MODULE 1

LINES

LINEAR FUNCTIONS

LEAST SQUARES CALCULATIONS

MODULE 1

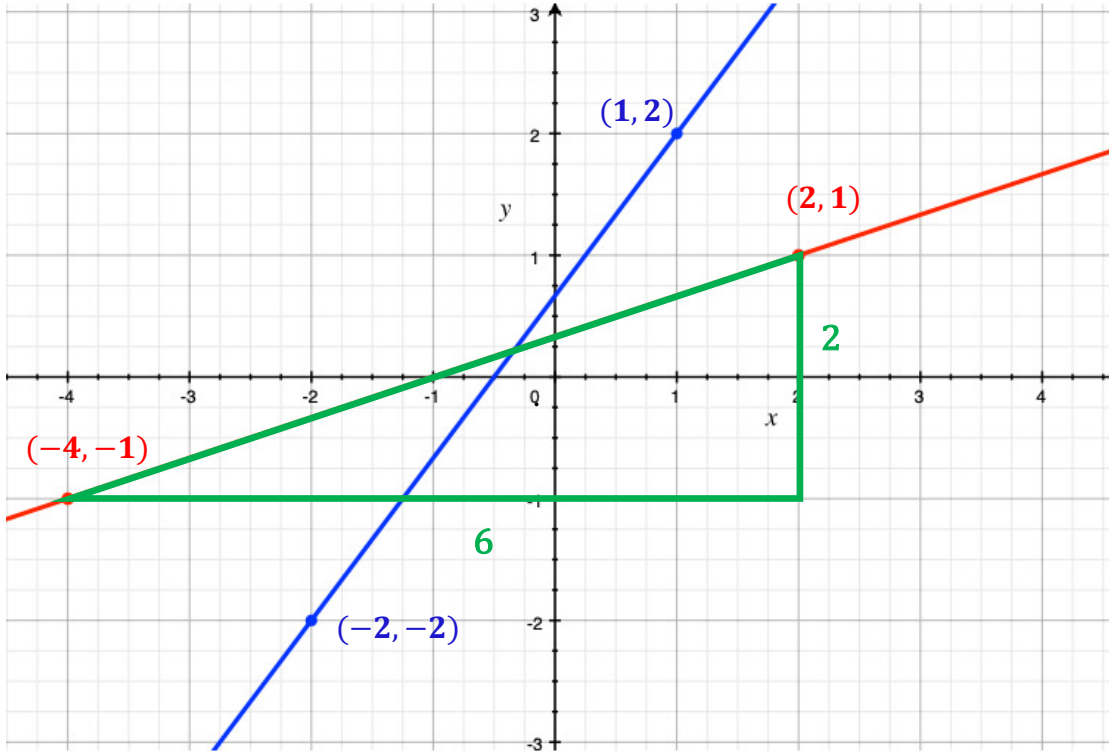
LINES

SLOPE

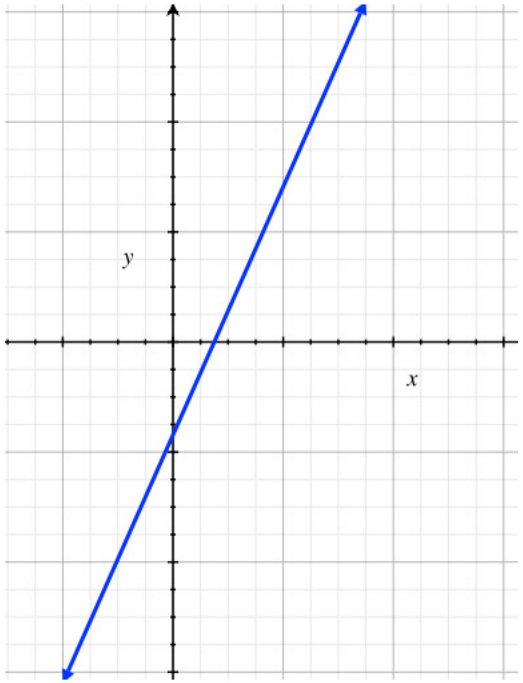
$$\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}}$$

$$m = \frac{2-(-2)}{1-(-2)} = \frac{4}{3}$$

$$m = \frac{1-(-1)}{2-(-4)} = \frac{2}{6} = \frac{1}{3}$$

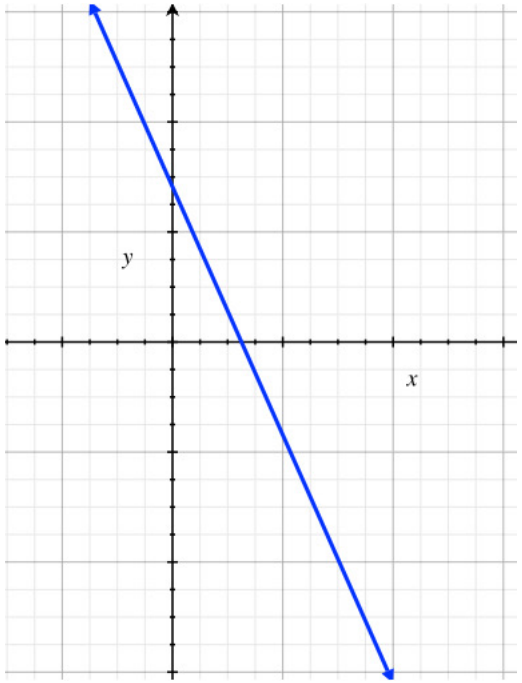


Positive slope



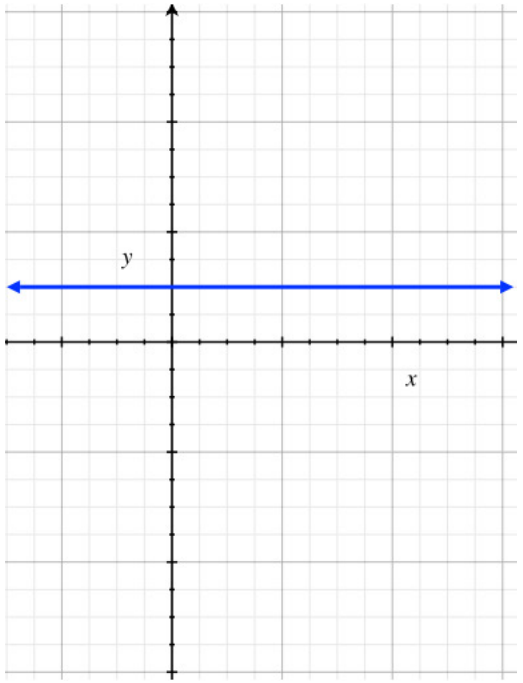
$y = mx + b$
$m > 0$

Negative slope



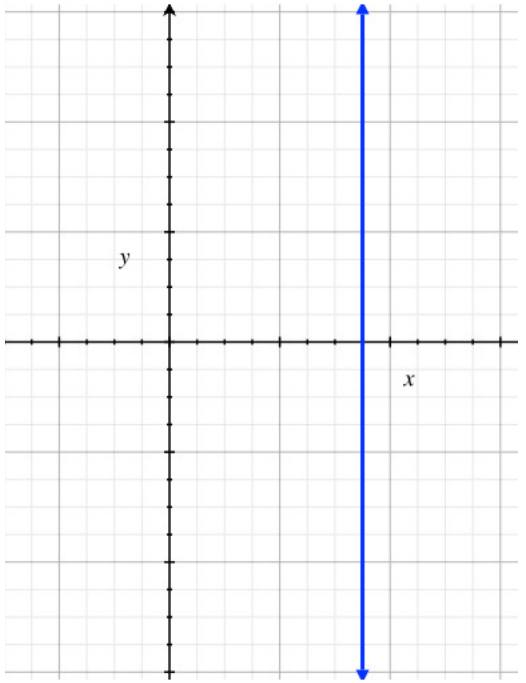
$y = mx + b$
$m < 0$

Zero slope



$y = b$

Undefined slope



$x = a$

LINEAR EQUATIONS

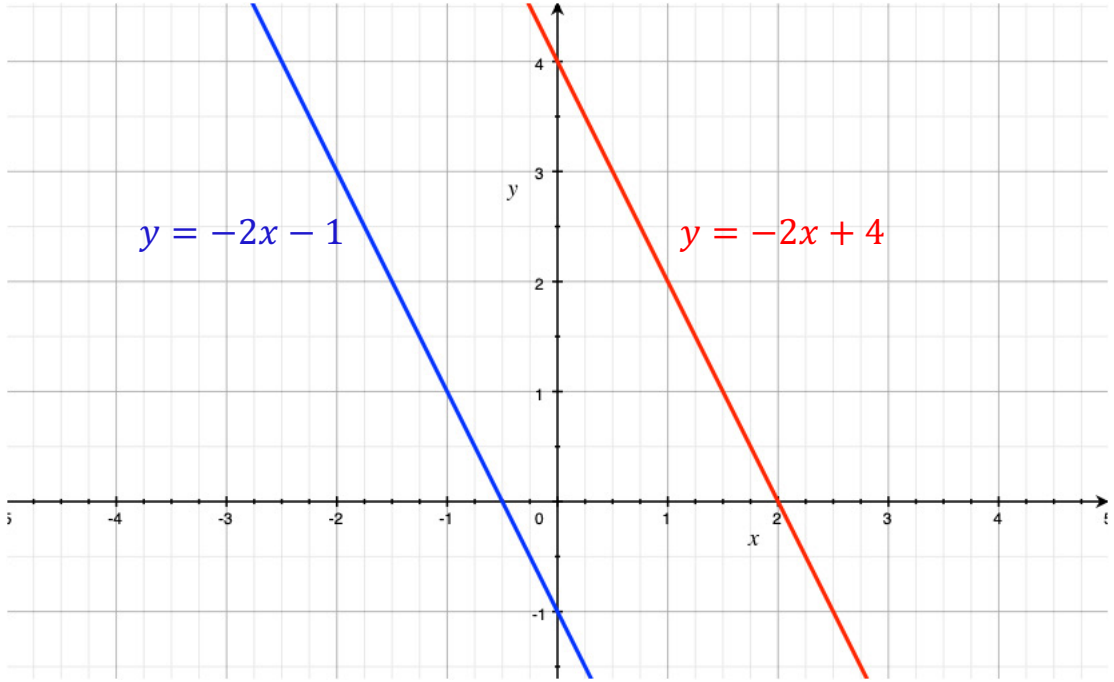
- Slope-intercept form: $y = mx + b$

- Standard form: $Ax + By = C$, where A and B are not both 0.

PARALLEL LINES

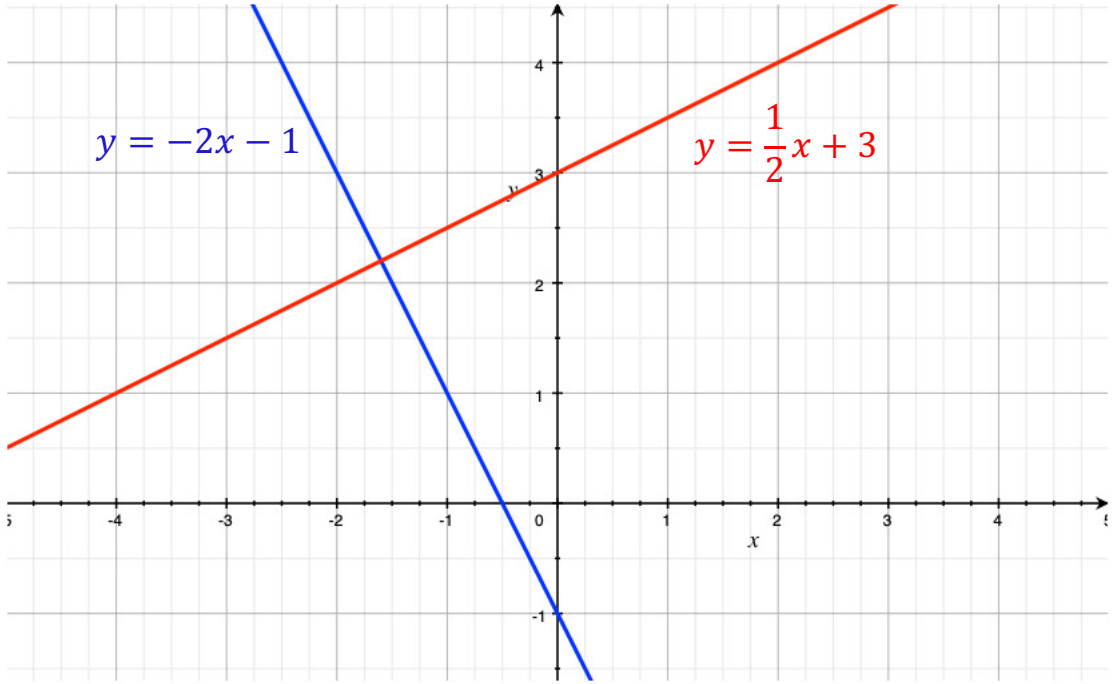
Same slope (positive, negative, zero) or

both vertical



PERPENDICULAR LINES

Product of slopes is –1 or one is vertical and the other horizontal



EXAMPLE

Passes through $(-\mathbf{2}, \mathbf{6})$ and is parallel to

$y = \frac{2}{3}x - \frac{5}{3}$

$$m = \frac{2}{3}$$

$$y = \frac{2}{3}x + b$$

$$\left(\frac{2}{3}\right)(-2) + b = 6$$

$$-\frac{4}{3} + b = 6$$

$$b = \frac{4}{3} + \frac{18}{3} = \frac{22}{3}$$

$$\boldsymbol{y = \frac{2}{3}x + \frac{22}{3}}$$

Passes through $(-\mathbf{2}, \mathbf{6})$ and is perpendicular to

$y = \frac{2}{3}x - \frac{5}{3}$

$$m = -\frac{3}{2}$$

$$y = -\frac{3}{2}x + b$$

$$\left(-\frac{3}{2}\right)(-2) + b = 6$$

$$3 + b = 6$$

$$b = 6 - 3 = 3$$

$$\boldsymbol{y = -\frac{3}{2}x + 3}$$

MODULE 1

LINEAR FUNCTIONS

FUNCTIONS

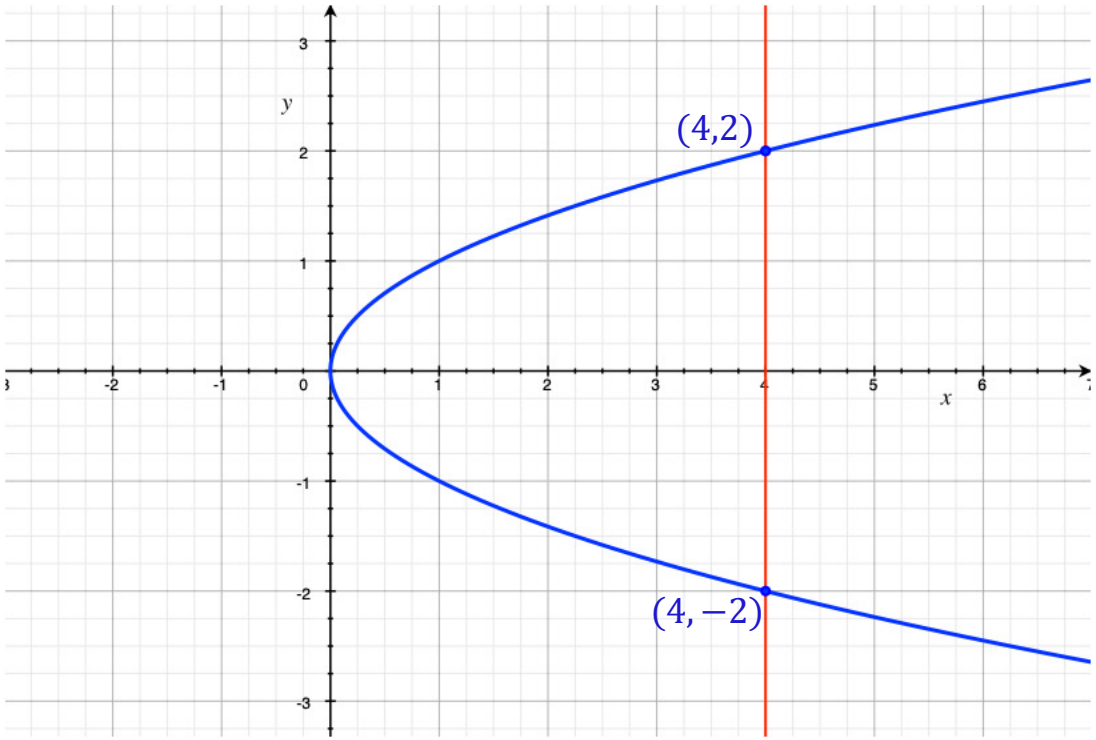
- Rule that assigns exactly one output to each input
- Letters f, g, and h are common names for functions
- $f(x) = 4x^2 - 1$, $g(x) = |x| + 5$, $h(x) = -x + 2$
- Vertical line test

$f(x) = 4x^2 - 1$ in words:
Square the input then multiply by four and subtract one.

$g(x) = |x| + 5$ in words:
Find the absolute value of the input then add five.

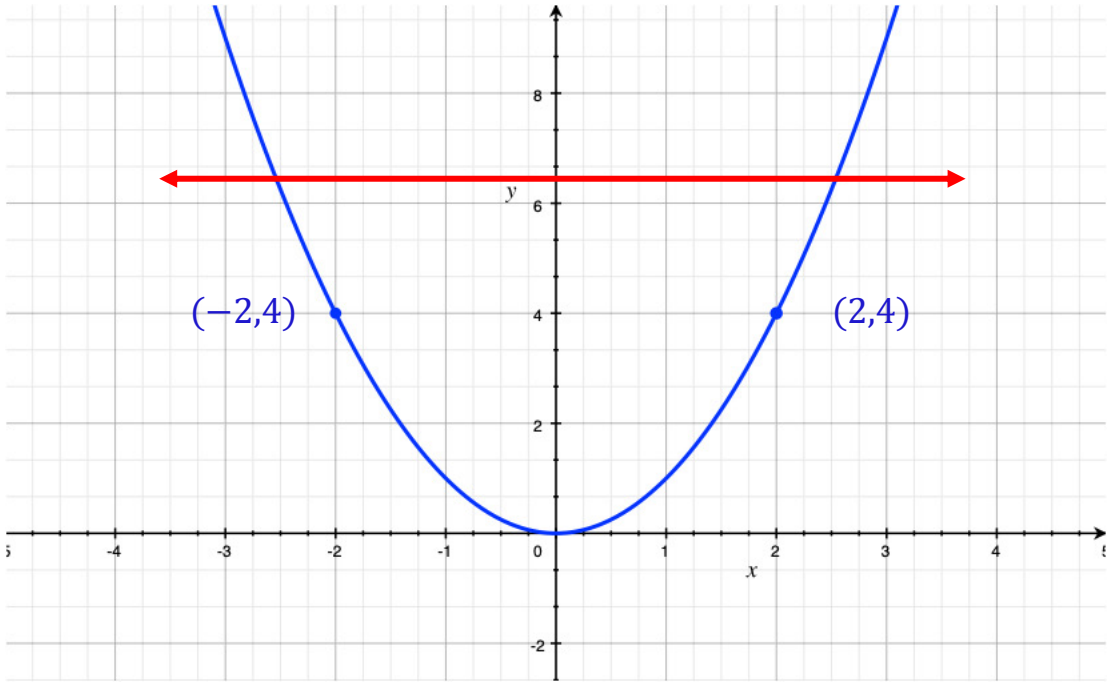
$h(x) = -x + 2$ in words:
Negate the input then add two.

VERTICAL LINE TEST



$x = y^2$

HORIZONTAL LINE TEST



$y = x^2$

LINEAR FUNCTIONS

- $f(x) = mx + b$ or $f(x) = b$
- Exponent of x is 0 or 1
- The graph of both $y = mx + b$ and $y = b$ are lines
- Vertical lines are not the graph of functions

EXAMPLE

Let $g(x) = -4x + 5$. Find $g(3)$, $g(0)$, $g(-2)$, and $g(b)$.

Solution:

To find $g(3)$, substitute 3 for x.

$$g(3) = -4(3) + 5 = -12 + 5 = -7$$

Ordered pair $(3, -7)$

Similarly,

$$g(0) = -4(0) + 5 = 0 + 5 = 5,$$

Ordered pair $(0,5)$

$$g(-2) = -4(-2) + 5 = 8 + 5 = 13,$$

Ordered pair $(-2,13)$

and

$$g(b) = -4b + 5$$

Ordered pair $(b, -4b + 5)$

BREAK-EVEN ANALYSIS

- Linear cost function, $C(x) = mx + b$

  m is the marginal cost, b is the fixed cost, x is the number of items produced

- Revenue function, $R(x) = px$

  p is the selling price per unit and x is the number of units sold

- Profit function, $P(x) = R(x) - C(x)$

- Break-even point: The point where $R(x) = C(x)$

  Occurs where the two lines intersect

EXAMPLE

The cost to produce x widgets is given by $C(x) = 105x + 6000$ and each widget sells for $\$250$. Determine the break-even quantity.

Solution:

$R(x) = 250x$

$250x = 105x + 6000$

$145x = 6000$

$x \approx 41.38$

$R(41) = 250(41) = 10250$

and

$C(41) = 105(41) + 6000 = 10305$

$R(42) = 250(42) = 10500$

and

$C(42) = 105(42) + 6000 = 10410$

Note: Selling 41 widgets is not enough.

The breakeven quantity is $\mathbf{42}$ widgets.

MODULE 1

LEAST SQUARES CALCULATIONS

LEAST SQUARES LINE

Minimize the sum of the squares of the vertical distances from the data points to the line

$$y = mx + b$$

Data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

and

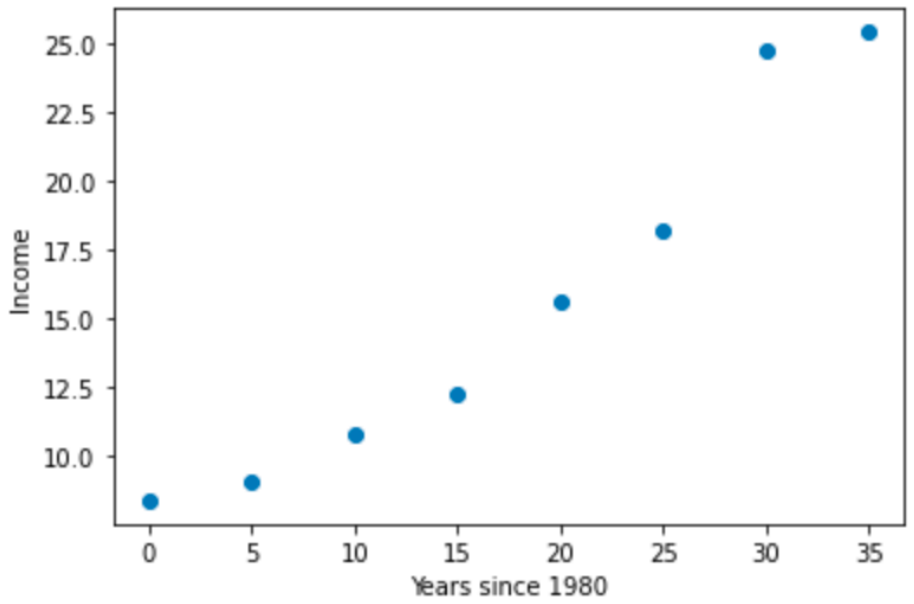
$$b = \frac{\sum y - m(\sum x)}{n}$$

SCATTERPLOT

Income from side business

Year	Income
1980	8,414
1985	9,124
1990	10,806
1995	12,321
2000	15,638
2005	18,242
2010	24,792
2015	25,436

Let x represent the number of years since 1980 and y represent the income in thousands of dollars



LEAST SQUARES CALCULATIONS

Year	Income
1980	8,414
1985	9,124
1990	10,806
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2000	15,638
2005	18,242
2010	24,792
2015	25,436

Least Squares Calculations				
x	y	xy	x^2	y^2
0	8.414	0	0	70.795396
5	9.124	45.62	25	83.247376
10	10.806	108.06	100	116.769636
15	12.321	184.815	225	151.807041
20	15.638	312.76	400	244.547044
25	18.242	456.05	625	332.770564
30	24.792	743.76	900	614.643264
35	25.436	890.26	1225	646.990096
140	**124.773**	**2741.325**	**3500**	**2261.57042**

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$= \frac{8(2741.325) - (140)(124.773)}{8(3500) - (140)^2}$$

$$= 0.5312$$

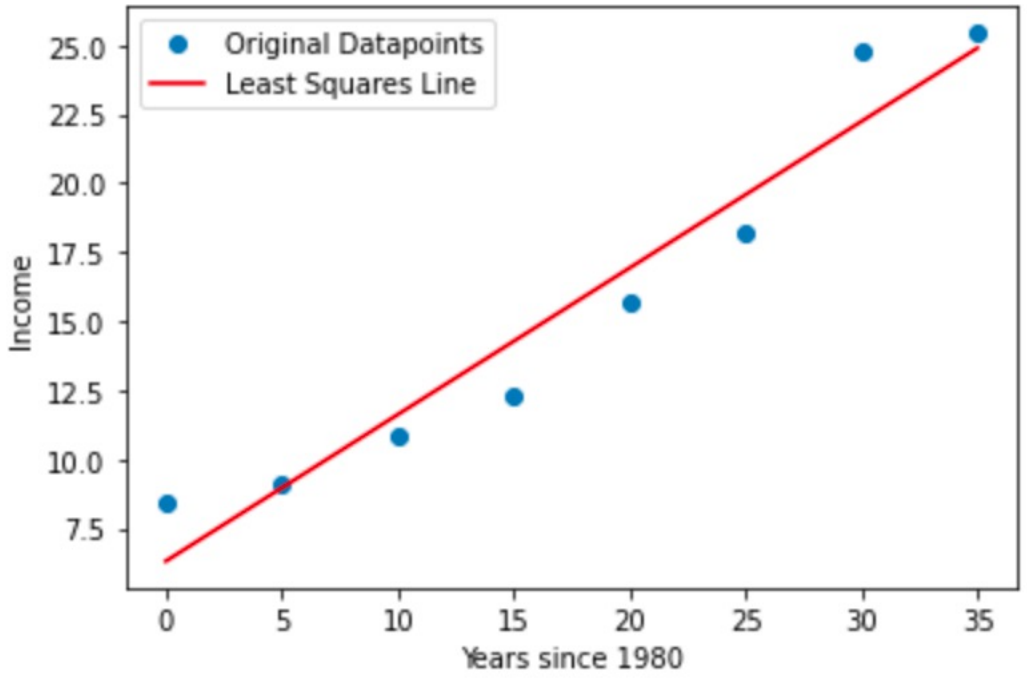
$$b = \frac{\sum y - m(\sum x)}{n}$$

$$= \frac{124.773 - (0.5312)(140)}{8}$$

$$= 6.3$$

$$\boldsymbol{y = 0.5312x + 6.3}$$

GRAPH OF LEAST SQUARES LINE



LEAST SQUARES LINE PREDICTION

Use the least squares line $y = 0.5312x + 6.3$ to predict income in 2025

Recall, x is the number of years since 1980, so $x = 45$ corresponds to 2025

$$y = (0.5312)(45) + 6.3 = 30.204$$

Since y is in thousands of dollars, the predicted income in 2025 is $\$30{,}204$

CORRELATION COEFFICIENT

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$$

$$= \frac{8(2741.325)(140)(124.773)}{\sqrt{8(3500) - (140)^2} \cdot \sqrt{8(2261.57042) - (124.773)^2}}$$

$$= \mathbf{0.9691}$$

Least Squares Calculations				
x	y	xy	x^2	y^2
0	8.414	0	0	70.795396
5	9.124	45.62	25	83.247376
10	10.806	108.06	100	116.769636
15	12.321	184.815	225	151.807041
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140	**124.773**	**2741.325**	**3500**	**2261.57042**

PYTHON

```
from scipy import stats
x = [0,5,10,15,20,25,30,35]
y = [8.414,9.124,10.806,12.321,15.638,18.242,24.792,25.436]
slope, intercept, r_value, p_value, std_err = stats.linregress(x, y)
print("slope = ", slope)
print("intercept = ", intercept)
print("correlation coefficient = ", r_value)
```

```
slope =  0.5312357142857143
intercept =  6.300000000000001
correlation coefficient =  0.9690801754643459
```

QUESTIONS?