Linear Regression- Gradient Descent

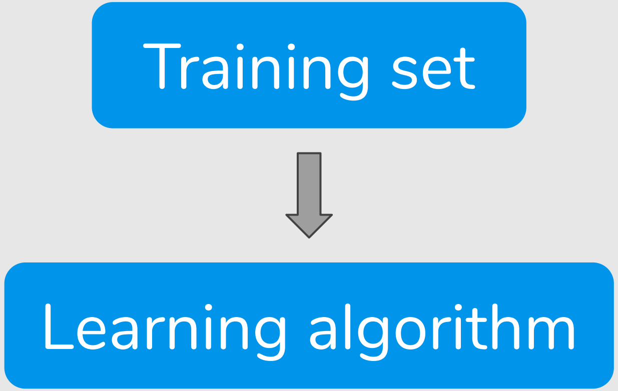
BY

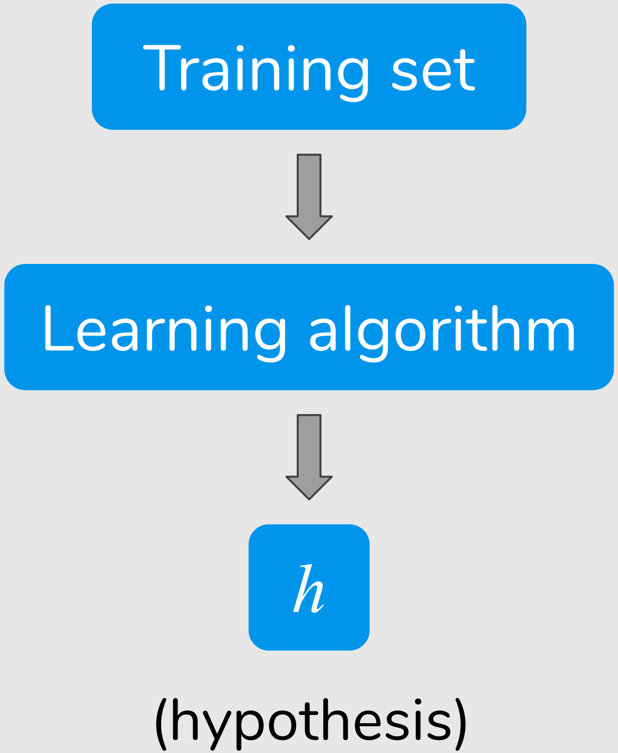
VINOD P

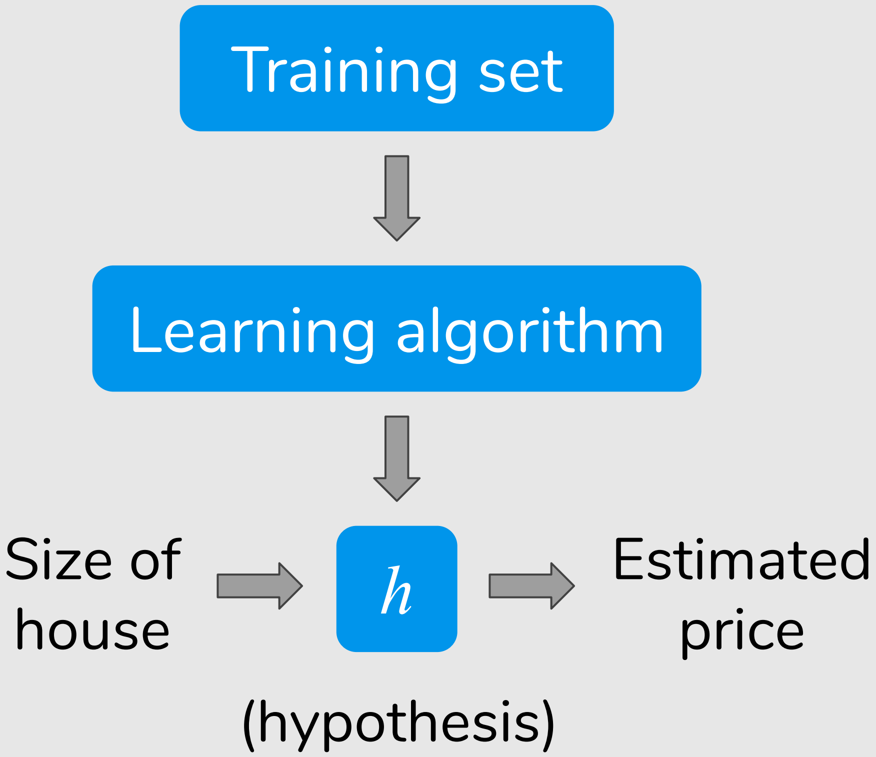
Acknowledgement:
Various resources collected from Internet

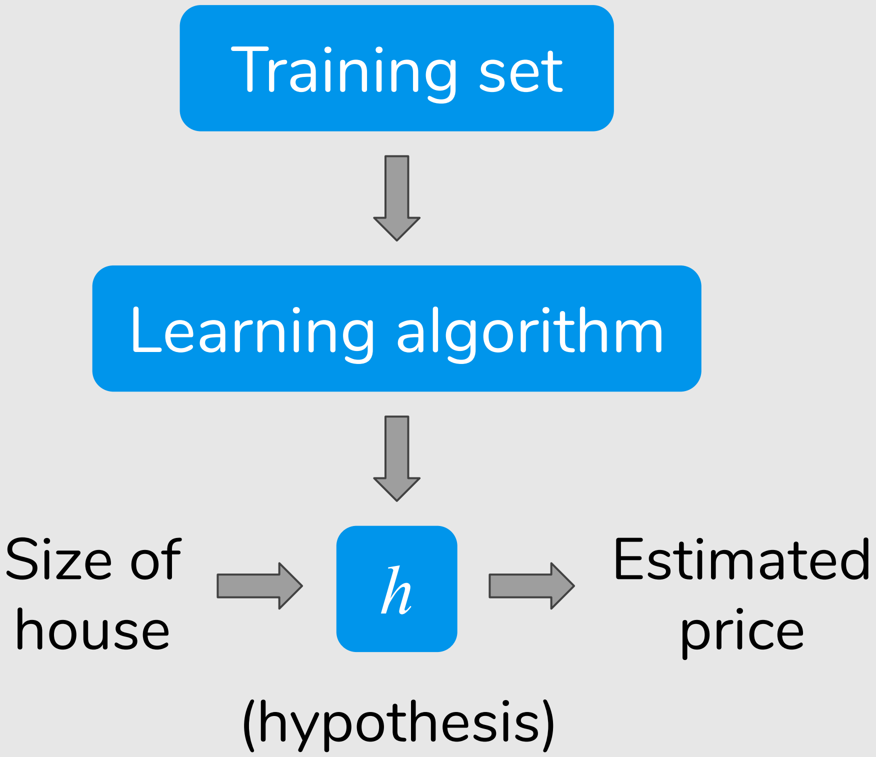
Model Representation

Training set


Training set
Learning algorithm

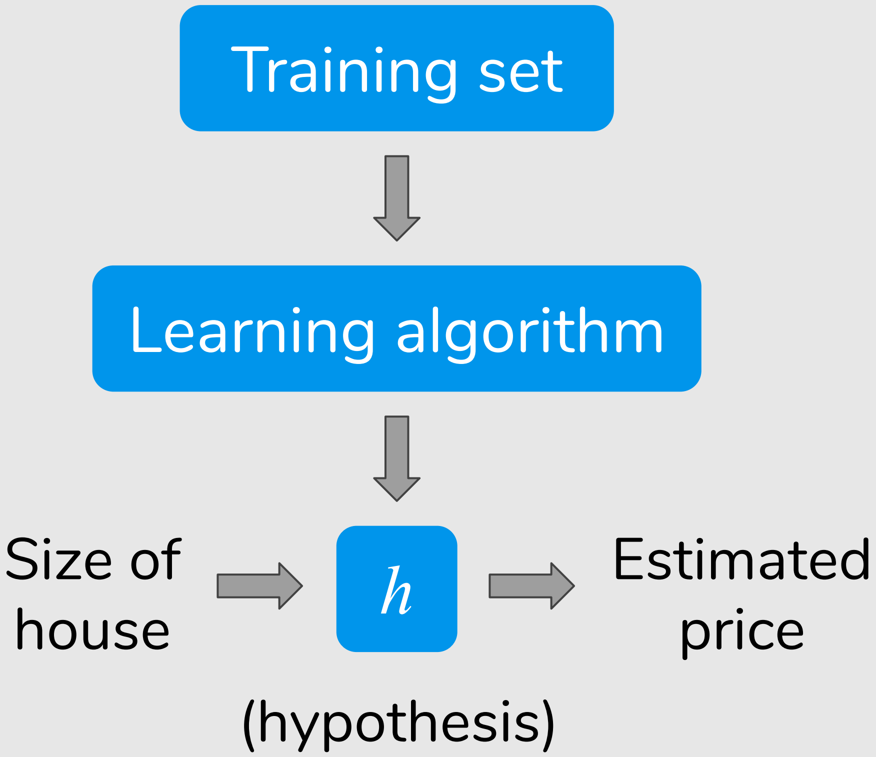

Training set
Learning algorithm
h
(hypothesis)


Training set
Learning algorithm
Size of house
h
Estimated price
(hypothesis)

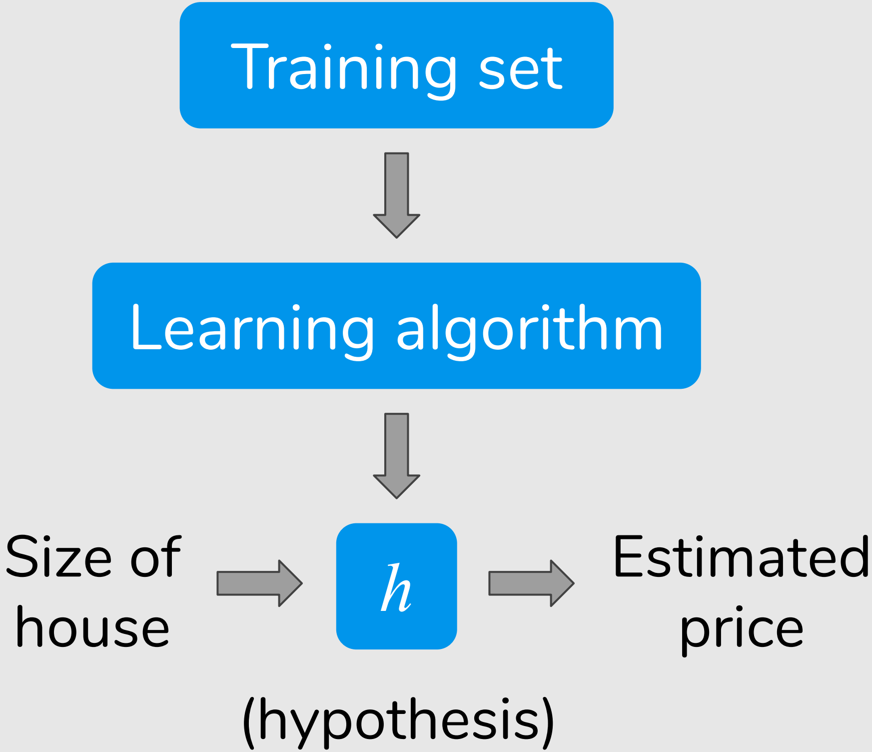


h maps x's to y's

How do we represent h ?



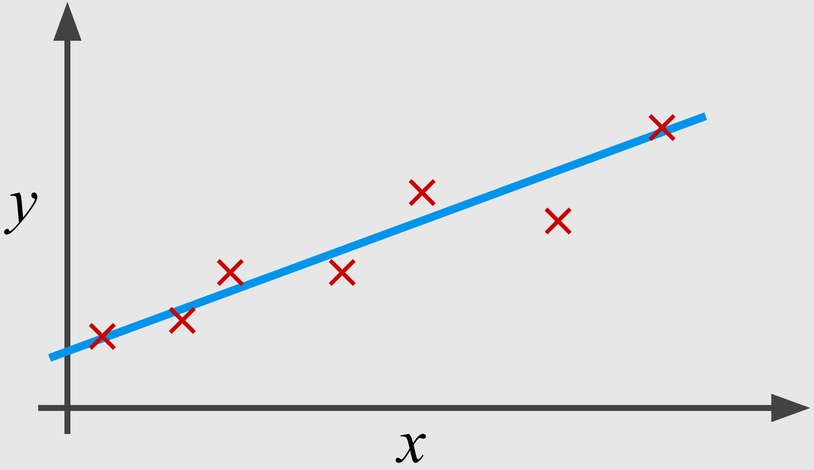
h maps x's to y's



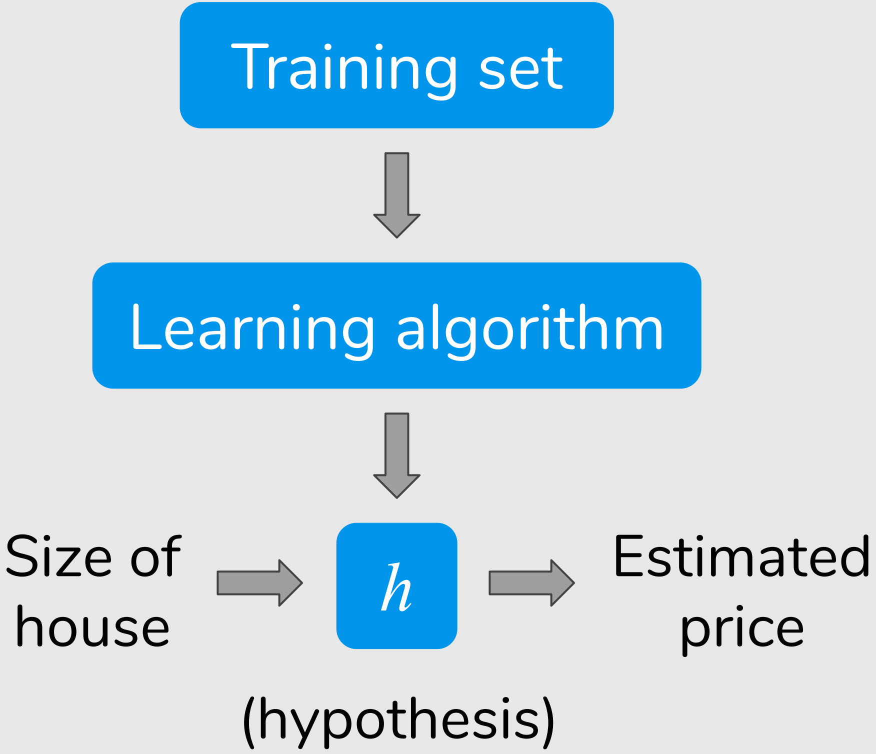
h maps x's to y's

How do we represent h ?

$$h_\theta(x) = \theta_0 + \theta_1 x$$

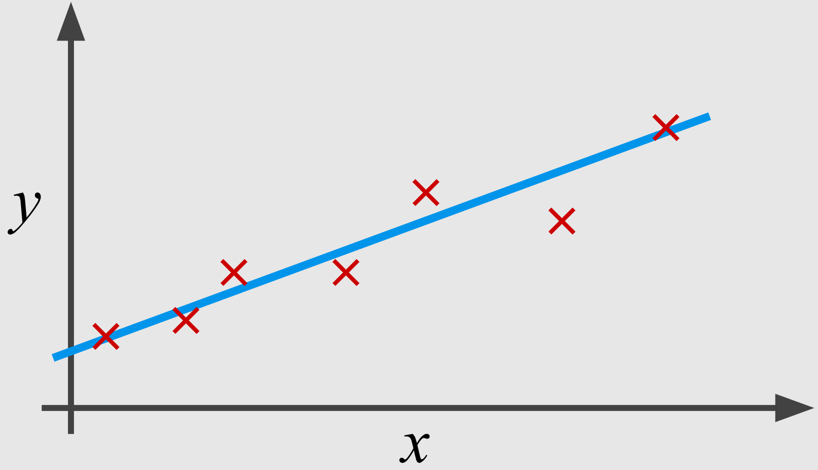


How do we represent h ?



h maps x's to y's

$$h_\theta(x) = \theta_0 + \theta_1 x$$



Linear regression with one variable.

Univariate linear regression.

Cost Function

Training Set

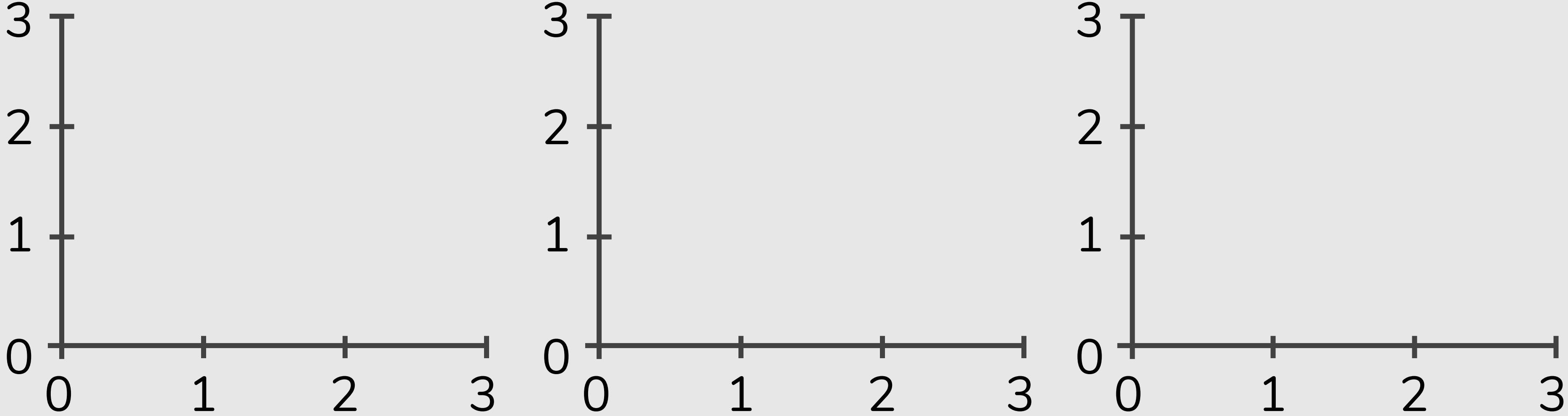
Size in feet^2 (x)	**Price (\$) in 1000's (y)**
2104	460
1416	232
1534	315
852	178
...	...

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

θ_i's: Parameters

How to choose θ_i's ?

$$h_\theta(x) = \theta_0 + \theta_1 x$$

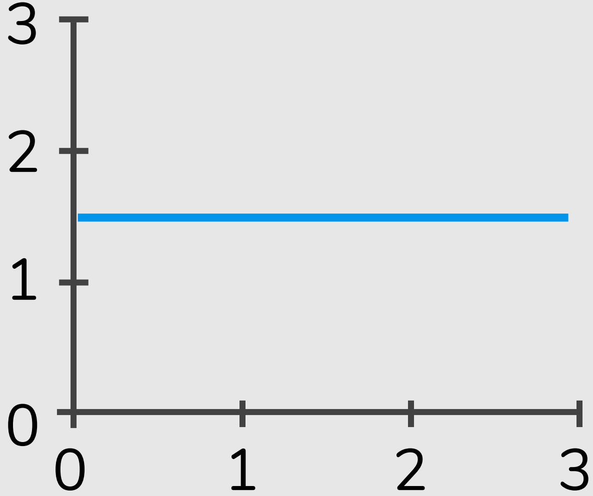


$\theta_0 = 1.5$
$\theta_1 = 0$

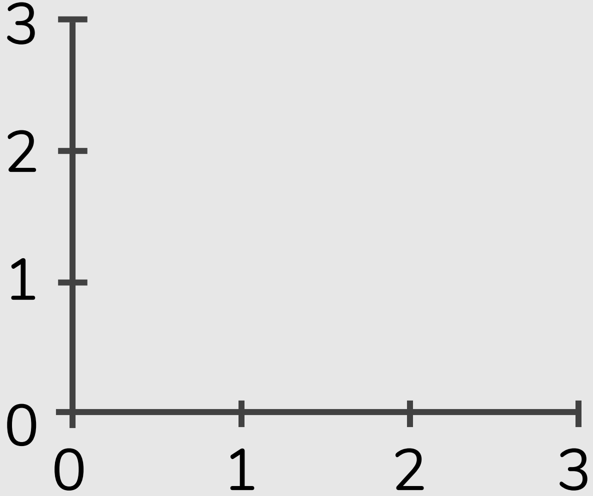
$\theta_0 = 0$
$\theta_1 = 0.5$

$\theta_0 = 1$
$\theta_1 = 0.5$

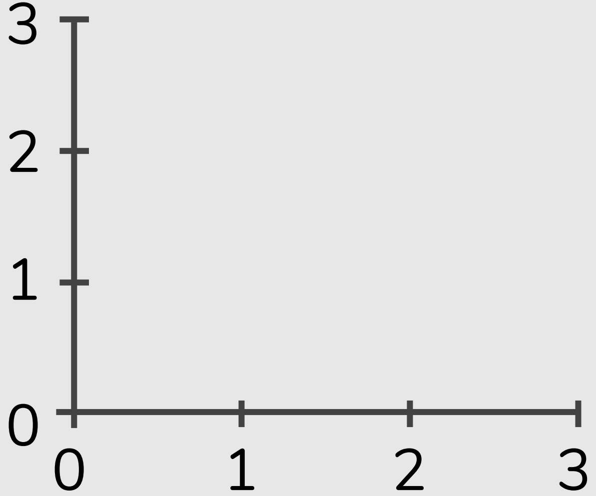
$$h_\theta(x) = \theta_0 + \theta_1 x$$



$\theta_0 = 1.5$
$\theta_1 = 0$

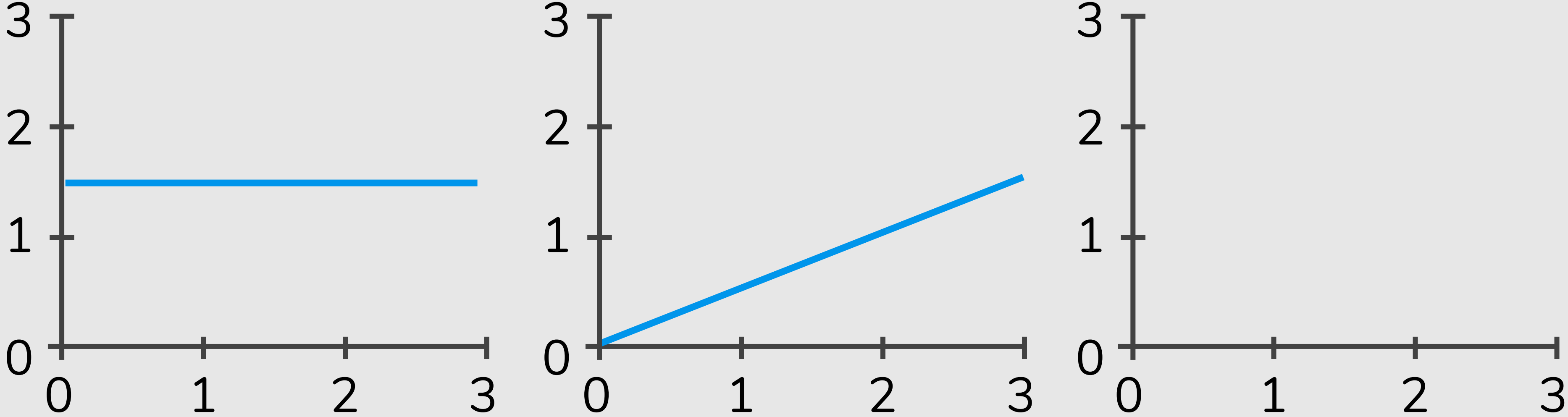


$\theta_0 = 0$
$\theta_1 = 0.5$



$\theta_0 = 1$
$\theta_1 = 0.5$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

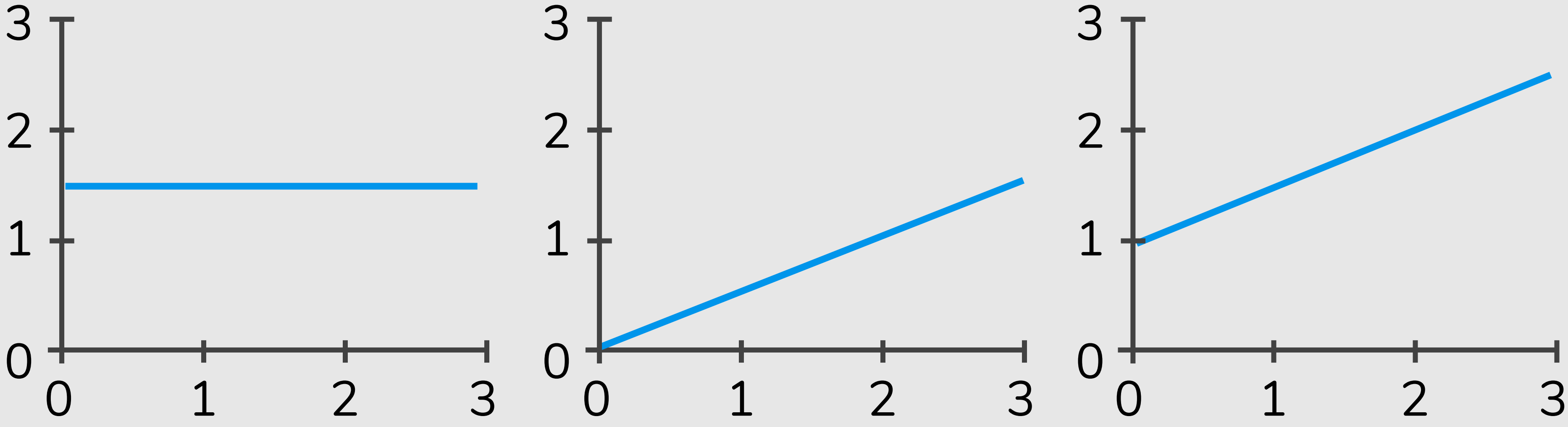


$\theta_0 = 1.5$
$\theta_1 = 0$

$\theta_0 = 0$
$\theta_1 = 0.5$

$\theta_0 = 1$
$\theta_1 = 0.5$

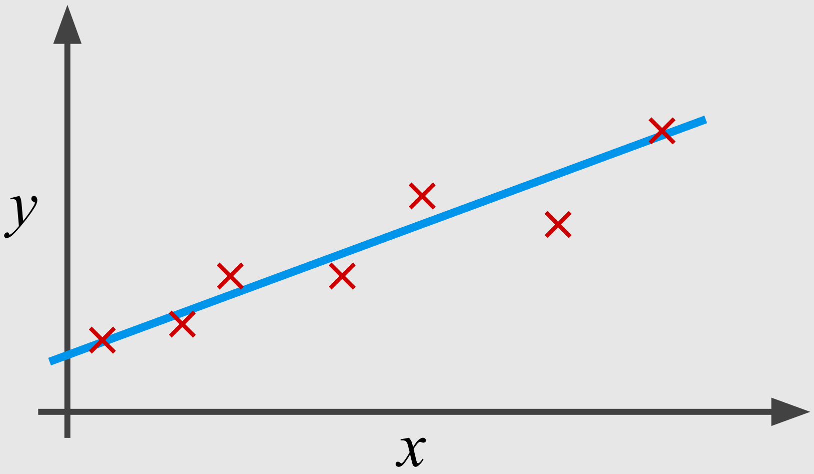
$$h_\theta(x) = \theta_0 + \theta_1 x$$



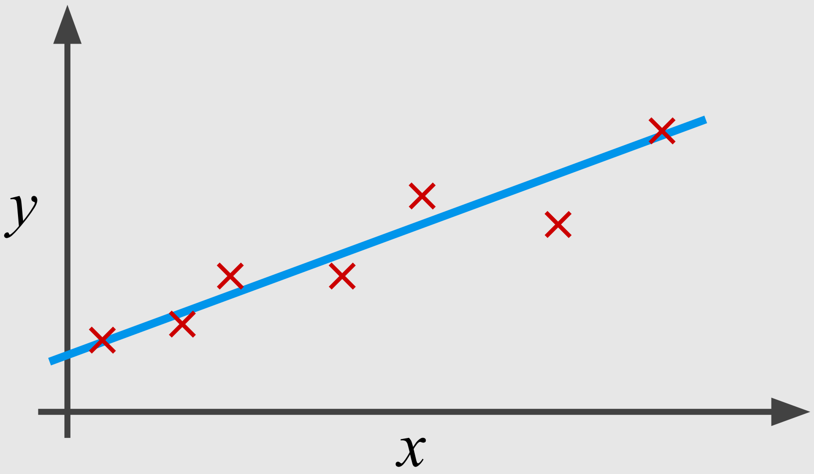
$\theta_0 = 1.5$
$\theta_1 = 0$

$\theta_0 = 0$
$\theta_1 = 0.5$

$\theta_0 = 1$
$\theta_1 = 0.5$

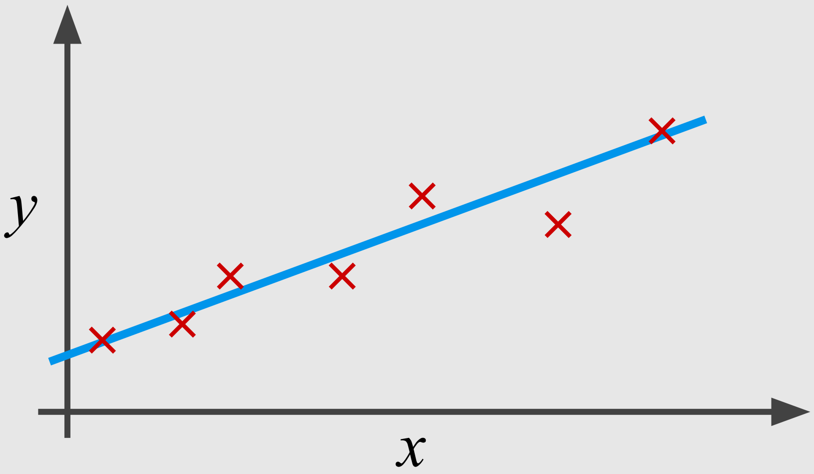


Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



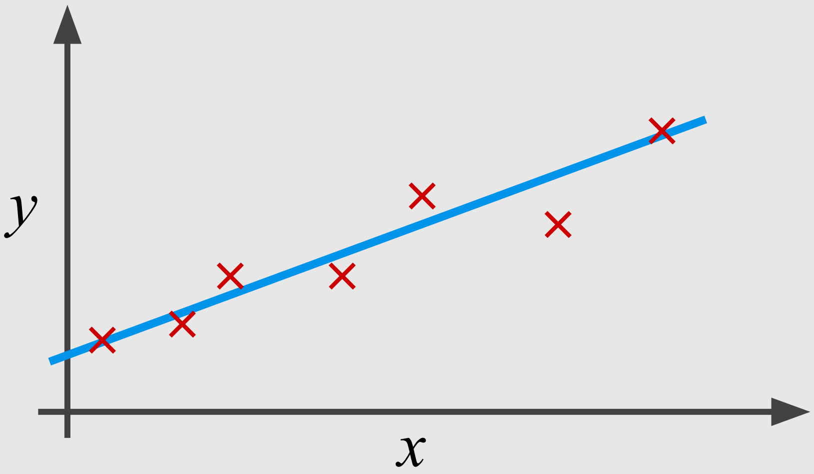
$$\underset{\theta_0,\theta_1}{\text{minimize}}$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



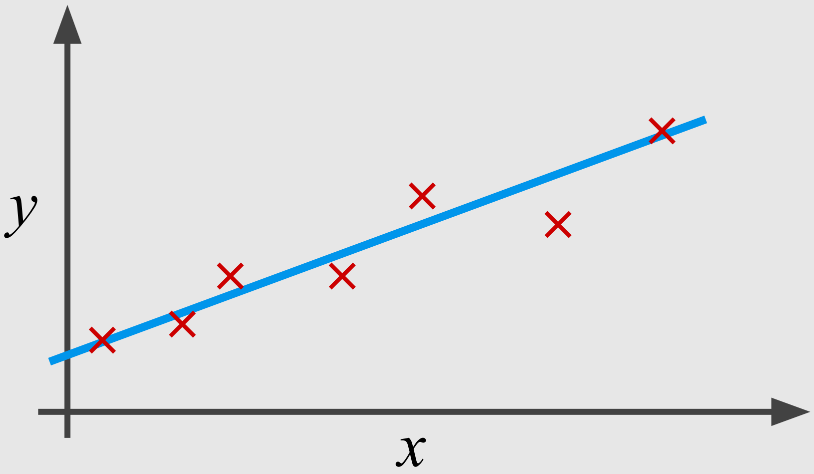
$$\underset{\theta_0,\theta_1}{\text{minimize}} \qquad (h_\theta(x \quad) - y \quad)^2$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



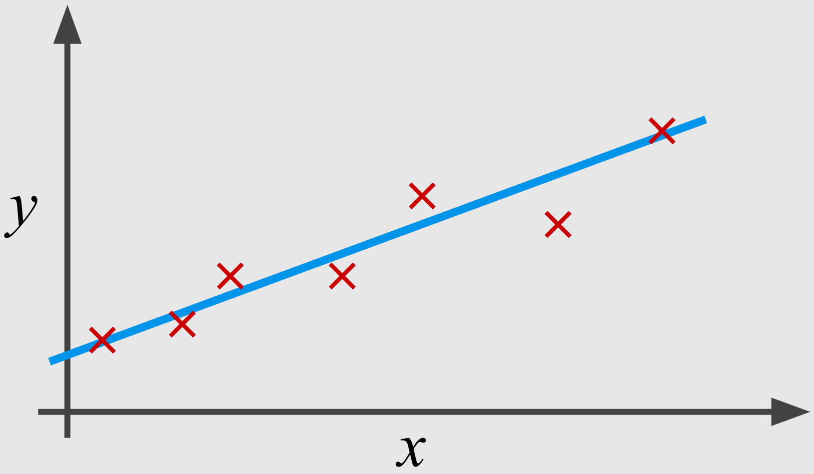
$$\underset{\theta_0,\theta_1}{\text{minimize}} \quad \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

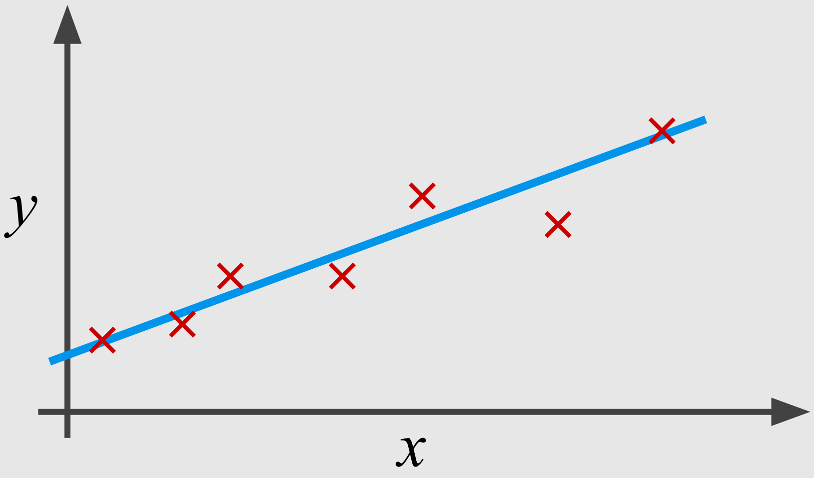
Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)

$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$



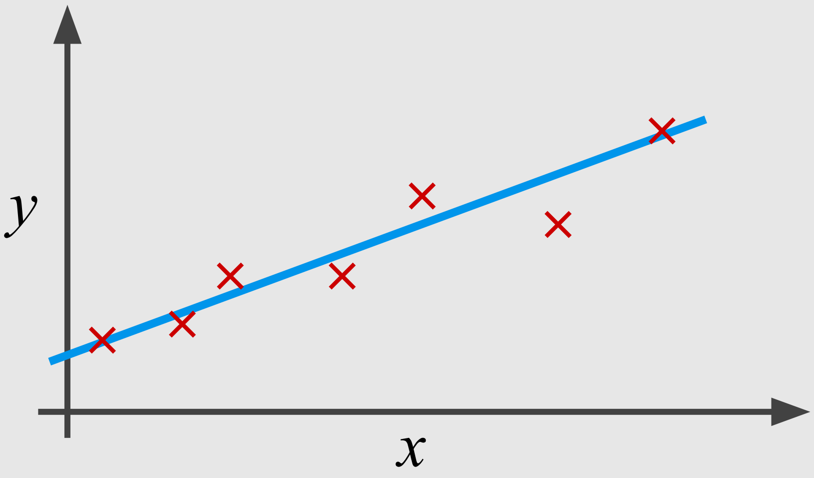
$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\Downarrow$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)

$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\underset{\theta_0,\theta_1}{\text{minimize}} \ J(\theta_0,\theta_1)$$

Cost function
(Squared error function)

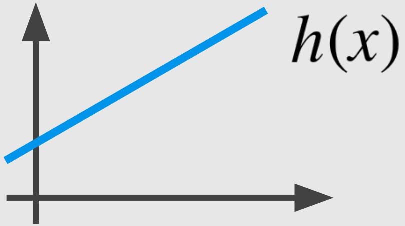
Cost Function Intuition I

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0 , \theta_1$$



Cost Function:

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

Goal:

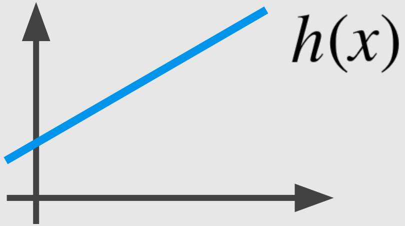
$$\underset{\theta_0,\theta_1}{\text{minimize}}\, J(\theta_0,\theta_1)$$

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0, \theta_1$$



Cost Function:

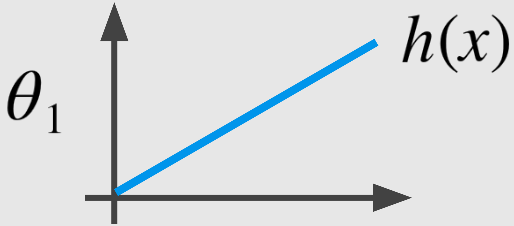
$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal:

$$\underset{\theta_0,\theta_1}{\text{minimize}}\ J(\theta_0,\theta_1)$$

<u>Simplified</u>

$$h_\theta(x) = \theta_1 x$$



$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$$

$h_\theta(x)$

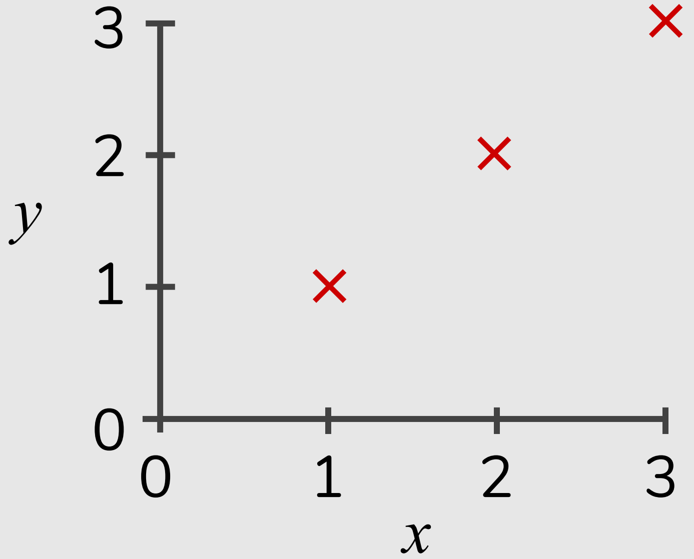
(for fixed θ_1, this is a function of x)

$J(\theta_1)$

(function of the parameters θ_1)

$h_\theta(x)$

(for fixed θ_1, this is a function of x)

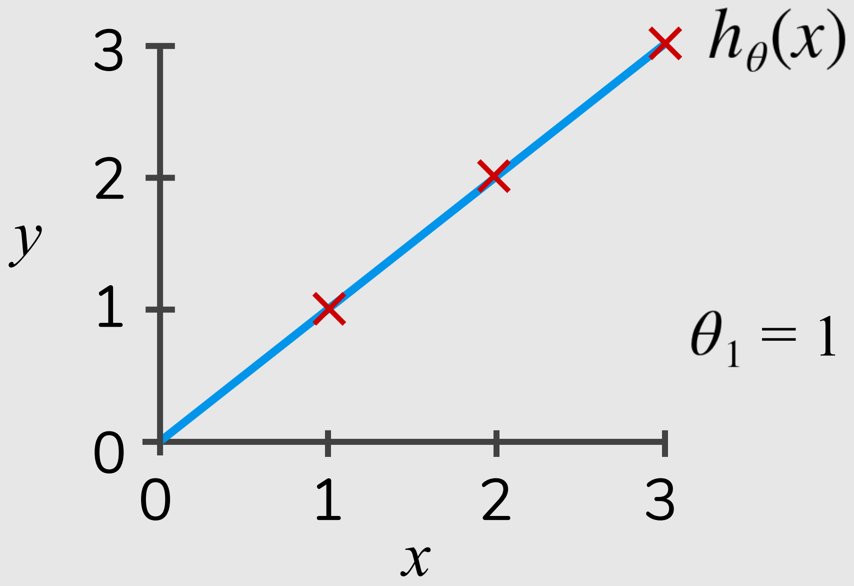


$J(\theta_1)$

(function of the parameters θ_1)

$h_\theta(x)$

(for fixed θ_1, this is a function of x)



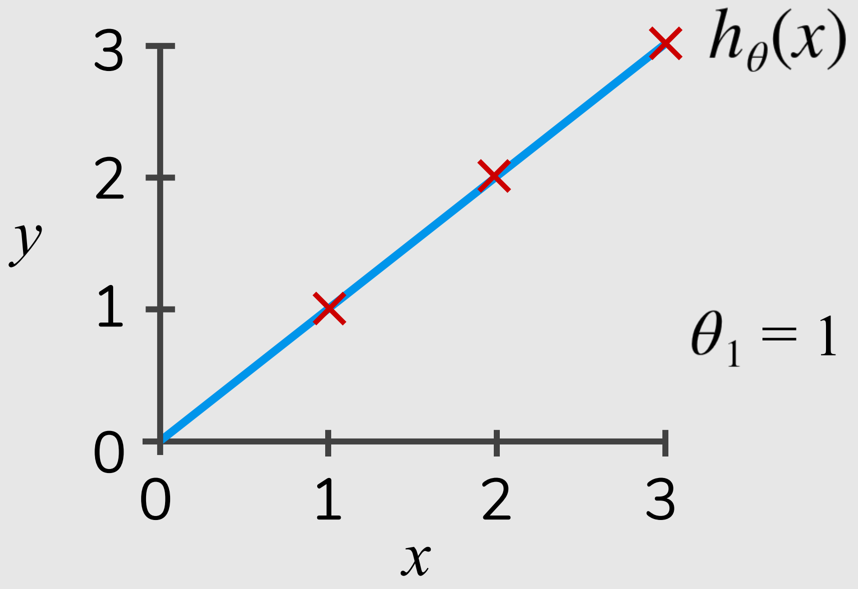
$J(\theta_1) = J(1) = ?$

$J(\theta_1)$

(function of the parameters θ_1)

$h_\theta(x)$

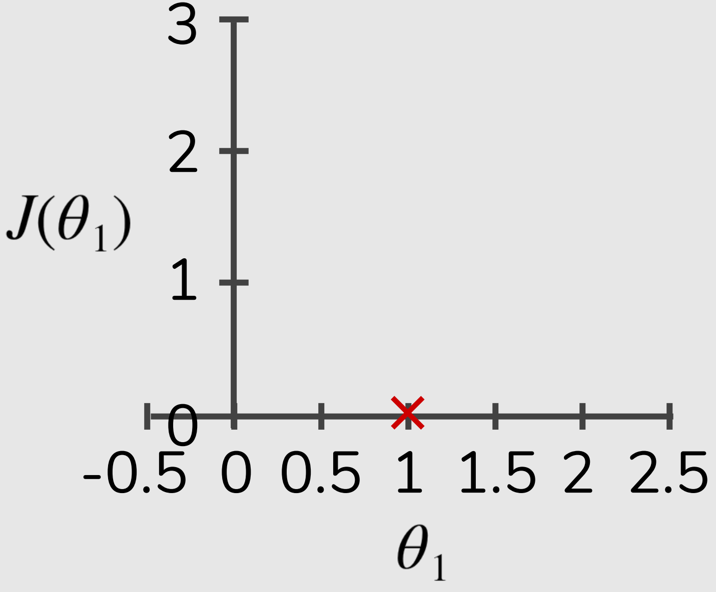
(for fixed θ_1, this is a function of x)



$J(\theta_1) = J(1) = 0$

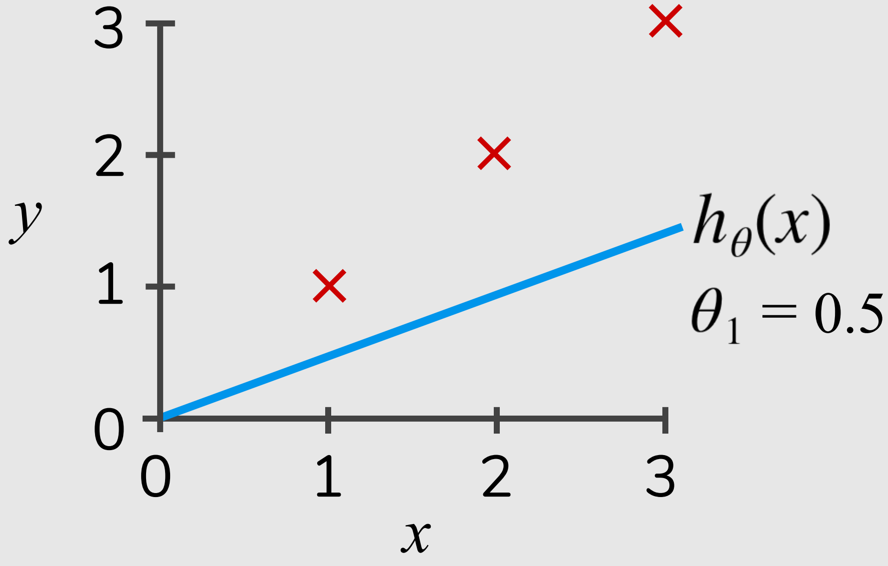
$J(\theta_1)$

(function of the parameters θ_1)



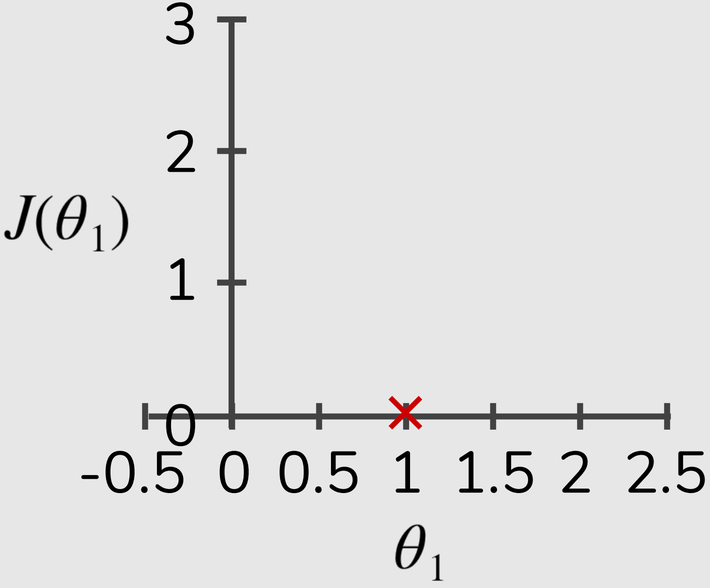
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



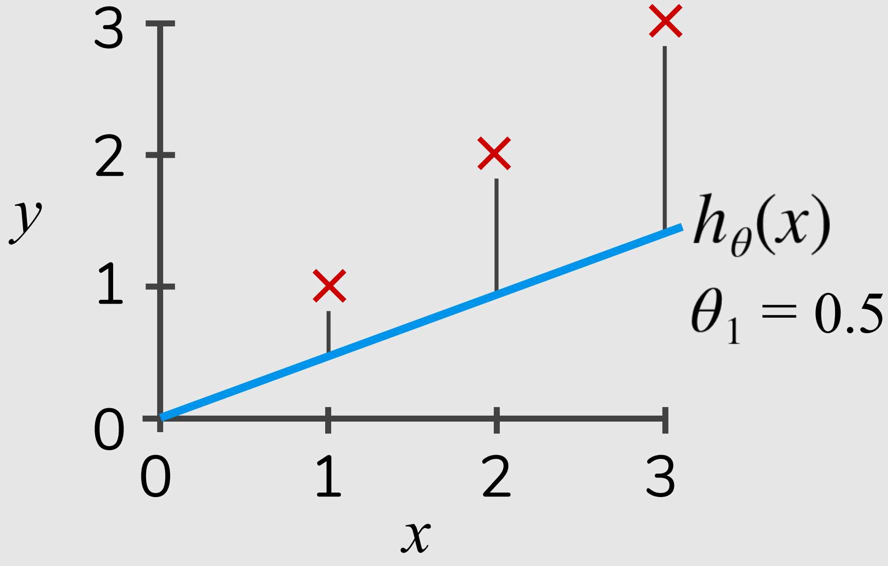
$J(\theta_1)$

(function of the parameters θ_1)



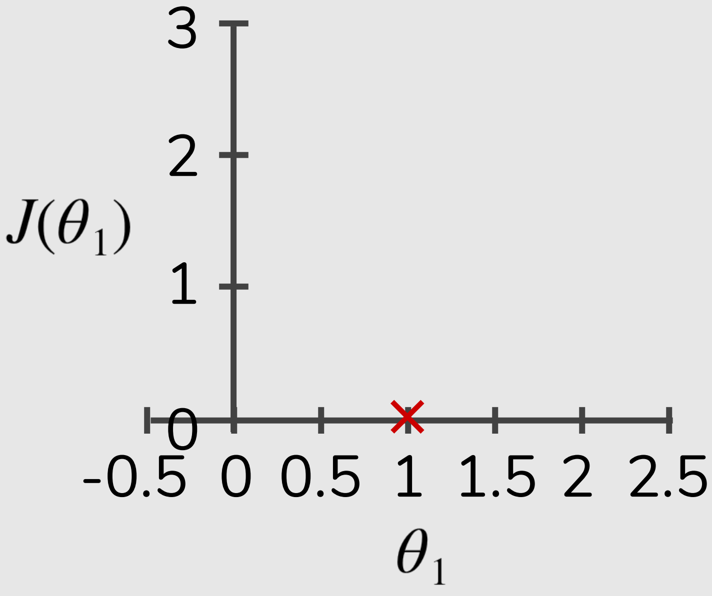
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



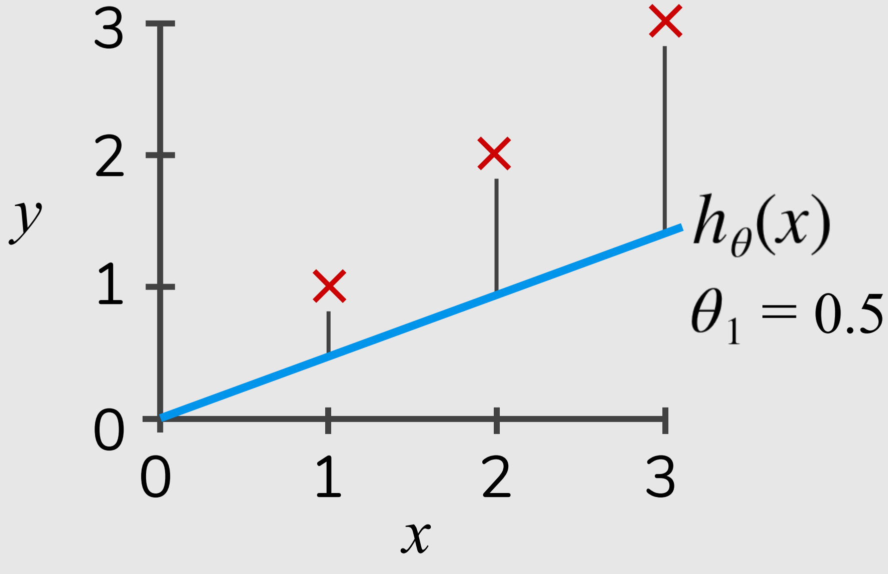
$J(\theta_1)$

(function of the parameters θ_1)



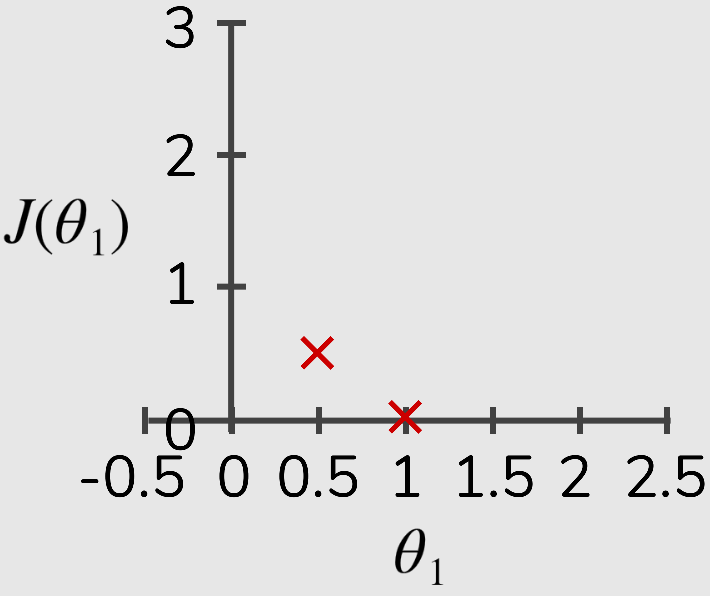
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



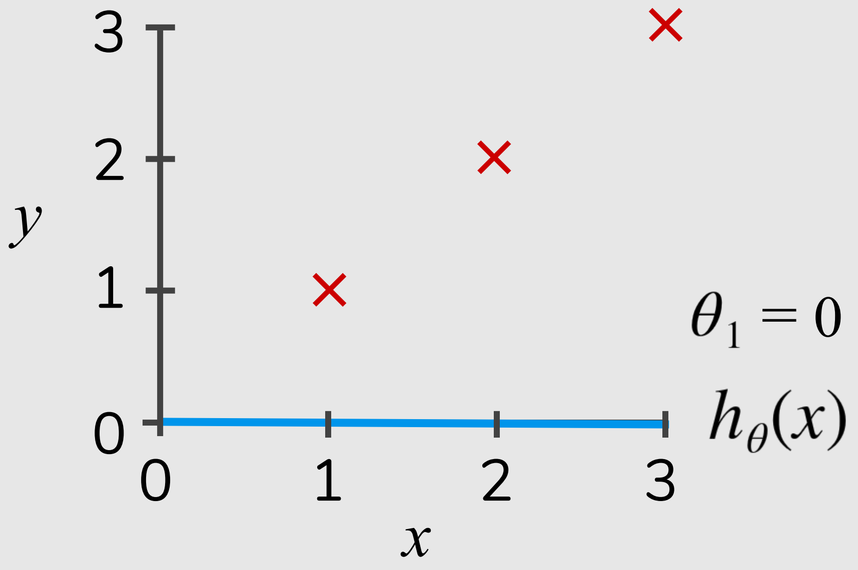
$J(\theta_1)$

(function of the parameters θ_1)



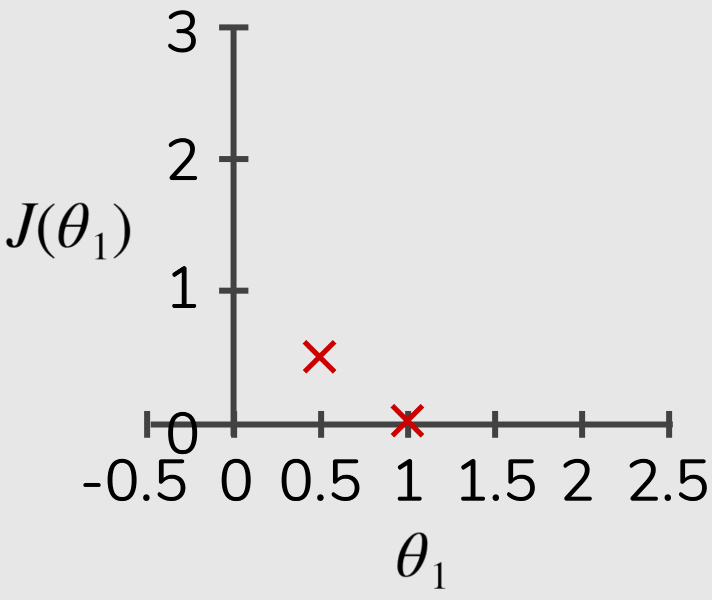
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



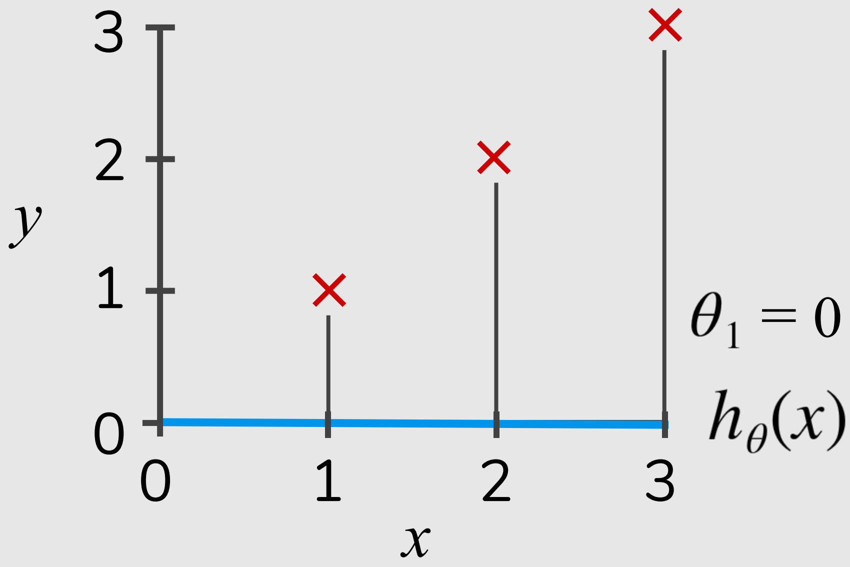
$J(\theta_1)$

(function of the parameters θ_1)



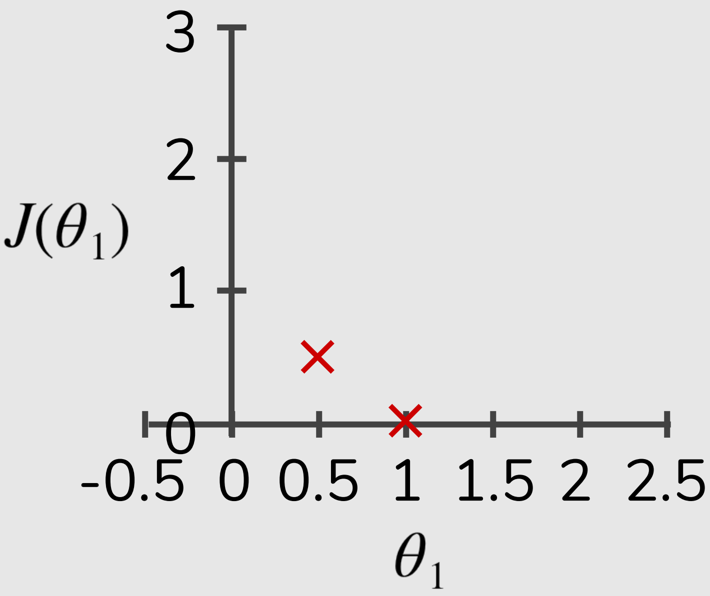
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



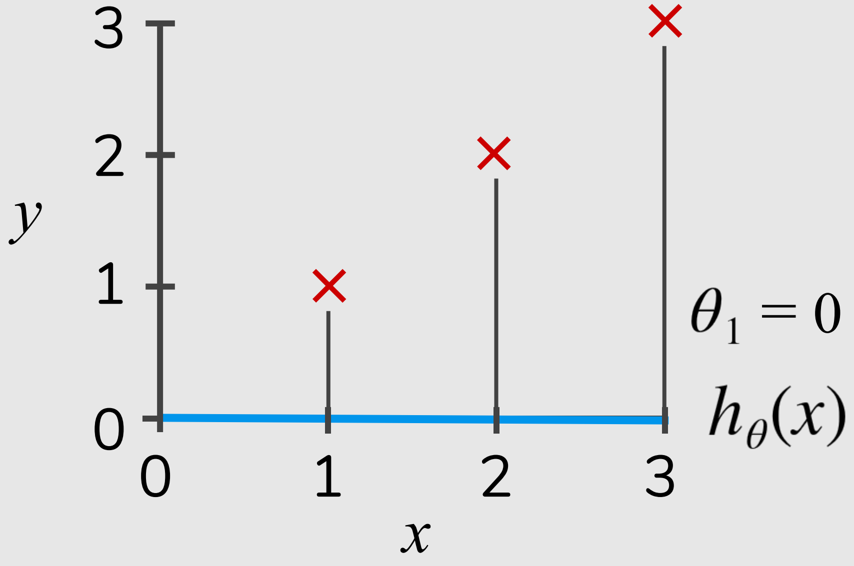
$J(\theta_1)$

(function of the parameters θ_1)



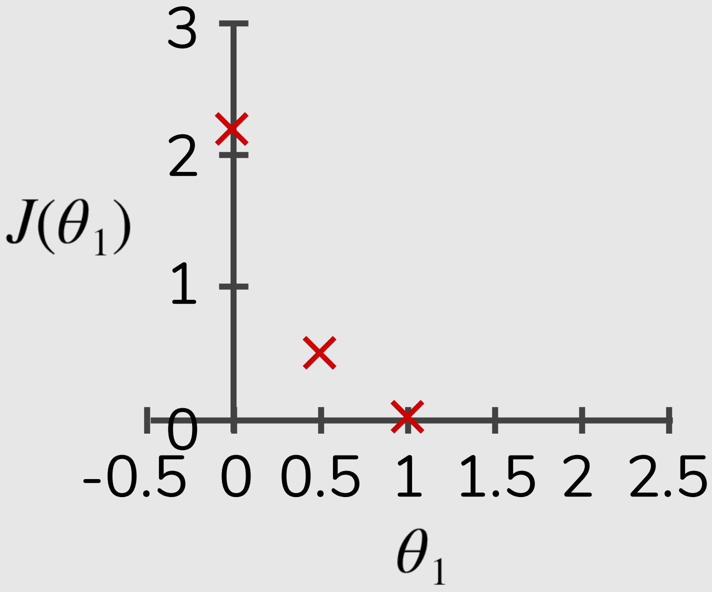
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



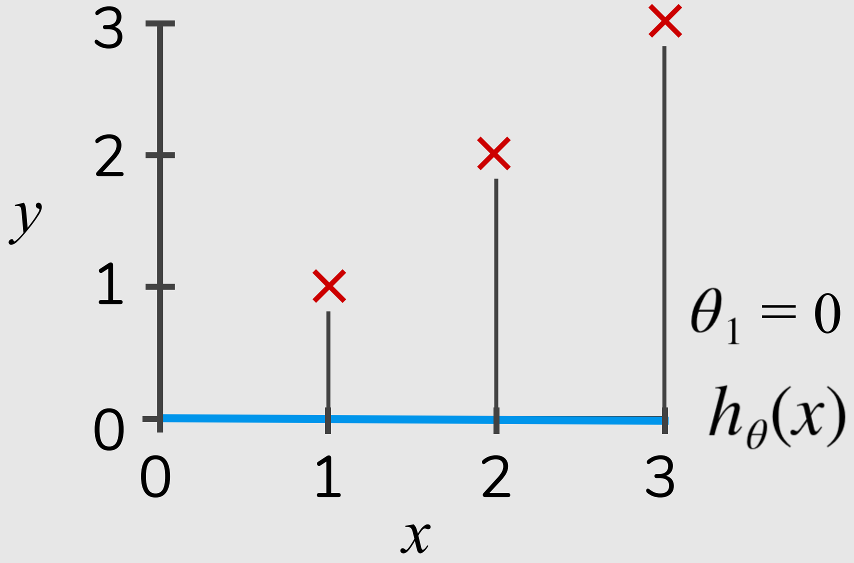
$J(\theta_1)$

(function of the parameters θ_1)



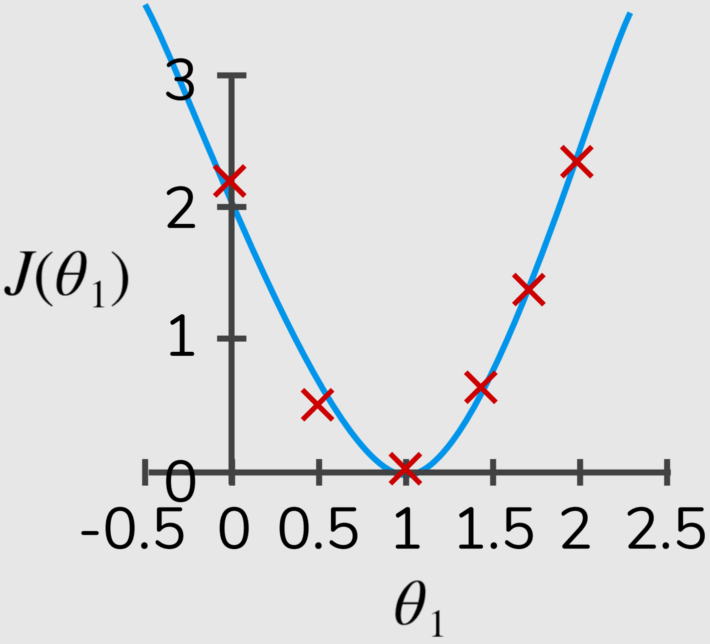
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameters θ_1)



Cost Function Intuition II

$h_\theta(x)$

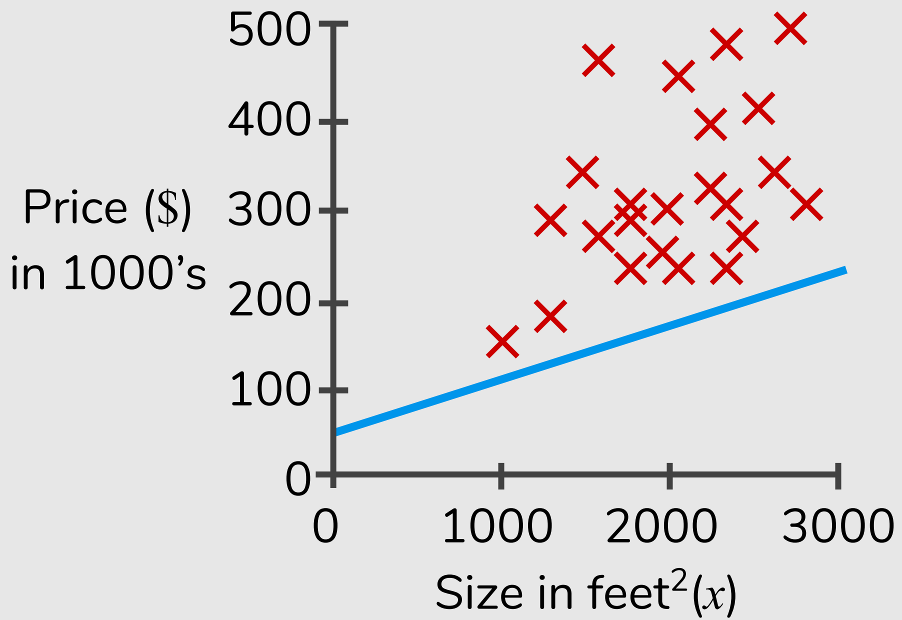
(for fixed θ_0, θ_1, this is a function of x)

$J(\theta_0,\theta_1)$

(function of the parameters θ_0, θ_1)

$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$\theta_0 = 50$

$\theta_1 = 0.06$

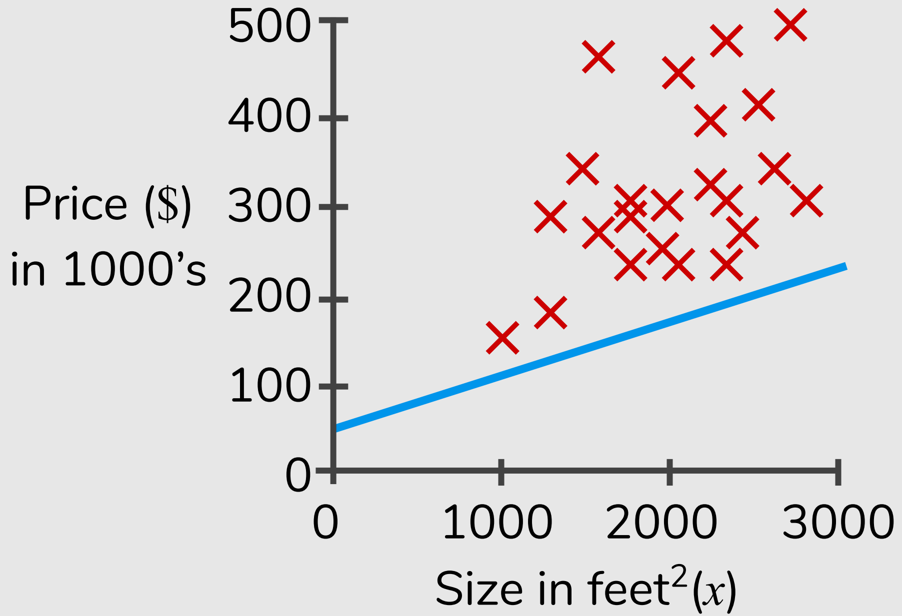
$$h_\theta(x) = 50 + 0.06x$$

$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)

$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



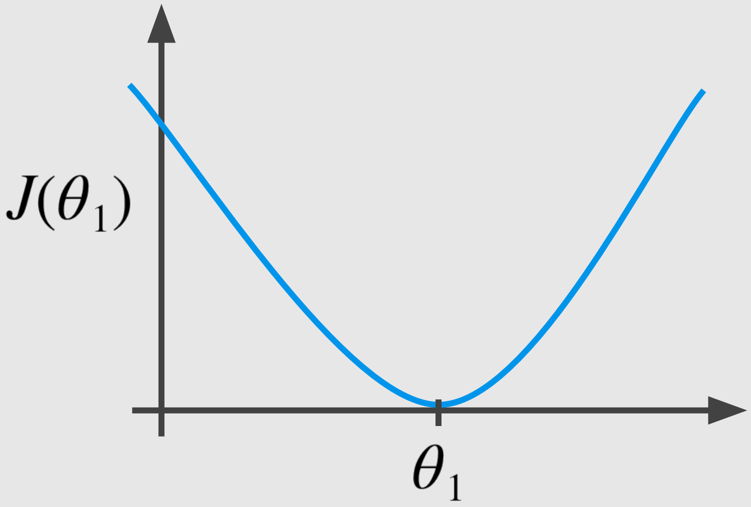
$$h_\theta(x) = 50 + 0.06x$$

$\theta_0 = 50$

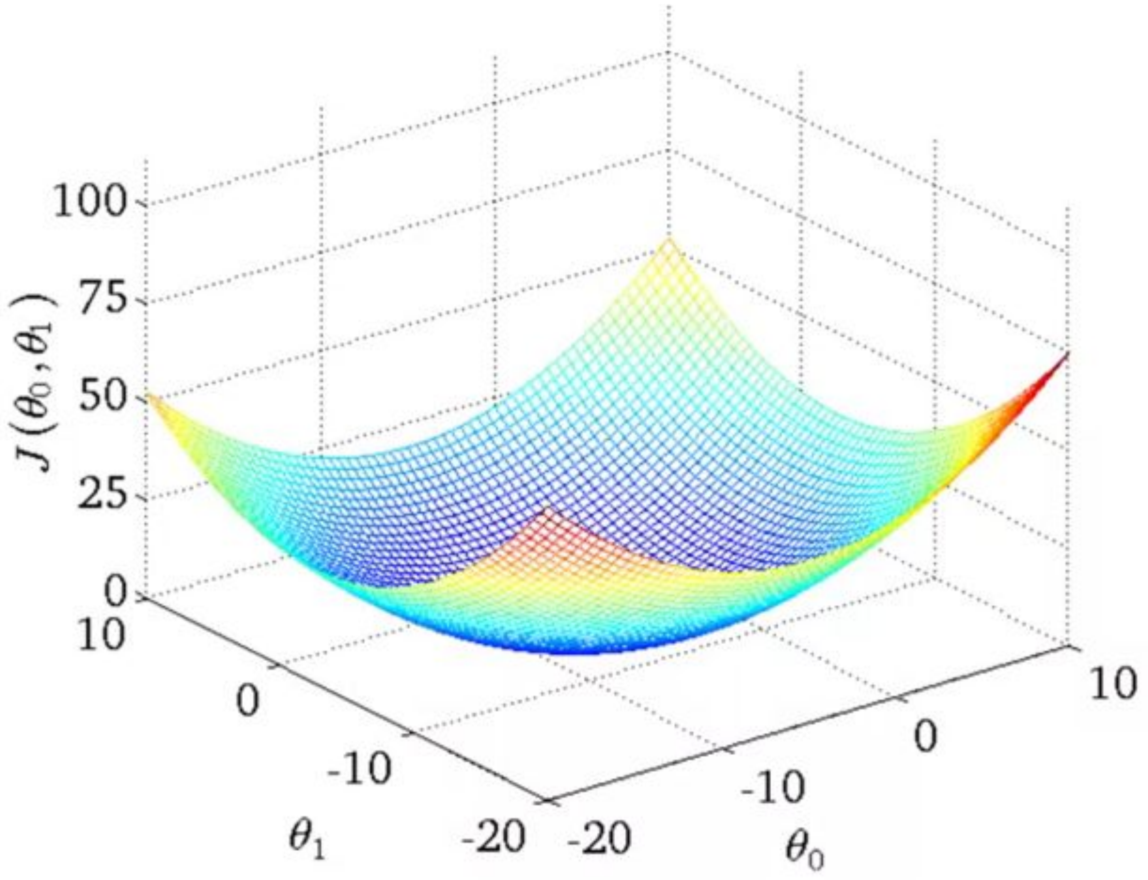
$\theta_1 = 0.06$

$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)

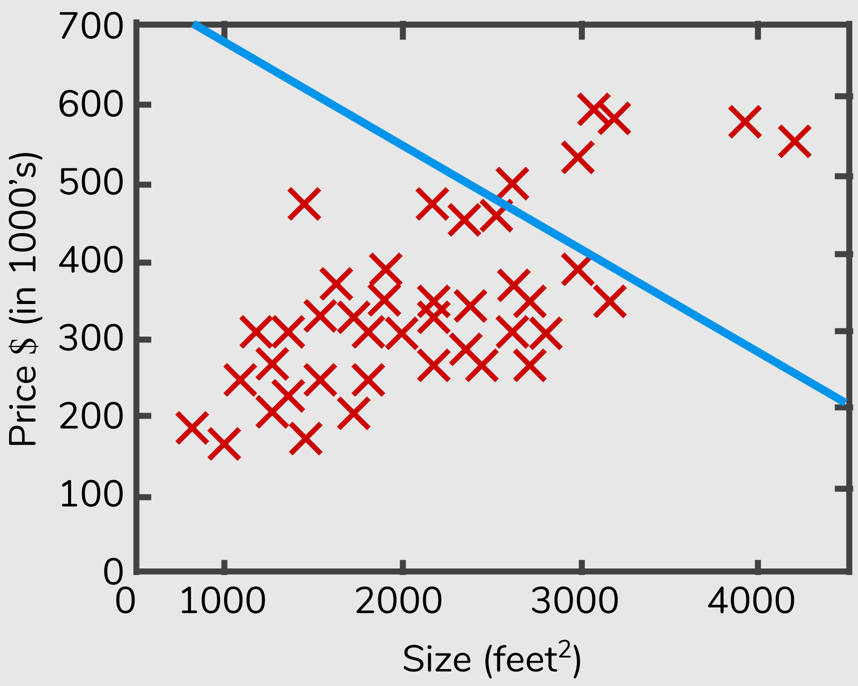


θ_0 and θ_1?


100
75
50
25
0
$J(\theta_0, \theta_1)$
10
0
-10
-20
θ_1
-20
-10
0
10
θ_0

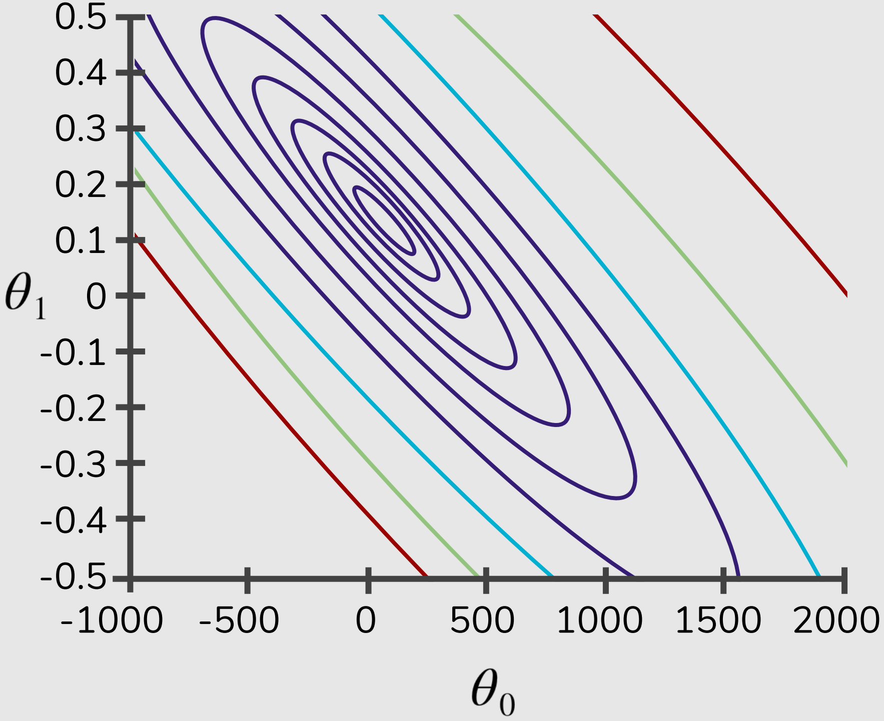
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



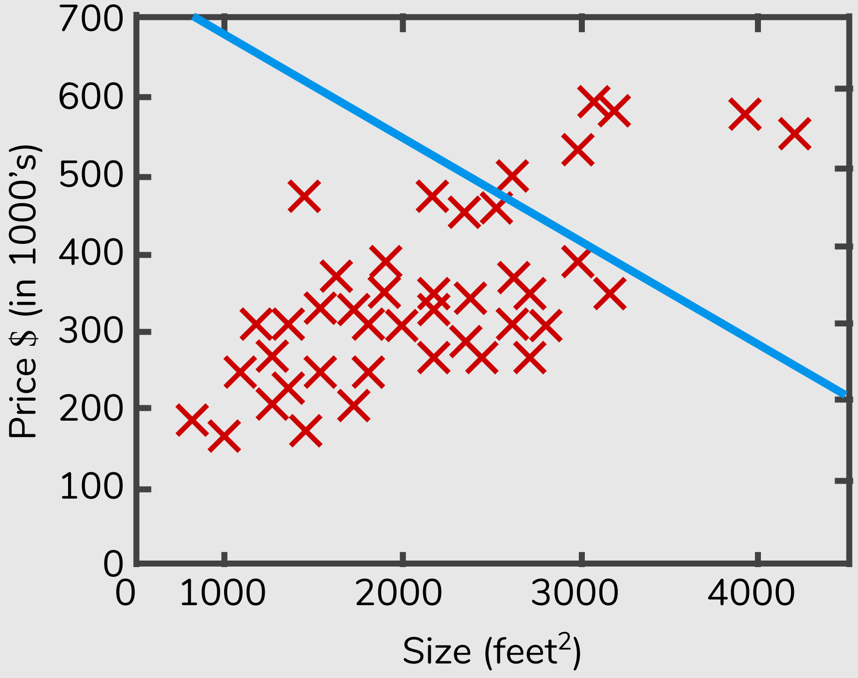
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



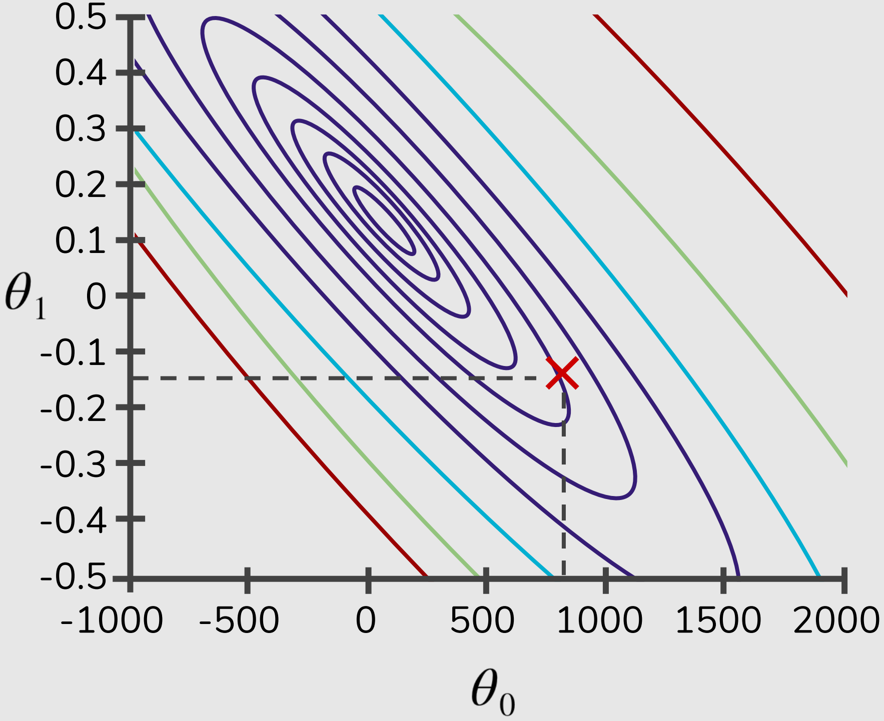
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



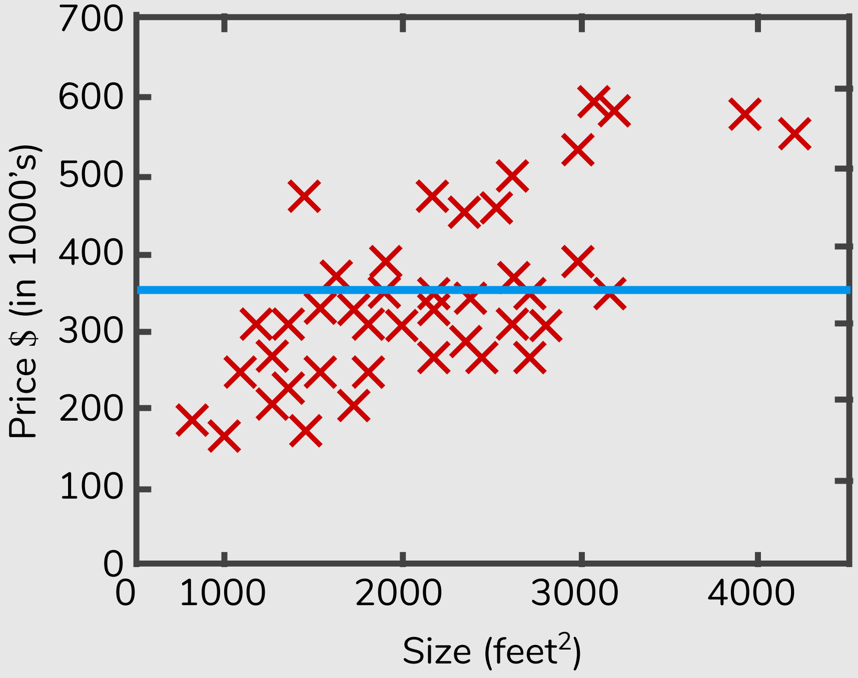
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



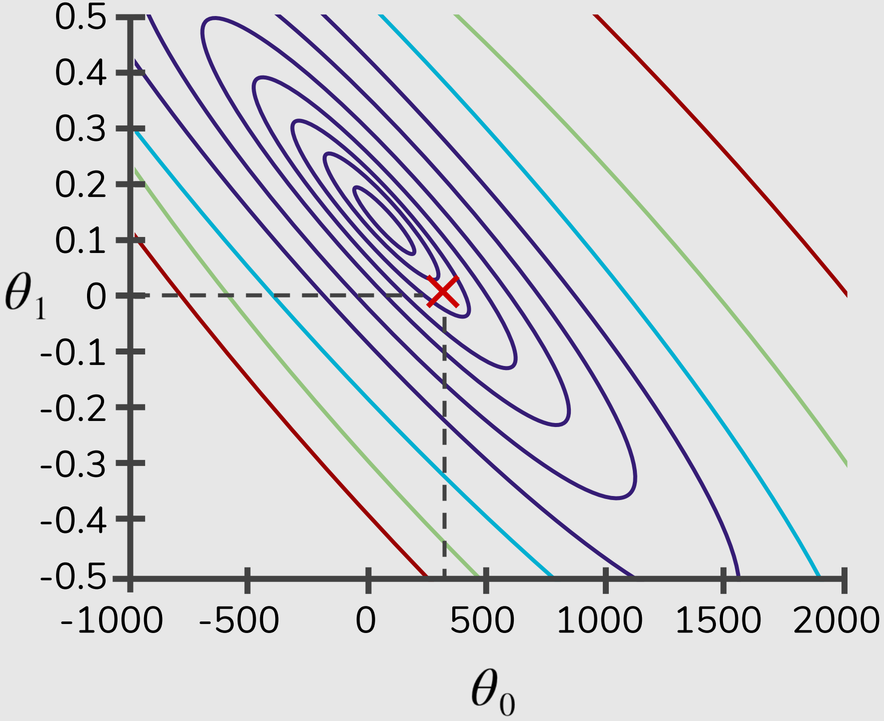
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



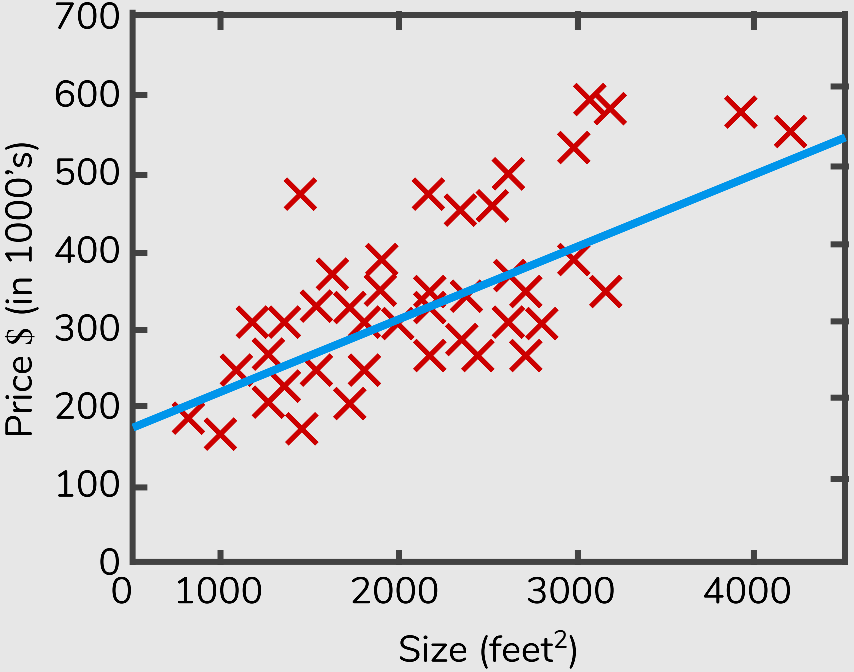
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



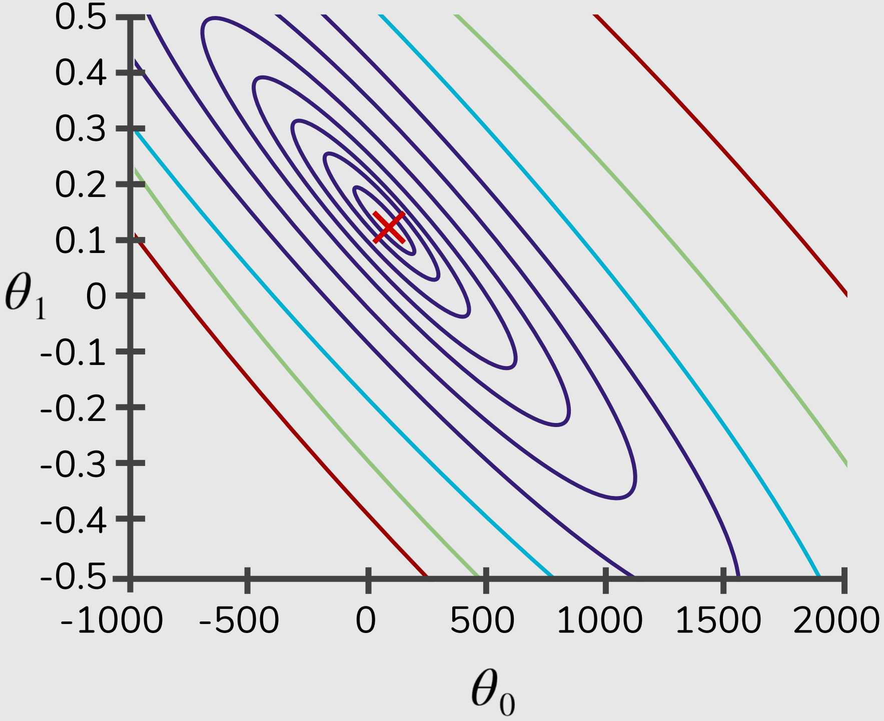
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



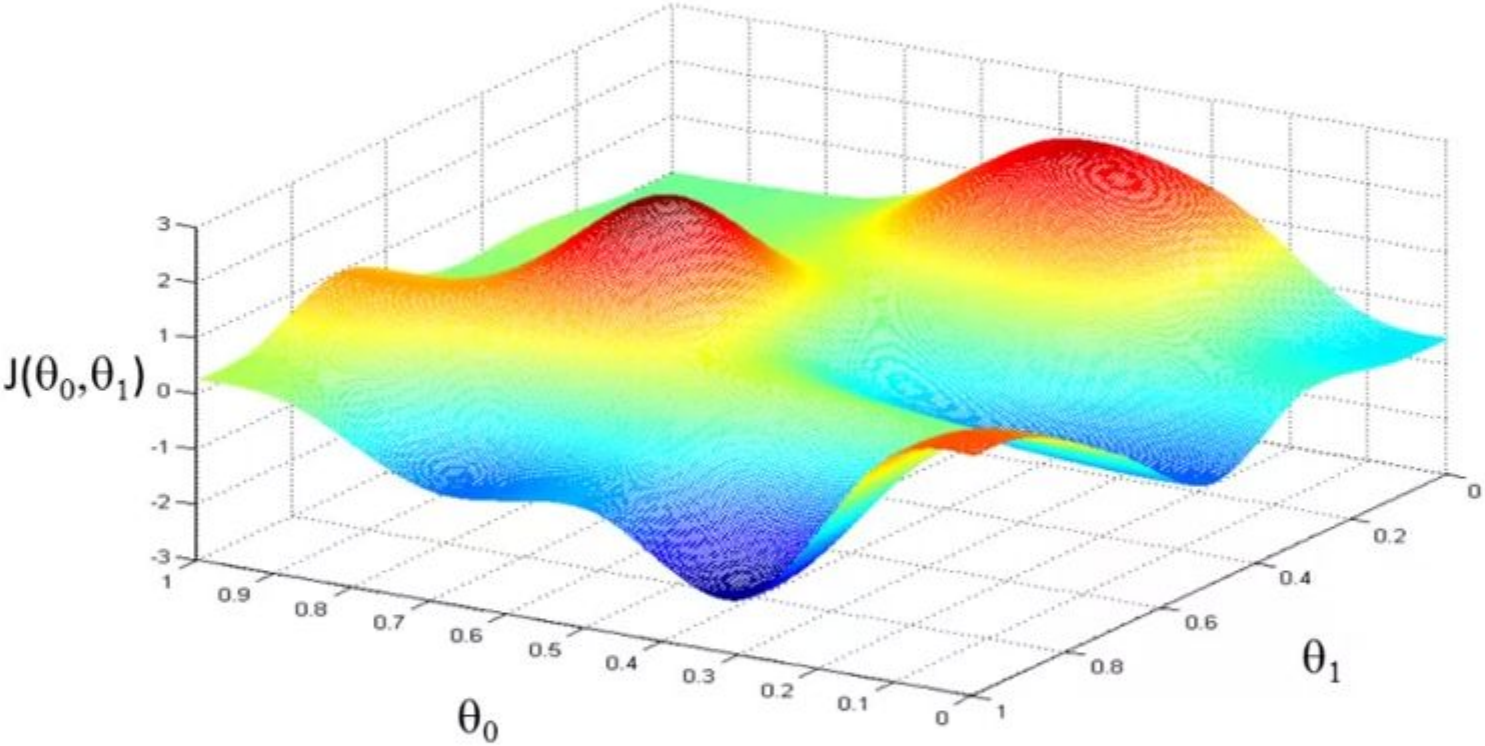
Gradient Descent

Have some function $J(\theta_0,\theta_1)$

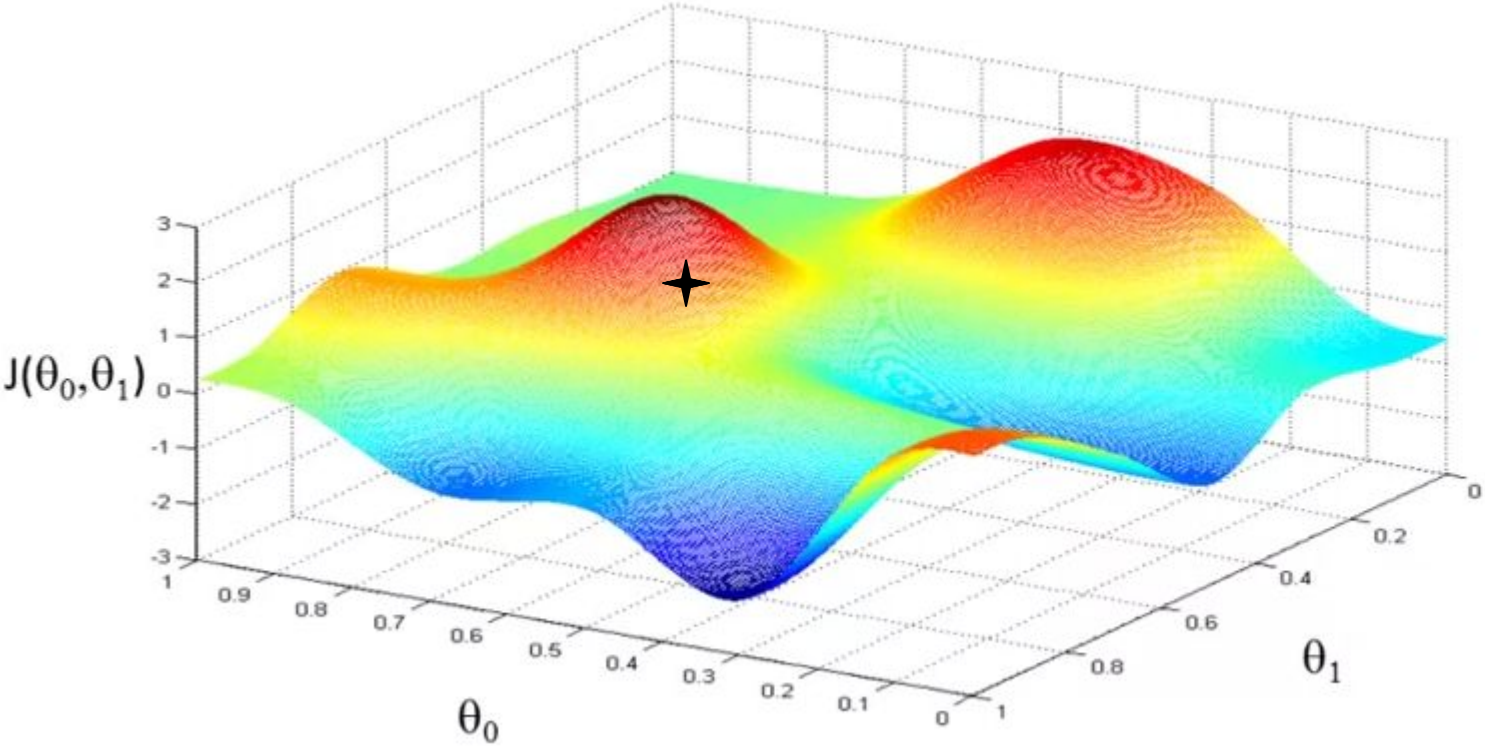
Want $\underset{\theta_0,\theta_1}{\text{minimize}}\, J(\theta_0,\theta_1)$

Outline:

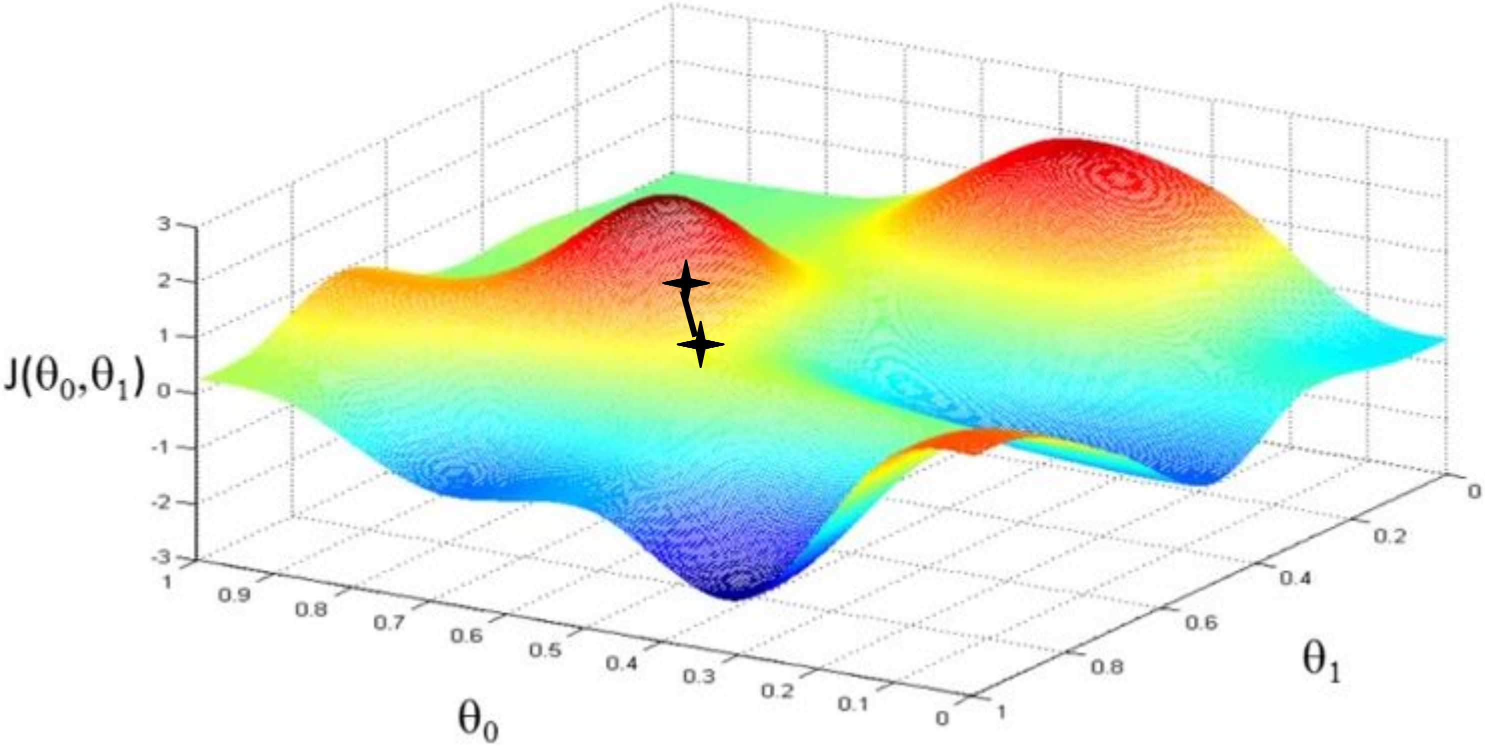
- Start with some θ_0, θ_1
- Keep changing θ_0, θ_1 to reduce $J(\theta_0,\theta_1)$ until we hopefully end up at a minimum



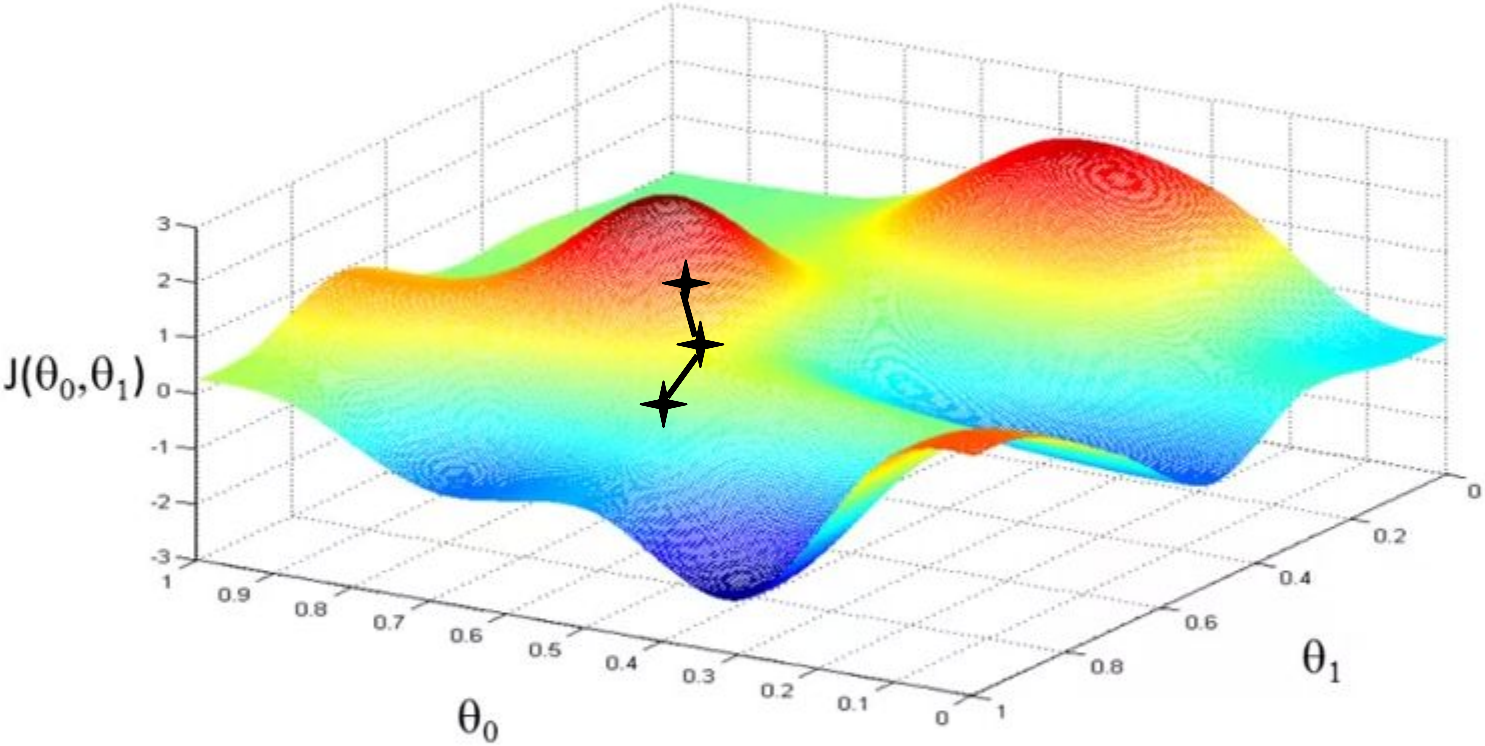
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



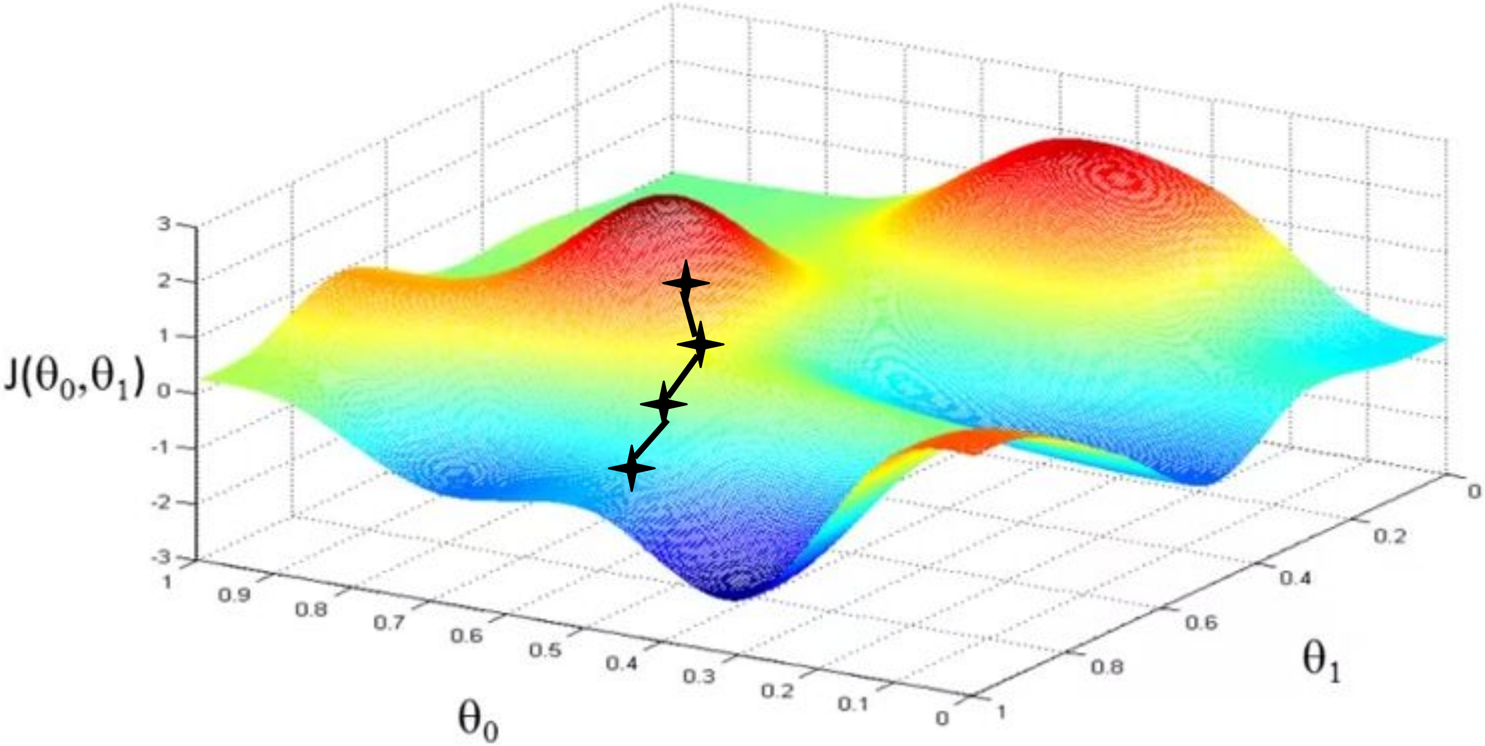
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



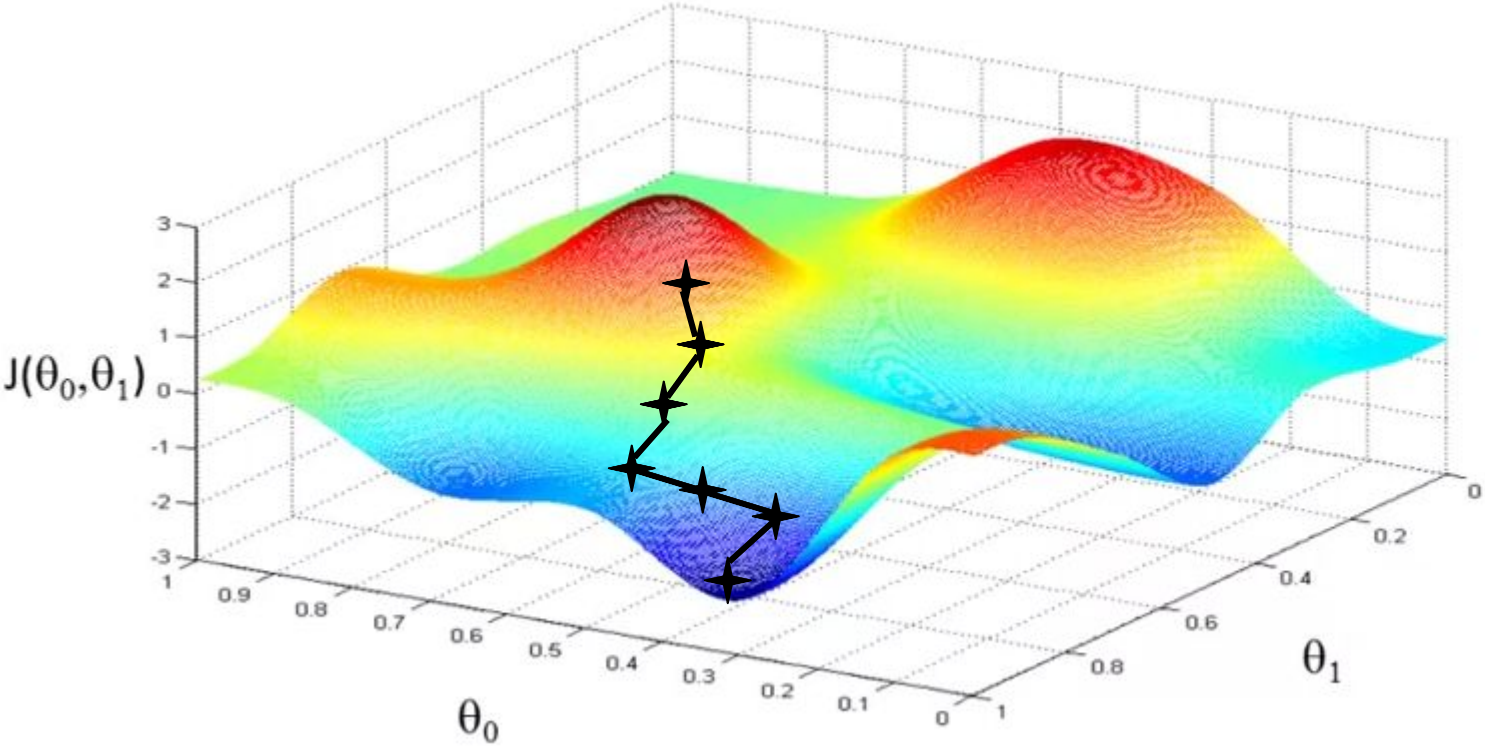
J(θ0,θ1)
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ0
0
0.2
0.4
0.6
0.8
1
θ1



$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



J(θ0,θ1)
θ0
θ1
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1



$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
θ_0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_1
0
0.2
0.4
0.6
0.8
1

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

(simultaneously update

$j = 0$ and $j = 1$)

}

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)}$$

(simultaneously update $j = 0$ and $j = 1$)

}

Learning rate

Derivative term

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad \text{(for } j = 0 \text{ and } j = 1\text{)}$$

}

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

$$\theta_0 := \text{temp0}$$

$$\theta_1 := \text{temp1}$$

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad \text{(for } j = 0 \text{ and } j = 1\text{)}$$

}

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

$$\theta_0 := \text{temp0}$$

$$\theta_1 := \text{temp1}$$

Incorrect

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

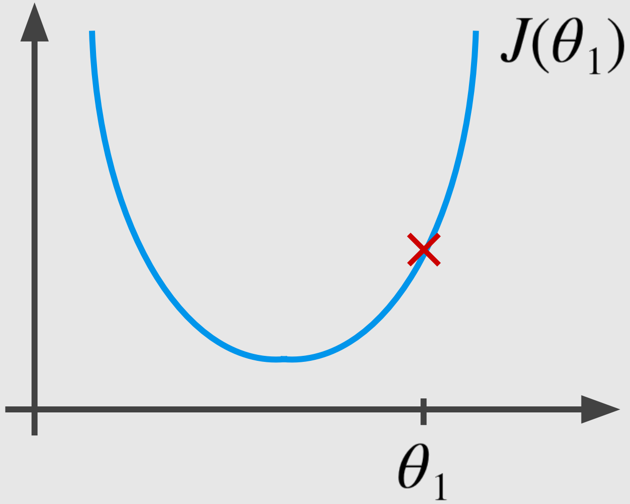
$$\theta_0 := \text{temp0}$$

$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

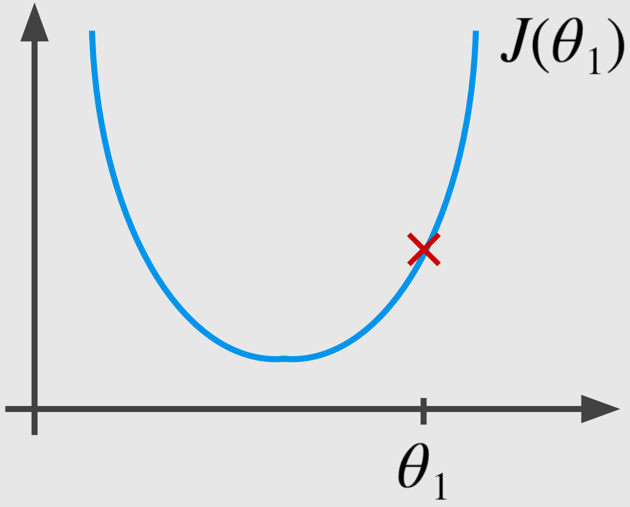
$$\theta_1 := \text{temp1}$$


$J(\theta_1)$

$\theta_1 \in \mathbb{R}$

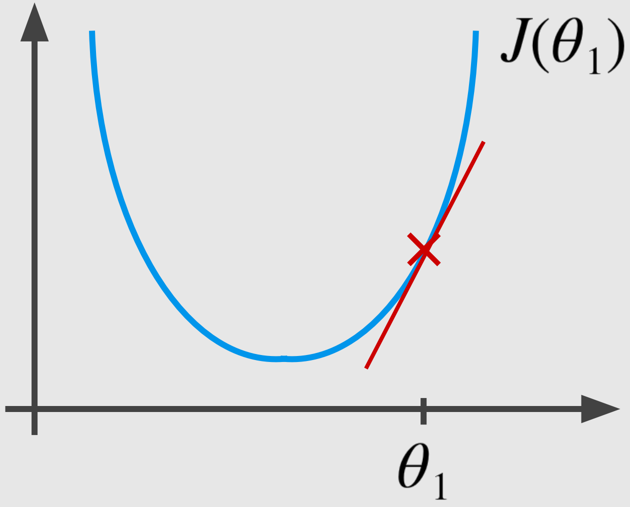


$$\theta_1 \in \mathbb{R}$$



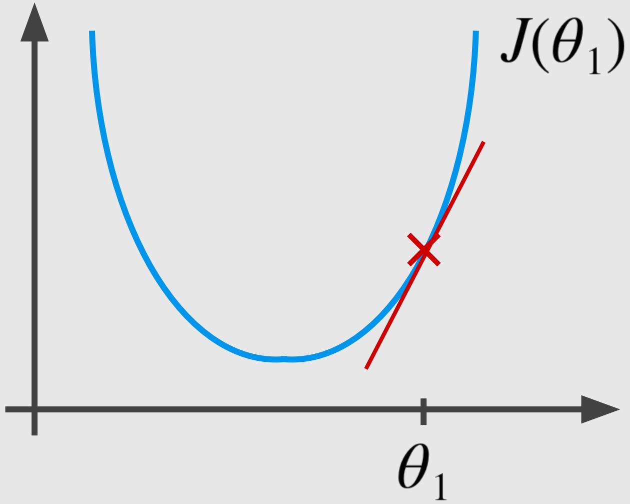
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

$\theta_1 \in \mathbb{R}$



$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

$\theta_1 \in \mathbb{R}$

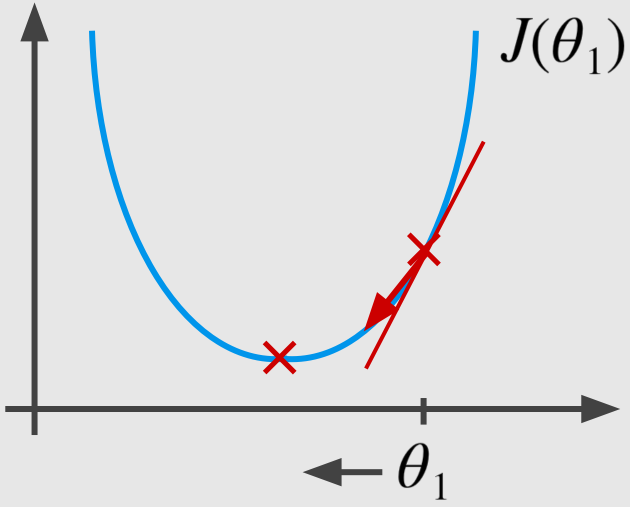


≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$\theta_1 := \theta_1 - \alpha \cdot$ (positive number)

$\theta_1 \in \mathbb{R}$

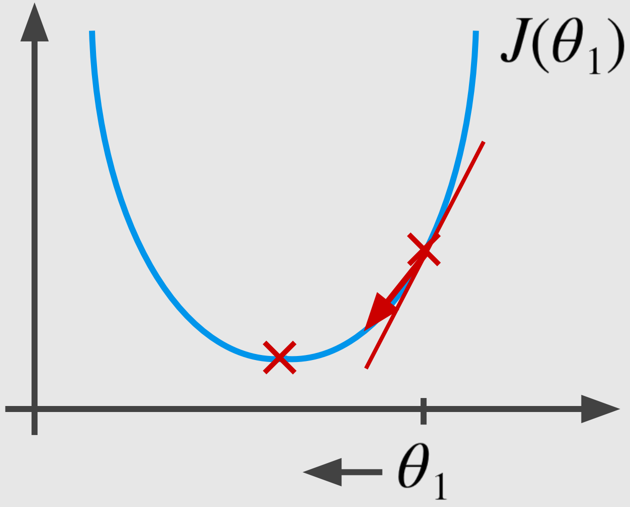


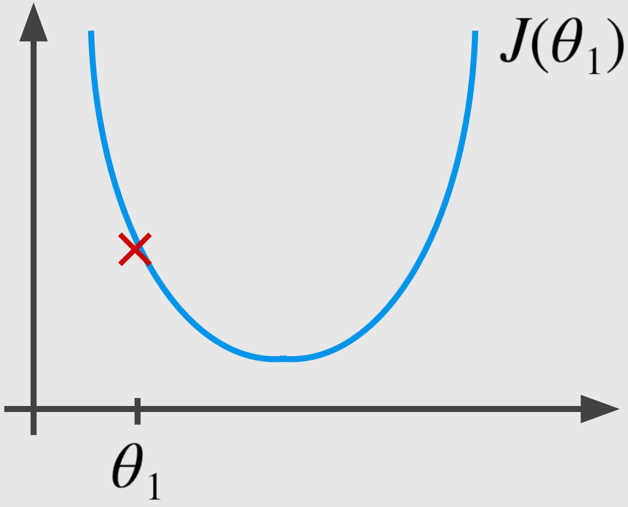
$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

≥ 0

$$\theta_1 := \theta_1 - \alpha \cdot \text{(positive number)}$$

$$\theta_1 \in \mathbb{R}$$



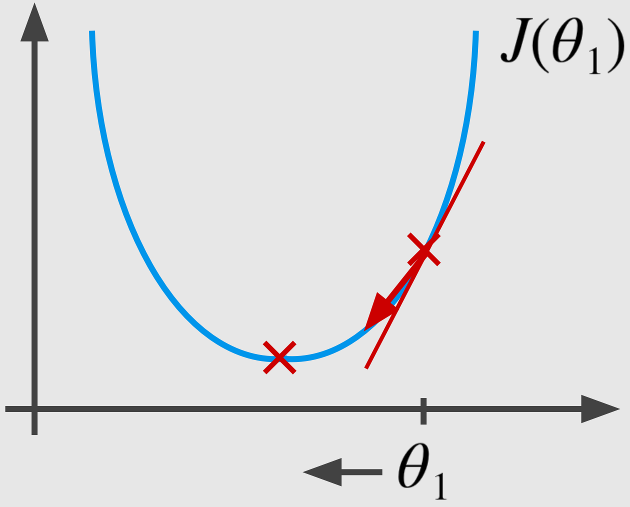


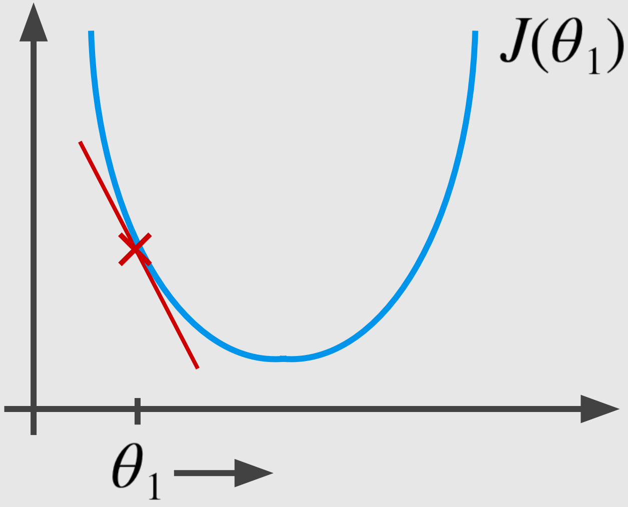
≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(positive number)}$$

$$\theta_1 \in \mathbb{R}$$





≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

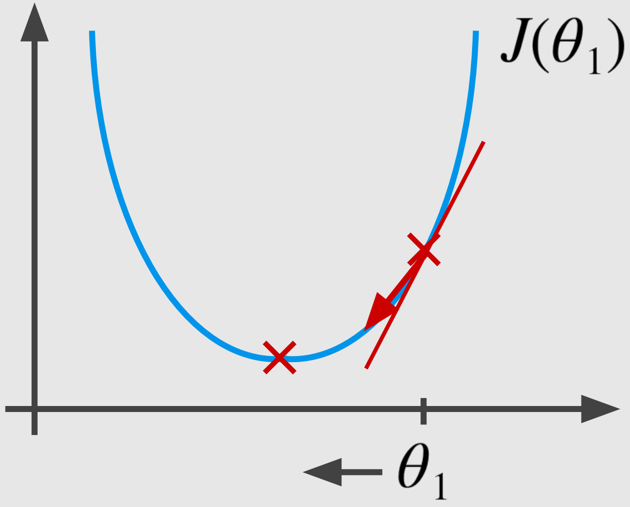
$$\theta_1 := \theta_1 - \alpha \cdot \text{(positive number)}$$

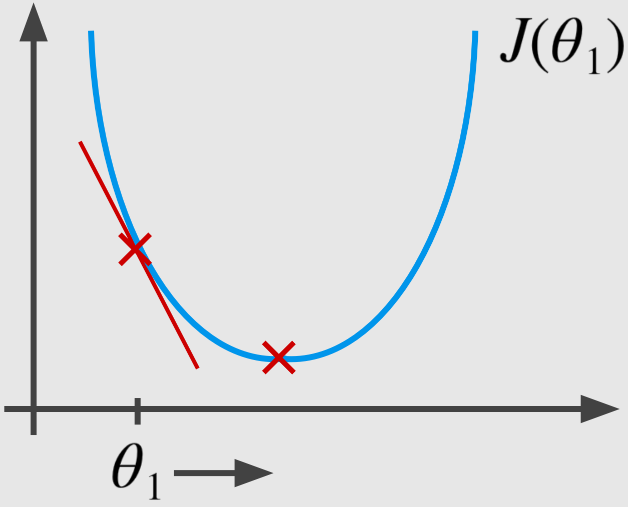
≤ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(negative number)}$$

$$\theta_1 \in \mathbb{R}$$





≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(positive number)}$$

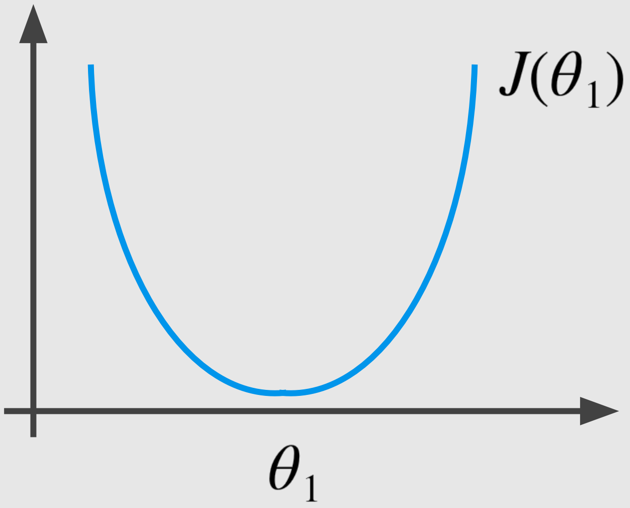
≤ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(negative number)}$$

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be ...



$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

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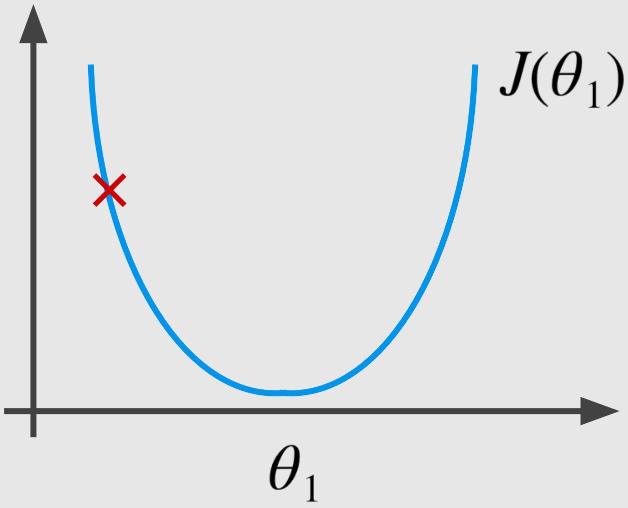
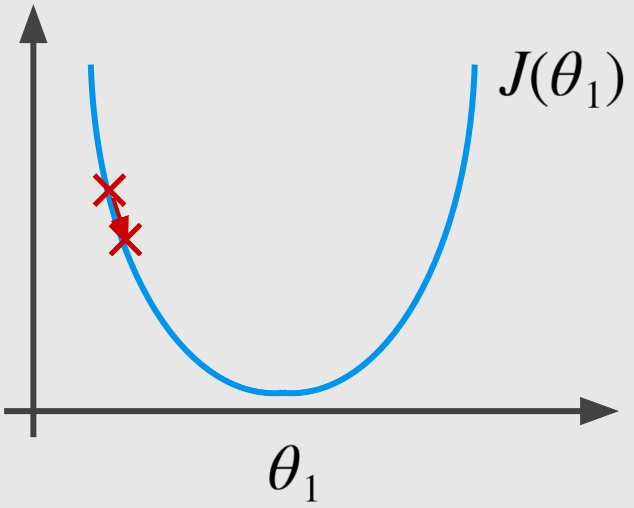


$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



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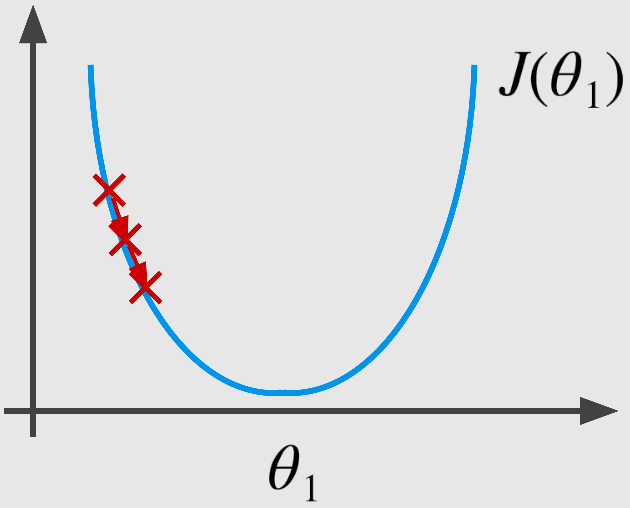

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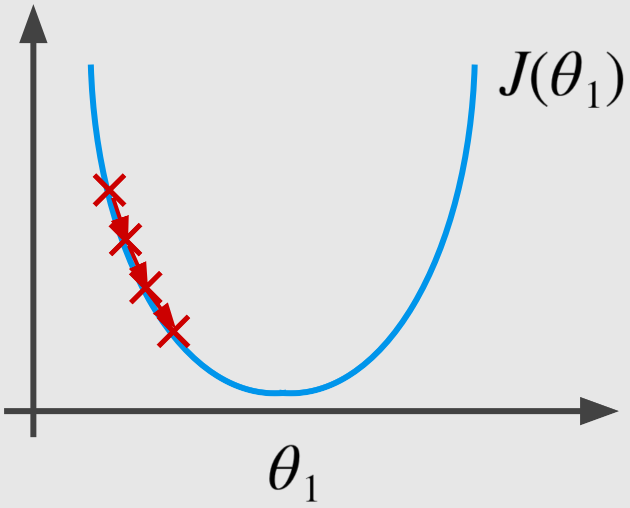

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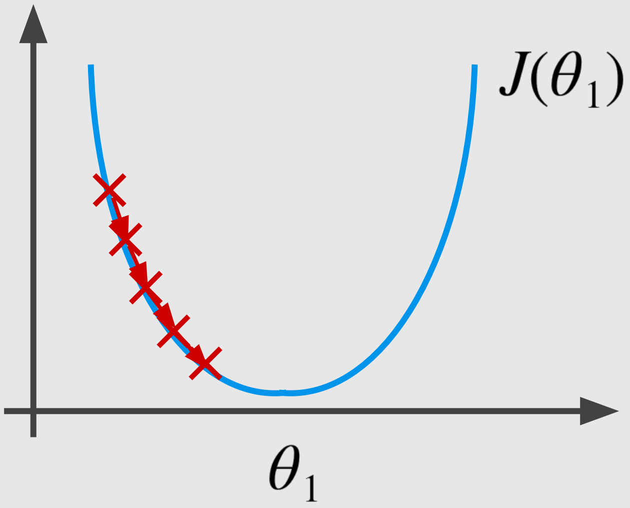

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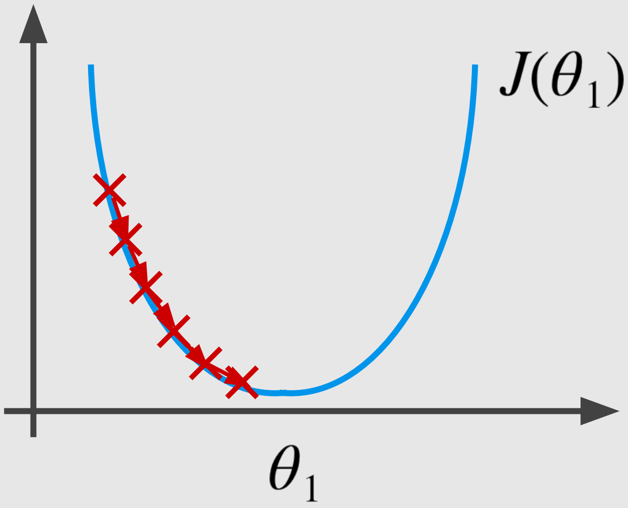

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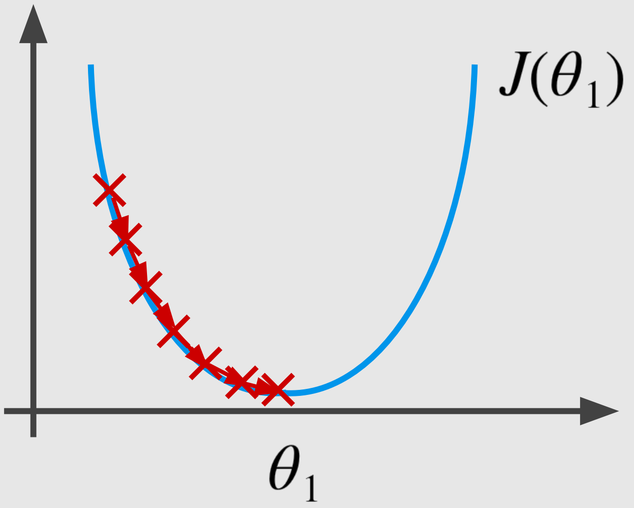

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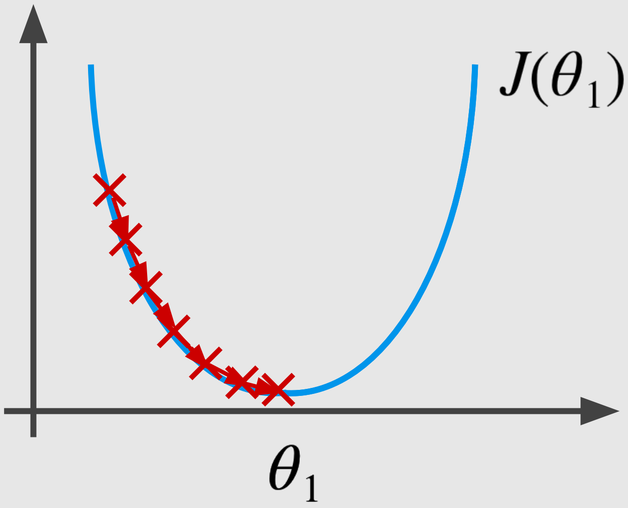
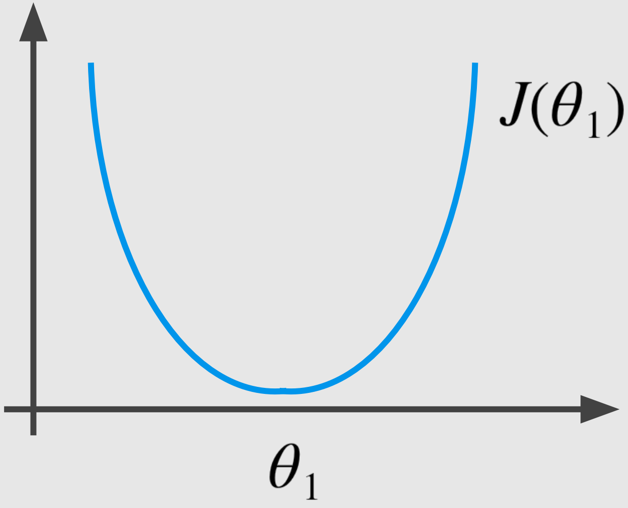


If α is too large, gradient descent can be ...



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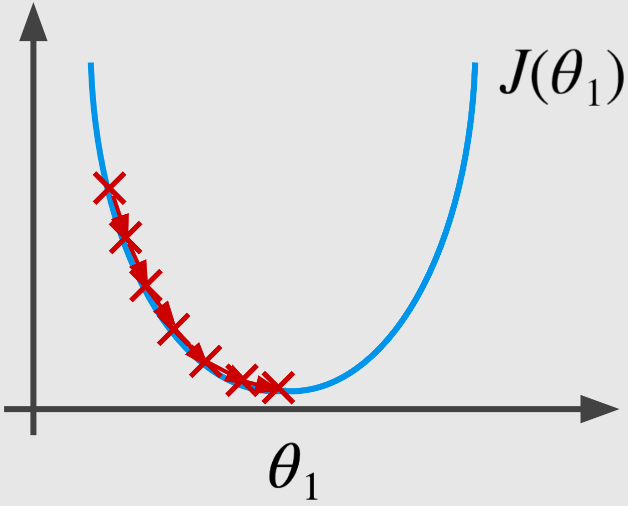

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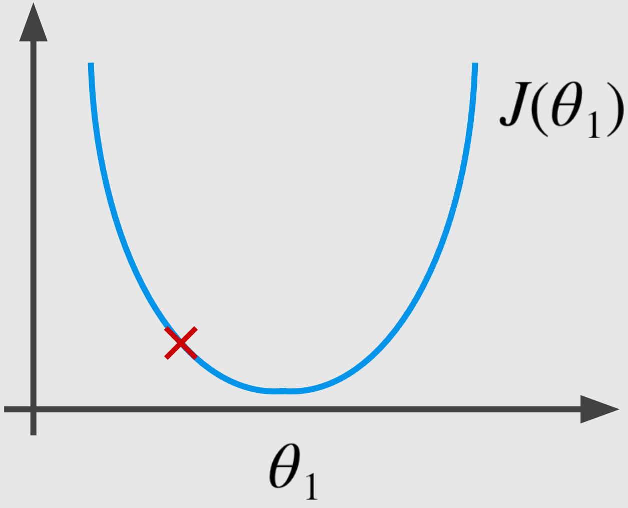

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

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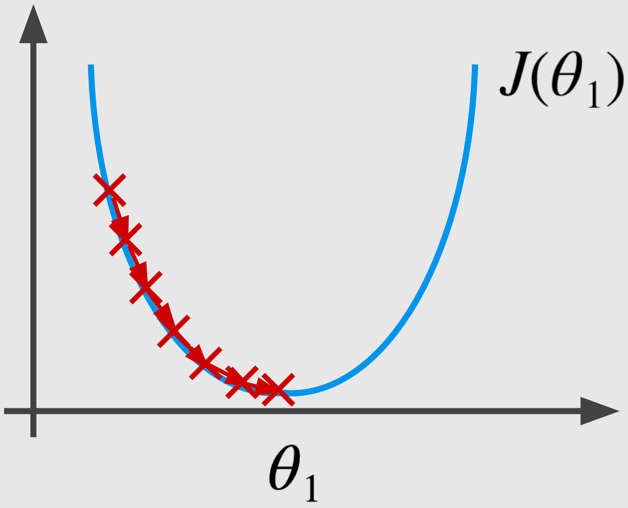
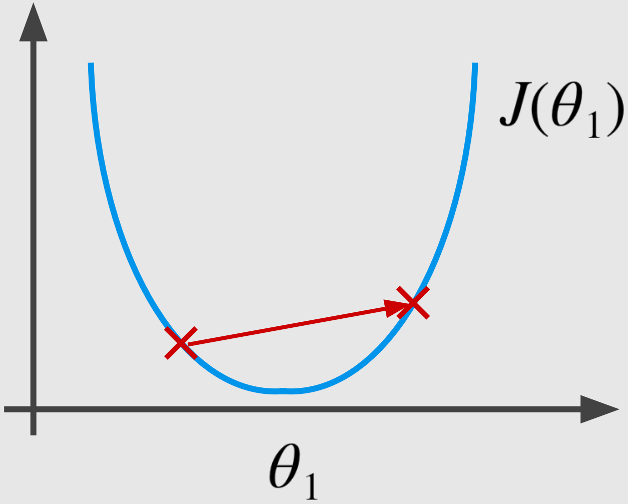
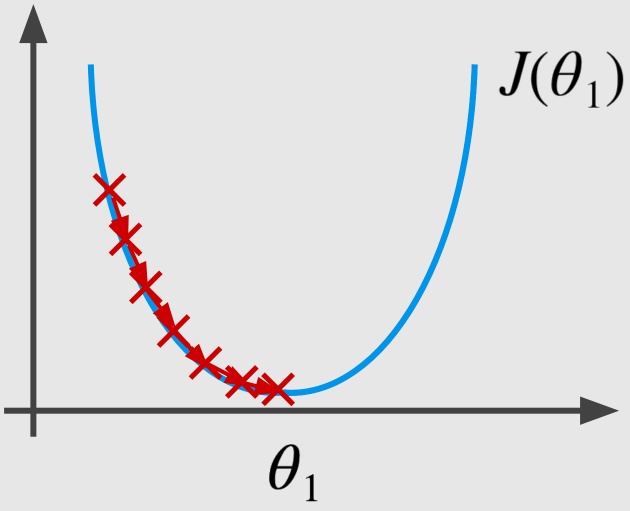


If α is too large, gradient descent can be overshoot the minimum. It may fail to converge, or even diverge.



$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$



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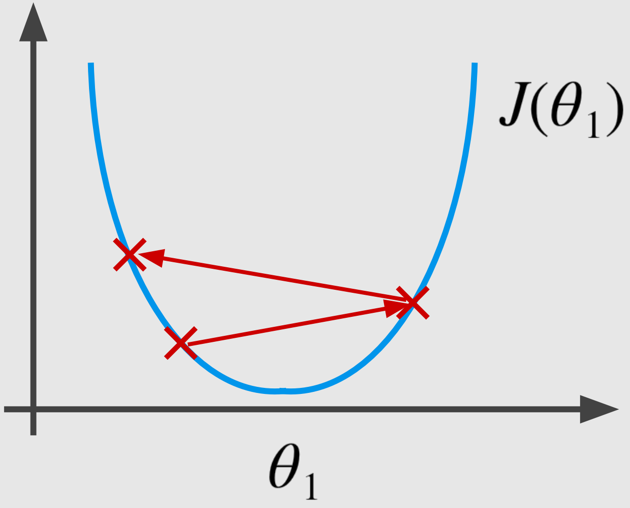

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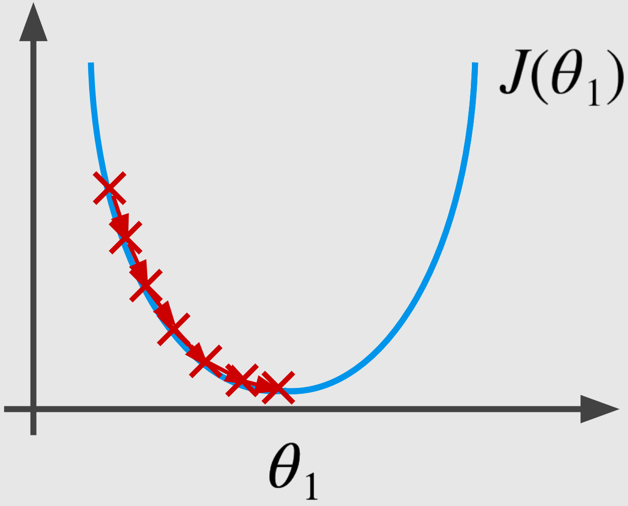

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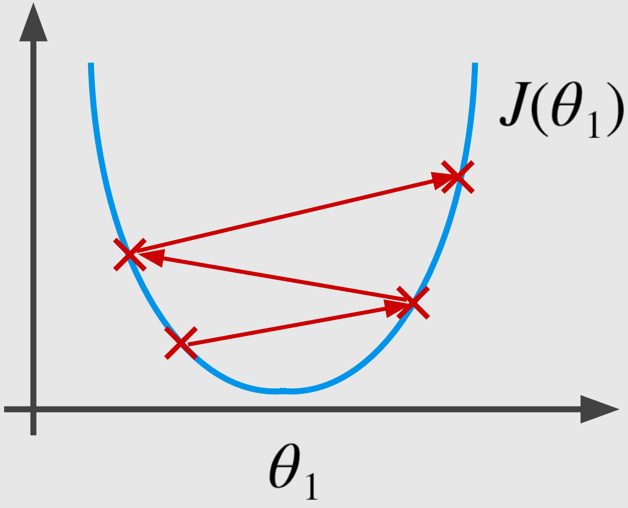
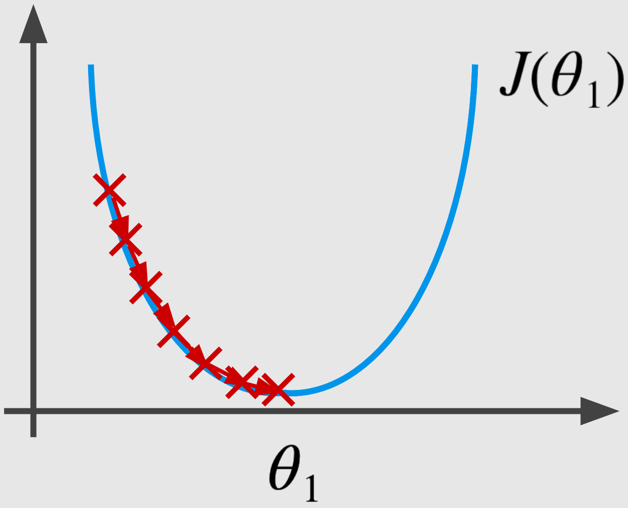


$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$



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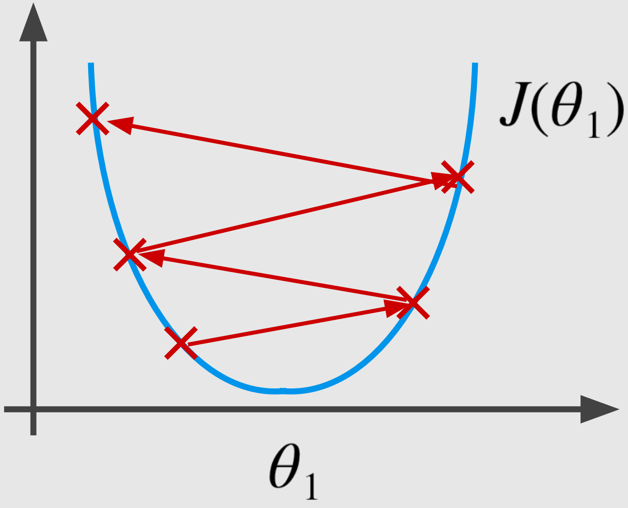
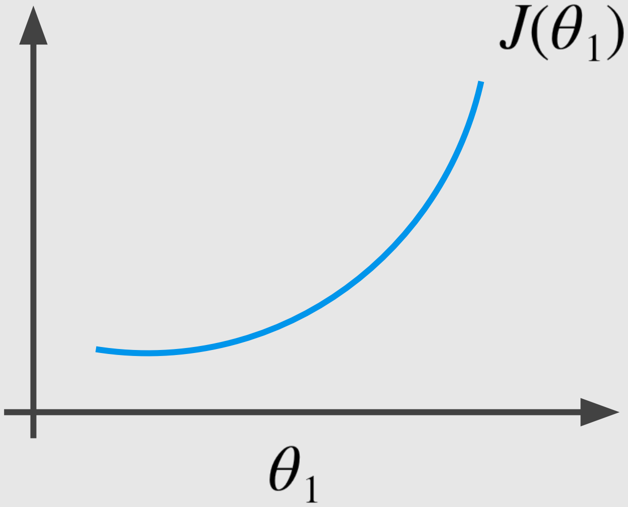


If α is too large, gradient descent can be overshoot the minimum. It may fail to converge, or even diverge.

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

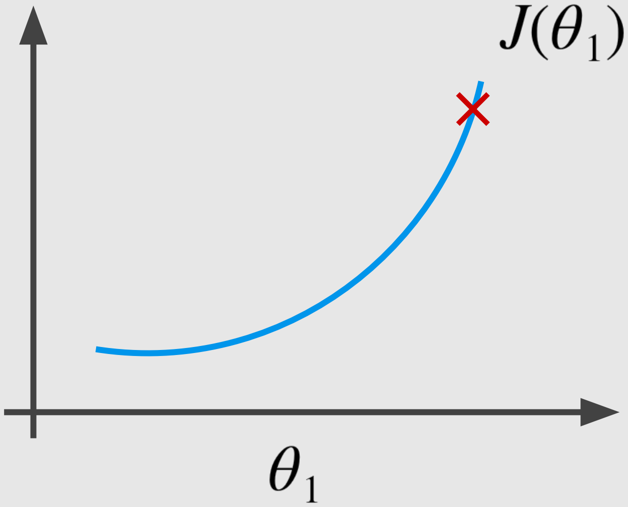
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$



As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$



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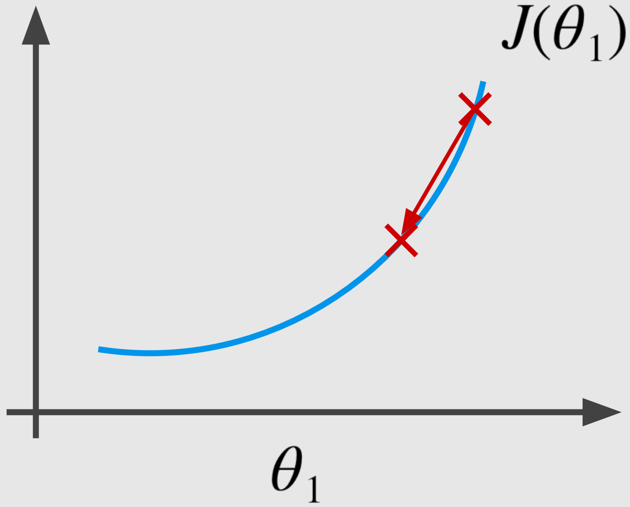

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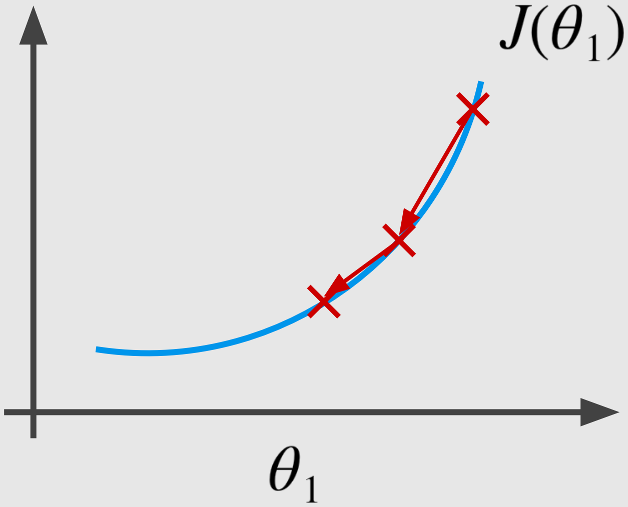

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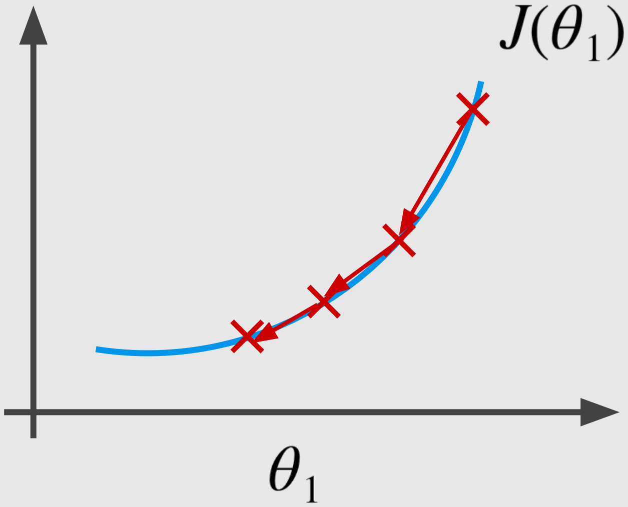

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.

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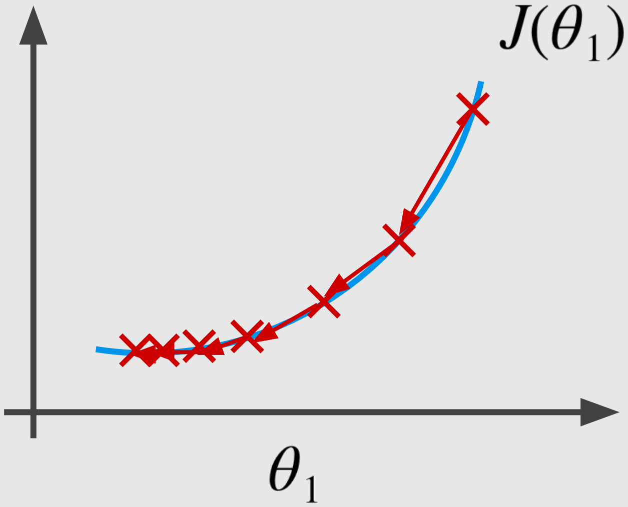

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Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

(for $j = 0$ and $j = 1$)

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

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$$\boxed{\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)}$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

$$\frac{\partial}{\partial\theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial\theta_j} \cdot \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

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$$j = 0: \frac{\partial}{\partial \theta_0} J(\theta_0,\theta_1) = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$$j = 1: \frac{\partial}{\partial \theta_1} J(\theta_0,\theta_1) = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})\, x^{(i)}$$

Gradient Descent algorithm

repeat until convergence {

$$\left.\begin{aligned} \theta_0 &:= \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) \\ \theta_1 &:= \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})\, x^{(i)} \end{aligned}\right\}$$

update θ_0 and θ_1 simultaneously

}