# Course List

of Abhijit Chowdhary (Applied Mathematics Ph.D. applicant for Fall—2020)

## Summer 2016

**Data Structures:** Use and design of data structures, which organize information in computer memory. Stacks, queues, linked lists, binary trees: how to implement them in a high-level language, how to analyze their effect on algorithm efficiency, and how to modify them. Programming assignments.

## Fall 2016

**Computer Systems Organization:** Covers the internal structure of computers, machine (assembly) language programming, and the use of pointers in high-level languages. Topics include the logical design of computers, computer architecture, the internal representation of data, instruction sets, and addressing logic, as well as pointers, structures, and other features of high-level languages that relate to assembly language. Programming assignments are in both assembly language and other languages.

**Honors Linear Algebra:** This honors section of Linear Algebra is a proof-based course intended for well-prepared students who have already developed some mathematical maturity and ease with abstraction. Its scope will include the usual Linear Algebra (MATH-UA 140) syllabus; however this class will be faster, more abstract and proof-based, covering additional topics. Topics covered are: Vector spaces, linear dependence, basis and dimension, matrices, determinants, solving linear equations, linear transformations, eigenvalues and eigenvectors, diagonalization, inner products, applications.

## Spring 2017

**Operating Systems:** Covers the principles and design of operating systems. Topics include process scheduling and synchronization, deadlocks, memory management (including virtual memory), input/output, and file systems. Programming assignments.

**Basic Algorithms:** Introduction to the study of algorithms. Presents two main themes: designing appropriate data structures and analyzing the efficiency of the algorithms that use them. Algorithms studied include sorting, searching, graph algorithms, and maintaining dynamic data structures. Homework assignments, not necessarily involving programming.

**Numerical Computing:** The need for floating-point arithmetic, the IEEE floating-point standard, and the importance of numerical computing in a wide variety of scientific applications. Fundamental types of numerical algorithms: direct methods (e.g., for systems of linear equations), iterative methods (e.g., for a nonlinear equation), and discretization methods (e.g., for a differential equation). Numerical errors: can you trust your answers? Uses graphics and software packages such as Matlab. Programming assignments.

## Summer 2017

**Complex Variables for Scientists and Engineers:** This course is an introduction to complex variables accessible to juniors and seniors in engineering, physics and mathematics. The algebra of complex numbers, analytic functions, Cauchy Integral Formula, theory of residues and application to the evaluation of real integrals, conformal mapping and applications to physical problems.

**Introduction to Number Theory:** Integers, divisibility, prime numbers, unique factorization, congruences, quadratic reciprocity, Diophantine equations and arithmetic functions.

## Fall 2017

**Honors Analysis I:** This is an introduction to the rigorous treatment of the foundations of real analysis in one variable. It is based entirely on proofs. Students are expected to know what a mathematical proof is and are also expected to be able to read a proof before taking this class. Topics include: properties of the real number system, sequences, continuous functions, topology of the real line, compactness, derivatives, the Riemann integral, sequences of functions, uniform convergence, infinite series and Fourier series. Additional topics may include: Lebesgue measure and integral on the real line, metric spaces, and analysis on metric spaces.

**Honors Algebra I:** Introduction to abstract algebraic structures, including groups, rings, and fields. Sets and relations. Congruences and unique factorization of integers. Groups, permutation groups, group actions, homomorphisms and quotient groups, direct products, classification of finitely generated abelian groups, Sylow theorems. Rings, ideals and quotient rings, Euclidean rings, polynomial rings, unique factorization.

**Numerical Methods I:** This course is part of a two-course series meant to introduce graduate students in mathematics to the fundamentals of numerical mathematics (but any Ph.D. student seriously interested in applied mathematics should take it). It will be a demanding course covering a broad range of topics. There will be extensive homework assignments involving a mix of theory and computational experiments, and an in-class final. Topics covered in the class include floating-point arithmetic, solving large linear systems, eigenvalue problems, interpolation and quadrature (approximation theory), nonlinear systems of equations, linear and nonlinear least squares, nonlinear optimization, and Fourier transforms. This course will not cover differential equations, which form the core of the second part of this series, Numerical Methods II.

## Spring 2018

**Honors Analysis II:** This is a continuation of MATH-UA 328 Honors Analysis I. Topics include: metric spaces, differentiation of functions of several real variables, the implicit and inverse function theorems, Riemann integral on Rn, Lebesgue measure on Rn, the Lebesgue integral.

**Honors Algebra II:** Principle ideal domains, polynomial rings in several variables, unique factorization domains. Fields, finite extensions, constructions with ruler and compass, Galois theory, solvability by radicals.

**Topology:** Set-theoretic preliminaries. Metric spaces, topological spaces, compactness, connectedness, covering spaces, and homotopy groups.

**Special Topics: Geometric Modeling:** Recent advances in 3D digital geometry processing have created a plenitude of novel concepts for the mathematical representation and interactive manipulation of geometric models. This course covers some of the latest developments in geometric modeling and digital geometry processing. Topics include surface modeling based on polygonal meshes, surface reconstruction, mesh improvement, mesh parametrization, discrete differential geometry, interactive shape editing, skinning animation, architectural and structure-aware geometric modeling, shape modeling with an eye on 3D printing. The students will learn how to design, program and analyze algorithms and systems for interactive 3D shape modeling and digital geometry processing.

## Summer 2018

**Introduction to Artificial Intelligence:** Introduces a range of ideas and methods in AI, varying semester to semester but chosen largely from: automated heuristic search, planning, games, knowledge representation, logical and statistical inference, learning, natural language processing, vision, robotics, cognitive modeling, and intelligent agents. Programming projects will help students obtain a hands-on feel for various topics.

**Partial Differential Equations:** Introduction to the subject of partial differential equations: first order equations (linear and nonlinear), heat equation, wave equation, and Laplace equation. Examples of nonlinear equations of each type. Qualitative properties of solutions. Method of characteristics for hyperbolic problems. Solution of initial boundary value problems using separation of variables and eigenfunction expansions. Some numerical methods.

## Fall 2018

**Partial Differential Equations:** A basic introduction to PDEs, designed for a broad range of students whose goals may range from theory to applications. This course emphasizes examples, representation formulas, and properties that can be understood using relatively elementary tools. We will take a broad viewpoint, including how the equations we consider emerge from applications, and how they can be solved numerically. Topics will include: the heat equation; the wave equation; Laplace's equation; conservation laws; and Hamilton-Jacobi equations. Methods introduced through these topics will include: fundamental solutions and Green's functions; energy principles; maximum principles; separation of variables; Duhamel's principle; the method of characteristics; numerical schemes involving finite differences or Galerkin approximation; and many more.

**Algebra I:** Basic concepts of groups, rings and fields. Symmetry groups, linear groups, Sylow theorems; quotient rings, polynomial rings, ideals, unique factorization, Nullstellensatz; field extensions, finite fields.

**Honors Theory of Probability:** Counting, Permutations and Combinations, Uncertainty and Probability. Calculating probabilities. Independence , conditional probability. Some discrete distributions. random variables, expectation and variance. Generating functions. Joint distributions. Covariance and correlation. Law of large numbers. Continuous distributions. Normal distribution. Central limit theorem. Charateristic functions. Inversion Theorem. Continuity Theorem. Multivariate distributions. Change of variables. Multivariate Normal and related distributions. Poisson Processes. Markov Chains. Brownian Motion.

## Spring 2019

**Numerical Methods II:** This course will cover fundamental methods that are essential for the numerical solution of differential equations. It is intended for students familiar with ODE and PDE and interested in numerical computing; computer programming assignments in MATLAB will form an essential part of the course. The course will introduce students to numerical methods for (1) ordinary differential equations, explicit and implicit Runge-Kutta and multistep methods, convergence and stability; (2) finite difference and finite element and integral equation methods for elliptic partial differential equations (Poisson eq.); (4) spectral methods and the FFT, exponential temporal integrators, and multigrid iterative solvers; and (5) finite difference and finite volume parabolic (diffusion/heat eq.) and hyperbolic (advection and wave) partial differential equations.

**Chaos and Dynamical Systems:** Topics will include dynamics of maps and of first order and second-order differential equations, stability, bifurcations, limit cycles, dissection of systems with fast and slow time scales. Geometric viewpoint, including phase planes, will be stressed. Chaotic behavior will be introduced in the context of one-variable maps (the logistic), fractal sets, etc. Applications will be drawn from physics and biology. There will be homework and projects, and a few computer lab sessions (programming experience is not a prerequisite).

**Advanced Topics In Numerical Analysis: High Performance Computing:** This class will be an introduction to the fundamentals of parallel scientific computing. We will establish a basic understanding of modern computer architectures (CPUs and accelerators, memory hierarchies, interconnects) and of parallel approaches to programming these machines (distributed vs. shared memory parallelism: MPI, OpenMP, OpenCL/CUDA). Issues such as load balancing, communication, and synchronization will be covered and illustrated in the context of parallel numerical algorithms. Since a prerequisite for good parallel performance is good serial performance, this aspect will also be addressed. Along the way you will be exposed to important tools for high performance computing such as debuggers, schedulers, visualization, and version control systems. This will be a hands-on class, with several parallel (and serial) computing assignments, in which you will explore material by yourself and try things out. There will be a larger final project at the end. You will learn some Unix in this course, if you don't know it already. Prerequisites for the course are (serial) programming experience with C/C++ (I will use C in class) or FORTRAN, and some familiarity with numerical methods.

## Fall 2019

**Advanced Topics In Numerical Methods: Finite Element Methods:** This course covers theoretical and practical aspects of finite element methods for the numerical solution of partial differential equations. The first part of the course will focus on theoretical foundations of the method (calculus of variations, Poincare inequality, Cea's lemma, Nitsche trick, convergence estimates). The second part targets practical aspects of the method, illustrates how it can be implemented and used for solving partial differential equations in two and three dimensions. Examples will include the Poisson equation, linear elasticity and, time permitting, the Stokes equations.

**Methods of Applied Math:** This is a first-year course for all incoming PhD and Masters students interested in pursuing research in applied mathematics. It provides a concise and self-contained introduction to advanced mathematical methods, especially in the asymptotic analysis of differential equations. Topics include scaling, perturbation methods, multi-scale asymptotics, transform methods, geometric wave theory, and calculus of variations

## Intended in Spring 2020

**Advanced Topics In Numerical Analysis: Nonsmooth Optimization:** Convex optimization problems have many important properties, including a powerful duality theory and the property that any local minimum is also a global minimum. Nonsmooth optimization refers to minimization of functions that are not necessarily convex, usually locally Lipschitz, and typically not differentiable at their minimizers. Topics in convex optimization that will be covered include duality, linear and semidefinite programming, CVX ("disciplined convex programming"), gradient and Newton methods, Nesterov's complexity bound, the alternating direction method of multipliers, the nuclear norm and matrix completion, the primal barrier method, primal-dual interior-point methods for linear and semidefinite programs. Topics in nonsmooth optimization that will be covered include subgradients and subdifferentials, Clarke regularity, and algorithms, including gradient sampling and BFGS, for nonsmooth, nonconvex optimization. Homework will be assigned, both mathematical and computational. Students may submit a final project on a pre-approved topic or take a written final exam.

**Basic Probability:** The one-semester course introduces the basic concepts and methods of probability. Topics include: probability spaces, random variables, distributions, law of large numbers, central limit theorem, random walk martingales in discrete time, and if time permits Markov chains and Brownian motion.

**Mechanics:** This course provides brief mathematical introductions to elasticity, classical mechanics, and statistical mechanics – topics at the interface where differential equations and probability meet physics and materials science. For students preparing to do research on physical applications, the class provides an introduction to crucial concepts and tools; for students of analysis the class provides valuable context by exploring some central applications. No prior exposure to mechanics or physics is assumed. The segment on elasticity (about 6 weeks) will include: one-dimensional models (strings and rods); buckling as a bifurcation; nonlinear elasticity for 3D solids; and linear elasticity. The segment on classical mechanics (about 5 weeks) will include: basic examples; alternative formulations including action minimization and Hamilton's equations; relations to the Calculus of Variations including Hamilton-Jacobi equations, optimal control, and geodesics; stability and parametric resonance. The segment on statistical mechanics (about 3 weeks) will include basic concepts such as the microcanonical and canonical ensembles, entropy, and the equilibrium distribution; some simple examples; and the numerical method known as Metropolis sampling.