Exponential Distribution

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Point Estimation: Methods

We compared the following point estimators:

- Maximum Likelihood Estimator
- Unbiased correction for the MLE
- Second Method of Moment Estimator

##add criteria for comparing estimators!

Maximum Likelihood Estimator

```
##code here
```

Unbiased correction for the MLE

```
##code here
```

Second Method of Moment Estimator

```
##code here
```

Confidence Intervals: Methods

We compared the following confidence intervals:

- Wald-based Confidence Interval
- Gamma-based Confidence Interval
- Score-based Confidence Interval
- Bootstrap Confidence Interval

##add criteria for comparing CIs!!

Wald Confidence Interval

```r
wald_ci <- function(N, rate, alpha = 0.05){
  x <- rexp(N, rate = rate)
  x_bar <- mean(x)
  se <- sd(x)/sqrt(N)
  ci <- x_bar + c(-1, 1)*qnorm(1 - (alpha / 2))
  return(ci)
}
```

Gamma Confidence Interval

If $X_1, X_2, ..., X_n \overset{\text{iid}}{\sim} \text{Exponential}(\lambda)$, then

$$\sum_i^n x_i \sim \text{Gamma}(n, \lambda) \implies \lambda\bar{x} \sim \text{Gamma}(n, n)$$

Let g_y be the yth percentile of this distribution. Then we can say:

$$1 - \alpha = P(g_{\alpha/2} \leq \lambda\bar{x} \leq g_{1-\alpha/2}) = P(g_{\alpha/2}/\bar{x} \leq \lambda\bar{x} \leq g_{1-\alpha/2}/\bar{x})$$

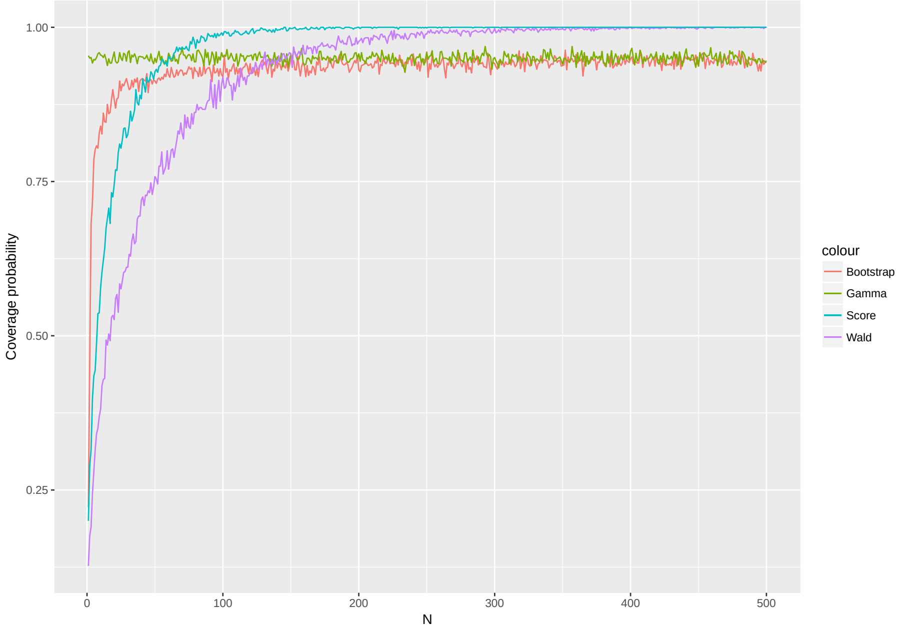
And therefore, a $(1 - \alpha)\%$ confidence interval for λ is

$$(g_{\alpha/2}/\bar{x},\, g_{1-\alpha/2}/\bar{x})$$

Score Confidence Interval

Bootstrap Confidence Interval

Coverage probabilities



Summary of Findings and Recommendations

```r
summary(cars)
```

```
##      speed           dist       
##  Min.   : 4.0   Min.   :  2.00  
##  1st Qu.:12.0   1st Qu.: 26.00  
##  Median :15.0   Median : 36.00  
##  Mean   :15.4   Mean   : 42.98  
##  3rd Qu.:19.0   3rd Qu.: 56.00  
##  Max.   :25.0   Max.   :120.00  
```