Linear Algebra

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Chapter 1

Introduction

1.1 Definitions

- **Matrix**: A matrix $A_{m\times n}$ is a rectangular array of real/complex numbers. More precisely,
$$A_{m\times n} = [a_{j,k}], 1 \leq j \leq m, 1 \leq k \leq n$$
- **Tensors**: A tensor $A_{i_1\times i_2\times \ldots i_k}$ is a k-dimensional array of real/complex numbers.
- **Transpose**: A matrix $B_{n\times m} = [b_{rs}]$ is the transpose of matrix $A_{m\times n} = [a_{jk}]$ iff
$$b_{rs} = a_{sr}; 1 \leq r \leq n, 1 \leq s \leq m$$
  - Denoted by $B = A^T$
- **Symmetric Matrix**: A matrix $A_{m\times m} = [a_{jk}]$ is symmetric iff
$$a_{jk} = a_{kj},\ \forall j,k\ 1 \leq j,k \leq m$$
  - Equivalently, $A = A^T$
  - **Skew-Symmetric Matrix**: $A = -A^T$
- **Scalar Multiplication**: $\lambda A = [\lambda a_{jk}]$
- **Dot Product**: Given column vectors $v_{n\times 1}, w_{n\times 1}$, the dot product is defined as
$$\langle v, w\rangle = \sum_i v_i w_i$$
  - Also denoted by $v \cdot w$
- **Matrix Product**: Given matrices $A_{p\times q}, B_{q\times r}$, the matrix product $C_{p\times r}$ is defined as
$$c_{i,k} = \sum_j a_{i,j} b_{j,k}; \quad 1 \leq i \leq p, 1 \leq j \leq q, 1 \leq k \leq r$$
  - Denoted by $C = AB$
- **Hadamard Product**: Given matrices $A_{p\times q}, B_{p\times q}$, the hadamard product $C = A \odot B$ is defined as
$$c_{i,j} = a_{i,j} b_{i,j}; \quad 1 \leq i \leq p, 1 \leq j \leq q$$
- **Matrix Inverse**: Given a square matrix A, the matrix inverse A^{-1} is defined such that $A^{-1}A = I_n$

1.2 Theorems

- **Associativity**: Given matrices $A_{p\times q}, B_{q\times r}, C_{r\times s}$,
$$A(BC) = (AB)C$$
- **Distributive**: Given matrices $A_{p\times q}, B_{q\times r}, C_{q\times r}$,
$$A(B + C) = AB + AC$$
- **Transpose of a Product**: $(AB)^T = B^T A^T$
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