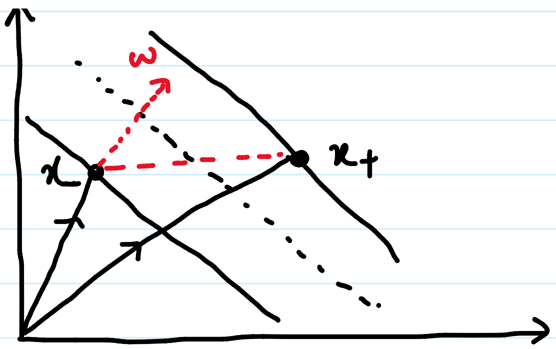
$y_i\,(w x_i + b) \leq -1$, $y_i \rightarrow -1$

$\Downarrow$

$y_i(w \cdot x_i + b) \geq 1$: Compact Represt^n

(3) $\quad y_i\,(w \cdot x_i + b) - 1 = 0$: x_i lies on dotted





$$\text{Width} = (x_+ - x_-) \cdot \frac{w}{\|w\|} \longrightarrow (4)$$

$$y_i \, (w \cdot x_+ + b) - 1 = 0$$

$$w \cdot x_+ = 1 - b$$

$$y_i \, (w \cdot x_- + b) - 1 = 0$$

$$-1 \, (w \cdot x_- + b) = 1$$

$$-w \cdot x_- - b = 1$$

$$-w \cdot x_- = 1 + b$$

$$(\underbrace{w \cdot x_+}_{1-b} \underbrace{- w \cdot x_-}_{1+b}) \cdot \frac{1}{\|w\|}$$

$$\text{width} = \frac{1 - b + 1 + b}{\|w\|} = \frac{2}{\|w\|} \Rightarrow \text{Maximize.}$$

Mathematical Simplicity

$$\max \frac{2}{\|w\|} \quad \text{or} \quad \max \frac{1}{\|w\|} \quad \text{or} \quad \min \|w\| \quad \text{or}$$

$$\min \|w\|^2 \quad \text{or} \quad \min \frac{1}{2} \|w\|^2$$

If we find extremum of funct^n w.r.t constraint.
LAGRANGE MULTIPLIER

$$L = \min \frac{1}{2} \|w\|^2 - \sum_i \alpha_i \left[y_i (w \cdot x_i + b) - 1 \right] \rightarrow (5)$$

unknown

unknown

$$\frac{\partial L}{\partial w} = 0 \qquad \text{Likewise} \quad \frac{\partial L}{\partial b} = 0$$

$$\frac{\partial L}{\partial w} = w - \sum_i \alpha_i y_i x_i = 0$$

$$\boxed{w = \sum_i \alpha_i y_i x_i} \longrightarrow (6)$$

$$\frac{\partial L}{\partial b} = \boxed{\sum_i \alpha_i y_i = 0} \longrightarrow (7)$$

$$L = \frac{1}{2}\left[\sum_i (\alpha_i y_i x_i) \underbrace{\sum_j (\alpha_j y_j x_j)}\right] - \sum_i (\alpha_i y_i x_i) \underbrace{(\alpha_j y_j x_j)}$$

$$- \sum \alpha_i y_i b + \sum \alpha_i$$

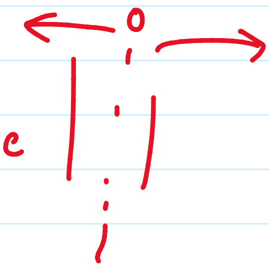
$$L = -\frac{1}{2} \sum_{i,j} (\alpha_i y_i x_i)(\alpha_j y_j x_j) - b \underbrace{\sum_i \alpha_i y_i}_{=0} + \sum_i \alpha_i$$

$$L = \sum_i \alpha_i - \frac{1}{2} \sum (\alpha_i y_i x_i)(\alpha_j y_j x_j)$$

$$L = \sum_i \alpha_i - \frac{1}{2} \sum \alpha_i \alpha_j y_i y_j \boxed{x_i \cdot x_j}$$

Dot product x_i and x_j

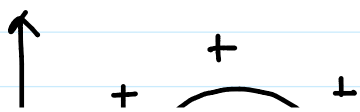
Predict u → +ve / −ve



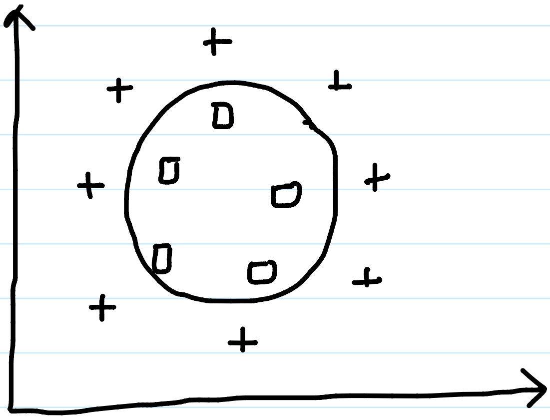
$w \cdot u + b \geq 0$ +ve

if $\underbrace{\sum \alpha_i y_i (x_i \cdot u) + b} \geq 0$ +ve

if ≤ 0 −ve



$\phi : +$ $\phi(+)$



ϕ : + $\phi(+)$

Kernel

Linear   Poly   Gaussian