Statistical Modeling

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Preface

This is a Quarto book.

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1 Parameter Estimation

1.0.1 Point Wise Estimation: Maximum Likelihood Estimators

1.0.2 Interval Estimates

- Consider a sample $X_1, X_2, \ldots, X_n$ drawn from a known distribution with an *unknown* mean μ.
- It is established that the sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ serves as the maximum likelihood estimator for μ.
- However, the sample mean $\bar{X}$ is not expected to be exactly equal to μ, but rather close to it.
- Therefore, instead of providing a single point estimate, it is often more useful to specify **an interval** within which we are confident that μ lies.
- To determine such an interval estimator, we utilize the probability distribution of the point estimator.

1.0.2.1 Normal Population with unknown Mean and known Variance

- Consider a sample $X_1, X_2, \ldots, X_n$ drawn from a normal distribution with an **unknown** mean μ and a **known** variance σ^2.
- The point estimator $\bar{X}$ is normal with mean μ and variance σ^2/n.
- Therefore, $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ follows a standard normal distribution.

> **Note**
>
> Consider that I want to find an interval around $\bar{X}$ such that the actual population mean μ falls within the interval, say 95 % of the times.

Tip

- For finding such an interval, I can use the Z-table. From the Z-table I can find:

$$P\left(-1.96 < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < 1.96\right) = 0.9750 - 0.0250 = 0.95$$

- Rewriting the above equation:

$$P\left(-1.96\frac{\sigma}{\sqrt{n}} < \bar{X}-\mu < 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$P\left(1.96\frac{\sigma}{\sqrt{n}} > \mu-\bar{X} > -1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$P\left(-1.96\frac{\sigma}{\sqrt{n}} < \mu-\bar{X} < 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$P\left(\bar{X}-1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{X}+1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

- We have P(Z < -1.96) = 0.025, similarly P(Z > 1.96) = 0.025. Usually 1.96 is represented generally as $z_{0.025}$. Thus, P(Z < -$z_{0.025}$) = 0.025 and P(Z > $z_{0.025}$) = 0.025.

- So, I can re write the equation as:

$$P\left(\bar{X}-z_{0.025}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X}+z_{0.025}\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$P\left(\bar{X}-z_{0.025}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X}+z_{0.025}\frac{\sigma}{\sqrt{n}}\right) = 1-2*(0.025)$$

- For a confidence level of $(1-\alpha)$, the corresponding critical value from the standard normal distribution is $z_{\alpha/2}$.
- The $(1-\alpha)$ confidence interval for μ is given by:

$$\left(\bar{X}-z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X}+z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

2 Summary

In summary, this book has no content whatsoever.

References