MODULE 6

MODULE 6 QUIZ REVIEW

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- Module 3
  - Linear programming models
- Module 4
  - Probability
  - Bayes' Theorem
- Module 5
  - Continuity
  - Average rate of change
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The authors of a best-selling textbook in finite mathematics are told that for the next edition of their book, each simple figure would cost the project $\$20$, each figure with additions would cost $\$35$, and computer-drawn sketch would cost $\$60$. They are limited to 400 figures, for which they are allowed to spend up to $\$2200$. The number of computer-drawn sketches must be no more than the number of the other two types combined, and there must be at least twice as many simple figures as there are figures with additions. If each simple figure increases the royalties by $\$95$, each figure with additions increases royalties by $\$200$, and each computer-drawn sketch increases royalties by $\$325$, how many of each type of figure should be included to maximize royalties, assuming all art costs are borne by the publisher?

Let x_1 = the number of simple figures, x_2 = the number of figures with additions, and x_3 = the number of computer-drawn sketches.

Maximize $z = 95x_1 + 200x_2 + 325x_3$

Subject to:
$$\begin{aligned} &20x_1 + 35x_2 + 60x_3 \le 2200 \\ &x_1 + x_2 + x_3 \le 400 \\ &-x_1 - x_2 + x_3 \le 0 \\ &-x_1 + 2x_2 \le 0 \\ &x_1, x_2, x_3 \ge 0 \end{aligned}$$

Set up the initial tableau

$$\begin{array}{c} \begin{array}{ccccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & s_4 & z & \end{array} \\ \left[\begin{array}{cccccccc|c} 20 & 35 & 60 & 1 & 0 & 0 & 0 & 0 & 2200 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 400 \\ -1 & -1 & \boxed{1} & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline -95 & -200 & -325 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

Determine the pivot

Recall: If there is a 0 in the right-hand column, do not disregard that row, unless the corresponding number in the pivot column is negative or 0.

Perform row operations and continue until you reach the final tableau

$$\begin{array}{c} \begin{array}{ccccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & s_4 & z & \end{array} \\ \left[\begin{array}{cccccccc|c} 255 & 0 & 0 & 2 & 0 & 120 & 0 & 0 & 4400 \\ 0 & 0 & 0 & -2 & 85 & -205 & -2 & 0 & 29600 \\ 0 & 0 & 170 & 2 & 0 & 290 & 0 & 0 & 4400 \\ 0 & 510 & 0 & 2 & 0 & 120 & 255 & 0 & 4400 \\ \hline 0 & 0 & 0 & 546 & 0 & 65910 & 0 & 102 & 1201200 \end{array}\right] \end{array}$$

Basic variables are x_1, x_2, x_3, and s_2.
All other variables are 0.

s_2 corresponds to the slack in the 2nd constraint

$$255x_1 = 4400 \rightarrow x_1 \approx 17.2549$$
$$510x_2 = 4400 \rightarrow x_2 \approx 8.6275$$
$$170x_3 = 4400 \rightarrow x_3 \approx 25.8824$$
$$85s_2 = 29600 \rightarrow s_2 \approx 348.2353$$

Rounded values are $x_1 = 17$, $x_2 = 9$, $x_3 = 26$, and $s_2 = 348$, however notice the 1st and 4th constraints fail

x_3 produces the highest royalty, so leave this at 26

x_2 produces the next highest royalty, but if $x_1 = 16$ and $x_2 = 9$, the 4th constraint still fails

So, $x_1 = 17$, $x_2 = 8$, $x_3 = 26$, and now $s_2 = 349$

The authors should include 17 simple figures, 8 figures with additions, and 26 computer-drawn sketches

```python
from pulp import LpVariable, LpProblem, LpMaximize, LpStatus, value, LpMinimize

# declare your variables
x1 = LpVariable("x1", 0, None) # x1>=0
x2 = LpVariable("x2", 0, None) # x2>=0
x3 = LpVariable("x3", 0, None) # x3>=0

# defines the problem
prob = LpProblem("problem", LpMaximize)

# defines the constraints
prob += 20*x1 + 35*x2 + 60*x3 <= 2200
prob += x1 + x2 + x3 <= 400
prob += -x1 - x2 + x3 <= 0
prob += -x1 + 2*x2 <= 0

# defines the objective function to maximize
prob += 95*x1 + 200*x2 + 325*x3

# solve the problem
status = prob.solve()
LpStatus[status]

# print the results
print(value(x1))
print(value(x2))
print(value(x3))
print(95*value(x1) + 200*value(x2) + 325*value(x3))
```

```
17.254902
8.627451
25.882353
11776.470614999998
```

Jose would like to start exercising to burn at least 1500 extra calories per week, but he does not have much free time to do so. After a bit of research, he knows he can burn an average of 3.5 calories per minute walking, 4 calories per minute cycling, and 8 calories per minute swimming. He would like his total time spent walking and cycling to be at least 3 times as long as he spends swimming. He would also like to walk at least 30 minutes per week. How much time should he spend on each activity to meet his goals but also minimize his total exercise time per week? What is his minimum exercise time per week?

Let y_1 = minutes spent walking each week, y_2 = minutes spent cycling each week, and y_3 = minutes spent swimming each week

Minimize $w = y_1 + y_2 + y_3$

Subject to:
$$3.5y_1 + 4y_2 + 8y_3 \geq 1500$$
$$y_1 + y_2 - 3y_3 \geq 0$$
$$y_1 \geq 30$$
$$y_2, y_3 \geq 0$$

Write the augmented matrix

$$\left[\begin{array}{ccc|c} 3.5 & 4 & 8 & 1500 \\ 1 & 1 & -3 & 0 \\ 1 & 0 & 0 & 30 \\ \hline 1 & 1 & 1 & 0 \end{array}\right]$$

Transpose the matrix for the dual

$$\left[\begin{array}{ccc|c} 3.5 & 1 & 1 & 1 \\ 4 & 1 & 0 & 1 \\ 8 & -3 & 0 & 1 \\ \hline 1500 & 0 & 30 & 0 \end{array}\right]$$

Write the dual problem

Maximize $z = 1500x_1 + 30x_3$

Subject to:
$$3.5x_1 + x_2 + x_3 \leq 1$$
$$4x_1 + x_2 \leq 1$$
$$8x_1 - 3x_2 \leq 1$$
$$x_1, x_2, x_3 \geq 0$$

Write the initial Simplex tableau

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \end{array} \\ \left[\begin{array}{ccccccc|c} 3.5 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 4 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ \boxed{8} & -3 & 0 & 0 & 0 & 1 & 0 & 1 \\ \hline -1500 & 0 & -30 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

Determine the pivot

Perform row operations and continue until you reach the final tableau

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \end{array} \\ \left[\begin{array}{ccccccc|c} 0 & 0 & 80 & 80 & -74 & 2 & 0 & 8 \\ 0 & 5 & 0 & 0 & 2 & -1 & 0 & 1 \\ 40 & 0 & 0 & 0 & 6 & 2 & 0 & 2 \\ \hline 0 & 0 & 0 & 240 & 1578 & 606 & 8 & 2424 \end{array}\right] \end{array}$$

Divide the bottom row by 8 and read solution

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \end{array} \\ \left[\begin{array}{ccccccc|c} 0 & 0 & 80 & 80 & -74 & 2 & 0 & 8 \\ 0 & 5 & 0 & 0 & 2 & -1 & 0 & 1 \\ 40 & 0 & 0 & 0 & 6 & 2 & 0 & 2 \\ \hline 0 & 0 & 0 & 30 & 197.25 & 75.75 & 1 & 303 \end{array}\right] \end{array}$$

<u>Solution:</u> Jose should spend 30 minutes walking, 197.25 minutes cycling, and 75.75 minutes swimming for a total of 303 minutes per week.

```python
from pulp import LpVariable, LpProblem, LpMaximize, LpStatus, value, LpMinimize

# declare your variables
y1 = LpVariable("y1", 0, None) # y1>=0
y2 = LpVariable("y2", 0, None) # y2>=0
y3 = LpVariable("y3", 0, None) # y3>=0

# defines the problem
prob = LpProblem("problem", LpMinimize)

# defines the constraints
prob += 3.5*y1 + 4*y2 + 8*y3 >= 1500
prob += y1 + y2 - 3*y3 >= 0
prob += y1 >= 30

# defines the objective function to minimize
prob += y1 + y2 + y3

# solve the problem
status = prob.solve()
LpStatus[status]

# print the results
print(value(y1))
print(value(y2))
print(value(y3))
print(value(y1) + value(y2) + value(y3))
```

```
30.0
197.25
75.75
303.0
```

Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. The table below lists the percentages in each category and the percentages who died.

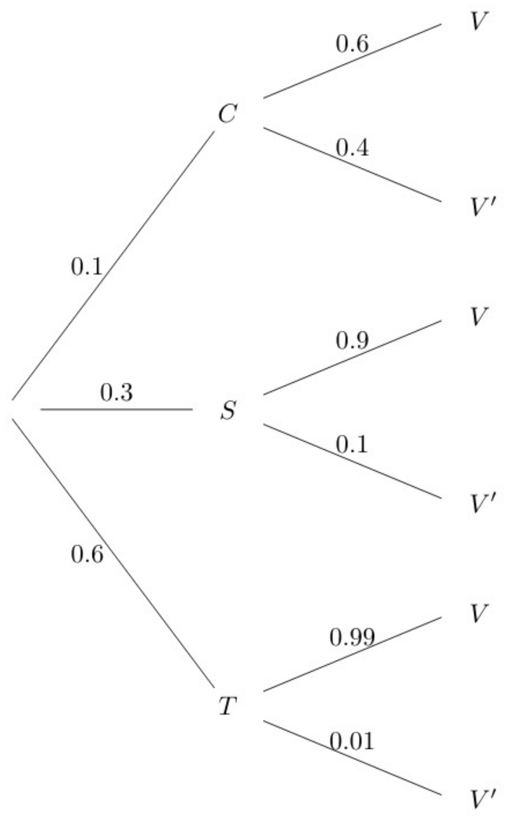
Find the probability that a patient who survived was categorized as serious upon arrival.

Category of Patient	Percent of Category in ER	Percent who did not Survive
Critical	10%	40%
Serious	30%	10%
Stable	60%	1%

Let C represent "patient was critical", S represent "patient was serious", T represent "patient was stable", and V represent "patient survived."

Create a new table to use for determining $P(S|V)$

Category of Patient	Portion of Category	Probability of Survival for Category
Critical	0.1	0.6
Serious	0.3	0.9
Stable	0.6	0.99

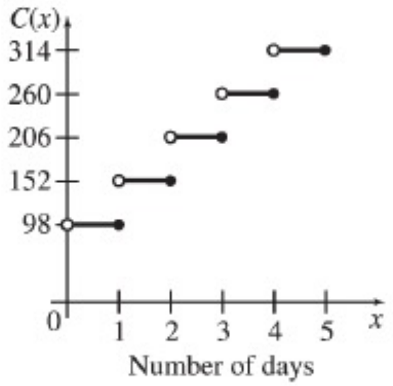


$$P(S|V) = \frac{P(S)P(V|S)}{P(C)P(V|C) + P(S)P(V|S) + P(T)P(V|T)}$$

$$= \frac{(0.3)(0.9)}{(0.1)(0.6) + (0.3)(0.9) + (0.6)(0.99)} \approx 0.29$$

The cost to rent a compact car is $\$54$ per day or fraction of a day. If the car is picked up in Pittsburgh and dropped off in Cleveland, there is a fixed $\$44$ drop-off charge. Let $C(x)$ represent the cost of renting the car for x days, taking it from Pittsburgh to Cleveland.

Graph $y = C(x)$



Determine the cost to rent the car for $2/3$ of a day

Since x is greater than 0 and less than or equal to $1 \rightarrow C(2/3) = 98$

Determine the cost to rent the car for 2.4 days

Since $x > 2$ and $x \leq 3 \rightarrow C(2.4) = 54(3) + 44 = 206$

Determine the cost to rent the car for 1 day

The cost up to one day is $54 + 44 = 98$

Find all x-values where the function is discontinuous on the interval $(0,5)$

Notice the jump discontinuities at $x = 1,2,3,4$

The average number of vehicles waiting in a line to enter a parking ramp can be modeled by the function

$$f(x) = \frac{x^2}{2(1-x)},$$

where x is a quantity between 0 and 1 known as the traffic intensity. Find the rate of change of the number of vehicles in line with respect to the traffic intensity for (a) $x = 0.1$ and (b) $x = 0.65$.

$$f'(x) = \frac{2x[2(1-x)] - x^2(-2)}{[2(1-x)]^2}$$

$$= \frac{2x(2-2x) + 2x^2}{4(1-x)^2}$$

$$= \frac{4x - 2x^2}{4(1-x)^2}$$

$$= \frac{2x - x^2}{2(1-x)^2}$$

$$f'(0.1) = \frac{2(0.1) - (0.1)^2}{2(0.9)^2} \approx 0.1173$$

The number of vehicles in line is increasing at a rate of 0.1173 vehicles when traffic intensity is 0.1

$$f'(0.65) = \frac{2(0.65) - (0.65)^2}{2(0.35)^2} \approx 3.5816$$

The number of vehicles in line is increasing at a rate of 3.5816 vehicles when traffic intensity is 0.65

Is the rate of change ever decreasing?

Since x is between 0 and 1, both the numerator and denominator of $f'(x)$ will always be positive, so $f'(x) > 0$ for all values of x in the domain.

Observations:

1. The rate of change in the number of vehicles in line is approaching 0 when traffic intensity is near 0
2. The rate of change in the number of vehicles in line is approaching ∞ when traffic intensity is near 1.

It has been observed that the following formula accurately models the relationship between the size of a breast tumor and the amount of time it has been growing.

$$V(t) = 1100(1023e^{-0.02415t} + 1)^{-4},$$

where t is in months and $V(t)$ is measured in cubic centimeters.

(a) Find the tumor volume at 240 months.
(b) Assuming the shape of a tumor is spherical, find the radius of the tumor from part (a).
(c) Find and interpret $\lim_{t\to\infty} V(t)$. Explain whether this makes sense.
(d) Calculate the rate of change of the volume of a tumor at 240 months and interpret the result.

$$V(240) = 1100\left(1023e^{-0.02415(240)} + 1\right)^{-4} \approx 3.857 \text{ cm}^3$$

Formula for the volume of a sphere: $V = (4/3)\pi r^3$

$$(4/3)\pi r^3 = 3.857$$

$$r^3 = \frac{3(3.857)}{4\pi}$$

$$r = \sqrt[3]{\frac{11.571}{4\pi}} \approx 0.973 \text{ cm}$$

$$\lim_{t\to\infty} 1100(1023e^{-0.02415t} + 1)^{-4}$$

$e^{-0.02415t} \to 0$ as $t \to \infty$

So, $1023e^{-0.02415t} + 1 \to 1$ as $t \to \infty$

$$\lim_{t\to\infty} 1100(1023e^{-0.02415t} + 1)^{-4} = 1100 \text{ cm}^3$$

1100 cm^3 corresponds to a sphere with radius

$$r = \sqrt[3]{\frac{3(1100)}{4\pi}} \approx 6.4 \text{ cm}$$

This makes sense that a tumor reaches a maximum volume of this size.

$$V'(t) = 1100(-4)(1023e^{-0.02415t} + 1)^{-5}(1023)(e^{-0.02415t})(-0.02415)$$

$$= 108793.98(1023e^{-0.02415t} + 1)^{-5}(e^{-0.02415t})$$

$$V'(240) \approx 0.282$$

At 240 months, the tumor is increasing in volume at a rate of $0.282 \text{ cm}^3/\text{month}$

In a particular region of the world, the percent of persons 65 years and over with a family income below the poverty level has declined and be approximated by the following function:

$$P(t) = 30.60 - 5.79 \ln t,$$

where t is the number of years since 1965. Find the percent of persons 65 years and over with family income below the poverty level and the rate of change in the years $1970, 1995,$ and 2018.

$$P(5) = 30.60 - 5.79 \ln 5 \approx 21.28$$

The percent of persons 65 years and over with family income below the poverty level in 1970 was approximately 21.28%

$$P'(t) = -\frac{5.79}{t}$$

$$P'(5) = -\frac{5.79}{5} = -1.158$$

The rate of change in 1970 was decreasing at a rate of -1.158% per year

$$P(30) = 30.60 - 5.79 \ln 30 \approx 10.91$$

The percent of persons 65 years and over with family income below the poverty level in 1995 was approximately 10.91%

$$P'(30) = -\frac{5.79}{30} = -0.193$$

The rate of change in 1995 was decreasing at a rate of -0.193% per year

What is happening to the rate of change over time?

Will the rate of change ever reach 0?

$$P(53) = 30.60 - 5.79 \ln 53 \approx 7.612$$

The percent of persons 65 years and over with family income below the poverty level in 2018 was approximately 7.612%

$$P'(53) = -\frac{5.79}{53} \approx -0.109$$

The rate of change in 2018 was decreasing at a rate of -0.109% per year

QUESTIONS?