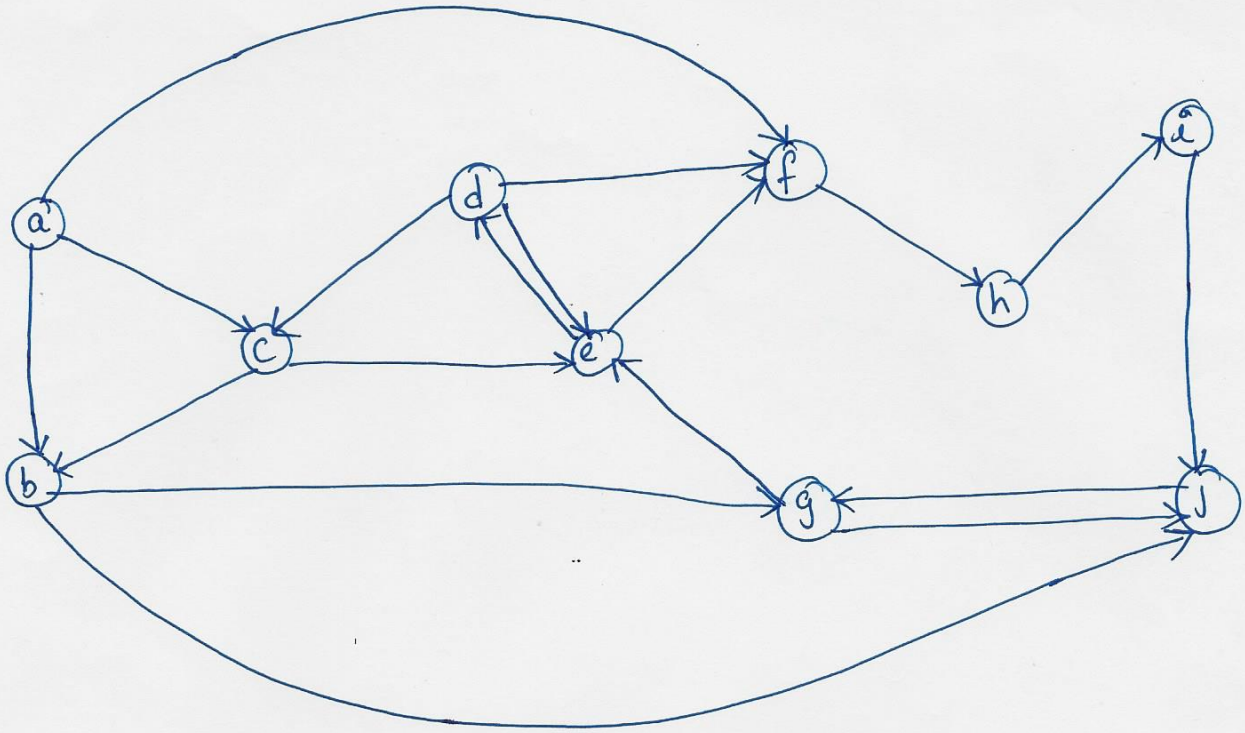
G



node:	a	b	c	d	e	f	g	h	i	j
group:	q_1	q_1	q_1	X	q_2	q_2	q_2	X	q_3	q_3

as group of node u

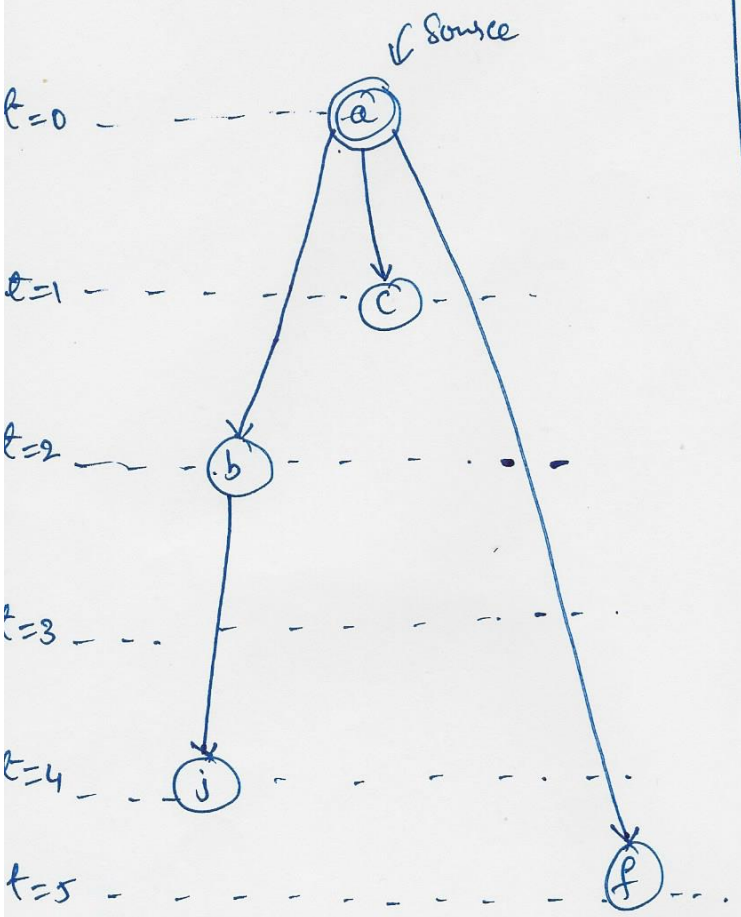
X: indicates that node u is not associated to any group (so we group them as q_4 and add a constraint that q_4 cannot be intervened at t_d)

Groups = $\{q_1, q_2, q_3\} \cup \{q_4\}$

q_4 → Set of all nodes that do not belong to any group. = $\{d, h\}$.

Latency period $l = 1$

Simulation Instance (S1)



After $t=5$,

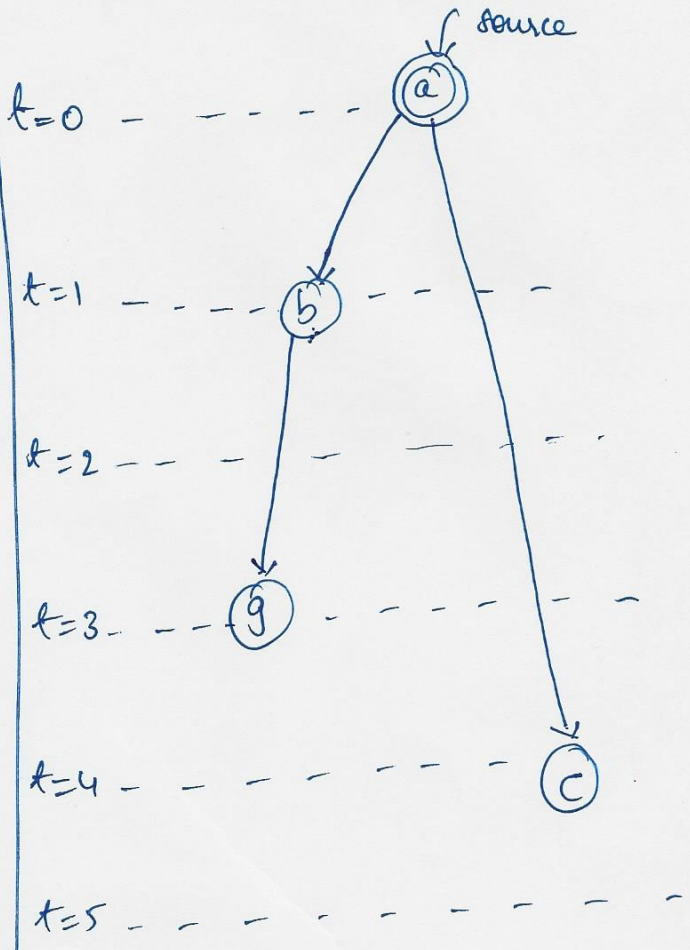
S: {d, e, g, h, i} Susceptible

I: {a, b, c, j} Infectious

E: {f} Exposed

#inf + #exposed = 5.

Simulation Instance (S2)



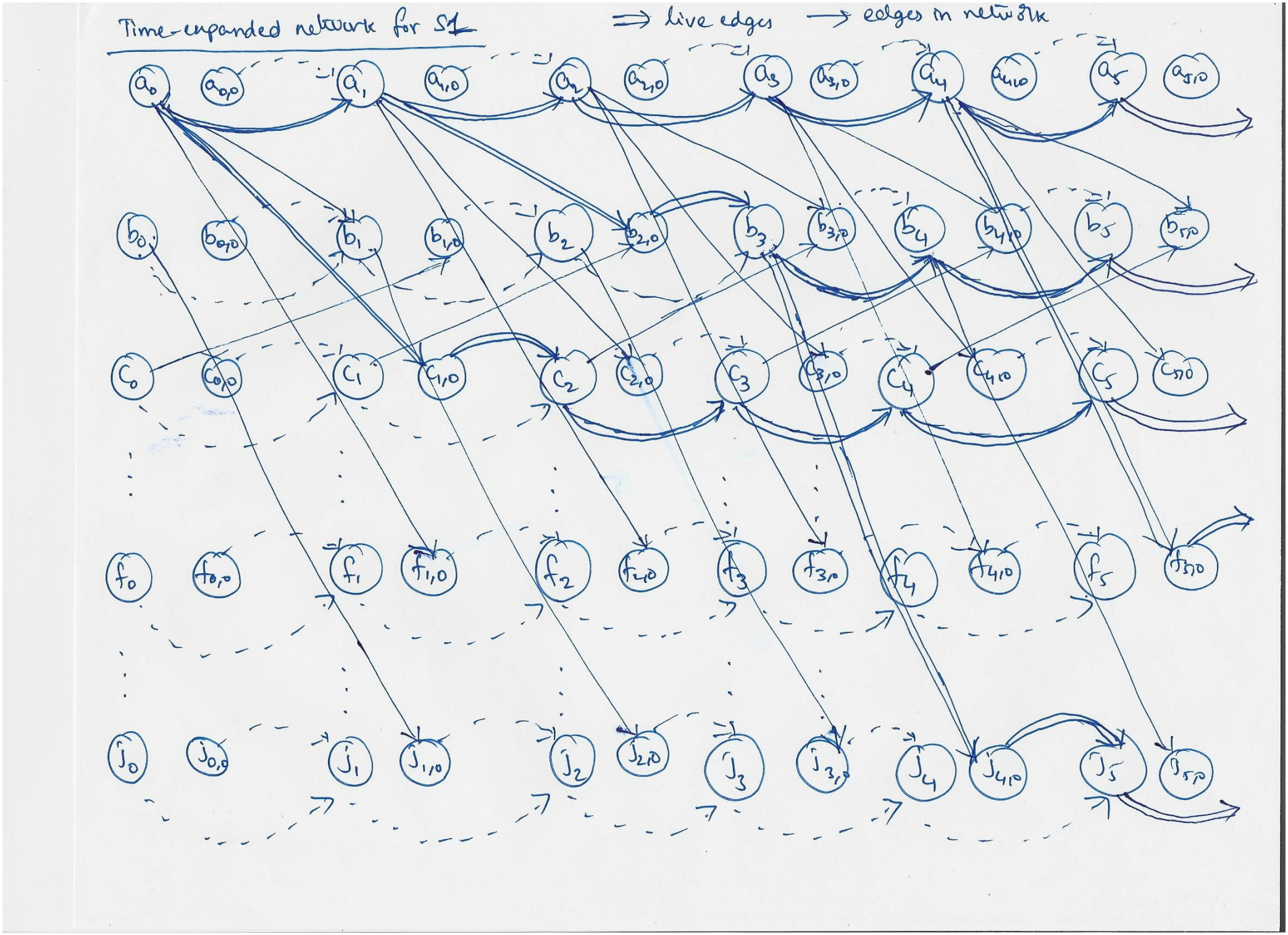
After $t=5$;

S: {d, e, f, h, i, j}

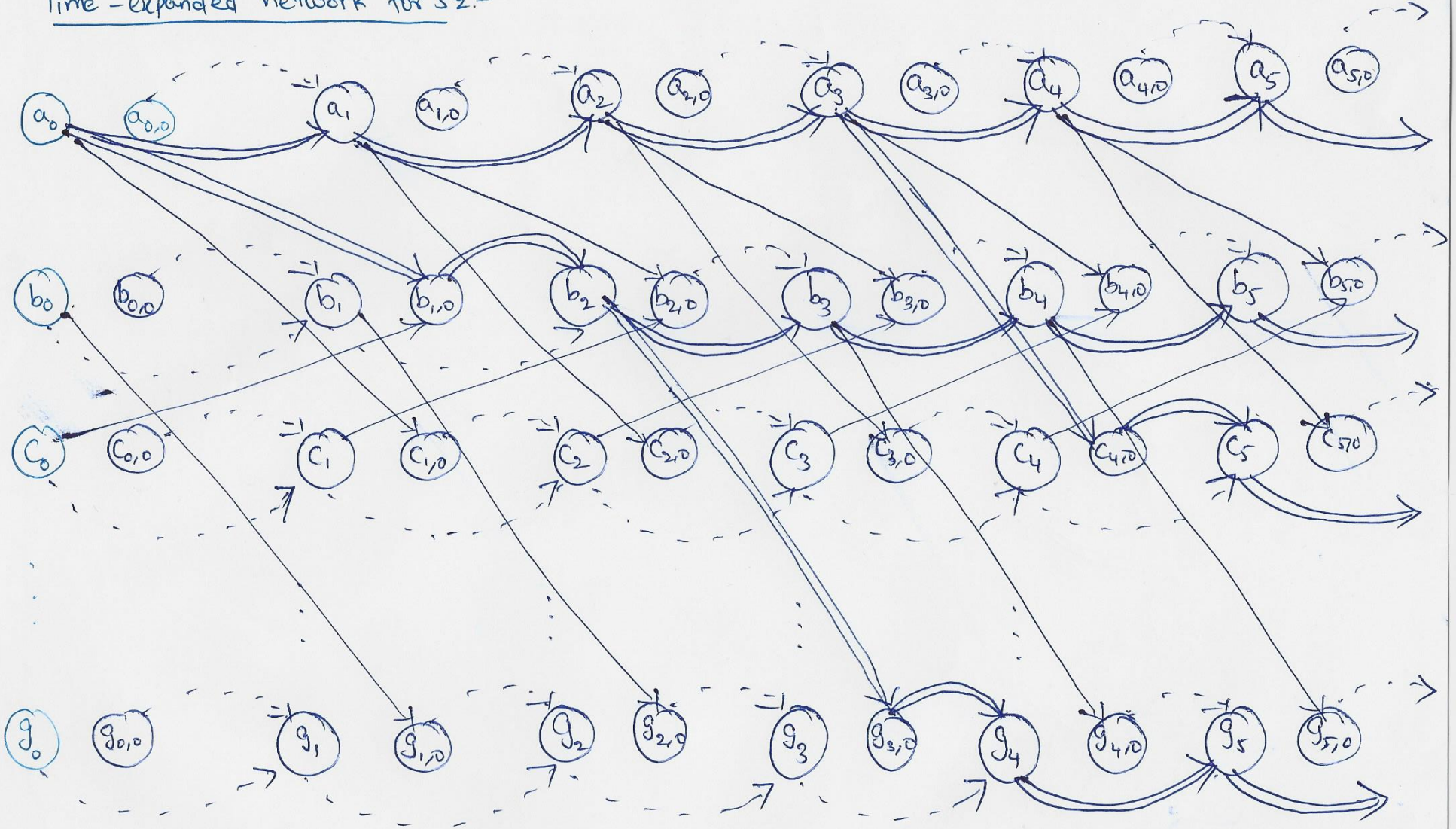
I: {a, b, c, g}

E: ϕ

#inf + # exposed = 4.


Time-expanded network for S1
⇒ live edges
→ edges in network
a_0 $a_{0,0}$ a_1 $a_{1,0}$ a_2 $a_{2,0}$ a_3 $a_{3,0}$ a_4 $a_{4,0}$ a_5 $a_{5,0}$
b_0 $b_{0,0}$ b_1 $b_{1,0}$ b_2 $b_{2,0}$ b_3 $b_{3,0}$ b_4 $b_{4,0}$ b_5 $b_{5,0}$
c_0 $c_{0,0}$ c_1 $c_{1,0}$ c_2 $c_{2,0}$ c_3 $c_{3,0}$ c_4 $c_{4,0}$ c_5 $c_{5,0}$
f_0 $f_{0,0}$ f_1 $f_{1,0}$ f_2 $f_{2,0}$ f_3 $f_{3,0}$ f_4 $f_{4,0}$ f_5 $f_{5,0}$
j_0 $j_{0,0}$ j_1 $j_{1,0}$ j_2 $j_{2,0}$ j_3 $j_{3,0}$ j_4 $j_{4,0}$ j_5 $j_{5,0}$

Time-expanded network for S2:-



$\tau_d = 2$

Constraints :-

$H' = (V_{te}^1, E_{te}^1)$, $H^2 = (V_{te}^2, E_{te}^2)$ — S1

C1: S to E constraints

$\forall i \geq \tau_d$, $\forall (v_{i-1}, u_{i,0}) \in E_{te}^j$: $y_{u,i,0}^j \geq y_{v,i-1}^j - x_{g(u),\tau_d}$

S1 ($H' = (V_{te}^1, E_{te}^1)$)

- $(a_1, b_{2,0}) \in E_{te}^1$:
  $y_{b,2,0}^1 \geq y_{a,1}^1 - x_{a_1,2}$
- $(b_3, j_{4,0}) \in E_{te}^1$:
  $y_{j,4,0}^1 \geq y_{b,3}^1 - x_{a_3,2}$
- $(a_4, f_{5,0})$:
  $y_{f,5,0}^1 \geq y_{a,4}^1 - x_{a_2,2}$

S2

- $(b_2, g_{3,0})$:
  $y_{g,3,0}^2 \geq y_{b,2}^2 - x_{a_1,2}$
- $(a_3, c_{4,0})$:
  $y_{c,4,0}^2 \geq y_{a,3}^2 - x_{a_1,2}$

C2: E to E constraints do not apply for $\ell = 1$

$\forall \ell \geq \tau_d$, $\forall (u_{i,r}, u_{i,r+1}) \in E_{te}^j$: $y_{u,i,r+1}^j \geq y_{u,i,r}^j - x_{g(u),\tau_d}$.

C3: E to I constraints

$\forall i \geq \tau_d$, $\forall (u_{i-\ell,\ell-1}, u_i) \in E_{te}^j$: $y_{u,i}^j \geq y_{u,i-\ell,\ell-1}^j - x_{g(u),\tau_d}$

S1

- $(c_{1,0}, c_2)$:
  $y_{c,2}^1 \geq y_{c,1,0}^1 - x_{a_1,2}$
- $(b_{2,0}, b_3)$:
  $y_{b,3}^1 \geq y_{b,2,0}^1 - x_{a_1,2}$
- $(j_{4,0}, j_5)$:
  $y_{j,5}^1 \geq y_{j,4,0}^1 - x_{a_3,2}$

S2

- $(b_{1,0}, b_2)$:
  $y_{b,2}^2 \geq y_{b,1,0}^2 - x_{a_1,2}$
- $(g_{3,0}, g_4)$:
  $y_{g,4}^2 \geq y_{g,3,0}^2 - x_{a_2,2}$
- $(c_{4,0}, c_5)$:
  $y_{c,5}^2 \geq y_{c,4,0}^2 - x_{a_1,2}$.

C4: $\forall i \geq \tau_d$, $\forall (u_{i-1}, u_i) \in E^j_{te}$: $y^j_{u,i} \geq y^j_{u,i-1} - x_{g(u),\tau_d}$

I to I constraints

S1:-

$y^1_{a,2} \geq y^1_{a,1} - x_{a_1,2}$

$y^1_{c,3} \geq y^1_{c,2} - x_{a_1,2}$

$y^1_{b,5} \geq y^1_{b,4} - x_{a_1,2}$

⋮

S2:-

$y^2_{a,3} \geq y^2_{a,2} - x_{a_1,2}$

$y^2_{b,3} \geq y^2_{b,2} - x_{a_1,2}$

$y^2_{g,5} \geq y^2_{g,4} - x_{a_2,2}$

⋮

$R(H^1) = \{a_0, a_1, a_2, a_3, a_4, a_5, b_{2,0}, b_3, b_4, b_5, c_{1,0}, c_2, c_3, c_4, c_5, j_{4,0}, j_5, \dots\}$

$R(H^2) = \{a_0, a_1, a_2, a_3, a_4, a_5, b_{1,0}, b_2, b_3, b_4, b_5, c_{4,0}, c_5, g_{3,0}, g_4, g_5, \dots\}$

C5: $\forall i \geq \tau_d$ $\forall u_i, u_{i,r} \in R(H^j)$: $y^j_{u,i} \leq 1 - x_{g(u),\tau_d}$

$y^j_{u,i,r} \leq 1 - x_{g(u),\tau_d}$.

S1

$y^1_{a,4} \leq 1 - x_{a_1,2}$

$y^1_{j,4,0} \leq 1 - x_{a_3,2}$

$y^1_{j,5} \leq 1 - x_{a_3,2}$

⋮

S2

$y^2_{a,5} \leq 1 - x_{a_1,2}$

$y^2_{c,4,0} \leq 1 - x_{a_1,2}$

$y^2_{g,5} \leq 1 - x_{a_2,2}$

⋮

C6: $\forall i < \tau_d$, $u_i, u_{i,r} \in R(H^j)$: $y^j_{u,i} = 1$, $y^j_{u,i,r} = 1$

S1

$y^1_{a,0} = 1$, $y^1_{a,1} = 1$

$y^1_{c,1,0} = 1$

S2

$y^2_{a,0} = 1$

$y^2_{b,1,0} = 1$

$y^2_{a,1} = 1$

C7: $\forall i < \tau_d$, $\forall u_i, u_{i,r} \in R(H^j)$: $z^j_u \geq y^j_{u,1}$, $z^j_u \geq y^j_{u,i,r}$

S1

$z^1_a \geq y^1_{a,3}$

$z^1_b \geq y^1_{b,2,0}$

$z^1_b \geq y^1_{b,3}$

⋮

S2

$z^1_a \geq y^2_{a,4}$

$z^2_b \geq y^2_{b,1,0}$

$z^2_g \geq z^2_{g,4}$

⋮

Additional:- q_4 corresponds to all nodes that do not belong to any group

$x_{a_4,2} = 0$

C8: $x_{a_1,2} + x_{a_2,2} + x_{a_3,2} + x_{a_4,2} \leq B$

Obj: $M=2$?

$$\frac{1}{2}\left(z^1_a + z^1_b + z^1_c + z^1_d + z^1_e + z^1_f + z^1_g + z^1_h + z^1_i + z^1_j + z^2_a + z^2_b + z^2_c + z^2_d + z^2_e + z^2_f + z^2_g + z^2_h + z^2_i + z^2_j \right)$$