(1) To meet after N steps, both drunkards have to take the same no. of right (and left) steps.

let $p(x) = P(\text{taking } x \text{ rights in N steps})$

$$= \binom{N}{x}\left(\frac{1}{2}\right)^{x} \cdot \left(\frac{1}{2}\right)^{N-x}$$

$$= \binom{N}{x}\left(\frac{1}{2}\right)^{N}$$

$\therefore$ For 2 drunkards to meet,

$$\text{probability} = \sum_{x=0}^{N} p(x) \cdot p(x)$$

$$= \sum_{x=0}^{N}\left[\binom{N}{x}\left(\frac{1}{2}\right)^{N}\right]^{2}$$

$$= \left(\frac{1}{2}\right)^{2N} \sum_{x=0}^{N}\binom{N}{x} \cdot \binom{N}{N-x}$$

$$\therefore \boxed{\text{probability} = \left(\frac{1}{2}\right)^{N} \cdot \binom{2N}{N}}$$

(2) To come back to origin, there must be an equal no. of right and left steps.

This is not possible for odd N.

$\therefore$ ~~P(A) = 0~~

P(coming back to origin after N steps) = 0 for odd N

For even N, there must be $N/2$ right steps.

$\therefore$ P(coming back to origin after N steps) $= \binom{N}{N/2}\left(\frac{1}{2}\right)^N$

$\therefore$ P(coming back to origin after N steps)

$$= \begin{cases} 0, & \text{if } N \text{ odd} \\ \binom{N}{N/2}\left(\frac{1}{2}\right)^N, & \text{if } N \text{ even} \end{cases}$$

3 let $S_i = \begin{cases} 1 & \text{if } i^{th} \text{ step to the right} \\ -1 & \text{if } i^{th} \text{ step to the left} \end{cases}$

let d = distance $= \sum_{i=1}^{N} S_i$

~~Ed~~ E
mean distance $= E[d]$

$$= \sum_{i=1}^{N} E[S_i]$$

B $E[S_i] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$

$\therefore \boxed{E[d] = 0}$

(4) let $D =$ square distance $= \left[\sum_{i=1}^{N} S_i\right]^2$

mean sq distance $= E[D]$

$= E\left[\left(\sum_{i=1}^{N} S_i\right)^2\right]$

$= \sum_{i=1}^{N} E[S_i^2] + \sum_{1 \le i < j \le N} E[S_i S_j]$

$E[S_i^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$

$E[S_i S_j] = \frac{1}{4}\Big((1)(1) + (1)(-1) + (-1)(1) + (-1)(-1)\Big)$

$= 0$

$\therefore$ mean square distance $= \sum_{i=1}^{N} 1 + \sum_{1 < i < j \le N} 0$

$\therefore$ mean square distance $= N$