IITM-CS6014 : Advanced Theory of Computation	Given on: Aug 28, 8am
Problem Set #2	Due on : Sep 11, 5pm

- The instructions remain the same as last problem set. Answers need not be written in LaTeX. However, from Problem Set 3 onwards, this will be compulsory. No PDFs will be accepted unless it is produced from a LaTeXfile.

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1. Consider the following three problems. Formulate the problems as languages and argue whether they are decidable or undecidable. (You can apply Rice's theorems wherever possible.)
   (a) An *unused state* of a Turing machine is a state that is never entered on any input string. The problem is to determine whether a Turing machine has any useless states.
   (b) Given a Turing machine M on input x, does the machine ever move its head left at any point during its computation on w.
   (c) Given two Turing machines, M_1 and M_2, check if they accept complimentary languages.
   (d) Given a Turing machine, test whether there exist a string on which the machine runs for ever.
   (e) Given a Turing machine, test whether it accepts at least one palindrome string.
   (f) Given a Turing machine, test whether it accepts only palindromes.
   (g) Given two Turing machines, M_1 and M_2, check if they accept a common string.
2. Kozen 104-110, 115.
3. Over a singleton alphabet, describe:
   (a) a language which is not regular, but is decidable.
   (b) a language which is semi-decidable but not decidable.
   (c) a language which is in Σ_2 but not in Σ_1.
4. (a) View $\leq_m$ and $\leq_T$ as relation defined over the set of languages. Argue whether they are reflexive, symmetric and transitive.
   (b) Show that L is decidable if and only if $L \leq_m 1^*0^*$.
5. Show that $FIN \leq_T REG$. Suppose you are given an oracle which will always answer questions of the form, "Is $L(M)$ a regular set?" truthfully. Show how to use such an oracle to decide questions of the form, "Is $L(M)$ finite?"