



To convert a non-decimal number to decimal, we need to add the weighted sum of each digit. For example, to convert the octal number  $720.5_{(8)}$  to decimal.

$$720.5_{(8)} = 7 \times 8^2 + 2 \times 8^1 + 0 \times 8^0 + 5 \times 8^{-1} = 464.625$$

#### Decimal to non-decimal

To convert a decimal number to a base-X non-decimal, we need to convert the integer part and the fractional part, separately.

To convert the integer part, we will integer divide it by X until it reaches 0, and record the remainder each time. Traversing the remainder in reverse order will give us the representation in base-X system.

For example, to convert 50 in base-10 to base-2:

$$50 / 2 = 25; 50 \% 2 = 0$$

$$25 / 2 = 12; 25 \% 2 = 1$$

$$12 / 2 = 6; 12 \% 2 = 0$$

$$6 / 2 = 3; 6 \% 2 = 0$$

$$3 / 2 = 1; 3 \% 2 = 1$$

$$1 / 2 = 0; 1 \% 2 = 1$$

Traversing the remainder in reverse order, we get  $1, 1, 0, 0, 1, 0$ , so 50 in decimal will become  $110010_{(2)}$  in binary.

To convert the fractional part, we will multiply the fractional part of the decimal number by X until it becomes 0 and record the integer part each time. Traversing the integer part in order will give us the representation in base-X system.

For example, to convert the decimal number 0.6875 to binary:

$$0.6875 \times 2 = 1.375 \text{ with integer } 1$$

$$0.375 \times 2 = 0.75 \text{ with integer } 0$$

$$0.75 \times 2 = 1.5 \text{ with integer } 1$$

$$0.5 \times 2 = 1 \text{ with integer } 1$$

Traversing the integer apart in order, we get  $1, 0, 1, 1$ , so 0.6875 in decimal will become  $0.1011_{(2)}$  in binary. Note that a finite fraction in one base may become an infinite recurring fraction in another base. For instance, the decimal number 0.2 will become  $0.\dot{0}01\dot{1}_{(2)}$

#### Conversion between other bases

The common practice is to convert between non-base-10 numbers to convert to decimal first and then convert to the target base. Under certain circumstances, we can perform the conversion without going through decimal.

For instance, for converting binary numbers to octal or hexadecimal, and converting octal or hexadecimal numbers to binary, there is no need to go through the intermediate decimal step. Each digit in an octal number can be represented as three digits in a binary number. Each digit in a hexadecimal number can be represented as four digits in a binary number.

Let's look at an example. We can group digits in a binary number  $101110010_{(2)}$  in groups of three as  $101 | 110 | 010$ , or in groups of four as  $1 | 0111 | 0010$ , which can be converted to  $562_{(8)}$  in octal and  $172_{(16)}$  in hexadecimal.