MSI exercise 3

Students: Abhijeet Nayak(5253170) and Rean Fernandes(5250057)

November 14, 2021

1 Task 1

1. $$\nabla f(x) = 2Qx + c, \nabla^2 f(x) = 2Q \tag{1}$$

2. Q should be invertible and Positive Semi-Definite

3. $$\begin{aligned} 2Qx^* + c &= 0 \\ x^* &= -\frac{1}{2}Q^{-1}C \end{aligned} \tag{2}$$

$$\begin{aligned} f(x^*) &= \frac{1}{4}c^T Q^{-1} Q Q^{-1} c - \frac{1}{2} c^T Q^{-1} c \\ f(x^*) &= -\frac{1}{4} c^T Q^{-1} c \end{aligned} \tag{3}$$

2 Task 2

$$\nabla f(x) = 2 \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} x + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 0 \tag{4}$$

$$x = -\frac{1}{2} \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -0.1429 \\ -0.9286 \end{bmatrix} \tag{5}$$

There exists a single value for x that satisfies the First order Necessary Condition. This is the Global Minimizer as well because the objective function is convex.

3 Task 3

$$
\begin{aligned}
E[S^2] &= E\left[\frac{1}{N-1}\sum_{n=1}^{N}(Y(n)-M(Y_N))^2\right] \\
&= \frac{1}{N-1}\sum_{n=1}^{N}E\left[Y(n)^2 - 2*Y(n)*M(Y_N) + M(Y_N)^2\right] \\
&= \frac{N}{N-1}\left(E[Y(n)^2] - 2E[Y(n)*M(Y_N)] + E[M(Y_N)^2]\right) \\
&= \frac{N}{N-1}\left(E[Y(n)^2] - 2E\left[Y(n)*\frac{(Y(1)+...+Y(n)+...+Y(N))}{N}\right] + E[M(Y_N)^2]\right) \\
&= \frac{N}{N-1}\left(\sigma_Y^2 + \mu_y^2 - 2\left(\frac{\sigma_Y^2}{N} + \mu_y^2\right) + \frac{\sigma_Y^2}{N} + \mu_y^2\right) \\
&= \frac{N}{N-1}\left(\sigma_Y^2 - \frac{\sigma_Y^2}{N}\right) \;=\; \sigma_Y^2
\end{aligned}
\tag{6}
$$

If we use the factor (N - 1) for the variance, we end up with an unbiased estimator, as shown.

4 Task 4

- **Part b**

$$
\Phi = \begin{bmatrix} 1 & i(1) \\ 1 & i(2) \\ . & . \\ . & . \\ 1 & i(N) \end{bmatrix}, y = \begin{bmatrix} u(1) \\ u(2) \\ . \\ . \\ u(N) \end{bmatrix}, \theta = \begin{bmatrix} E \\ R \end{bmatrix} \tag{7}
$$

$$
m = \Phi\theta \tag{8}
$$

- **Part e** The second dataset is noisier. The first dataset assumes Gaussian Noise. The second dataset assumes Uniform Noise.