CS6014 : Advanced Theory of Computation — Given on: Nov 6, 7pm

Problem Set #6 — Due on : Nov 14, 5pm

1. (10 points) Oracle Queries.
   (a) Argue that $\mathsf{P}^{\mathsf{P}} = \mathsf{P}$. Why does not the same argument work for NP and give $\mathsf{NP}^{\mathsf{NP}} = \mathsf{NP}$?
   (b) If $\mathsf{NP} = \mathsf{coNP}$, argue that $\mathsf{PH} = \Sigma_1^p$.

2. (10 points) (a) Reading Assignment : Read the proof (Section 3.4, Theorem 3.21, Page 93) of the claim : $\mathsf{DTIME}(2^{O(s(n))} \subseteq ASPACE(s(n))$. Determine an upper bound on the number of children for any universal configuration in the alternating Turing machine produced in the construction.
   (b) Conclude that $AL = \mathsf{P}$. Show that all CFLs are in P by giving an alternating Turing machine running in log space for checking membership. (Assume that the CFL is given at the input in the CNF form.).

3. (10 points) Dene the language:

$$\textsc{ShortestPath} = (G, k, s, t) | \text{ the shortest path from } s \text{ to } t \text{ in } G \text{ has length } k$$

   (a) (5 points) Prove that $\textsc{ShortestPath}$ is in NL.
   (b) (5 points) Prove that $\textsc{ShortestPath}$ is in L if and only if $\mathsf{L} = \mathsf{NL}$.

4. (15 points) An undirected graph is bipartite if its nodes can be divided into two sets such that all edges go from a node in one set to a node in the other set. Show that a graph is bipartite if and only if it does not contain a cycle that contains an odd number of nodes. Let

$$\textsc{Bipartite} = \{G \mid G \text{ is a bipartite graph }\}$$

   Show that $\textsc{Bipartite}$ is in NL. (Hint : Use Immerman-Szelepsinyi theorem !).

5. (25 points) We define the product of two $n \times n$ Boolean matrices A and B as another $n \times n$ Boolean matrix C such that $C_{ij} = \bigvee_{k=1}^{n} (A_{ik} \wedge B_{kj})$.
   (a) (5 points) Show that boolean matrix multiplication can be done in logarithmic space.
   (b) (5 points) Using repeated squaring, argue that A^p can be computed in space $O(\log n \log p)$.
   (c) (5 points) Show that if A is the adjacency matrix of a graph, then $(A_{ij}^k = 1$ if and only if there is a path of length at most k from the vertex i to vertex j and is 0 otherwise.
   (d) (5 points) Use the above to give an alternative proof that $\mathsf{NL} \subseteq \mathsf{DSPACE}(\log^2 n)$. We originally proved it using Savitch's theorem.