Table of Contents


```
% Astrobee Model
```

System

```
% A Matrix

load Matrices/A_matrix.mat
A = A_matrix

% B Matrix: Stowed

load Matrices/B_stowed.mat
B = B_stowed

% Full-State Feedback

Cf = eye(12);

Df = [zeros(12, 6)];

sys_full = ss(A, B, Cf, Df);

tf_full = tf(sys_full);

syms s

tf_full_sym = simplify(Cf * inv(s * eye(12) - A) * B + Df);
pretty(tf_full_sym)
```

```
A =
     0     0     0     0     0     0     0     0     0     0     0
  0
     0     0     0     0     0     0     0     0     0     0     0
  0
     0     0     0     0     0     0     0     0     0     0     0
  0
     0     0     0     0     0     0     0     0     0     0     0
  0
     0     0     0     0     0     0     0     0     0     0     0
  0
```

```
     0     0     0     0     0     0     0     0     0     0     0
0
     1     0     0     0     0     0     0     0     0     0     0
0
     0     1     0     0     0     0     0     0     0     0     0
0
     0     0     1     0     0     0     0     0     0     0     0
0
     0     0     0     1     0     0     0     0     0     0     0
0
     0     0     0     0     1     0     0     0     0     0     0
0
     0     0     0     0     0     1     0     0     0     0     0
0
B =
    0.0630         0         0         0         0         0
         0    0.0630         0         0         0         0
         0         0    0.0630         0         0         0
         0         0         0    5.4054         0         0
         0         0         0         0    4.9505         0
         0         0         0         0         0    5.3191
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
/   500                                              \
| ------,    0,      0,     0,      0,     0   |
| 7939 s                                           |
|                                                  |
|           500                                    |
|    0,   ------,    0,     0,      0,     0   |
|         7939 s                                   |
|                                                  |
|                    500                           |
|    0,      0,   ------,   0,      0,     0   |
|                 7939 s                           |
|                                                  |
|                            200                   |
|    0,      0,      0,    ----,    0,     0   |
|                           37 s                   |
|                                                  |
|                                    500           |
|    0,      0,      0,     0,    -----,   0   |
|                                   101 s          |
|                                                  |
|                                            250 |
|    0,      0,      0,     0,      0,     ---- |
|                                           47 s |
|                                                  |
|   #1,      0,      0,     0,      0,     0   |
|                                                  |
|    0,     #1,      0,     0,      0,     0   |
```

```
|                                                |
|    0,       0,      #1,      0,       0,      0   |
|                                                |
|                              200                 |
|    0,       0,       0,    -----,     0,      0   |
|                                 2                |
|                             37 s                 |
|                                                |
|                                      500         |
|    0,       0,       0,      0,    ------,   0   |
|                                         2        |
|                                     101 s        |
|                                                |
|                                             250  |
|    0,       0,       0,      0,       0,   ----- |
|                                               2  |
\                                            47 s  /

where

            500
   #1 == -------
                2
         7939 s
```

Section 1: Translation Controller Design -> Marginally Stable Pole at the Origin

```
% Top-half matrix (t1):

% Satisfaction of the first interpolation condition

translation_full = [tf_full_sym(1:3, 1:3); tf_full_sym(7:9, 1:3)];
pretty(translation_full);

Gp_t1 = translation_full(1:3, 1:3);
P_t1 = Gp_t1 * s;

[UL_t1, UR_t1, S_t1] = smithForm(P_t1, s)

Mp_t1 = S_t1/s

K_t1 = 1;
tp_t1 = 100;

Y1 = (K_t1 * s)/(tp_t1 * s + 1);
Y2 = Y1;
Y3 = Y1;

My_t1 = diag([Y1 Y2 Y3])
```

```
Mt = Mp_t1 * My_t1

Gc_t1_sym = simplify((UR_t1 * inv(eye(size(My_t1 * Mp_t1)) - My_t1 *
 Mp_t1) * My_t1 * UL_t1))

Gc_t1 = tf(double(Gc_t1_sym));

% % Convert to a string
% Gc_t1_arr = [zeros(1, 3)];
% Gc_t1_str = [];
% for i = 1:size(Gc_t1_sym, 1)
%         Gc_t1_str = char(Gc_t1_sym(i, i));
%         % Define ?s? as transfer function variable
%         s = tf('s');
%         % Evaluate the expression:
%         eval(Gc_t1_arr(i) == Gc_t1_str)
% end
%
% Gc_t1 = diag(Gc_t1_arr)

/    500                     \
|  ------,    0,       0     |
|  7939 s                    |
|                            |
|              500           |
|    0,     ------,    0     |
|           7939 s           |
|                            |
|                       500  |
|    0,       0,     ------ |
|                    7939 s |
|                            |
|   500                      |
| -------,    0,       0     |
|       2                    |
| 7939 s                     |
|                            |
|             500            |
|    0,    -------,    0     |
|                2           |
|          7939 s            |
|                            |
|                      500   |
|    0,       0,    ------- |
|                          2 |
\                  7939 s   /

UL_t1 =
[ 0, 0, 1]
[ 0, 1, 0]
[ 1, 0, 0]
UR_t1 =
[        0,         0, 7939/500]
```

```
[         0, 7939/500,         0]
[ 7939/500,         0,         0]
S_t1 =
[ 1, 0, 0]
[ 0, 1, 0]
[ 0, 0, 1]
Mp_t1 =
[ 1/s,   0,   0]
[   0, 1/s,   0]
[   0,   0, 1/s]
My_t1 =
[ s/(100*s + 1),             0,             0]
[             0, s/(100*s + 1),             0]
[             0,             0, s/(100*s + 1)]
Mt =
[ 1/(100*s + 1),             0,             0]
[             0, 1/(100*s + 1),             0]
[             0,             0, 1/(100*s + 1)]
Gc_t1_sym =
[ 7939/50000,          0,          0]
[          0, 7939/50000,          0]
[          0,          0, 7939/50000]
```

Section 2: Translation Controller Design -> Unstable Double-Pole at the Origin

```
% Bottom-half matrix (t2):

% Run this section first to calculate 'tz' to ensure that the second
 interpolation condition is satisfied

% d^k(T)/ds^k|(s=0) = 0, where k = 1 (since there is a double unstable
 pole
% (multiplicity ap = 2) in the plant at s = 0; k = ap - 1) -> 2nd
% interpolation condition

C_t2 = 500/7939; % Constant
Wn = 0.01; % Natural Frequency of the Control System
K = Wn^2/C_t2; % Controller Gain
Z = 2^-0.5; % Damping Ratio
tp = 1/(10*Wn); % Time constant (of the included pole)

syms s tz

TF = ((K*C_t2)*(tz*s + 1))/((s^2 + 2*Z*Wn*s + Wn^2)*(tp*s + 1))
dTF = diff(TF,s)
eqn = subs(dTF,s,0) == 0;
tz = solve(eqn,tz)

TF =
((s*tz)/10000 + 1/10000)/((10*s + 1)*(s^2 + (2^(1/2)*s)/100 +
 1/10000))
```

```
dTF =
tz/(10000*(10*s + 1)*(s^2 + (2^(1/2)*s)/100 + 1/10000)) -
 (10*((s*tz)/10000 + 1/10000))/((10*s + 1)^2*(s^2 + (2^(1/2)*s)/100 +
 1/10000)) - (((s*tz)/10000 + 1/10000)*(2*s + 2^(1/2)/100))/((10*s +
 1)*(s^2 + (2^(1/2)*s)/100 + 1/10000)^2)
tz =
100*2^(1/2) + 10
```

Section 3: Translation Controller Design -> Unstable Double-Pole at the Origin

```
% Youla Control Design

s = tf('s');

% Constants & Design Parameters
C_t2 = 500/7939; % Constant
Wn = 0.01; % Natural Frequency of the Control System
K = Wn^2/C_t2; % Controller Gain
Z = 2^-0.5; % Damping Ratio
tp = 1/(10*Wn); % Time Constant of the added pole
tz = 100*2^(1/2) + 10;

% Plant TF, 'Gp'
Gp = zpk(minreal(C_t2/s^2))

% Chosen Youla Parameter, 'Y' -> Y(0) = 0
Y = zpk(minreal(((K*s^2)*(tz*s + 1)/((s^2 + 2*Z*Wn*s + Wn^2)*(tp*s +
 1))),1e-05))

% Complementary Sensitivity TF, 'T' -> T(0) = 1 (1st interpolation
% condition)
T = zpk(minreal((Y*Gp),1e-05))

% Sensitivity TF, 'S'
S = zpk(minreal((1-T),1e-05))

% Controller TF, 'Gc'
Gc = zpk(minreal((Y/S),1e-05))

% Return Ratio, 'L'
L = zpk(minreal((Gc*Gp),1e-05))

GpS = zpk(minreal((Gp*S),1e-05))

% Internal stability check
Y_stability = isstable(Y)
T_stability = isstable(T)
S_stability = isstable(S)
GpS_stability = isstable(GpS)

M2 = 1/getPeakGain(S) % M2-margin
```

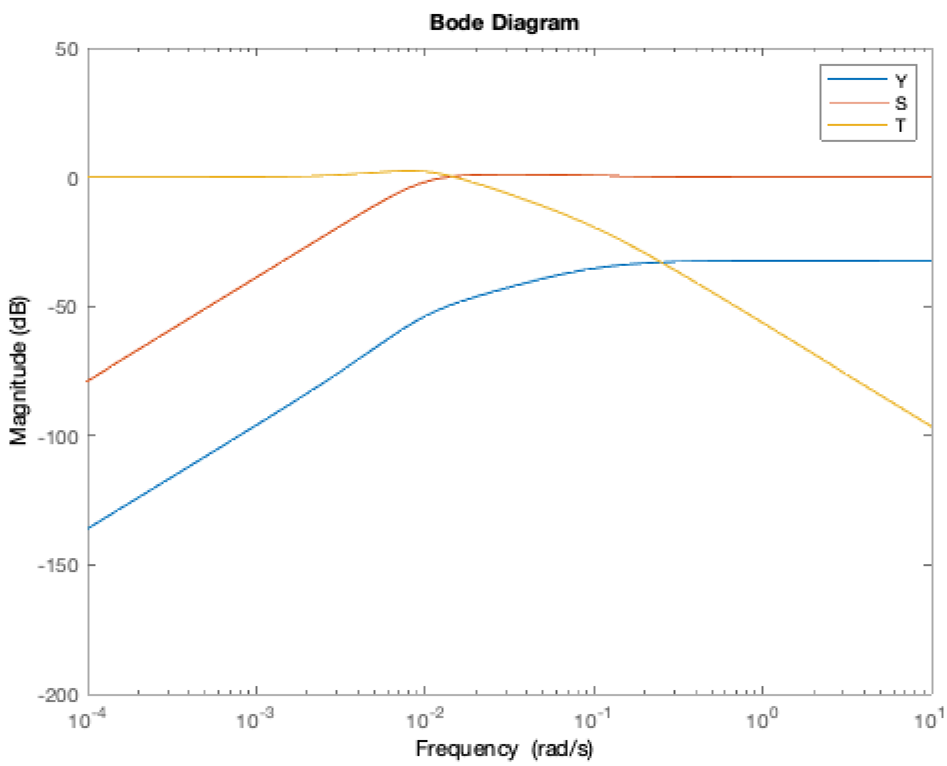
```
BW = bandwidth(T) % Bandwidth of the closed-loop
AE = getPeakGain(Y) % Maximum actuator effort

figure(1)
bodemag(Y, S, T);
legend('Y','S','T');

Gc_t2 = [tf(Gc) 0 0; 0 tf(Gc) 0; 0 0 tf(Gc)]

% Convert to symbolic matrix
[Num,Den] = tfdata(tf(Gc), 'v')
syms s
Gc_t2_sym_term = poly2sym(Num, s)/poly2sym(Den, s)
Gc_t2_sym = diag([Gc_t2_sym_term Gc_t2_sym_term Gc_t2_sym_term])

Gc_t = [Gc_t1_sym Gc_t2_sym]
```

```
Gp =

  0.06298
  -------
    s^2

Continuous-time zero/pole/gain model.


Y =

      0.024043 s^2 (s+0.006604)
  ---------------------------------
  (s+0.1) (s^2 + 0.01414s + 0.0001)

Continuous-time zero/pole/gain model.


T =

       0.0015142 (s+0.006604)
  ---------------------------------
  (s+0.1) (s^2 + 0.01414s + 0.0001)

Continuous-time zero/pole/gain model.


S =

           s^2 (s+0.1141)
  ---------------------------------
  (s+0.1) (s^2 + 0.01414s + 0.0001)

Continuous-time zero/pole/gain model.
```

```
Gc =

  0.024043 (s+0.006604)
  ---------------------
       (s+0.1141)

Continuous-time zero/pole/gain model.


L =

  0.0015142 (s+0.006604)
  ----------------------
      s^2 (s+0.1141)

Continuous-time zero/pole/gain model.


GpS =

         0.06298 (s+0.1141)
  ---------------------------------
  (s+0.1) (s^2 + 0.01414s + 0.0001)

Continuous-time zero/pole/gain model.

Y_stability =
  logical
   1
T_stability =
  logical
   1
S_stability =
  logical
   1
GpS_stability =
  logical
   1
M2 =
    0.8909
BW =
    0.0214
AE =
    0.0240

Gc_t2 =

  From input 1 to output...
       0.02404 s + 0.0001588
   1:  ---------------------
            s + 0.1141

   2:  0
```

```
   3:  0

  From input 2 to output...
   1:  0

       0.02404 s + 0.0001588
   2:  ---------------------
            s + 0.1141

   3:  0

  From input 3 to output...
   1:  0

   2:  0

       0.02404 s + 0.0001588
   3:  ---------------------
            s + 0.1141

Continuous-time transfer function.

Num =
    0.0240    0.0002
Den =
    1.0000    0.1141
Gc_t2_sym_term =
((3464915774230499*s)/144115188075855872 +
 5857948048047205/36893488147419103232)/(s +
 4112403835698463/36028797018963968)
Gc_t2_sym =
[ ((3464915774230499*s)/144115188075855872 +
 5857948048047205/36893488147419103232)/(s +
 4112403835698463/36028797018963968),

                     0,

     0]
[

             0, ((3464915774230499*s)/144115188075855872
 + 5857948048047205/36893488147419103232)/(s +
 4112403835698463/36028797018963968),

                   0]
[

       0,

               0, ((3464915774230499*s)/144115188075855872
 + 5857948048047205/36893488147419103232)/(s +
 4112403835698463/36028797018963968)]
Gc_t =
```

```
[ 7939/50000,           0,            0,
 ((3464915774230499*s)/1441151880758558720 +
 5857948048047205/36893488147419103232)/(s +
 4112403835698463/36028797018963968),

                     0,

    0]
[          0, 7939/50000,            0,

                             0, ((3464915774230499*s)/1441151880758558720
 + 5857948048047205/36893488147419103232)/(s +
 4112403835698463/36028797018963968),

                   0]
[          0,           0, 7939/50000,

                               0,

                           0, ((3464915774230499*s)/1441151880758558720
 + 5857948048047205/36893488147419103232)/(s +
 4112403835698463/36028797018963968)]
```

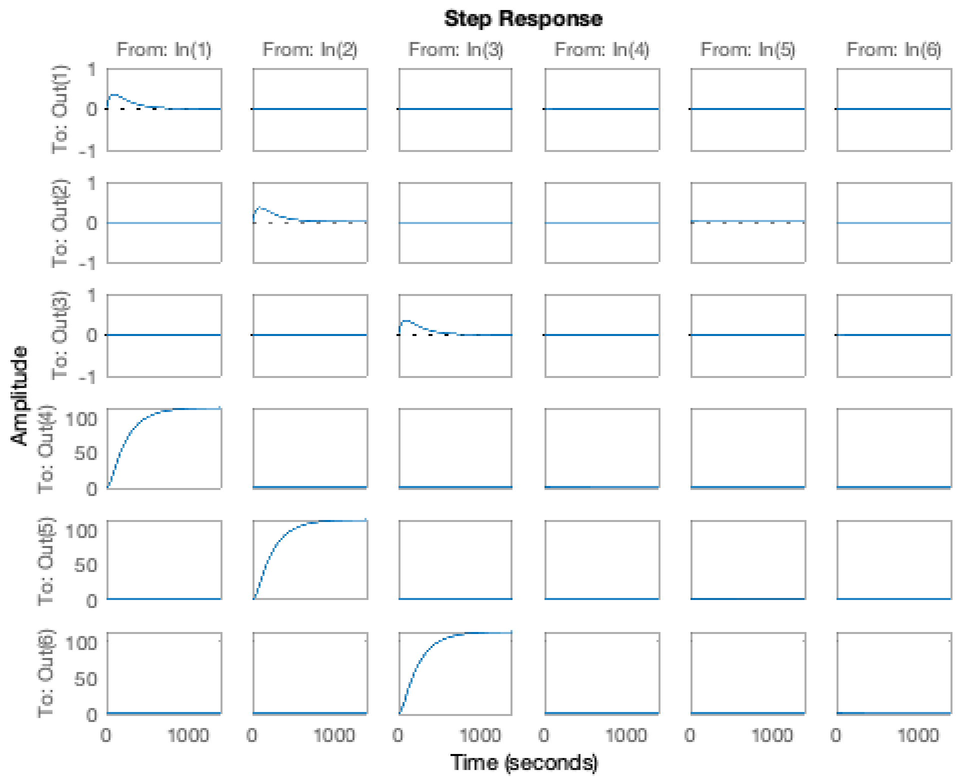


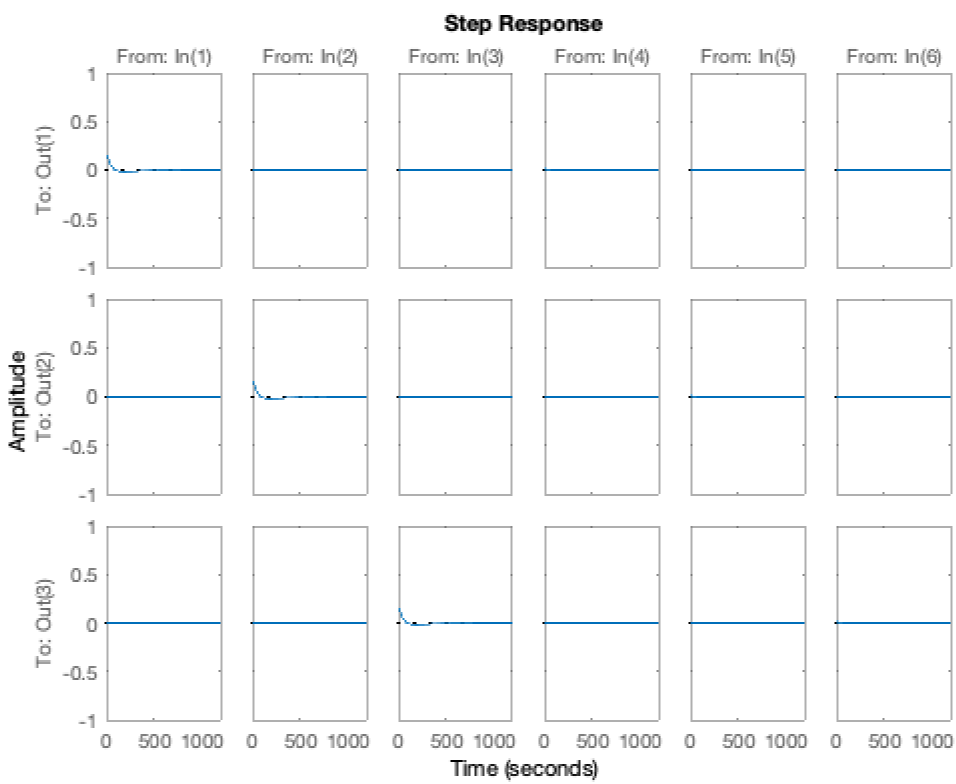
Simulation

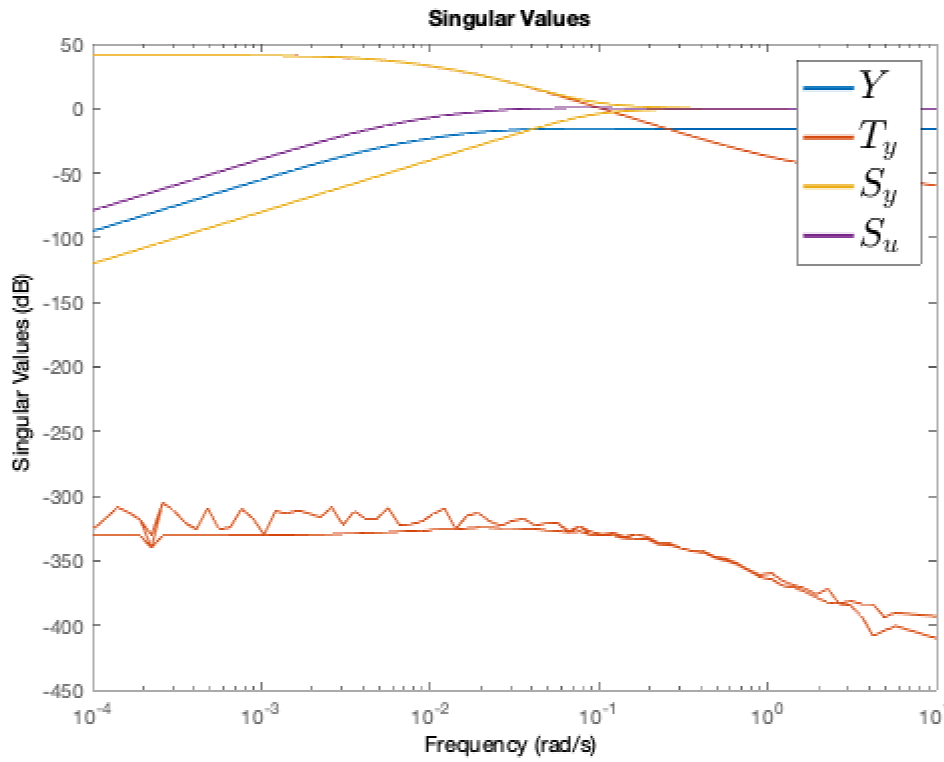
```
Gp = minreal([tf_full(1:3, 1:3); tf_full(7:9, 1:3)], 1e-05);
```

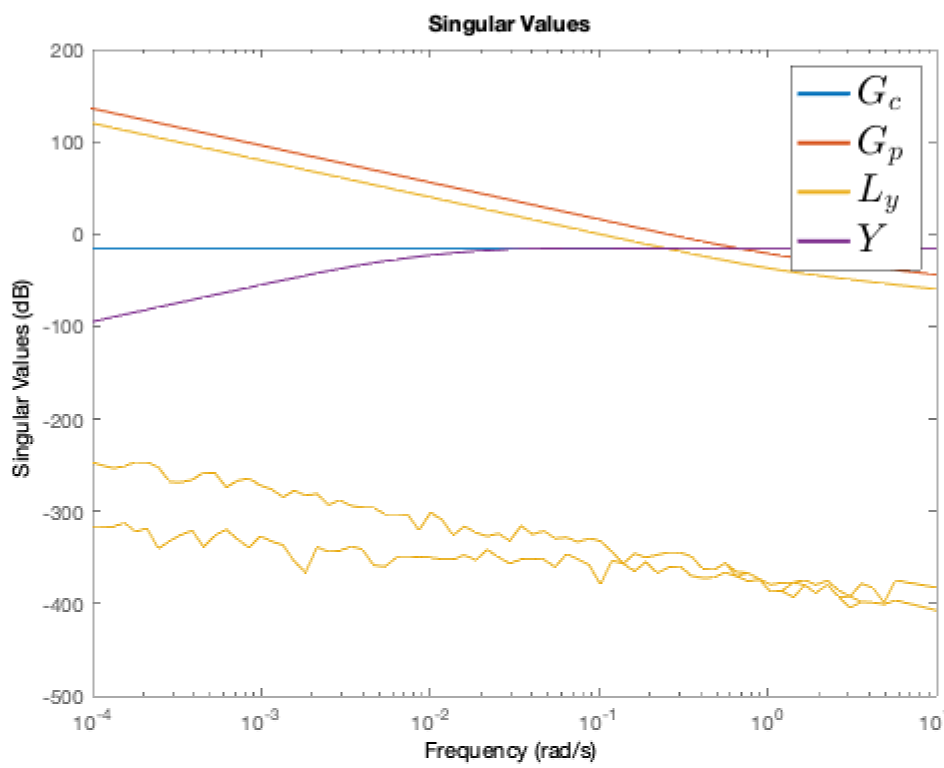
```
Gc = [Gc_t1 Gc_t2]
Lu = minreal(Gc * Gp, 1e-05);
Ly = minreal(Gp * Gc, 1e-05);
Y = minreal(inv(eye(3) + Lu) * Gc);
Ty = minreal(inv(eye(6) + Ly) * Ly);
Sy = minreal(inv(eye(6) + Ly), 1e-05);
Su = minreal(inv(eye(3) + Lu), 1e-05);

figure
step(Ty);

figure
step(Y);

figure
sigma(Y, Ty, Sy, Su)
[l, hObj] = legend('$Y
$', '$T_{y}$', '$S_{y}$', '$S_{u}$','Interpreter','latex','FontSize',
 20);
set(l,'string',{'$Y$', '$T_{y}$', '$S_{y}$', '$S_{u}$'});
hL = findobj(hObj,'type','line');
set(hL,'linewidth', 2);

figure
sigma(Gc, Gp, Ly, Y)
[l, hObj] = legend('$G_{c}$', '$G_{p}$', '$L_{y}$', '$Y
$','Interpreter','latex','FontSize', 20);
set(l,'string',{'$G_{c}$', '$G_{p}$', '$L_{y}$', '$Y$'});
hL = findobj(hObj,'type','line');
set(hL,'linewidth', 2);


Gc =

  From input 1 to output...
   1:  0.1588

   2:  0

   3:  0

  From input 2 to output...
   1:  0

   2:  0.1588

   3:  0

  From input 3 to output...
   1:  0

   2:  0

   3:  0.1588
```

```
  From input 4 to output...
       0.02404 s + 0.0001588
   1:  ---------------------
            s + 0.1141

   2:  0

   3:  0

  From input 5 to output...
   1:  0

       0.02404 s + 0.0001588
   2:  ---------------------
            s + 0.1141

   3:  0

  From input 6 to output...
   1:  0

   2:  0

       0.02404 s + 0.0001588
   3:  ---------------------
            s + 0.1141

Continuous-time transfer function.
```


Step Response
From: In(1)
From: In(2)
From: In(3)
From: In(4)
From: In(5)
From: In(6)
To: Out(1)
To: Out(2)
To: Out(3)
To: Out(4)
To: Out(5)
To: Out(6)
Amplitude
Time (seconds)


Step Response
From: In(1)
From: In(2)
From: In(3)
From: In(4)
From: In(5)
From: In(6)
To: Out(1)
To: Out(2)
To: Out(3)
Amplitude
Time (seconds)


Singular Values
50
0
-50
-100
-150
-200
-250
-300
-350
-400
-450
Singular Values (dB)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)
Y
T_y
S_y
S_u


Singular Values
200
100
0
-100
-200
-300
-400
-500
Singular Values (dB)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)
G_c
G_p
L_y
Y

Coordinate Feedback

```
% Cc = [zeros(6, 12)];
% Cc(1:6, 1:6) = eye(6);
%
% Dc = [zeros(6, 6)];
%
% sys_coord = ss(A, B, Cc, Dc);
%
% tf_coord = tf(sys_coord);
%
% syms s
%
% tf_coord_sym = simplify(Cc * inv(s * eye(12) - A) * B + Dc);
% pretty(tf_coord_sym)
%
% translation_coord = [tf_coord_sym(1:3, 1:3); tf_coord_sym(7:9,
 1:3)];
% pretty(translation_coord)
%
% attitude_coord = [tf_coord_sym(4:6, 4:6); tf_coord_sym(10:12, 4:6)];
% pretty(attitude_coord)
```

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