

---

# Table of Contents

.....	1
System .....	1
Section 1: Translation Controller Design -> Unstable Double-Pole at the Origin .....	3
Section 3: Translation Controller Design -> Unstable Double-Pole at the Origin .....	4
Simulation .....	7
Coordinate Feedback .....	12

% Astrobe Model

## System

```
% A Matrix

load Matrices/A_matrix.mat
A = A_matrix

% B Matrix: Stowed

load Matrices/B_stowed.mat
B = B_stowed

% % Full-State Feedback
%
% Cf = eye(12);
%
% Df = [zeros(12, 6)];
%
% sys_full = ss(A, B, Cf, Df);
%
% tf_full = tf(sys_full);
%
% syms s
%
% tf_full_sym = simplify(Cf * inv(s * eye(12) - A) * B + Df);
% pretty(tf_full_sym)

% Coordinate Feedback

Cc = [zeros(6, 12)];
Cc(1:6, 7:12) = eye(6);

Dc = [zeros(6, 6)];

sys_coord = ss(A, B, Cc, Dc);

tf_coord = tf(sys_coord);

syms s
```

---

```

tf_coord_sym = simplify(Cc * inv(s * eye(12) - A) * B + Dc);
pretty(tf_coord_sym)

% translation_coord_sym = [tf_coord_sym(4:6, 7:9); tf_coord_sym(7:9,
7:9)];
% pretty(translation_coord_sym)

% attitude_coord_sym = [tf_coord_sym(4:6, 4:6); tf_coord_sym(10:12,
4:6)];
% pretty(attitude_coord_sym)

```

```

A =
    0    0    0    0    0    0    0    0    0    0    0
    0
    0    0    0    0    0    0    0    0    0    0    0
    0
    0    0    0    0    0    0    0    0    0    0    0
    0
    0    0    0    0    0    0    0    0    0    0    0
    0
    0    0    0    0    0    0    0    0    0    0    0
    0
    0    0    0    0    0    0    0    0    0    0    0
    0
    1    0    0    0    0    0    0    0    0    0    0
    0
    0    1    0    0    0    0    0    0    0    0    0
    0
    0    0    1    0    0    0    0    0    0    0    0
    0
    0    0    0    1    0    0    0    0    0    0    0
    0
    0    0    0    0    1    0    0    0    0    0    0
    0
    0    0    0    0    0    1    0    0    0    0    0
    0
    0

```

```

B =
    0.0630    0    0    0    0    0
    0    0.0630    0    0    0    0
    0    0    0.0630    0    0    0
    0    0    0    5.4054    0    0
    0    0    0    0    4.9505    0
    0    0    0    0    0    5.3191
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0

```

```

/      500      \
/  -----,  0,  0,  0,  0,  0  /
/      2      /
/ 7939 s      /
/              /

```

$$\begin{array}{c}
 / \\
 / \quad 0, \quad \frac{500}{s^2}, \quad 0, \quad 0, \quad 0, \quad 0 \quad / \\
 / \quad \quad \quad 7939 \quad s \quad / \\
 / \\
 / \quad 0, \quad 0, \quad \frac{500}{s^2}, \quad 0, \quad 0, \quad 0 \quad / \\
 / \quad \quad \quad 7939 \quad s \quad / \\
 / \\
 / \quad 0, \quad 0, \quad 0, \quad \frac{200}{s^2}, \quad 0, \quad 0 \quad / \\
 / \quad \quad \quad \quad \quad 37 \quad s \quad / \\
 / \\
 / \quad 0, \quad 0, \quad 0, \quad 0, \quad \frac{500}{s^2}, \quad 0 \quad / \\
 / \quad \quad \quad \quad \quad \quad \quad 101 \quad s \quad / \\
 / \\
 / \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad \frac{250}{s^2} \quad / \\
 / \quad \quad \quad \quad \quad \quad \quad \quad \quad 47 \quad s \quad / \\
 \backslash
 \end{array}$$

## Section 1: Translation Controller Design -> Unstable Double-Pole at the Origin

```

% Run this section first to calculate 'tz' to ensure that the second
% interpolation condition is satisfied

% d^k(T)/ds^k|(s=0) = 0, where k = 1 (since there is a double unstable
% pole
% (multiplicity ap = 2) in the plant at s = 0; k = ap - 1) -> 2nd
% interpolation condition

C = 250/47; % Constant
Wn = 1; % Natural Frequency of the Control System
K = Wn^2/C; % Controller Gain
Z = 2^-0.5; % Damping Ratio
tp = 1/(10*Wn); % Time constant (of the included pole)
tpx = 0.5; % Time constant (of the pole included to drop Youla at high
% frequencies)

syms s tz

T_eqn = ((K*C)*(tz*s + 1)/((s^2 + 2*Z*Wn*s + Wn^2)*(tp*s + 1)*(tpx*s +
1)^2));
dT_eqn = diff(T_eqn,s);
eqn = subs(dT_eqn,s,0) == 0;
tz = double(solve(eqn,tz))

```

---

```
tz =  
    2.5142
```

## Section 3: Translation Controller Design -> Unstable Double-Pole at the Origin

```
% Youla Control Design  
  
s = tf('s');  
  
% Constants & Design Parameters  
% C = 500/7939; % Constant  
% Wn = 3.25; % Natural Frequency of the Control System  
% K = Wn^2/C; % Controller Gain  
% Z = 2^-0.5; % Damping Ratio  
% tp = 1/(10*Wn); % Time Constant of the added pole  
% tz = (4*2^(1/2))/13 + 2/65; % 100*2^(1/2) + 10;  
  
% Plant TF, 'Gp'  
Gp = minreal(C/s^2)  
  
% Chosen Youla Parameter, 'Y' -> Y(0) = 0  
Ys = minreal(((K*s^2)*(tz*s + 1)/((s^2 + 2*Z*Wn*s + Wn^2)*(tp*s + 1)*(tp*s + 1)^2)),1e-04)  
  
% Complementary Sensitivity TF, 'T' -> T(0) = 1 (1st interpolation condition)  
T = minreal((Ys*Gp),1e-04)  
  
% Sensitivity TF, 'S'  
S = minreal((1-T),1e-04)  
  
% Controller TF, 'Gc'  
Gc = minreal((Ys/S),1e-04)  
  
% Return Ratio, 'L'  
L = minreal((Gc*Gp),1e-04)  
  
GpS = minreal((Gp*S),1e-04)  
  
% Internal stability check  
Ys_stability = isstable(Ys)  
T_stability = isstable(T)  
S_stability = isstable(S)  
GpS_stability = isstable(GpS)  
  
M2 = 1/getPeakGain(S) % M2-margin  
BW = bandwidth(T) % Bandwidth of the closed-loop  
AE = getPeakGain(Ys) % Maximum actuator effort  
  
figure(1)  
bodemag(Ys, S, T);
```

---

```
legend('Ys','S','T');
```

```
Gp =
```

$$\frac{5.319}{s^2}$$

Continuous-time transfer function.

```
Ys =
```

$$\frac{18.91 s^3 + 7.52 s^2}{s^5 + 15.41 s^4 + 64.8 s^3 + 116.2 s^2 + 100.6 s + 40}$$

Continuous-time transfer function.

```
T =
```

$$\frac{100.6 s + 40}{s^5 + 15.41 s^4 + 64.8 s^3 + 116.2 s^2 + 100.6 s + 40}$$

Continuous-time transfer function.

```
S =
```

$$\frac{s^5 + 15.41 s^4 + 64.8 s^3 + 116.2 s^2 - 2.274e-13 s - 1.847e-13}{s^5 + 15.41 s^4 + 64.8 s^3 + 116.2 s^2 + 100.6 s + 40}$$

Continuous-time transfer function.

```
Gc =
```

$$\frac{18.91 s + 7.52}{s^3 + 15.41 s^2 + 64.8 s + 116.2}$$

Continuous-time transfer function.

```
L =
```

$$\frac{100.6 s + 40}{s^5 + 15.41 s^4 + 64.8 s^3 + 116.2 s^2}$$

---

Continuous-time transfer function.

GpS =

$$\frac{5.319 s^3 + 81.99 s^2 + 344.7 s + 618.2}{s^5 + 15.41 s^4 + 64.8 s^3 + 116.2 s^2 + 100.6 s + 40}$$

Continuous-time transfer function.

Ys\_stability =

logical

1

T\_stability =

logical

1

S\_stability =

logical

1

GpS\_stability =

logical

1

M2 =

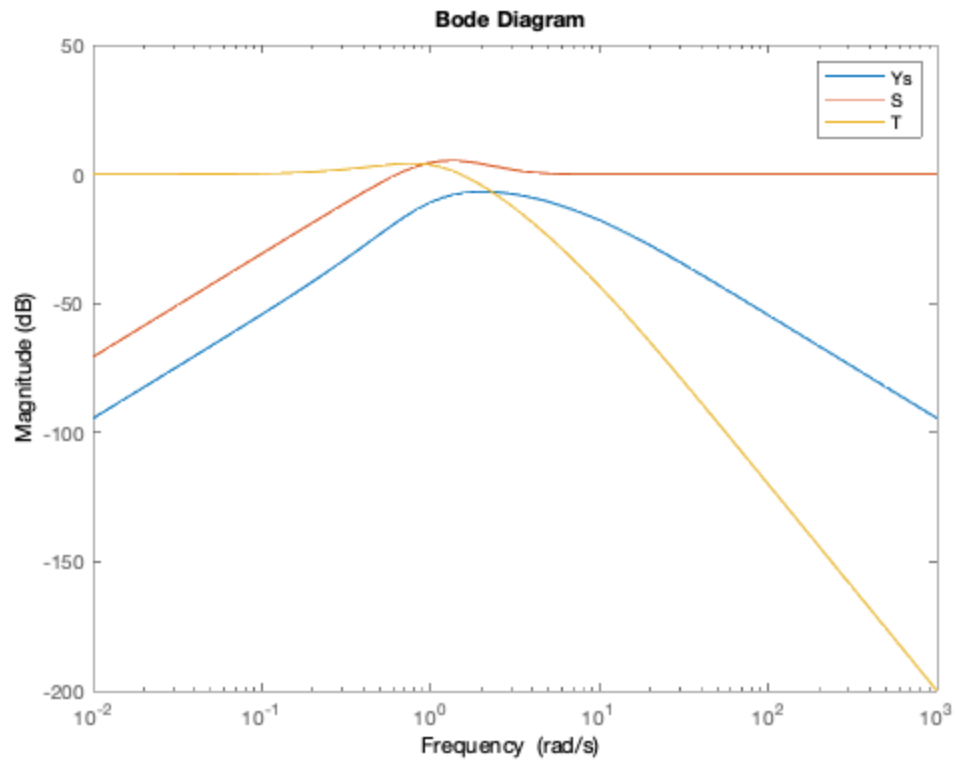
0.5469

BW =

1.8487

AE =

0.4587



## Simulation

```
load Gcs.mat

Gp = minreal(tf_coord, 1e-04);
Gc = minreal([Gc1 0 0 0 0 0; 0 Gc2 0 0 0 0; 0 0 Gc3 0 0 0; 0 0 0 Gc4 0
0; 0 0 0 0 Gc5 0; 0 0 0 0 0 Gc6], 1e-04);
Lu = minreal(Gc * Gp, 1e-04);
Ly = minreal(Gp * Gc, 1e-04);
Y = minreal(inv(eye(6) + Lu) * Gc);
Ty = minreal(inv(eye(6) + Ly) * Ly);
Sy = minreal(inv(eye(6) + Ly), 1e-04);
Su = minreal(inv(eye(6) + Lu), 1e-04);

figure
step(Ty);

figure
step(Y);

figure
sigma(Y, Ty, Sy, Su)
[1, hObj] = legend('$Y$', '$T_{y}$', '$S_{y}$', '$S_{u}$', 'Interpreter', 'latex', 'FontSize',
12);
```

---

```

set(l,'string',{ '$Y$', '$T_{y}$', '$S_{y}$', '$S_{u}$' });
hL = findobj(hObj,'type','line');
set(hL,'linewidth', 2);

figure
sigma(Gc, Gp, Ly, Y)
[l, hObj] = legend('$G_{c}$', '$G_{p}$', '$L_{y}$', '$Y$', 'Interpreter','latex','FontSize', 12);
set(l,'string',{ '$G_{c}$', '$G_{p}$', '$L_{y}$', '$Y$' });
hL = findobj(hObj,'type','line');
set(hL,'linewidth', 2);

figure
sigma(Gc, Gp, Y)
[l, hObj] = legend('$G_{c}$', '$G_{p}$', '$Y$', 'Interpreter','latex','FontSize', 12);
set(l,'string',{ '$G_{c}$', '$G_{p}$', '$Y$' });
hL = findobj(hObj,'type','line');
set(hL,'linewidth', 2);

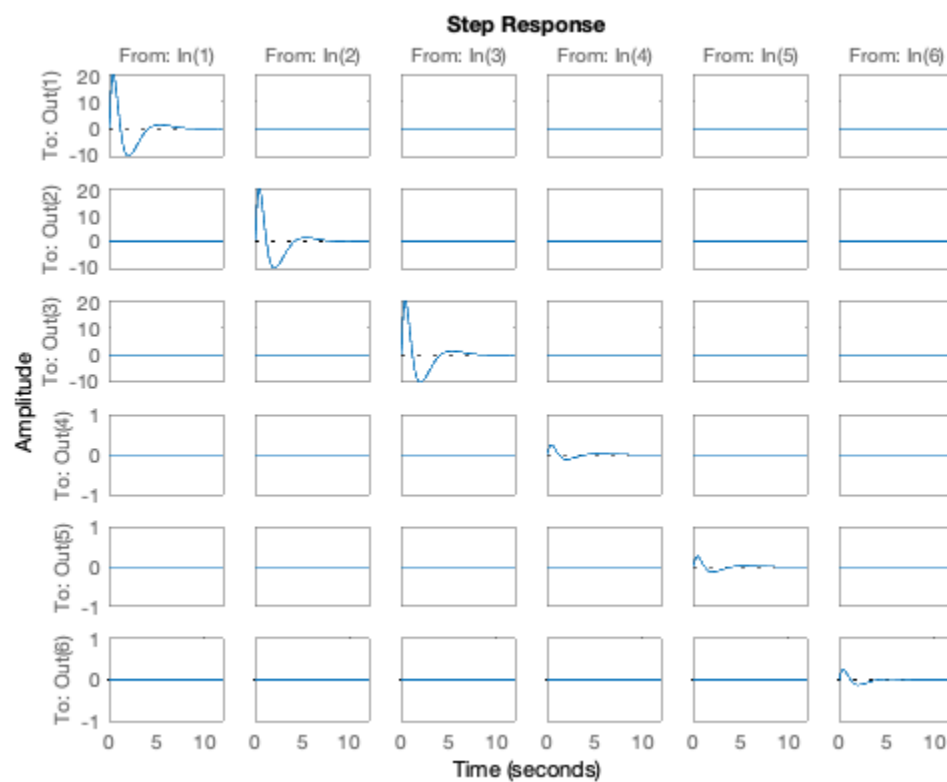
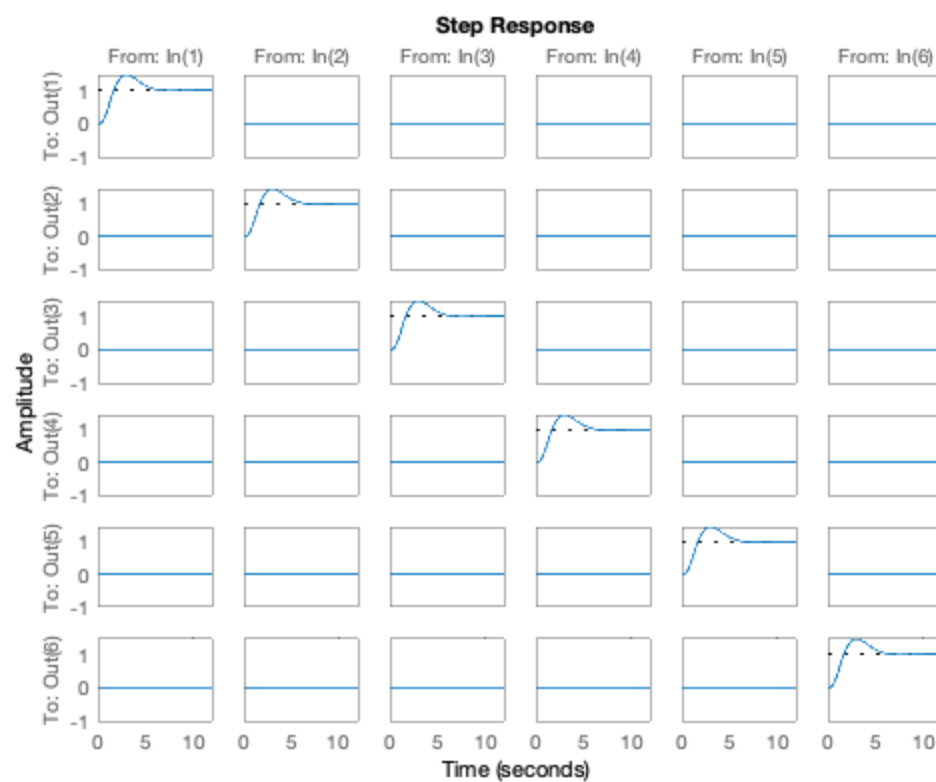
figure
sigma(Ly, Sy, Ty)
[l, hObj] =
    legend('$L_{y}$', '$S_{y}$', '$T_{y}$', 'Interpreter','latex','FontSize',
    12);
set(l,'string',{ '$L_{y}$', '$S_{y}$', '$T_{y}$' });
hL = findobj(hObj,'type','line');
set(hL,'linewidth', 2);

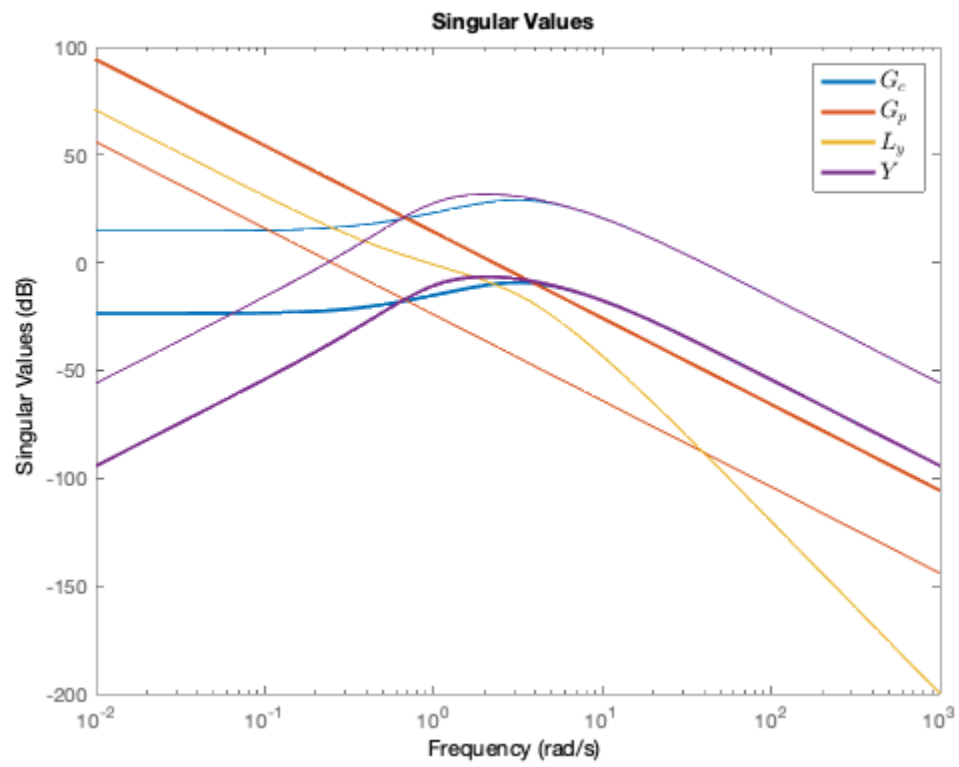
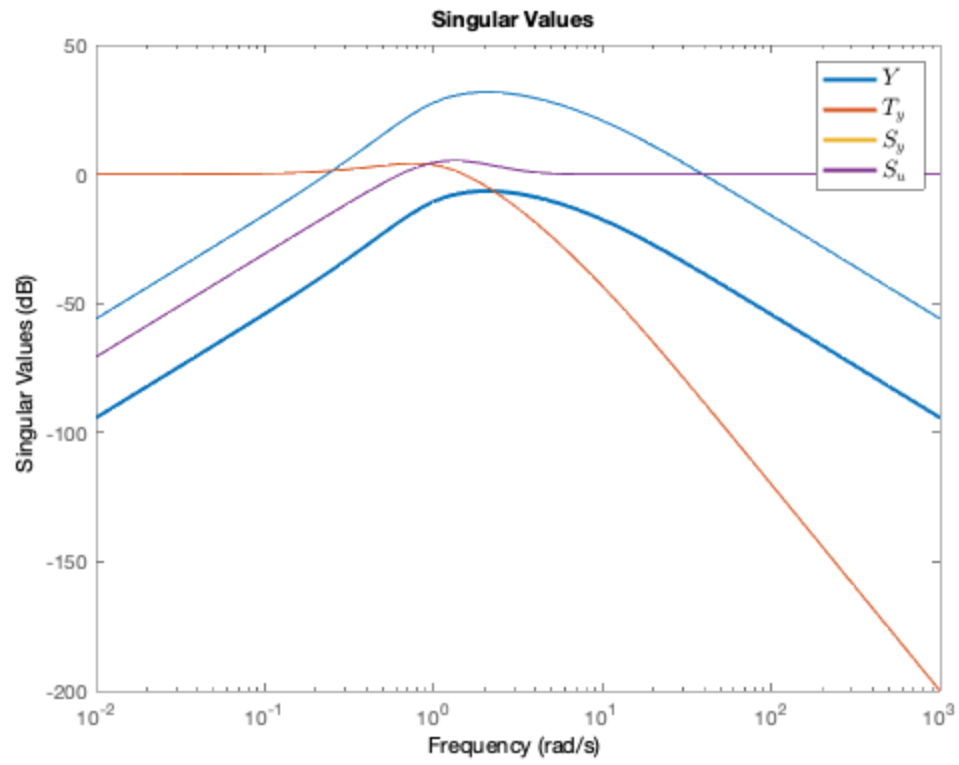
figure
sigma(Sy, Su)
[l, hObj] =
    legend('$S_{y}$', '$S_{u}$', 'Interpreter','latex','FontSize', 12);
set(l,'string',{ '$S_{y}$', '$S_{u}$' });
hL = findobj(hObj,'type','line');
set(hL,'linewidth', 2);

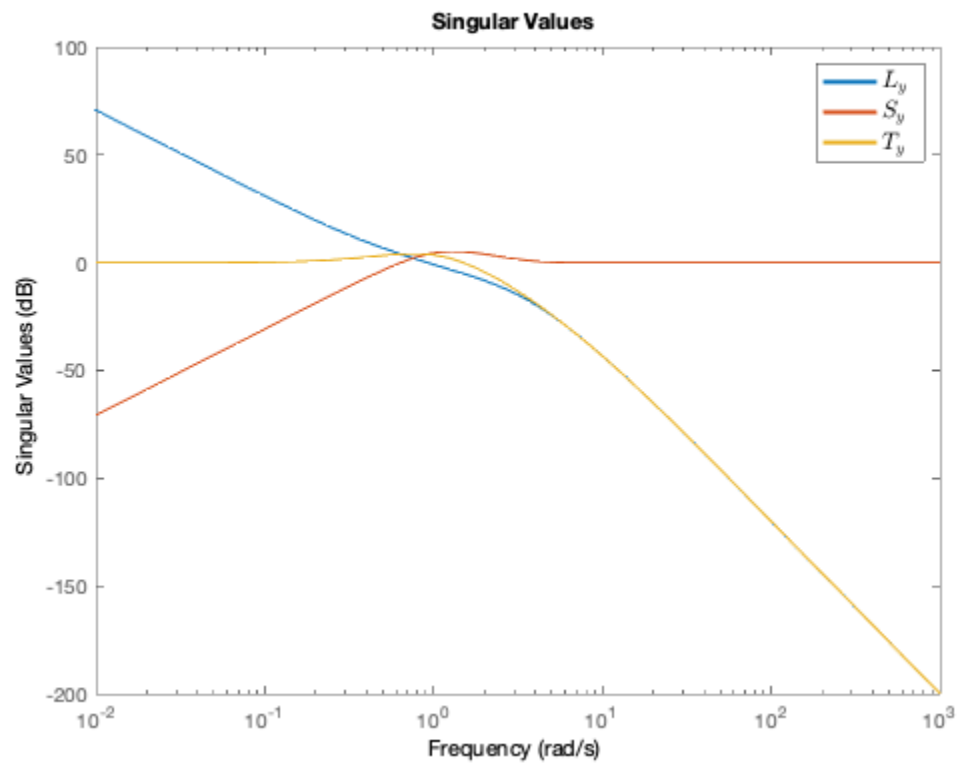
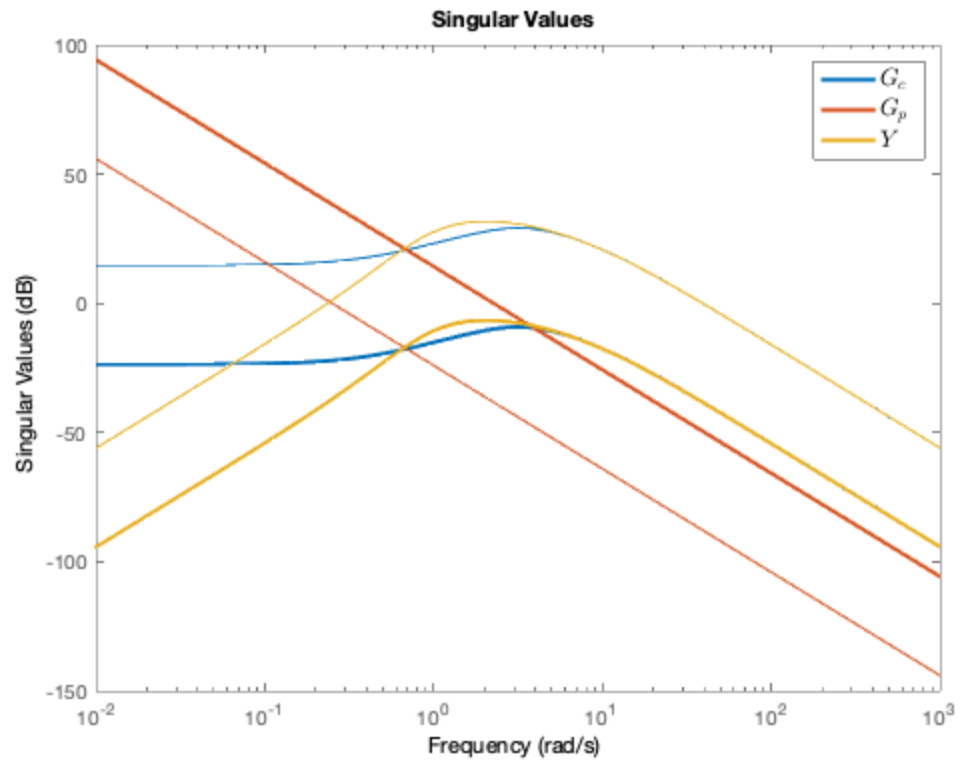
```

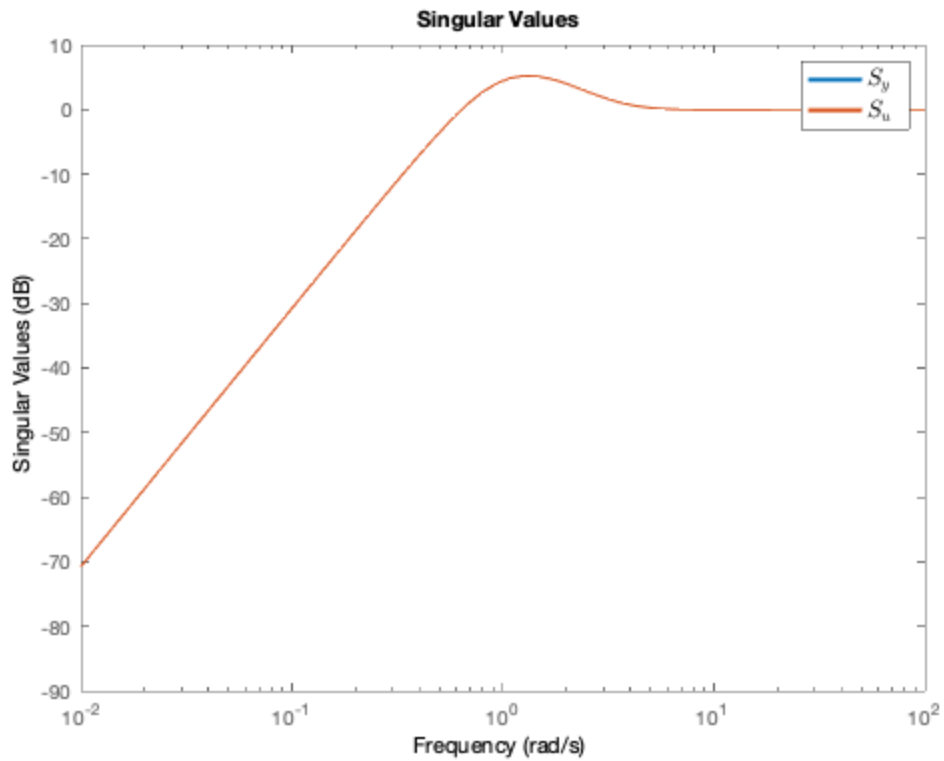
---











## Coordinate Feedback

```
% Cc = [zeros(6, 12)];
% Cc(1:6, 1:6) = eye(6);
%
% Dc = [zeros(6, 6)];
%
% sys_coord = ss(A, B, Cc, Dc);
%
% tf_coord = tf(sys_coord);
%
% syms s
%
% tf_coord_sym = simplify(Cc * inv(s * eye(12) - A) * B + Dc);
% pretty(tf_coord_sym)
%
% translation_coord = [tf_coord_sym(1:3, 1:3); tf_coord_sym(7:9,
% 1:3)];
% pretty(translation_coord)
%
% attitude_coord = [tf_coord_sym(4:6, 4:6); tf_coord_sym(10:12, 4:6)];
% pretty(attitude_coord)
```

*Published with MATLAB® R2019b*