MODULE 9

MODULE 9 QUIZ REVIEW

MODULE 9 QUIZ

- Module 6
  - Applications of the derivative
  - Derivatives and graphing
- Module 7
  - Integration
  - Continuous probability models
- Module 8
  - Partial derivatives
  - Lagrange Multipliers

Remember to add your work for every problem

The number of salmon swimming upstream to spawn is approximated by

$$S(x) = -x^3 + 3x^2 + 360x + 5000, \qquad 6 \leq x \leq 20$$

where x represents the temperature of the water in degrees Celsius. Find the water temperature that produces the maximum number of salmon swimming upstream.

Use the Extreme Value Theorem

Find the critical values in $(6,20)$

$$S'(x) = -3x^2 + 6x + 360$$

$$-3x^2 + 6x + 360 = 0$$
$$-3(x^2 - 2x - 120) = 0$$

$$-3(x + 10)(x - 12) = 0$$

$$x = -10, 12$$

$x = -10$ is not in the domain

Find the function value for $x = 12$

$$S(12) = -(12)^3 + 3(12)^2 + 360(12) + 5000 = 8024$$

Find the function values for the endpoints $x = 6$ and $x = 20$

$$S(6) = -(6)^3 + 3(6)^2 + 360(6) + 5000 = 7052$$

$$S(20) = -(20)^3 + 3(20)^2 + 360(20) + 5000 = 5400$$

$S(x)$ is maximized when $x = 12$

The temperature that produces the maximum number of salmon swimming upstream is $12°$ C

The average annual increment in the horn length (in centimeters) of bighorn rams born in recent years can be approximated by

$$y = 0.1762x^2 - 3.986x + 22.68$$

where x is the ram's age (in years) for x between 3 and 9. Find the total increase in the length of a ram's horn during this time.

Use the Fundamental Theorem of Calculus

$$\int_3^9 (0.1762x^2 - 3.986x + 22.86)dx = \left(\frac{0.1762}{3}x^3 - \frac{3.986}{2}x^2 + 22.86x\right)\Bigg|_3^9$$

$$= 34.8948 \text{ cm}$$

The total increase in the length of a ram's horn from age 3 years to 9 years is about 34.9 cm

A machine produces screws and the lengths of the screws produced are approximately normally distributed with a mean length of 2.5 cm and a standard deviation of 0.2 cm. Find the probabilities that a screw produced by this machine has lengths as follows.

(a) Greater than 2.7 cm
(b) Within 1.2 standard deviations of the mean

Standardize $X = 2.7$ using the z-scores theorem

$$z = \frac{2.7 - 2.5}{0.2} = 1$$

$$P(X > 2.7) = P(z > 1) = 0.1587$$

Use table from readings

Within 1.2 standard deviations of the mean:

$$z = \pm 1.2$$

$$P(2.26 \leq X \leq 2.74) = P(-1.2 \leq z \leq 1.2)$$

$$= P(z \leq 1.2) - P(z \leq -1.2)$$

$$= 0.8849 - 0.1151 = 0.7698$$

```python
from sympy.stats import Normal, P
from sympy import N, And

X = Normal("X",2.5,0.2)
print(f"P(X > 2.7) = {round(N(P(X>2.7)),4)}")
print(f"P(2.26 < X < 2.74) = {round(N(P(And(X>2.26,X<2.74))),4)}")
```

```
P(X > 2.7) = 0.1587
P(2.26 < X < 2.74) = 0.7699
```

The proportion of the times (in days) between major earthquakes in the north-south seismic belt of China is a random variable that is exponentially distributed with $a = 1/609.5$.

(a) Find the expected number of days and the standard deviation between major earthquakes for this region.
(b) Find the probability that the time between a major earthquake and the next one is more than 1 year.

Exponential distribution:

$f(x) = ae^{-ax}$ for x in $[0, \infty)$

$\mu = 1/a$ and $\sigma = 1/a$

$$f(x) = \frac{1}{609.5} e^{-x/609.5}$$

Since $a = 1/609.5$, the expected number of days and the standard deviation between major earthquakes are both 609.5

To find the probability that the time between a major earthquake and the next one is more than one year, integrate $f(x)$ from 365 to ∞

$$\int_{365}^{\infty} \frac{1}{609.5} e^{-x/609.5} dx = \lim_{b \to \infty} \int_{365}^{b} \frac{1}{609.5} e^{-x/609.5} dx$$

$$= \lim_{b \to \infty} -e^{-x/609.5} \Big|_{365}^{b}$$

$$= \lim_{b \to \infty} -e^{-b/609.5} - \left(-e^{-365/609.5}\right) = 0 + e^{-365/609.5} \approx 0.5494$$

The probability that the time between a major earthquake and the next one is more than one year is approximately 0.5494

Suppose that the profit (in hundreds of dollars) of a certain firm is approximated by

$$P(x, y) = 1500 + 36x - 1.5x^2 + 120y - 2y^2$$

where x is the cost of a unit of labor and y is the cost of a unit of goods. Find the values x and y that maximize profit. Find the maximum profit.

Use the test for relative extrema

$$P_x(x, y) = 36 - 3x$$
$$P_y(x, y) = 120 - 4y$$

Set each equal to 0 and solve

If $P_x = 0$, then $x = 12$
If $P_y = 0$, then $y = 30$

So, $(12,30)$ is a critical point

$$P_{xx} = -3$$
$$P_{yy} = -4$$
$$P_{xy} = 0$$

$$D = (-3)(-4) - 0^2 = 12 > 0$$

$$P_{xx}(12,30) = -3 < 0$$

Therefore, $P(12,30)$ is a relative maximum

$$P(12,30) = 1500 + 36(12) - 1.5(12)^2 + 120(30) - 2(30)^2 = 3516$$

The maximum profit is $\$351{,}600$ when the cost of a unit of labor is $\$12$ and the cost of a unit of goods is $\$30$

A manufacturing firm estimates that its total production of automobile batteries in thousands of units is

$$f(x,y) = 3x^{1/3}y^{2/3}$$

where x is the number of units of labor and y is the number of units of capital utilized. Labor costs are $\$80$ per unit and capital costs are $\$150$ per unit. How many units each of labor and capital will maximize production if the firm can spend $\$40{,}000$ for these costs?

Use the method of Lagrange multipliers

Maximize $f(x,y) = 3x^{1/3}y^{2/3}$ subject to $80x + 150y = 40000$

Step 1: (write the constraint)

$$g(x,y) = 80x + 150y - 40000$$

Step 2: (form the Lagrange function)

$$F(x,y,\lambda) = 3x^{1/3}y^{2/3} - \lambda(80x + 150y - 40000)$$

Steps 3 & 4: (find 1st partials and form the system)

$$F_x(x,y,\lambda) = x^{-2/3}y^{2/3} - 80\lambda = 0$$
$$F_y(x,y,\lambda) = 2x^{1/3}y^{-1/3} - 150\lambda = 0$$
$$F_\lambda(x,y,\lambda) = -80x - 150y + 40000 = 0$$

Step 5: (solve the system)

$$\lambda = \frac{x^{-2/3}y^{2/3}}{80}$$

$$\lambda = \frac{2x^{1/3}y^{-1/3}}{150}$$

$$\frac{x^{-2/3}y^{2/3}}{80} = \frac{2x^{1/3}y^{-1/3}}{150}$$

$$x = \frac{15y}{16}$$

$$-80\left(\frac{15y}{16}\right) - 150y + 40000 = 0$$

$$y \approx 178$$

If $y = 178$, then $x \approx 167$

Use about 167 units of labor and 178 units of capital to maximize production

The production function for one country is given by

$$z = x^{0.65}y^{0.35}$$

where x represents units of labor and y represents units of capital. At present, 50 units of labor and 29 units of capital are available. Use differentials to estimate the change in production if the number of units of labor is increased to 52 and capital is decreased to 27 units.

What we know:

$z = x^{0.65}y^{0.35}$
$x = 50, y = 29$
$dx = 52 - 50 = 2$
$dy = 27 - 29 = -2$

$$f_x(x,y) = 0.65x^{-0.35}y^{0.35} = 0.65\left(\frac{y}{x}\right)^{0.35}$$

$$f_y(x,y) = 0.35x^{0.65}y^{-0.65} = 0.35\left(\frac{x}{y}\right)^{0.65}$$

$$dz = f_x(x,y)dx + f_y(x,y)dy$$

$$= 0.65\left(\frac{y}{x}\right)^{0.35} dx + 0.35\left(\frac{x}{y}\right)^{0.65} dy$$

$$dz = 0.65\left(\frac{29}{50}\right)^{0.35}(2) + 0.35\left(\frac{50}{29}\right)^{0.65}(-2) \approx 0.07694$$

The change in production is approximately 0.07694 units

FINAL EXAM TOPICS

- Module 1
  - Linear functions
  - Least squares calculations
- Module 2
  - Systems of linear equations
- Module 3
  - Linear programming models
- Module 4
  - Conditional probability
  - Bayes Theorem
- Modules 5 & 6
  - Derivatives and the graph of a function
  - Applications of the derivative
- Module 7
  - Fundamental Theorem of Calculus
  - Continuous probability distributions
- Module 8
  - Lagrange multipliers
- Module 9
  - Minimum spanning trees

Suggestion 1:
Create a Jupyter notebook with sample code for:
1. Least squares calculations (M1)
2. Matrix inverses (M2)
3. LP Models (M3)
4. Anything else you feel you would want

Suggestion 2:
1. Find problems on these topics from PS and Quizzes that you struggled with.
2. Schedule tutoring sessions with the Math Place to go over those problems and concepts.

FINAL EXAM

- Due on the last day of class at 11:59pm CT
- 2-hour time limit
- 10 questions
- Comprehensive
- Register with Examity
- Schedule exam time

- Handwritten notes, scrap paper
- Files on computer
- PDF files of readings
- Copies of assignments
- Handheld calculator
- Python, R, Excel

Add your work to every problem

QUESTIONS?