UNIT NO. 01

# MATHEMATICAL LOGIC AND SET THEORY

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## # Proposition :-

Proposit$^n$ is a declarative sentence which is either true or false but not both.

eg :-

1) Delhi is a capital of India – True (1)
2) Nagpur is a capital of Maharashtra – false (0)
3) Good Morning – Not a proposition.
4) $x+5=8$ – Not a proposition ($x$ is not given)
5) $2+3=8$ – False

## # Propositional variables :-

Proposition are represented by variab-les which are denoted by small letters.

## # Propositional logic :-

1) Compound Proposition

logic Operation.

1) Negation ($\sim$ / $-$ / $\neg$)
2) Conjunction ($\wedge$)
3) Disjunction ($\vee$)
4) Exclusive OR ($\oplus$)
5) Implication / Conditional Operator ($\rightarrow$ / $\Rightarrow$)
6) Bi implication / Biconditional ($\leftrightarrow$ / $\Leftrightarrow$)

### 1) Negation ($\neg$) :-

e.g. P : Akshay's PC runs linux.

$\neg P$ : Akshay's PC does not run linux

| P | $\neg P$ |
|---|---|
| T | F |
| F | T |

2) Conjunction ($\wedge$) : AND / but.

| P q | $P \wedge q$ |
|---|---|
| F F | F |
| T F | F |
| F T | F |
| T T | T |

p : I bought the lottery ticket

q : I won the jackpot.

$\therefore$ $p \wedge q$ : I bought the lottery ticket & I won the jackpot

3) Disjunction ($\vee$) : OR

| P q | $P \vee q$ |
|---|---|
| F F | F |
| T F | T |
| F T | T |
| T T | T |

p = Students who have taken 'calculus' can attend this class.

q = Students who have taken computer can attend this class.

$p \vee q$ = Students who have taken 'calculus' can attend this class or Students who have taken computer can attend this class.

4) Ex - dis conjunction :-

The exclusive OR is true when exactly one of P & q are true.

| P | q | $P \oplus q$ |
|---|---|---|
| F | F | F |
| T | F | T |
| F | T | T |
| T | T | F |

5) Conditional / implication ($\rightarrow$ / $\Rightarrow$) :-

The conditional state $p \rightarrow q$ is false, when p is True & q is false & true otherwise.

| p | q | p→q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

eg : P = If I am detected.
q = I will lower the tax.
$p \rightarrow q$ = If I am elected, I will lower the taxe

Following are the ways to express conditional

1) If p then q
2) if p, q
3) p is sufficient for q
4) q if p
5) q when p
6) p implies q
7) p only if q.

6) Biconditional / Biimplication ($\leftrightarrow$ / $\Leftrightarrow$) :-

The statement $p \leftrightarrow q$ when p&q have the same truth values.

| p | q | p↔q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

eg: p = you can go by train.
q = you by a ticket

$p \leftrightarrow q$ : you can go by train if you buy a ticket.

Following ways are to represent $p \leftrightarrow q$ :

1) P is necessary & sufficient condition.
2) if p then q / if q then p
3) p if + q

Ex –

Que: let P & q be the proposition
p : It is below freezing
q : It is snowing.
Write these propositions using P & q & logic connectives.

→

1) $p \wedge q$ : It is below freezing & snowing.

2) It is below freezing but not snowing
$\Rightarrow p \wedge \sim q$

3) It is not belowing freezing and not snowing
$\Rightarrow \sim p \wedge \sim q$

4) It is either snowing ...

5) If it is below freezing it is also snowing.

$\Rightarrow p \to q$.

6) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing

$\Rightarrow (p \vee q) \wedge (\sim q \to p)$

Que 2 Let p & q are propositions.

p: Swimming at new jersey store is allowed

q: Sharks have been spotted near the show

express each of the following comp. propositions ans english stat.

→ 1) $\sim q$: ~~Sharking~~ sharks have not been spotted near the store

2) $p \wedge q$: Swimming at New jersey store is allowed and sharks have been spotted near the store.

3) $\sim p \vee q$: Swimming at New jersey store is not allowed or sharks have been spotted

6) $\sim p \rightarrow \sim q$ : If swimming at New jersey Store Store is not allowed then sharks have been spotted near the show.

7) $p \rightarrow \sim q$ : Swimming at new jersey show is allowed if sharks have not been spotted near the show.

8) $\sim p \wedge (p \vee q)$ : Swimming at new jersey show is not allow & either swimming at new jersey store is allowed at sharks have not been spotted near the store.

Que: Determine wheather each of the conditional state are true & false.

1) If $1+1=2$ then $2+2=3$
$\Rightarrow$ $T \rightarrow F = F$

2) If $1+1=3$ then $2+2=4$
$\Rightarrow$ $F \rightarrow T = T$

3) If $1+1=3$ then $2+2=3$
$\Rightarrow$ $F \rightarrow F = F$

4) If monkey can fly then $1+1=3$
$\Rightarrow$ $F \rightarrow F = T$

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**Que** For each of the statements state what the stat. means if the local connective VR is on inclusive or verses exclusive OR.

1) To take discrete mathematics you must have taken calculus or cource in computer science.
→ Inclusive OR ( any one state condo).

2) When you buy a new car you will get Rs 250 as a cash-back or a 2% car loan.
→ Exclusive OR (not get both things)

3) Dinner for 2 includes 2 item from column A or 3 items from column B.
→ Exclusive OR (only one thing)

4) School is closed if more than 2 feet of snow falls or if the wind is two high.
→ Inclusive OR ( if both are there school is closed).

Q. Construct the truth table for each of the following compound preposition.

1) $(p \vee q) \rightarrow (p \oplus q)$
2) $(p \oplus q) \rightarrow (p \cap q)$
3) $(p \vee q) \rightarrow \oplus (p \cap q)$
4) $(p \leftrightarrow q) \oplus (\sim p \rightarrow q)$
5) $(p \leftrightarrow q) \oplus (\sim p \leftrightarrow \sim r)$
6) $(p \oplus q) \rightarrow (p \oplus \cap q)$

ex// Construct the T.T

$(p \leftrightarrow q) \leftrightarrow (r \rightarrow s)$ $(2^4 = 16)$

| p | q | r | s | $p \leftrightarrow q$ | $r \leftrightarrow s$ | $A \leftrightarrow B$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | T | F | T | F | F |
| T | T | F | T | T | F | F |
| T | T | F | F | T | T | T |
| T | F | T | T | F | T | F |
| T | F | T | F | F | F | T |
| T | F | F | T | F | F | T |
| T | F | F | F | F | T | F |
| F | T | T | T | F | T | F |
| F | T | T | F | F | F | T |
| F | T | F | T | F | F | T |
| F | T | F | F | F | T | F |
| F | F | T | T | T | T | T |
| F | F | T | F | T | F | F |
| F | F | F | T | T | F | F |
| F | F | F | F | T | T | T |

—x—

Que. find bitwise OR bitwise AND bitwise XOR

① 1011110 , 0100001

| OR | AND | XOR |
|---|---|---|
| 1011110 | 1011110 | 1011110 |
| 0100001 | 0100001 | 0100001 |
| 1111111 | 0000000 | 1111111 |

Q. Construct the T.T $((p \leftrightarrow q) \rightarrow r) \rightarrow S$.

| $p$ | $q$ | $r$ | $S$ | $p \leftrightarrow q$ | $A \rightarrow r$ | $B \oplus S$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | T | F | T | T | F |
| T | T | F | T | T | F | T |
| T | T | F | F | T | F | T |
| T | F | T | T | F | T | T |
| T | F | T | F | F | T | F |
| T | F | F | T | F | T | T |
| T | F | F | F | F | T | F |
| F | T | T | T | T | T | T |
| F | T | T | F | T | T | F |
| F | T | F | T | T | F | T |
| F | T | F | F | T | F | T |
| F | F | T | T | T | T | F |
| F | F | T | F | T | T | F |
| F | F | F | T | T | F | T |
| F | F | F | F | T | F | T |

—x—

② 0001110001 , 1001001000

OR XOR

# Proof Methods :-

Direct

Indirect
① Contropositive
② Contradiction.

eg :- If $n$ is an odd integer. then $n^2$ is also an odd integer.

① **Contropositive** :-
If $p \to q$ is logically equivalent to its contropositive
$\neg q \to \neg p$.

② **Contradiction** :-
If $p \to q$ then consider $p$ & $\neg q$ are true & prove that $\neg q$ is false.

# Principle of mathematical induction

1. To prove $P(n)$ is true for all the +ve integers '$n$' where $p(n)$ is a propositional function.

① **Basic Step** : We verify basis step by considering the value of $n=0$ or 1 or 2

② **Inductive Step** : We show that the conditional statement
$p(k) \to p(k+1)$ is true for all positive integers $k$.

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To compute the inductive step we assume that $p(k)$ is true & under that assumption we prove that $p(k+1)$ must be true.

Que: Prove the following if 'n' is a tve integer.

(1) $1+2+\cdots+n = \frac{n(n+1)}{2}$

**Basis Step**: for $n=1$

$$p(k) : \cancel{11} \quad 1 = 1\frac{(1+1)}{2} = 1 = 1$$

Inductive Step:

$$p(k) : 1+2+\cdots+k = \frac{k(k+1)}{2} \quad \text{--- (1)}$$

L.H.S

for $p(k+1) : 1+2+\cdots+k+k+1 =$

$$: \frac{k(k+1)}{2} + k+1 \quad \cdots\cdots \quad (\text{from (1)})$$

$$: \frac{k(k+1)+2(k+1)}{2}$$

$$p(k+1) = \frac{(k+2)(k+1)}{2}$$

R.H.S : $\frac{(k+1)(k+2)}{2}$

$$\boxed{\therefore \text{L.H.S} = \text{R.H.S}}$$

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2) $1^3+2^3+3^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

→ Basic step for $p(1)$

$p(1): 1 = \left(\frac{1(1+1)}{2}\right)^2$

$\therefore 1 = 1$

Industive Step :

for k

$p(k) = 1^3+2^3+3^3+\dots+k^3 = \left(\frac{k(k+1)}{2}\right)^2$ —— ①

for (k+1)

$L.H.S = 1^3+2^3+3^3+\dots+k^3+(k+1)^3$

$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^2$ ---- from ①

$= \frac{\left(\frac{k(k+1)}{2}\right)^2 + 8(k+1)^2}{8}$

$L.H.S = \frac{(k+1)^2(k^3+8)}{4} = \left(\frac{(k+1)(k+2)}{4}\right)^3$

$R.H.S = \left(\frac{(k+1)(k+2)}{2}\right)^2$

$L.H.S = R.H.S = \left(\frac{(k+1)(k+2)}{2}\right)^2$

$p(a)$ : $1 = 2^{0+1} - 1$

$1 = 1.$

Inductive step

for $p(k)$

$$1+2+2^2+\dots+2^k = 2^{k+1}-1$$

for $p(k+1)$

$$L.H.S = 1+2+2^2+\dots+2^k+2^{(k+1)}$$
$$= (2^{(k+1)}-1)+2^{k+1}$$
$$= 2^{k+1}+2^{k+1}-1$$
$$= 2^{k+2}-1$$

$$= 2^{1k}-2^1+2^k-2-1$$
$$= 2^k(2^k)-1$$
$$= 2k(2+2)-1$$
$$= 2^2-2^k-1$$
$$= 2^{k+2}-1$$

$$R.H.S = 2^{(k+1+1)}-1$$
$$= 2^{k+2}-1$$
$$= 2^{k+2}-1$$

$$L.H.S = R.H.S$$

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4) $1+3+5+\dots+(2n-1)=n^2$

$\rightarrow$ Basic step for $p(1)$

$p(1) \therefore 1 = \left(\frac{1(1+1)}{2}\right)^2$

$\therefore 1 = 1.$

Inductive Step:

for $k_1$

$P(k) = 1^3+2^3+3^3+\dots+k^3 = \left(\frac{k(k+1)^2}{2}\right)$ — ①

for $(k+1)$

$L.H.S = 1^3+2^3+3^3+\dots+k^3+(k+1)^3$

$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^2$ — from ①.

$= \left(\frac{k(k+1)^2 + 8(k+1)^2}{8}\right)$

$L.H.S = \frac{(k+1)^2(k^3+8)}{4} = \left(\frac{(k+1)(k+2)}{4}\right)^3$

$R.H.S = \left(\frac{(k+1)(k+2)}{2}\right)^2$

$L.H.S = R.H.S = \left(\frac{(k+1)(k+2)}{2}\right)^2$

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8) $1+2+2^2+\ldots\ldots+2^n = 2^{n+1}-1$

→ Basic step for -

$p(0)$ : $1 = 2^{0+1}$

5) $1+3+5+\cdots+(2n-1) = n^2$

Basis step for -

$p(1) = 1.$

Inductive step

for $p(k)$

$$1+3+5+\cdots+(2k-1) = k^2 \quad \text{— (1)}$$

for $p(k+1)$

$$L.H.S = 1+3+5+\cdots+(2k-1)+(2(k+1)-1)$$

~~for p~~ $= 1+3+5+\cdots+2k-1+2k+2-1$

$1+3+5+\cdots+k^2+2k+1$

$= k^2+2k+1$

$= (k+1)^2$

$R.H.S = n^2$

$= (k+1)^2$

$L.H.S = R.H.S.$

6) $p(n) = 1^2+2^2+\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

→ Basic step :

$P(1) = 1 = 1$

Inductive Step :-

$$p(k) = 1^2+2^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

$p(k+1)$

$$L.H.S = 1^2+2^2+\ldots k^2+(k+1)^2.$$

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$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)\left(k(2k+1) + 6(k+1)\right)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$R.H.S = \frac{(k+1)(k+2)\left(2(k+1)+1\right)}{6}$$

$$= \frac{(k+1)(k+2)(2k+2+1)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$R.H.S = \frac{(k+2)(2(k+1)+1)(2(k+1)+3)}{3}$$

$$= \frac{(k+2)(2k+3)(2k+5)}{3}$$

$$1+3+3^2+\dots+3^{n+1} = \frac{(3^n-1)}{2}$$

Basis $P(1)$

$$\therefore 1 = 1$$

Inductive step $= 1+3+3^2+\dots+3^{k-1} = \frac{(3^k-1)}{2}$

$P(k+1)$

$$L.H.S = 1+3+3^2+\dots+3^{k-1}+3^{k+1-1}\ 3k$$

$$= \left(\frac{3^k-1}{2}\right)+3^k$$

$$= \frac{3^k-1+(2)(3^k)}{2} = \frac{3^{k+1}-1}{2}$$

$$R.H.S = \frac{3^{k+1}-1}{2}.$$

# SETS

Set builder & Roster / Tabular Method.

$A = \{1, 2, 3, 4, 5\}$

$A = \{x \mid x$ is a prime no less than $100\}$

1. Finite sets : eg : $A = \{1, 2, 3, 4, 5\}$
2. Infinite sets : $A = \{$ set of natural no. $\}$
3. Singleton set : single element eg. $A = \{6\}$
4. Empty set eg. $A = \{\ \}$ , $A = \phi$
   but $A = \{\phi\}$ is not a empty set.
5. Equal sets = $A\{1, 2, 3, 4\}$
   $B\{1, 2, 3, 4, 5\}$
6. Equivalent sets eg $A = \{1, 2, 3, 4, 5\}$
   $B = \{6, 7, 8, 9, 10\}$ – no. of element same.

sets
- empty (countable)
- Non-empty
  - Finite (countable)
  - Infinite
    - countable
    - uncountable.

# Countable Infinite Set :-

If there is ~~no~~ one to one mapping between elements of set & natural no.'s then it is countable infinite set.

# Uncountable Infinite set.

If there cannot be one to one mapping between element of set & natural no's then it is unforgetable infinite sets.

# Power Set :-

let A be the infinite set then, the set of all subsets of A is called as power set of A.

e.g : $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

If $\boxed{|A| = N \text{ then } |P(A)| = n^2}$

1. $A \subseteq P(A) \rightarrow$ False ( P(A) does not contain element of A it contain {A} )
2. $A \in P(A) \longrightarrow$ True.
3. $\phi \subseteq P(A) \longrightarrow$ True.
4. $\phi \in P(A) \longrightarrow$ True.
5. $\phi \subseteq A \longrightarrow$ True
6. $\phi \in A \longrightarrow$ False.

eg 1) $A = \{5, \{6\}, \{7\}\}$, $P(A) = ?$

$\rightarrow$ $P(A) = \{\phi, \{5\}, \{\{6\}\}, \{\{7\}\}, \{5\{6\}\}, \{5, \{7\}\}, \{\{6\}, \{7\}\}, \{5, \{6\}, \{7\}\}\}$.

1. $\{5, \{6\}\} \in P(A) \longrightarrow$ True.
2. $\{5, \{6\}\} \subseteq P(A) \longrightarrow$ False
3. $\phi \subseteq P(A) \rightarrow$ True.

# Application of propositions :-

eg - You are not allowed to drive vehicle if your age is less than 18 years or you have no age proof.

→ p : You are not allowed to drive vehicle.
q : If your age is less than 18 years.
r : you have no age proof.

$\Rightarrow (\sim r \vee q) \rightarrow p$.

# Consistent Propositions :-

1. The diagnostic message is stored in the buffer or it is retransmitted.
2. The diagnostic message is not stored in the buffer.
3. The diagnostic message is stored in the buffer then it is retransmitted.

Here.
p = The dignostic message is stored in the buffer.
q = it is retransmitted.

① $p \vee q$ ② $\sim p$ ③ $p \rightarrow q$.

| $p$ | $q$ | $p \vee q$ | $\sim p$ | $p \rightarrow q$ | |
|---|---|---|---|---|---|
| T | T | T | F | T | |
| T | F | T | F | F | |
| F | T | T | T | T | → consistent |
| F | F | F | T | T | |

ex.2. The diagnostic message is retransmitted ~q.

| p | q | p∨q | ~p | p→q | ~q |
|---|---|---|---|---|---|
| T | T | T | F | T | F |
| T | F | T | F | F | T |
| F | T | T | T | T | F |
| F | F | F | T | T | T |

above eg is inconsistent (is none row is all true).

ex- 1. The system is in multiuser state if & only if it is operating normally.

2. If the system is operating normally, then the kernel is functioning.

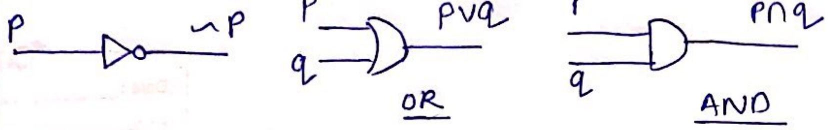
3. The kernel is not functioning or the system is in interrupt mode.

4. If the system is not in multiuser state then it is in interrupt mode.

5. The system is not in interrupt mode.

if p : The system is in multiuser state.
q : The system is operating normally.
r : The kernel is functioning.
s : The system is in interrupt mode.

# exp1)






1.) $p \leftrightarrow q$ 2.) $q \rightarrow r$ 3) $\sim r \vee s$ 4) $\sim p \rightarrow s$ 5) $\sim s$.

if, $s = F$, so 5) is true

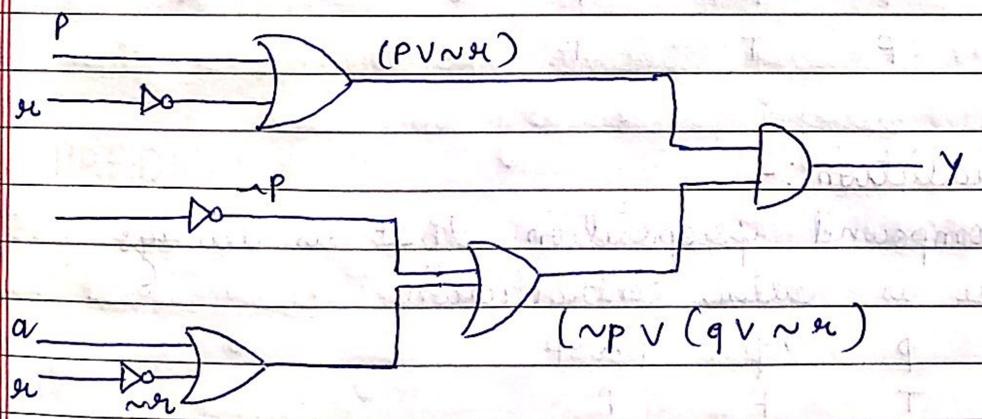
4) if we want 4th st. true then $\sim p \rightarrow F$
then $p = T$.

If we want 3th true then $\sim \vee F$ $r = F$.
if we want 2nd true then $q \rightarrow r = q \rightarrow F$
then $q = F$

But if $p = T$ & $q = F$ 1st state is not true
Hence, above ex is inconsistent.

# logical Circuit :-

exp 1) Built a digital circuit that produces the O/P $(p \vee \sim r) \wedge (\sim p \vee) (q \vee \sim r)$ when given I/P bits p q.



# Propositional Equivalance :-

The compound proposition P & q are called logically equivalent if $p \leftrightarrow q$ is a technology ($\equiv$)

Classification of Compound Proposition are done in 3 =

1. Tautology : (All are true)
2. Contradiction : All are false
3. Contingency : Neither tautology nor contradiction.

① Tautology :-
A compound proposition always true called as tautology

eg

| $p$ | $\sim p$ | $p \vee \sim p$ |
|---|---|---|
| T | F | T |
| F | T | T |

② Contradiction :-
A Compound proposition that is always false is called contradiction.

eg

| $p$ | $\sim p$ | $p \wedge \sim p$ |
|---|---|---|
| T | F | F |
| F | T | F |

③ Contigency :- A Compound proposition that is neither tautology nor contradiction.

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Q. Show that $\sim(p \vee q)$ & $(\sim p \wedge \sim q)$ are logically equation

| p | q | ~p | ~q | p∨q | ~(p∨q) | (~p∨~q) |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | T | F | F |
| F | T | T | F | T | F | F |
| F | F | T | T | F | T | T |

Here column a & b are same

$\therefore \sim(p \vee q) \equiv (\sim p \wedge \sim q)$

Q. Prove that :- 1) De-morgan's law 2) Commutative law 3) Associative law 4) Distributive law 5) Dempotent law.

Q. $(p \rightarrow q) \cap (p \rightarrow r) \equiv p \rightarrow (q \cap r)$, p.t :

Q. $\sim(p \leftrightarrow q) \equiv p \leftrightarrow \sim q$.

# PREDICATES & QUANTIFIERS :-

QUANTIFIERS.

1. Universal Quantifiers -

$\forall \Rightarrow$ for all, for every

$\forall x P(x)$ : for all $x$ P(x) or for every $x(P(x))$

2. Existential Quantifiers -

$\exists \Rightarrow$ there exist

- "There exit an element $x$ in the domain such that $p(x)$
- $\exists x P(x)$ : for the existential Qualification $p(x)$

Example 1) $p(x)$ : $x$ spends more than 5 hrs every week day in class.

Domain : all Students.

# Explain the following Quanlifications in data english.

1. $\exists x P(x)$ : There is a student who spends more than 5 hrs every week day in class

2. $\forall x P(x)$ : Every student spends more than 5 hrs every week day in class.

3. $\exists x \sim P(x)$ : There is a student who does not spend more than 5 hrs every week day in class.

4. $\forall x \sim p(x)$ : Every Student spends less than or equal to 5 hrs every week day in cls are no students spend more than 5 hours every week day in class.

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Q. Translate the statement in english.

where $C(x) \longrightarrow$ " $x$ is a comedian "
$f(x) \longrightarrow$ " $x$ is funny "
Domain : consists of all people.

1. $\forall x\,(C(x) \rightarrow f(x))$ : for every $x$, if $x$ is a comedian then $x$ is funny.
$\Rightarrow$ " Every comedian is funny ".

2. $\forall x\,(C(x) \wedge f(x))$ : " Every comedian are funny " or " Every person is a funny comedian

3. $\exists x\,(C(x) \rightarrow f(x))$ : " There exist a person such that if she/he is a comedian then he/she is funny.

4. $\exists x\,(C(x) \wedge f(x))$ : " There exists a funny comedian "
<u>or</u> " Some comedians are funny "
or " Some funny people or comedians.

Q. $P(x)$ : $x$ can speak Russian.
$Q(x)$ : $x$ know the computer Engineering language C++.
Domain : All student at your school.

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2. There is a student at your school who can speak Russian but doesn't know C++

→ $\exists x\,(p(x) \cap \sim Q(x))$

3. Every student at your school either can speak Russian or knows C++.

→ $\forall x\,(p(x) \vee Q(x))$

4. No student at your school can speak Russian or know C++.

→ $\forall x\,(\sim p(x) \vee \sim Q(x))$

Q. $p(x) - x = x^2$ if the domain consist of int what are these truth values?

1. $p(0) \longrightarrow T$
2. $p(1) \longrightarrow T$
3. $p(2) \longrightarrow$ false
4. $p(7) \longrightarrow T$
5. $\exists x P(x) \longrightarrow T$
6. $\forall x P(x) \longrightarrow$ false

Q. Translate following specialization in english.

E(P) : Printer 'P' is out of service.

B(P) : Printer 'P' is busy.

L(j) : print job 'j' is lost.

Q(j) : print job 'j' is queued.

1. $\exists p\,(E(P) \wedge B(p)) \longrightarrow \exists j\,L(j))$

→ If there exist a printer 'p' which is out of service & busy then print job 'j' is lost.

$\forall p \; B(p) \longrightarrow \exists j \, Q(j)$

If every printer is busy then there job 'j' is queued.

$\exists j \, ( Q(j) \wedge L(j)) \rightarrow \exists p \; F(p).$

If there is a job 'j' both lost & queued then some printers are out of service.

$(\forall p \; B(p) \wedge \forall j \; Q(p)) \longrightarrow \exists j \, L(j)$

If all printer are busy & all jobs are queued then some jobs are lost.

# Negation :-

1. $\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$
2. $\neg(\exists x\, P(x)) \equiv \forall x\, \neg P(x)$

Q. "There is an honest politician".
- $H(x)$ : "$x$ is honest".
- "There is an honest politician" is represented by $\exists x\, H(x)$.

Negation :- All politicians are not honest.

$\neg(\exists x\, P(x)) \equiv \forall x\, \neg p(x)$

1. Proof : $\sim(\forall x\, P(x)) \equiv \exists x\, \sim p(x)$

$L.H.S = \forall x\, P(x)$

$= p(x_1) \wedge p(x_2) \wedge \cdots \wedge p(x_n)$

$\sim(\forall x\, p(x)) = \sim[p(x) \wedge p(x_2) \wedge \cdots \wedge p(x_n)]$

$= \sim p(x_1) \vee \sim p(x_2) \vee \cdots \vee \sim p(x_n)$

... De-morgan's law.

$\sim(\forall x\, P(x)) = \exists x\, \sim p(x)$

2. $\sim(\exists x\, P(x)) \equiv \forall x \sim P(x))$

→ $\exists x\, P(x) = P(x_1) \vee p(x_2) \vee \cdots \vee p(x_n)$

$L.H.S = \sim(\exists x\, p(x))$

$= \sim[p(x_1) \vee p(x_2) \vee \cdots \vee p(x_n)]$

$= \sim p(x_1) \wedge \sim p(x_2) \wedge \cdots \wedge \sim p(x_n)$

.... De-morgan's law.

$= \forall x\, \sim p(x)$

$= R.H.S$

Q. Some old dogs can learn new tricks.

→ given = $\exists x\, P(x)$

Negation = $\sim(\exists x P(x)) \equiv \forall x \sim P(x)$

∴ All old dogs cannot learn new tricks.

2. No rabbit know Calculus

→ let $C(x)$ = $x$ know Calculus

Domain = Rabbit

given stat = $\sim \exists x\, (C(x))$

Negation = $\exists x\, (C(x))$

= There exists a rabbit who know calculus

3. Every bird can fly

→ let $f(x)$ = $x$ can fly

domain = Birds

given = $\forall x\, f(x)$

Negation = $\exists x \sim F(x)$

= There exists a bird who cannot fly.

Q. Express the negation of following & express in english

1. Some drivers do not obey the spend limit.
→ $f(x) = x$ obey the speed limit.
Domain = drivers.
Given = $\sim \exists x \, f(x)$
Negation = $\exists x \, f(x)$
= There exist some drivers who obey the speed limit

2. All swedish movies are serious
→ $f(x)$ : $x$ are serious.
Domain : Swedish movies
Given : $\forall x \, f(x)$
Negation : $\exists x \sim f(a)$
= There exist a swedish movies which is not serials.

3. No one can keep a secret.
→ $f(x)$ : $x$ can keep a secret.
Domain : No one
Given :
Negation : There exists some who can keep a secret.

4. There is someone in this class does not have a good attitude.
→ Neg : Everyone in their class have a good attitude.

# Nested Quantifiers :-

- Domain variables $x$ & $y$ consists : all real no
- The statement $\forall x \, \forall y \, (x+y = y+x)$
  - Says that $x+y = y+x$ for all real number $x$ & $y$.
- $\forall x \, \exists y \, (x+y = 0)$
  - Says that for every real number $x$ there is a real no $y$ such that $x+y=0$.

1. $Q(x,y)$
   $x$ has sent an email to $y$
   Domain $x$ & $y$ - All student in your class.

2. $\exists x \, \exists y \, Q(x,y)$
   → There are some student in class who have sent email to some student.

3. $\exists x \, \forall y \, Q(x,y)$
   → There exist some student in class for all students in class who have sent email.

4. $\forall x \, \forall y \, Q(x,y)$
   → All students in class have sent an email to all student in your class.

Date: __/__/__
Page: ____

# Rules of Inference :-

Rules of inference provides a template for constructing valid arguments.

1. <u>Modus pones</u> :-

$p \to q$ is true & $P$ is also true then $q$ has to be true

$$\begin{array}{ll} p \to q & T \\ \underline{p} & T \\ q & \text{has to be True} \end{array}$$

~~or $[(p \to q) \wedge \sim q] \to \sim q$~~

2. <u>Modus Tollens</u> :-

$$\begin{array}{ll} p \to q & T \\ \underline{\sim q} & T \\ \sim p & \text{has to be True} \end{array}$$

or $[(p \to q) \wedge \sim q] \to \sim p$.

3. <u>Hypothetical Syllogism</u> :-

$$\begin{array}{ll} p \to q & T \\ \underline{q \to r} & T \\ p \to r & \text{has to be True} \end{array}$$

or $[(p \to q) \cap (q \wedge r)] \to (p \to$

4. <u>Disjunctive syllogism</u> :-

$$\begin{array}{ll} p \vee q & T \\ \underline{\sim q} & T \\ q & T \end{array}$$

or $[(p \vee q) \wedge \sim p] \to q$.

5. Addition :

$$\begin{array}{ll} \underline{p} & T \\ p \vee q & T \end{array}$$

or $p \to p \vee q$

$$\begin{array}{ll} p & T \\ q & T \\ \hline p \wedge q & T \end{array}$$

or $[(p) \wedge (q)] \rightarrow (p \wedge q)$

Resolution -

$$\begin{array}{ll} p \vee q & T \\ \sim p \vee r & T \\ \hline q \vee r & T \end{array}$$

"Raja work hard" , "If Raja work hard, that he is a dully boy" & "if Raja is a dull boy then he will not get the job."
We need to conclude that "Raja will not get job".

H = Raja works hard
D = Raja is a dull boy
J = Raja will get job.

1) Raja works hard - H
2) If Raja works hard, then he is a dull boy

3) If Raja is a dull boy then he will not the job.

$D \rightarrow \sim j$.

Use 2 premiis at a time

If we solve first D & third premise $(D \rightarrow \sim j)$ we will get $\sim j$ (By modus pone)

$\sim j$ : Raja will not get the job.

2. "If it does not rain or it is not foggy the sailing race will be held & life saving demo will go on."
"If the sailing race is held, then the trophy will be awarded."
"The trophy was not awarded."

Conclusion : It rained.

→ R - It is raining
G - It is foggy
H - sailing race will be held
J - life saving demo will go on.
K - trophy will be awarded.

Premises -

1. $(\sim r \vee \sim g) \longrightarrow (h \vee j)$
2. $h \rightarrow k$
3. $\sim k$.

UNIT NO. 02

# RELATIONS AND FUNCTIONS

## # Relations :-

– Any possible subset of a cartesian product is a relation.

let $A = \{a, b, c\}$ & $B = \{1, 2\}$

then

$$A \times B = \{(a,1), (a,2), (b,1), (b,2), (1,1), (1,2)\}$$

then

Relations $R_1 = \{(a,1), (c,2)\}$

$R_2 = \{(b,1), (a,2)\}$

## # Representing relations :-

1) Matrix (rows)

2) Digraph. (~~row~~ column)

exp $A = \{1, 2, 3\}$ , $B = \{1, 2\}$

$R = \{(2,1), (3,1), (3,2)\}$

$\Rightarrow$ Matrix rep$^n$ =

$$\begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \end{array}$$

exp : $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3, b_4, b_5\}$

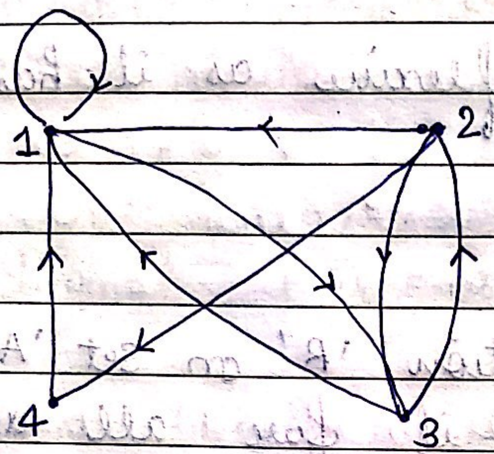
$$m_R = \begin{array}{c c} & \begin{array}{ccccc} b_1 & b_2 & b_3 & b_4 & b_5 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

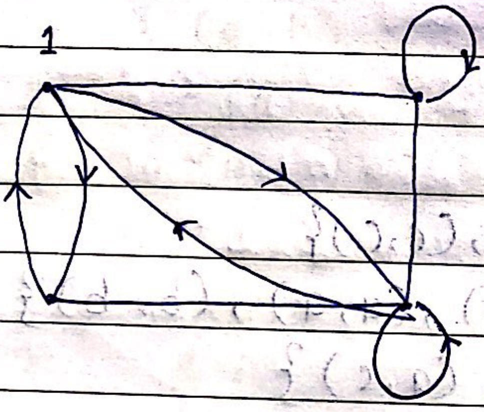
$\Rightarrow$ ordered pairs = $\{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_5), (a_3,$

## Diagraph :-

$R_1 = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\}$

$A = \{1, 2, 3, 4\}$




$\Rightarrow D_1 = \{(1,3), (1,4), (2,2), (2,1), (2,3), (3,3), (3,1), (3,4), (4,1)\}$

## Properties of relation :-

1. Reflexive.
2. Irreflexive
3. Symmetric
4. Anti - Symmetric.
5. Asymmetric :
6. Transitive :

Reflexive :-

3. $R_3 = \phi$

4. $R_4 = \{(a,a), (b,b), (c,c), (a,b), (b,c)\}$

5. $R_5 = \{(a,b), (b,a), (a,a), (b,b)\}$

→ $R_1, R_2$ & $R_4$ are reflexive as it has $(a,a)$ $(b,b)$ & $(c,c)$.

2. Irreflexive :-

A relation 'R' on set 'A' is said to be irreflexive if for all $a \in R$, $(a,a) \notin R$

e.g - $A = \{a,b,c\}$

1. $R_1 = \phi$
2. $R_2 = A \times A$
3. $R_3 = \{(a,a), (b,b), (c,c)\}$
4. $R_4 = \{(a,b), (b,a), (a,a), (b,b)\}$
5. $R_5 = \{(a,b), (b,c), (a,c)\}$

Here. $R_1$ & $R_5$ are irreflexive.

3. Symmetric :-

A relation 'R' on set 'A' is said to be symmetric $\forall (a,b) \in R$ if $(b,a) \in R$ then $(a,b) \in R$.

eg :- $A = \{a,b,c\}$

1. $\{(a,b), (b,a)\}$
2. $\{(b,c), (c,b), (b,b), (c,c)\}$
3. $\{(a,a), (b,b), (c,c)\}$
4. $\phi$
5. $A \times A$
6. $\{(a,b), (b,c), (a,c)\}$
7. $\{(a,b), (a,a), (a,c)\}$

Symmetric are : $R_1, R_2, R_3, R_4, R_5$.

4. Anti - Symmetric :-

(symmetric not allow but diagonal allowed)

A relation 'R' on set 'A' is said to be anti - symmetric $\forall a, b \in R$, $(a,b) \in R$, $(b,a) \in R$ then $a = b$.

e.g : $A = \{a, b, c\}$

diagonal pair = $(a,a), (b,b), (c,c)$

1. $\{(a,b), (b,c), (a,c)\}$
2. $\{(a,b), (a,a), (b,b)\}$
3. $\{(a,a), (b,b), (c,c)\}$
4. $\{(a,b), (b,a), (b,c), (c,c)\}$
5. $A \times A$
6. $\{(a,b), (b,c), (a,c), (c,a), (a,a), (c,c)\}$
7. $\phi$.

Anti Symmetric are $R_1, R_2, R_3, R_4$.

5. Asymmetric :-

(symmetric & diagonal both are not allowed)

A relation 'R' on set 'A' is asymmetric if $(\forall a, b \in R)$ if $(a,b) \in R$ & $(b,a) \notin R$

eg : $A = \{a, b, c\}$

1. $\{(a,b), (b,c), (a,c)\}$
2. $\{(a,b), (a,a), (b,b)\}$
3. $\{(a,a), (b,b), (c,c)\}$
4. $\{(a,b), (b,a), (b,c), (c,c)\}$
5. $A \times A$

2. $R_3 = \{(a,b)\} \longrightarrow$ Transitive

3. $R_4 = \{(a,b), (c,b)\} \longrightarrow$ Transitive

eg. $A = \{1,2,3,4\}$

1. $R_1 = \{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$
2. $R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,2)\}$
3. $R_3 = \{(2,4), (4,2)\}$
4. $R_4 = \{(1,2), (2,3), (3,4)\}$
5. $R_5 = \{(1,1), (2,2), (3,3), (4,4)\}$
6. $R_6 = \{(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)\}$

$\longrightarrow$ Reflexive $= R_5, R_2$

Irreflexive $= R_3, R_4, R_6$

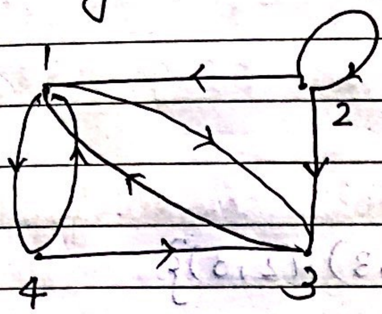
Symmetric $= R_3, R_5, R_2$

Anti - Symmetric $= R_4, R_5$

Assymetric = $R_4$

Transitive = $R_2, R_4, R_1$.

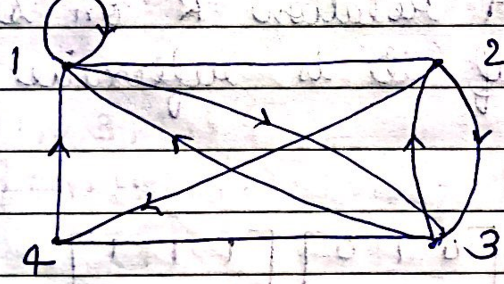
# Diagraph :-



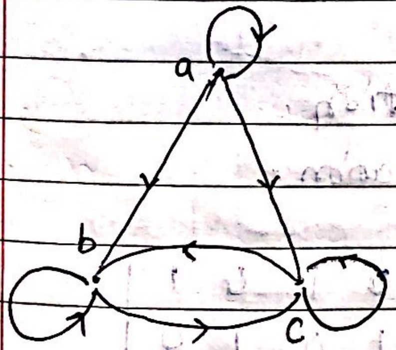
$A = \{1,2,3,4\}$

$R_1 = \{(1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,3), (4,1), (4,3)\}$

Q. $R = \{(1,1), (1,3), (2,1), (2,4), (3,1), (3,2), (4,1)\}$ on set $A = \{1,2,3,4\}$ Draw diagraph.



Q. Write ordered Pair :-



ordered Pair :

$\{(a,a), (a,b), (b,b), (b,c), (c,a), (c,c), (c,b)\}$

$$\begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

anti-symmetric & transitive :-

$A = \{1,2,3\}$

$R_1 = \emptyset$

$R_2 = \{(1,1), (2,2), (3,3)\}$

$R_3 = \{(1,1), (2,2), (3,3), (1,3), (2,3)\}$

$R_4 = A \times A$.

# Equivalence Relation :-

A relation 'R' on set 'A' said to be equivalent if it is reflexive, symmetric & transitive.

1]

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

→ $R_1$ = equivalence relation.

$R_2$ = not POR nor equation.

$R_3$ = not POR nor equation.

2]

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

→ $R_1$ = not POR nor eqn

$R_2$ = None

$R_3$ = None

$A = \{1,2,3,4\}$

$R_1 = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

$R_2 = \{(1,1), (1,4), (2,2), (3,3), (4,1)\}$

$R_3 = \{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1),$
$(3,2), (3,4), (4,1), (4,2), (4,3)\}$

$R_4 = \{(2,4), (3,1), (3,2), (3,4)\}$

## Equivalence class & Partition :-

To find the equivalence class & partitions relation. The equivalence class of '$x$' is denote by $|x|$.

$|x| = \{y \mid y \in A \text{ and } (x,y) \in R\}$

1) $A = \{1,2,3,4,5\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1),$
$(4,5), (5,4)\}$

→ Equivalence ~~partioning~~ class means the element with which an element is attached.

Here, R is equivalance relation.

$|1| = \{1,2\}$ ] $P1$
$|2| = \{2,1\}$

$|3| = \{3\} \rightarrow P2$

$|4| = \{4,5\}$ ] $P3$
$|5| = \{4,5\}$

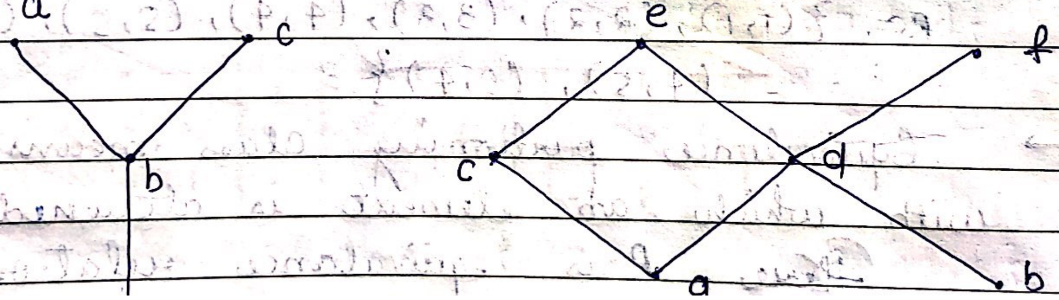
~~Partition~~ Partition means group having same class

$\therefore P1 = \{1,2\}$ $P2 = \{3\}$ $P3 = \{4,5\}$

2. Minimal:-

Element not related to it.

ex

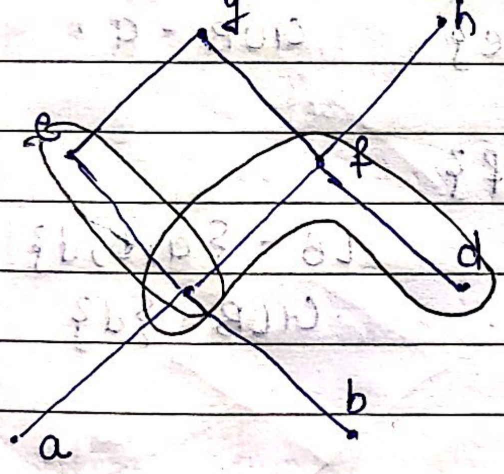


maximal = {a, c, l, m, n}

minimal = {g, i, k, b, e}

**UB**:

If [A, R] - post $a \in A$ is called upper bound of subset B ($B \subset A$) if $xRa$, $\forall x \in B$.




A & B are given.

A = {a, b, c, d, e, f, g, h}

B = {e, c}

UB = {e, g};

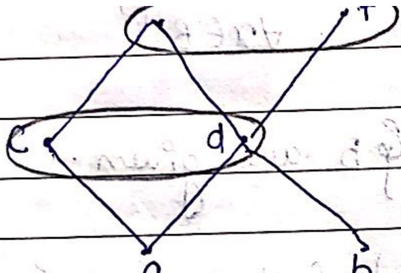
LB = {a, b, c};

B is subset

B = {c, f, d}

UB {f, g, h}  L.B = $\phi$

1]




B = {c, d}

UB = {e}

LUB = {e}

LB = {a}

GUB = a.

B = {e, f}

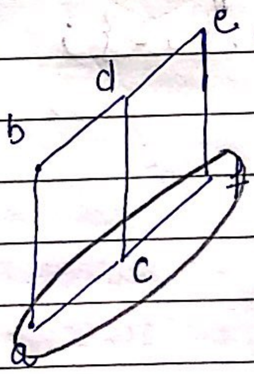
UB = $\phi$

LUB = $\phi$

LB = {a, b, d}

GLB = {d}

2]




B = {a, c, f}

UB = {f, e}

LUB = f

LB = a

GLB = a

B = {d, c}

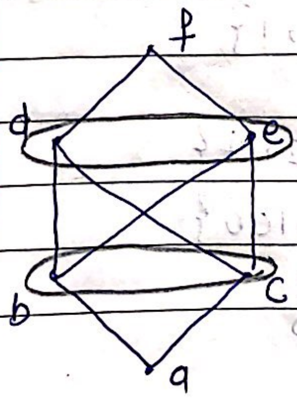
UB = e, d

LUB = d

LB = a, c

GLB = c.

3]




B = {d, e}

UB = f

LB = {b, c, a}

LUB = f

GLB = $\phi$

B = {b, c}

UB = {d, e, f}

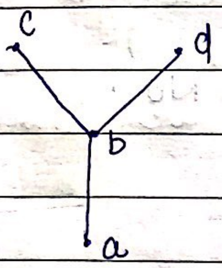
LUB = $\phi$

LB = {a}

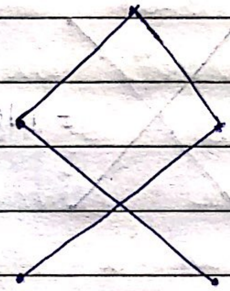
GLB = {a}

## Meet Semi lattice :-

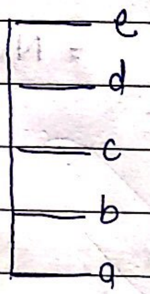
In a poset if GLB exist for every pair of element then that poset is called meet semi lattice.



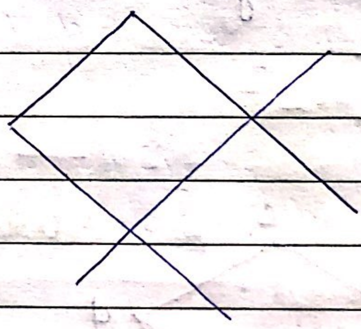

a = meet semi lattice.



= No.




= yes



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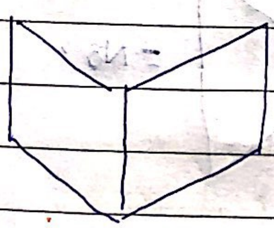
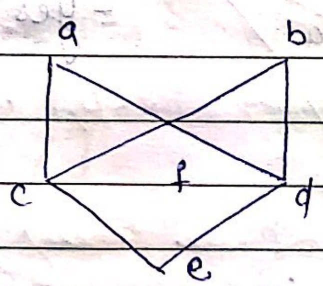
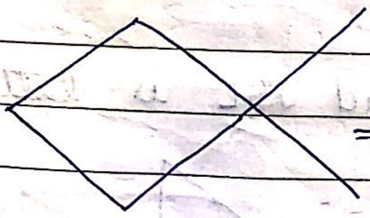


= No.



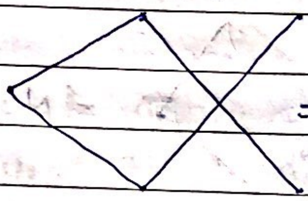

No
here to point are related.



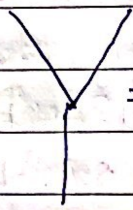
= No.

# Join Semi lattice :-

In a poset if LUB present for every element then its join semi lattice.



= NO



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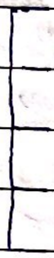

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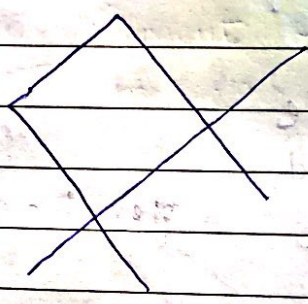

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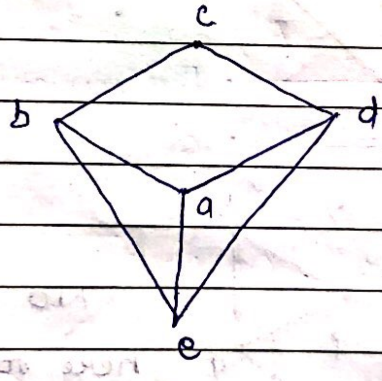
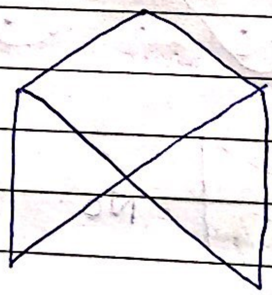



= yes.

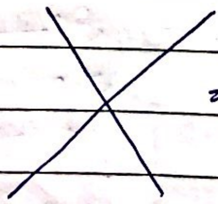


= No.

# lattice :-

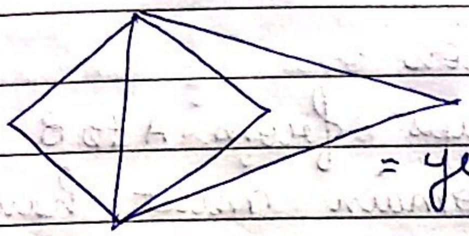
Both are present MSL and JSL is lattice.

= yes.



= No.

= No.



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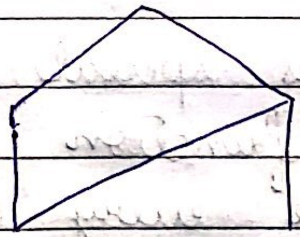

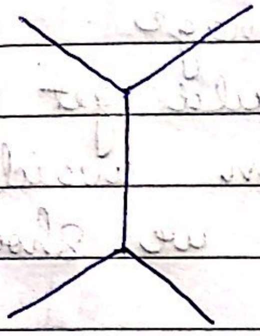

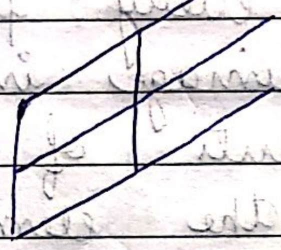

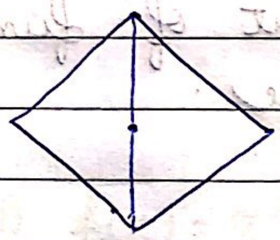

OR// questions are asked like·

ex// $\{ D_{110} , / ]$

1. $\{1,2,5,10,11,22,55,110\}$
2. $[ \{1,2,3,6 , 5,18 \} , / ]$
3. $[ \{2,3,4,6,12\} , / ]$

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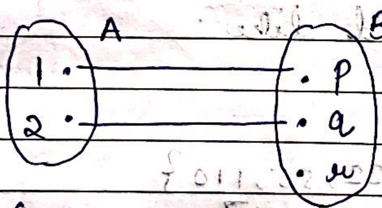
1. It is special type of relation.
2. A function F which maps from A to B ($F: A \to B$) where every element of domain must have a mapping.
3. Domain is also called as pre-image.
4. For every pre-image we should get unique pre-image in Relation we can avoid some elements of a but in funⁿ we should use all the elements of set A.
5. Range is a subset of Co-domain.
Range is nothing but the element of funcⁿ & range also called as image.

→ $A = \{1, 2\}$ $B = \{p, q, r\}$ →

$F: A \longrightarrow B$.

Domain

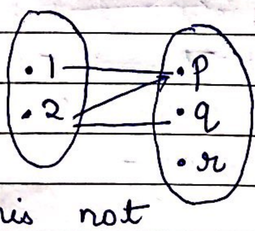



Co-domain

$F: (1, p), (2, q)$.

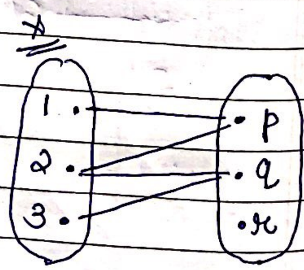
Domain have only one unique co-domain / image not.

Domain



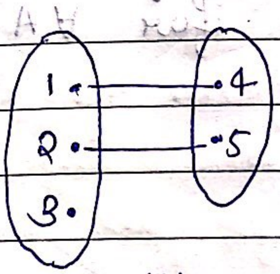

co-domain

this not allow.

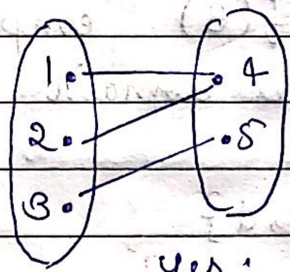



this allow -

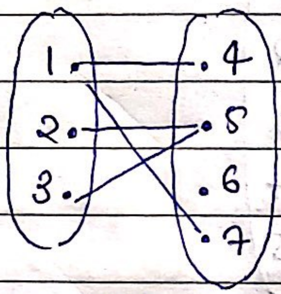
$F_1 = A \to B$.

① 

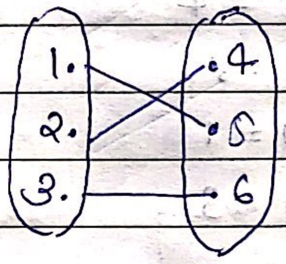

yes

② 


yes.

③ 


No

④ 


yes.

1) $f(x) = x^2$

$R \to R$

yes, valid function.

2) $f(x) = \sqrt{x}$

$R \to R$

No.

Types of Functions :-

1. One to One (Injective)
2. Onto (Subjective)
3. One to one & onto (Bijective)

<u>One to One</u> :-

Distinct element of A $\longrightarrow$ Distinct element of B.

A function F is said one to one if an only $f(A) = f(B)$ emplies that $A = B$ for $\forall A$ & $B$ in the domain of $f$.

→ $\{a, b, c, d\}$

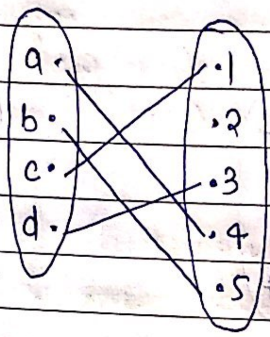
$\{1, 2, 3, 4, 5\}$

$f(a) = 4$

$f(b) = 5$

$f(c) = 1$

$f(d) = 3$




→ $f(x) x^2$ int to int

No.

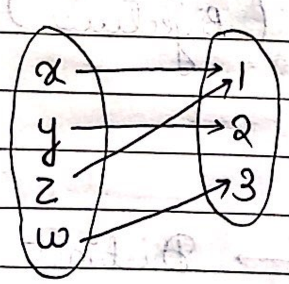
→ $f(x) = (x+1)$ Real No. to Real.

yes.

(2) <u>Onto.</u>

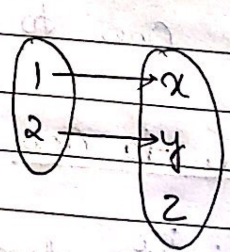
Every image in y must have a pre-image in X.

ex//




= yes

ex//



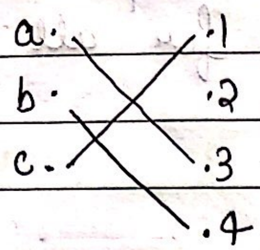

= yes.

→ $f(x) = x^2$ int to int

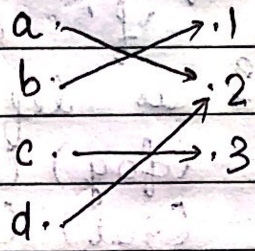
No.

## Bijective :-

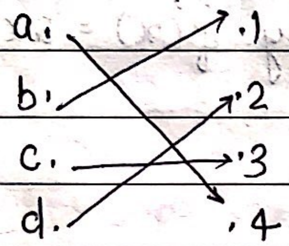
If its is both one to one & onto.

1] 

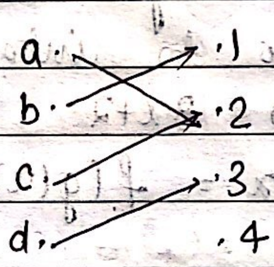

One to One

2] 

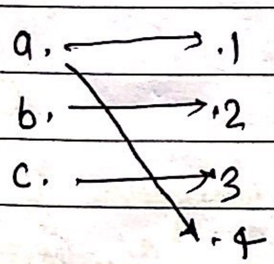

B onto

3] 


Bijective

4] 


function

5) 


Not function

# Composition function :-

let 'g' be the function from A to B & 'f' be the function B to C The composition of function 'f' & 'g' denated for all $a \in A$ by fog & is define by.

$$(f \circ g)(a) = f(g(a))$$
$$(g \circ f)(a) = g(f(a))$$

eg let f & g with the function form set of integer to integer define by $f(x) = 2x+3$ &
$g(x) = 3x+2$

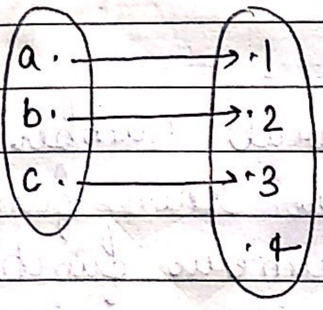
$(f \circ g)x = f(g(x))$

$(f \circ g)x = f(g(x))$
$= f(3x+2)$
$= 2 \cdot (3x+2)+3$
$= 6x+7$

## Inverse of function :-

given function $F : A \to B$ (from) if Relation o' inverse $f^{-1} : B \to A$ is a function then it is called inverse function.

A B




$\therefore f : A \to B$

$f^{-1} : B \to A$ Not invertible.

$f^{-1}(1) = a$

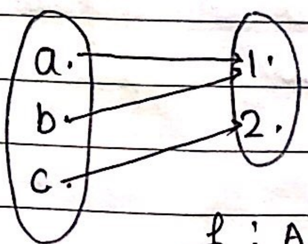
$f^{-1}(2) = b$

$f^{-1}(3) = c$

NOTE

If original function is not onto & one to one then it is not invertible function.

A B



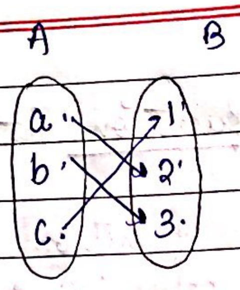

Not invertible

$f : A \to B$

E

If you want to get ~~involve~~ inverte of function then it is ~~bij~~ bijective function.

Q. $A = \{a, b, c\}$
$B = \{1, 2, 3\}$
$f(a) = 2$
$f(b) = 3$
$f(c) = 1$




yes it is invertible.

→ Discrete numeric function :-

let N be the set of natural number & R is the set of real number then function $f : N \to R$ then is called numeric function

write in
We practice describe DNF as (a) sequence like
$(f(0), f(1), f(2), \dots)$

→ The sequence can be written as
$a = a_0, a_1, a_2, a_3, \dots a_r$.

eg let A be the discrete numeric function :
$a_r = 7r^2 + 1, \quad r \geq 0$.

$a_0 = 7 \cdot (0)^2 + 1 = 1$
$a_1 = 7 \cdot (1)^2 + 1 = 8$
$a_2 = 7 \cdot (2)^2 + 1 = 29$

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Page: __

NOTE DNA function can define by different formulas.

$$d_r = \begin{cases} 2+r & 0 \leq r \leq 5 \\ 2-r & r > 5 \text{ \& } r \text{ is odd} \\ 2/r & r > 5 \text{ \& } r \text{ is even.} \end{cases}$$

## Generating function :-

A generating function is way of encoding a infinite sequence of numbers (an) by tracting them as the coefisions of power series.

like $a_0, a_1, \ldots \infty$

be a series of real no. denoted by $\{a_n\}$ then a series in power of $x$ such that

$$g(x)^2 = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + \infty.$$

Q. Find GF for following series :

1. 1, -1, 1, -1, 1, ....

$g(x) = a_0 + a_1 x + a_2 x^2 + \cdots$

$a_0 = 1$

$a_1 = -1$

$a_2 = 1$

$g(x) = 1 - x + x^2 + \cdots$

2. Find GF for following ~~relation~~ series.

ex: $2^0, 2^1, 2^2, 2^3, \ldots$

$= 2^0 + 2^1 x + 2^2 x^2$

$$\left\{ g(n) = \sum_{n=0}^{\infty} 2^n x^n \right\}$$

# Recurrence Relation :-

1. It is recursively define sequence.
2. A recurrence relation is an equation that recursively define a sequence where next term depend a previous term.

eg : fibonacci series.

## First Order linear recurrence relation :-

ex : $a_{n+1} = 2a_n$ , $a_0 = 1$

RR :

$$\boxed{a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}}$$

r format :

$r^n = c_1 r^{n-1} + c_2 r^{n-2} + \dots + c_k r^{n-k}$ .

# linear R.R.

1. Convert equation into the r - format.
2. divide by lowest value of R.
3. find the value of R.
4. find the value of constant.
5. eg. A & set no 4 is subtitute the value of R & A in this equation

$$a_n = A\cdot(r^n).$$

eg

$a_{n+1} = 2a_n$ , $a_0 = 1$.

1. $r^{n+1} = 2r^n$
2. $r^n$ divide $\Rightarrow r = 2$
3. $a_n = A\cdot(R^n)$
4. $a_0 = A\cdot(r^0)$ , $a=1$

$a_1 = A\cdot(r^1)$ , $\boxed{A=1}$

4. $\boxed{a_n = 1\cdot(2^n).}$ $\boxed{a_n = A\cdot(R^n)}$

# 2^n^ order linear Homogenious recurrence relation.

1. if we have

$$Ba_{n+2} + Ca_{n+1} + Da_n = 0.$$

Convert in r format.

2. Divide by lowest R term : $r^2 + r + 1 = 0$.
3. find the roots

$r_1$ & $r_2$.

4. Solve

$$a_n = A \cdot (R_1)^n + B(R_2)^n.$$

5. find A & B

6. Put value of $r_1$ & $r_2$, A & B in the equation

$a_n = 5a_{n-1} - 6a_{n-2}$, $a_0 = 1$, $a_1 = 1$.

1. $a_n - 5a_{n-1} - 6a_{n-2} = 0$

2. $r^n - 5a^{n-1} + 6Ca^{r-2} = 0$.

3. lowest value of $r = a^{r-2}$.

4. $r^n - 5r + 6 = 0$ $\quad r_1 = 3$

$r_2 = 2$

5. $a_n = A \cdot (3)^n + B \cdot (2)^n$

$\therefore a_0 = 1$

$a_0 = A \cdot (3)^0 + B(2)^0$ $\quad a_1 = 1$

$a_1 = A(3)^1 + B(2)^1$

$\boxed{A + B\phi = 1}$

$3A + 2B = 1$

$3A + 2(1-A) = 1$

$3A + 2 - 2A = 1$

$A + 2 = 1$

$A = 1 - 2$

$\boxed{A = 1}$

$A + B = 1$

$-1 + B = 1$

$B = 1 + 1$

$\boxed{B = 2}$

UNIT NO. 03

GRAPHS

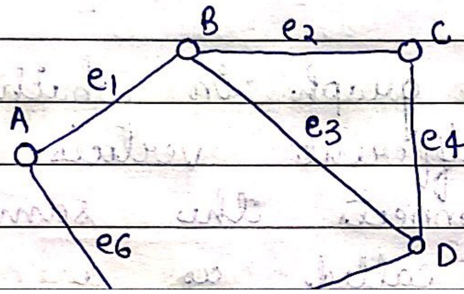
Graph :-

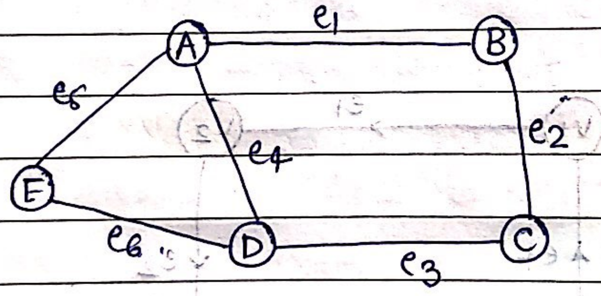
$$G = (V, E)$$

$V = \{v_1, v_2, v_3, v_4, \ldots, v_n\}$

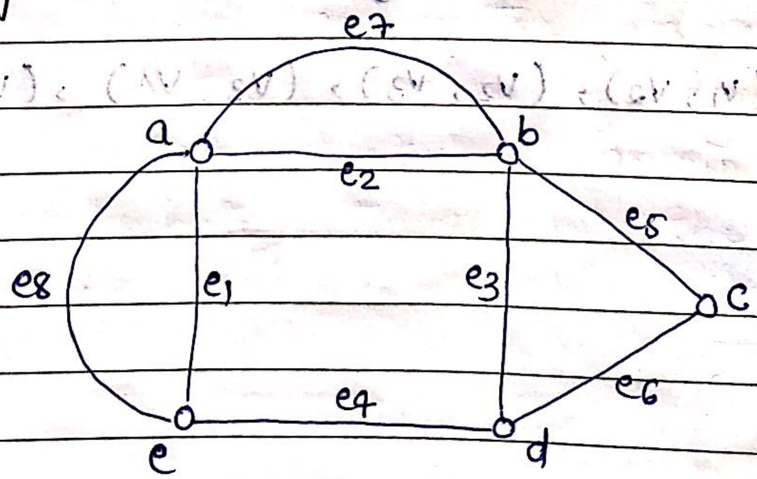
$E = \{e_1, e_2, e_3, e_4, \ldots, e_n\}$

$= \{(A,B), (B,C), (B,D), (C,D), (E,D), (A,E)\}$



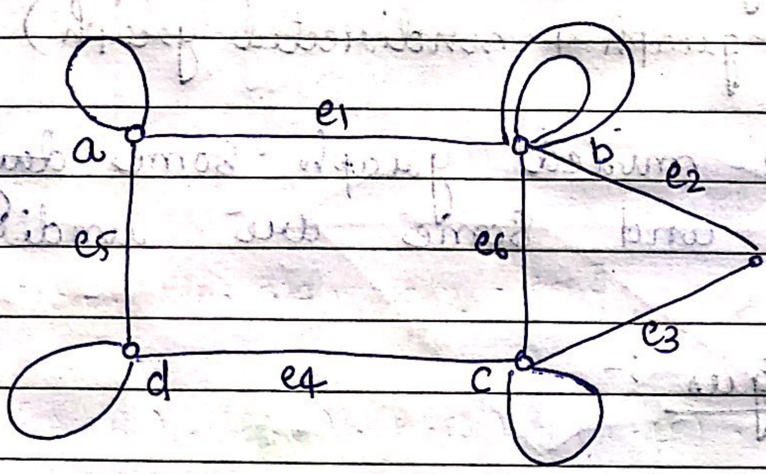



2] Multi graph.




So many edges is called Multigraph.

Pseudo Graph :-

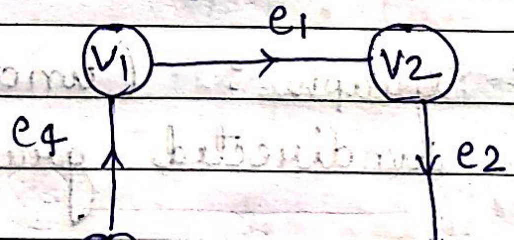



Pseudo graph is multiple edges & self loop.

NOTE We may even have more than one loop in a single vertex.

Simple Directed graph :-

No loops and No multiple edges & have directions.

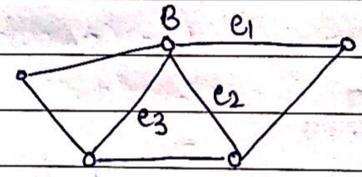


Mixed graph :-
~~(directed graph + undirected graph)~~

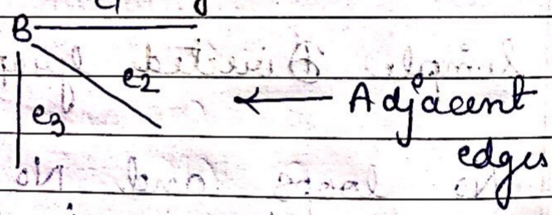
In mixed graph some ~~day~~ edges are directed and some are undirected.

Terminologies :-

Adjacent vertex
If two vertices are joined by the same edge then they are called adjacent edges.



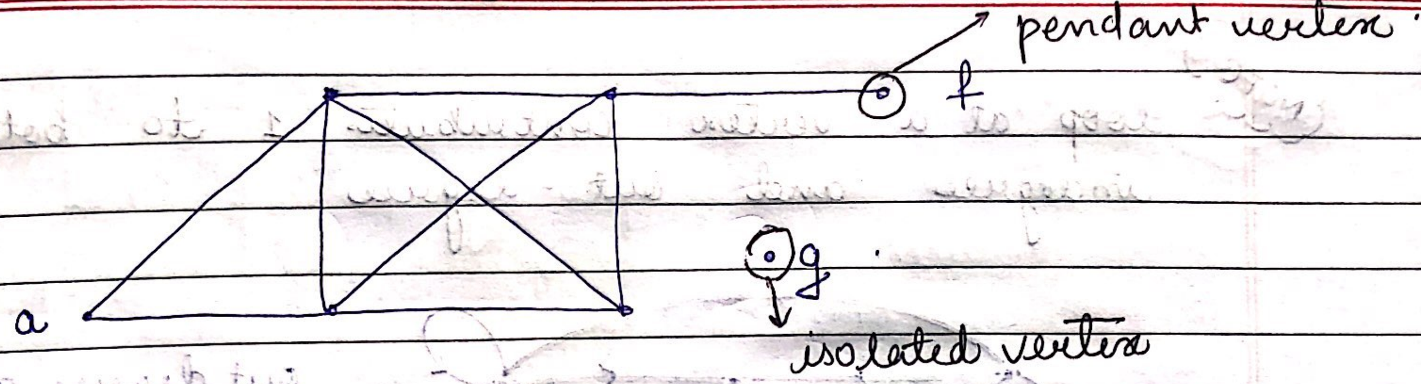

Adjacent edges.




If two edges are incident on same vertex are called as adjacent vertex.

Degree of the graph : (undirected graph)
degree of undirected graph.

The degree of vertex in an undirected graph is the number of edges incident to it except that a group loop at a vertex contribute twice to the degree of a vertex.

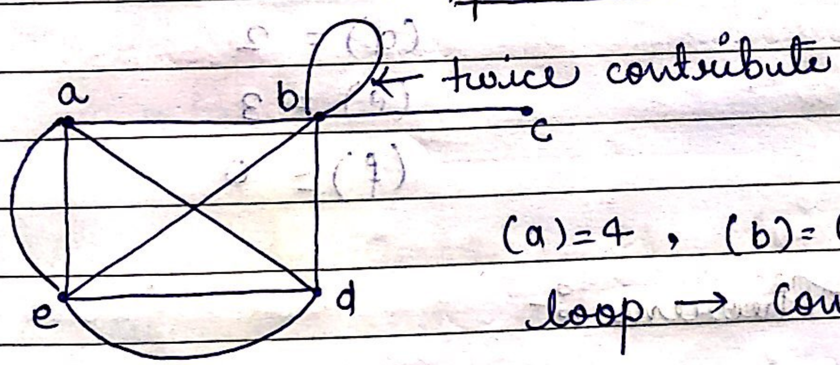



No. of edges = degree.

deg (a) = 2
(b) = 4
(c) = 4
(d) = 3
deg (e) = 4
(f) = 1
(g) = 0.

NOTE

1. A vertex of degree zero is called **isolated**.
2. A vertex of degree 1 is called **pendant**.




(a)=4 , (b)=6 , (c)=1 , (d)=5 , (e)=6.

loop → Contribute twice.

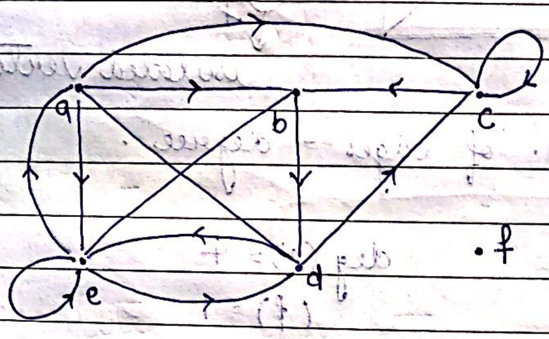
## Degree of a Directed graph

Their are two types of degrees in directed graphs.

- Indegree: deg(−)
- Out-Degree: deg (+)

degree denoted by $\left\{ \begin{array}{l} i \to \text{no. of vertices} \\ d(v_i) \quad deg(v_i) \end{array} \right.$

NOTE loop at a vertex contributes 1 to both in indegree and out-degree



out degree going out from vertex.

Indegree.

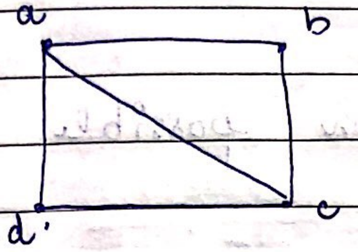
| Indegree | Outdegree. |
|---|---|
| deg(a) = 1 | deg(a) = 3 |
| (b) = 2 | (b) = 1 |
| (c) = 2 | (c) = 3 |
| (d) = 2 | (d) = 2 |
| (e) = 3 | (e) = 3 |
| (f) = 0 | (f) = 0 |

# Hand Shaking Theorem :-

let, $G = (V, E)$
be an undirected graph.
with $\underline{m}$ edges.

then,

$$\left\{ 2m = \sum_{v \in V} \text{degree}(v) \right\}$$ Handshaking Theorem.




$V = \{a, b, c, d\} = 4$.

$deg(a) = 3$.
$deg(b) = 2$
$deg(c) = 3$
$deg(d) = 2$

$V = 3+2+3+2 = 10$

$m =$ no. of edges 5.

$\boxed{10 = 10}$

eg. A simple graph G has 24 edges and the degree of each vertex is 4.
Find the total no. of vertices.

$e = 24$

$\boxed{v = 12}$

A graph contains 21 edges and 3 vertices of degree 4 and all the other vertices of degree 2.
Find total no. of vertices.

$$2m = \sum_{v \in V} deg(V)$$

3 — deg (4)
— deg (2)

$3 \rightarrow 4$ sum = 2
$(n-3)$ deg = 2.

$42 = 2(n-3) + 12$
$30 = 2(n-3)$
$15 = n-3$
$\boxed{n = 18}$

Scanned with CamScanner

NOTE → n.

what is the simple graph possible with n vertices

→ $\boxed{2^{\frac{n(n-1)}{2}}}$

(4) $2^{4(4-1)/2} = 2^{4(3)/2} = 2^{12/2} = 2^{6}$.

# Complete Graph :-

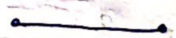
A complete graph on 'n' vertices is denoted by $k_n$ ( k suffix n)

It is a simple graph that contains exactly one edge between each pair of distinct vertices.

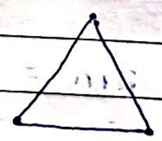
eg

$n=1$, $k_1$

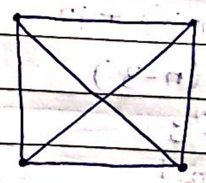
$n=2$, $k_2$



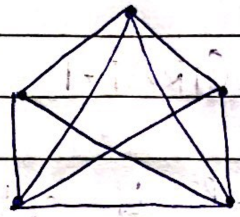
$n=3$, $k_3$



$n=4$, $k_4$



$n = 5$ , $K_5$



# Cycles

A cycle $C_n$ , $n \geq 3$. consits of $n$ vertices.

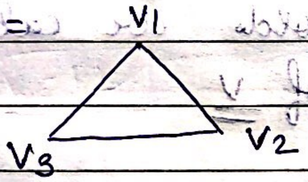
$v_1$ to $v_n$ ; $v_1 \ldots v_n$

and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, ...., $\{v_{n-1}, v_n\}$ and $\{v_n, v_1\}$

$\boxed{n=3}$

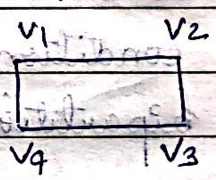
eg. $n=3$

$C_n =$

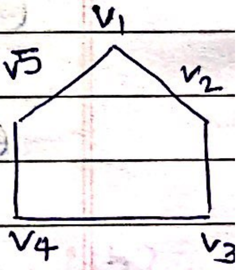



$n = 4$

$C_n =$



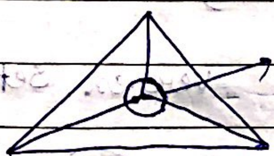

$n = 5$ , $C_n = C_5$




# Wheels

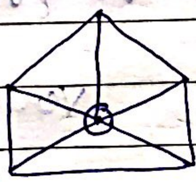
We obtain a whell $W_n$ when we add an additional vertex to a cycle and connect this new vertex to each of the $n$ vertices of $C_n$

eg.

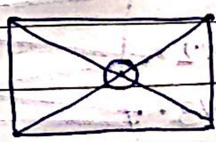



$= W_3$

$W_5 =$



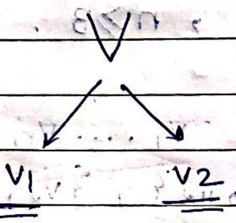
$W_4 =$



Imp

# BIPARTITE GRAPH.

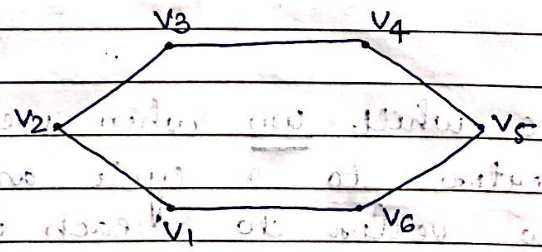
1. A simple graph is called bipartite if its vertex set $V$ can be partitioned into two disjoint sets $V_1$ & $V_2$




Such that every edge in the graph connects an vertex in $V_1$ & vertex in $V_2$
(so that no edge in $G$ connects

When this condition holds we call the pair $V_1, V_2$ a bipartition of $V$

eg

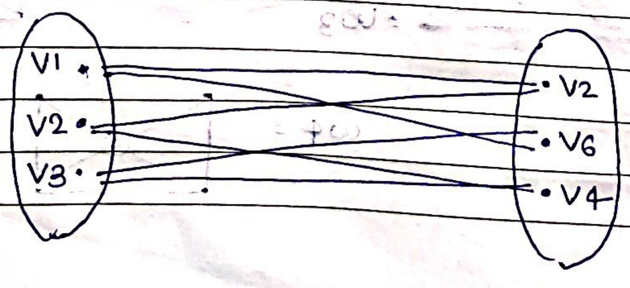



Draw the biparted graph for W6/C6.

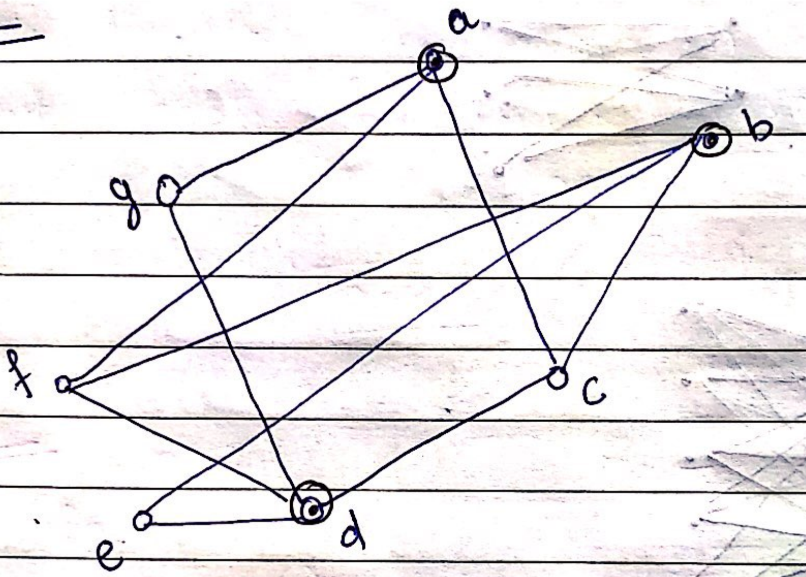
$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

Vertex set 1

vertex set 2

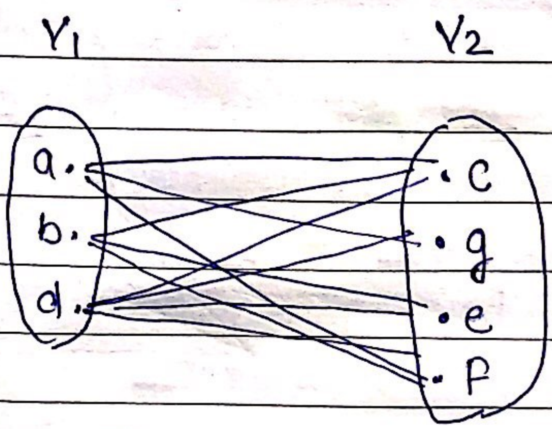


eg




V= {a,c} , {c,b} , {c,d}, {e,d} , {e,b} , {f,b} ,
{f,a} , {g,d} , {g,a}

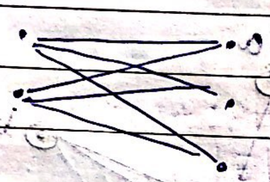
V= {a, b, c, d, e, f, g}



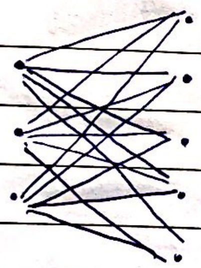
## Complete Bipartite Graph :-

All vertices of Set A $\xrightarrow{\text{Should be connected to}}$ All vertices of Set B

eg. $K_{2,3}$



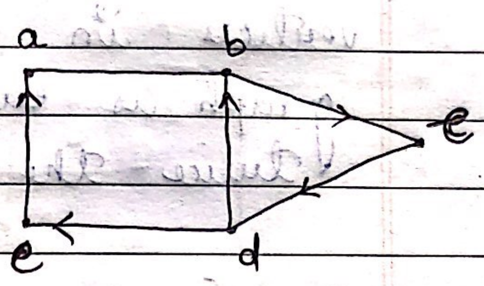
$K_{3,5}$



# HANDSHAKING THEOREM

Theorem : Directed graph.

If GB is a directed graph then the sum of out-degree of the vertices of a dia graph = Sum of in degree's of the vertices = no of edges in a di-graph.




$$\sum_{i=1}^{n} \text{Indeg}(V_i) = \sum_{i=1}^{n} \text{Outdegree}(V_i) = \text{no. of edges } n(e)$$

G (V, E)

V = {a, b, c, d, e}

E = {(a,b), (b,e), (e,d), (d,e), (e,a)}

| | a | b | c | d | e |
|---|---|---|---|---|---|
| Indeg | 1 | 2 | 1 | 1 | 1 |
| Outdeg | 1 | 1 | 1 | 2 | 1 |

$\sum$ Indeg = 6

$\sum$ Outdeg = 6

No. of Edge's = 6.

Only for Directed graph.

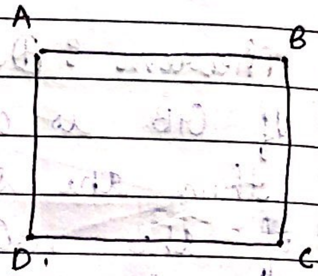
Hence proved.

Scanned with CamScanner

undirected .

II ~~Indegree~~ Theorem :-

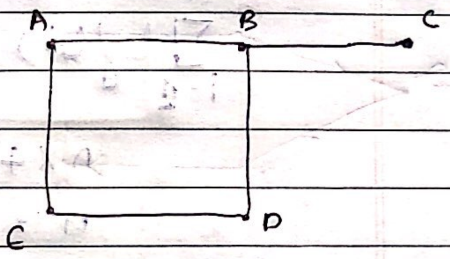
If G be an undirected graph with E edges, the sum of degree of vertices in an undirected graph is even or exactly twice the no' of edges.




$$\left\{ \sum_{i=1}^{n} deg(v_i) = 2 \times \text{no. of Edges } |E| \right\}$$

**Que.** Prove that the no. of vertices of odd degree in a graph is always even.

(by default : undirected graph).



deg(A) = 2

deg(B) = 2

deg(C) = 1

deg(D) = 2

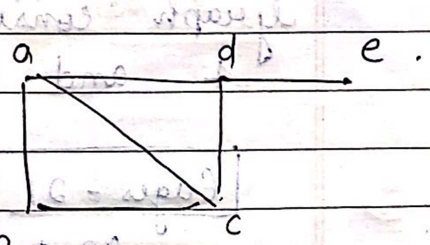
deg(E) = 2

$\cancel{\sum deg(v_i)}$ $\sum$ degree = 2+2+2+2+1 = 7.

No. of Edges = 5.

$$\sum_{i=1}^{5} deg(v_i) = 2 \times 5 = \boxed{10}$$

=X=



$$\sum_{i=1}^{n} deg(v_i) = \sum_{odd} deg(v_i) + \sum_{even} deg(v_i)$$

= a, c, d, e, b, ~~d, e~~ b

# A graph has 24 edges and degree of each vertex is 4.
find the total no of vertices.

$$\sum_{i=1}^{n} deg(v_i) = 2|E|$$

$|E| = 24$.
$deg(v_i) = 4$.

$n \times 4 = 2|24|$

$n = \frac{\cancel{2} \times 24}{\cancel{4}\,2}$

$\boxed{n = 12}$

Q. A graph consist of 21 edges and 3 vertices of degree 4 and all the other vertices of degree 2.
find the total no of vertices.

Graph consist of 21 edges and 3 vertices of degree 4 and all other vertices of degree 2.

$\boxed{Edges = 21}$

on putting formulas,

$$\sum_{i=1}^{n} deg(v_i) = 2 \times |E|$$
$$= 2 \times 21$$
$$= 42 \quad \ldots (A)$$

3 ~~edges~~ vertices of degree 4 $= 3 \times 4 = 12$

$x$ ~~edges~~ vertices of degree 2 $= 2x$.

$\therefore \quad 12 + 2x = 42 \quad \ldots$ from (A)

A graph has 24 edges and degree of each vertex is k then which of the following is possible no. of vertices?

a) 20

b) 15

c) 10

~~d)~~ 8.

$n \times k = 2|E|$

Edge = 24

deg = k.

when,

$|E| = 8$

then

no. of edges = 21

8)48

$\boxed{\text{P.T.O}}$

Mathematical proving of Bipartite graph & its Application.

$2x = 42 - 12$

$2x = 30.$

$\boxed{x = 15}$

The graph would have 15 vertices of degree 2

∴ Total no of ~~edges~~ vertices = 3+15

= 18 ← Ans

+ edges = {A, B, C, D}  $\boxed{\text{Total edges} = 4}$

deg(A) = 2
deg(B) = 2
deg(C) = 2
deg(D) = 2  $\boxed{\text{Total degree} = 8}$

48 × 1

24 × 2

12 × 4

6 × 8 → consider this.

4 × 12

2 × 14

1 × 48

repeted possiblities of vertex

$\boxed{48, 24, 12, 8}$

Ans is <u>8</u>

Mathematical proving of Bipartite Graph and its Application

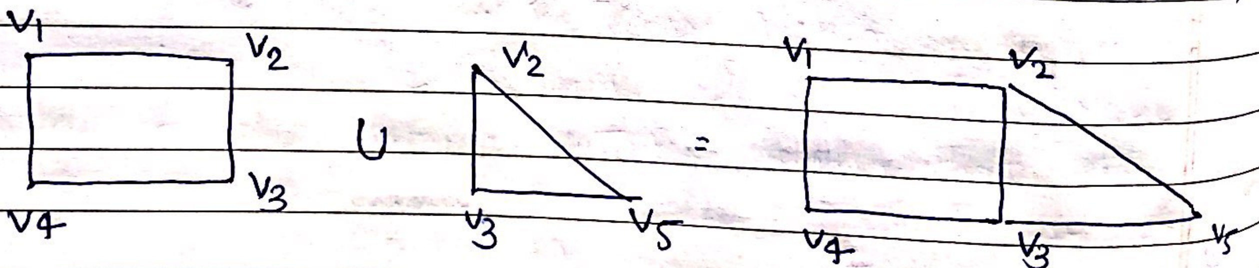
OPERATION ON GRAPH :-

1. Union $\cup$
2. Intersection $\cap$
3. Ring Sum $\oplus$
4. Decomposition
5. Deletion
6. Fusion

UNION ($\cup$)

$G_1(V_1, E_1)$ and $G_2(V_2, E_2)$

$G_3(V_3, E_3) = G_1(V_1, E_1) \cup G_2(V_2, E_2)$




$V_3 = V_1 \cup V_2$

$E_3 = E_1 \cup E_2$

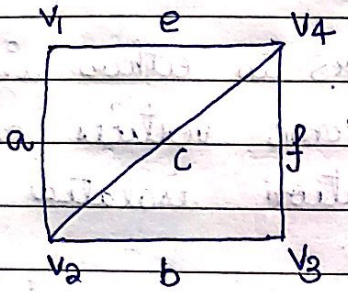
INTERSECTION ($\cap$)

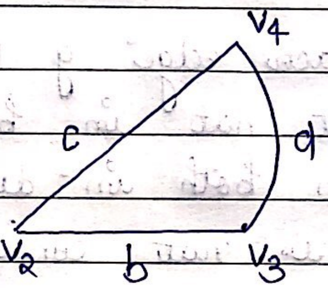
= Common Part

= $V_2$ $V_3$

# RING SUM ⊕

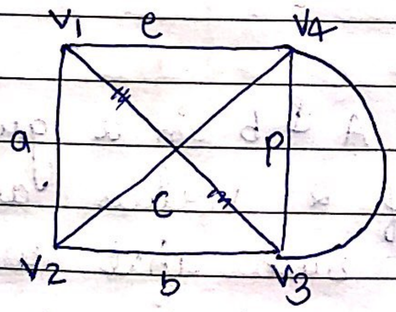
Ring Sum is a graph consisting of the vertex set $V_1 \cup V_2$ & edges that are either in $G_1$ & $G_2$ but Not in both.



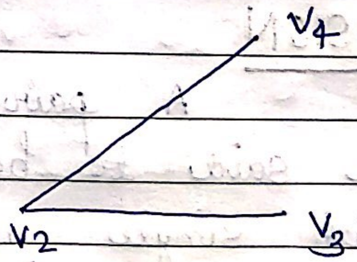

1] 1st perform union
then intersection
then subtract that is ring sum.

① UNION



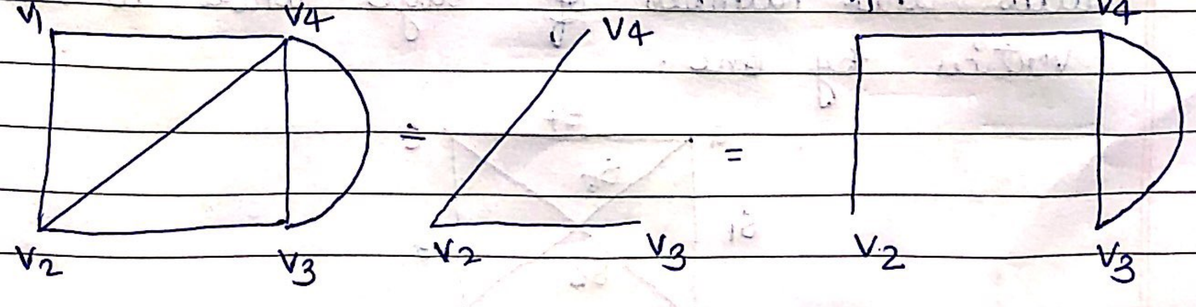

$G_1 \cup G_2$

② INTERSECTION

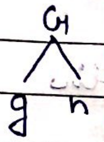



$G_1 \cap G_2$

③ SUBTRACT -

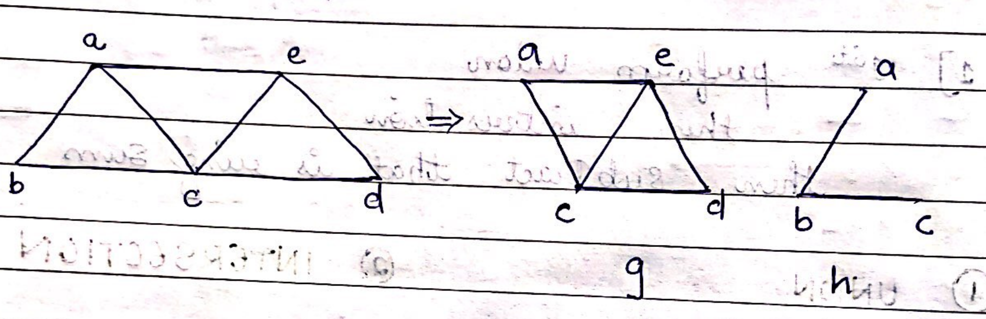


# DECOMPOSITION




A graph G is said to have accomposed into 2 subgraph 'G' & 'g' if $g \cap h$ is equal to G & if $g \cap h = \phi$

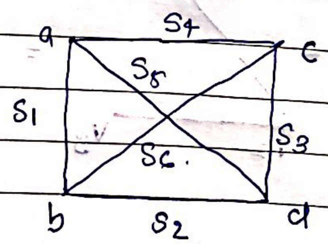
i.e:- each edge g G occurs in either in g or h but not in both, some vertices may occur in both in decomposition isolated vertex is not considered.




# FUSION

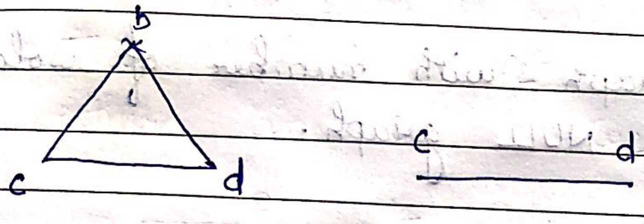
A pair of vertices A & b in a graph are said to be fused if 2 vertices are repla with single new vertex such that every edge that was incident on either a or b or in both is incedent on new vertex

So fusion of 2 vertices does not alter the number of edge where it reduce vertices by one.

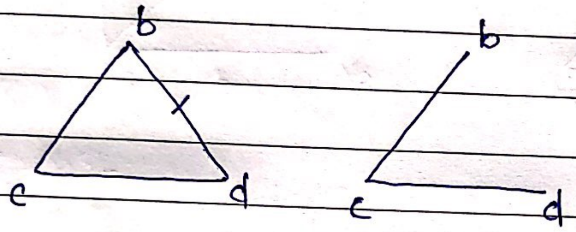


## DELETION

in which we either delete edge or vertices.




if delete vertice




if delete edge

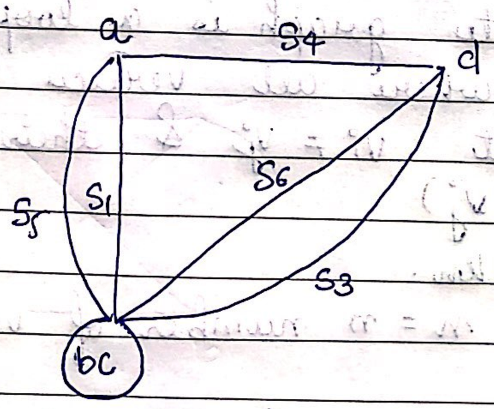



fig of fusion operation.

The graph after performing fusion.

# TYPES OF GRAPH :-

## 1. NULL GRAPH

A graph with number of isolated vertices is called NULL graph.

eg :

$G(V, E)$

$E = \phi$

$V \neq \phi$

## 2. COMPLETE GRAPH.

A complete graph is a loop free undirected graph where all vertices $v_i$, $v_j$ belongs to $V$ but $v_i \neq v_j$ & their exist $\exists e(v_i, v_j)$

denoted by $k_n$.

$n = n$ numb