UNIT - 5

# Counting

* Principle of product and sum
They are fundamental principle of counting

→ Principle of product : Outcome of random exp. occur indipendently one after the other

→ Principle of sum : Outcome
→ sometimes there 2 prin. has to combine depending on the prob.

~> If one exp has $m$ ways to perform and another exp can occur in $n$ ways then there are $m \times n$ possible outcomes

~> If one exp has $m$ possible outcom another has $n$ then there are $m+n$ possible outcomes when one of this experiment takes place

Eg   CS - 6 books    two diff category books.
Math - 3 books
Chem - 2 book

CS & M + CS & Chem + M & Chem
$= 18 + 12 + 6$
$= 36$ Ans.

* Permutation

If there are $n$ objects then a sequence of 'r' diff. objects is called permutation of $n$ object taken 'r' at a time

$$\boxed{{}^{n}P_{r} \cong P(n, r)}$$

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

* Combi~~n~~ation :

* Important deduction :

→ $P(n, r) = \dfrac{n!}{(n-r)!}$

→ $P(n, r) = n!$ if $r = n$

→ $P(n, n-1) = n!$

→ $0! = 1.$

→ Let A containing n object then the permutation of elements of a taken r at a time allowing repetaⁿ is $n^r$. Each such permutⁿ is also called a sequence of length r

→ The no. of permutⁿ of n object of which 'p' objects are of one type 'q' objects are of diff type and 'r' of 3rd type and remaining are of diff types is given by

$$\boxed{\frac{n!}{p!\,q!\,r!}}$$

REP. ALLOWED

Que Find out how many 5 digit no gr. than 30000 can be formed from 1, 2, 3, 4, 5

Ans REP ALL.

$3 \times 5 \times 5 \times 5 \times 5$

$= 1875$

REP NOT $^4P_4 = 4!$

$3 \times 4 \times 3 \times 2 \times 1$

$= 72.$

$^3P_1$

Que Consider a set {a, b, c, d} in how many ways can we select two of this letter rep not allowed.

$\Rightarrow {}^4P_2 = 12.$

Que Find out the no. of permutⁿ from letter of word ENGINEERING

$\frac{11!}{3!\,3!\,2!\,2!}$

$P(n,r) = \frac{n!}{(n-r)!}$ ORDER MATTERS

$C(n,r) = \frac{n!}{r!(n-r)!}$ ✗

Camlin Page Date / /

Que How many permutⁿ from letter of word MISSISSIPPI : taken all together.
ii) In how of these the vowels occupying the even places.

Ans $\frac{11!}{4!4!2!} = 34650$

ii) Vowels = I I I I

MISSISSIPPI
O E O E O E O E O E O
5 - E
6 - O

$^5C_4 \times \frac{7!}{4!2!}$ $\left( \frac{5!}{4!} \times \frac{7!}{4!2!} \right)$

Que2 No of permutation from ~~COS~~ CONSTITUTION
b) Vowels occur together
(b) Consonant and vowel occur alternatively
(d) N at the beginning and end.

(a) $= \frac{12!}{2!2!3!2!}$

7 + (O I U I O) Pack

(b) $\frac{7!}{2!3!} \times \frac{5!}{2!2!}$ even. ✗

$\frac{8!}{2!3!} \times \frac{5!}{2!2!}$
N T

(c) → GAP method.
→ Repetition is must.

(d) N _ _ _ _ _ _ _ _ _ _ N fixed.

$= \frac{10!}{2!3!2!}$

Ans. CO..N has 12 characters of which there are 2 N, 3 T, O-2, 1 U

∴ Total no of permutations

$$= \frac{12!}{2!\,2!\,3!\,2!}$$

(b) vowels occur together

There are seven consonants

CNNSTTT

and 5 vowels

OIUIO

To find no of perm in which 5 v. occur tog. consider them as one pack

∴ There are 7 of const. and 1 of vowels total 8 for permutation of which there are 2N and 3T ∴ Total permutⁿ are

$$= \frac{8!}{2!\,3!}$$

but 5 v. which was considered as a group contains 2O and 2I can be permuted themselve in $\frac{5!}{2!\,2!}$. Hence total no of permⁿ are

$$\frac{5!}{2!\,2!} \times \frac{8!}{2!\,3!}$$

(c) $\frac{7!}{2!\,3!} \times \frac{5!}{2!\,2!}$

(d)

Que In how many ways can the letter of the word ALLAHABAD can be arranged.
How many of these have
→ 2 L's occur together
→ 2 L's don't occur together.

Ans (a) No of words - 9
A - 4
L - 2

$\therefore$ Total no of permutation = $\frac{9!}{4!\,2!}$

(b) 2 L's occur together (consider as one grp)
$\Rightarrow \frac{9!}{4!}$

(c) $\Rightarrow$ Total - 2L's together
$= \frac{9!}{4!\,2!} - \frac{9!}{2!}$

Combⁿ

Que A candidate is req. to answer 6 of 10 questions which are divided into two groups containing 5 questions each and he is permited to attend not more than 4 questions from any group. In how many ways can he select the questions.

Ans No of choices

| | | | |
|---|---|---|---|
| = | 2 | 4 | 50 |
| | 3 | 3 | 100 |
| | 4 | 2 | 50 |
| | | | 100 |

COMBN: DON'T MATTER IN WHICH SEQ. HE IS ANSG JUST ANS from 1 and 2.

→ ORDER DOESN'T MATTER

In how many ways a cricket team of 11 players from 14 (a) How many of them will include a particular player

(b) Exclude a particular player.

Ans (0) = $\frac{14!}{11! \times 3!}$ ← ($C_{n,r}$)

(a) $^{13}C_{10}$ (one is fixed)

(b) $^{13}C_{11}$ (options reduced)

* Generating Permutation

We have learned $n!$ permutⁿ of $\{1, 2, 3, \ldots n\}$ it is important to generate a list of such permⁿ. If we have complete list of permⁿ of $\{1, 2 \ldots n-1\}$ then we can obtain complete list of permutation for $1 \to n$ by inserting $n$ in $n$ ways to each permutation of the list $(1, \ldots, n-1)$

for $n = 1$, the list is 1 ← inserting 2 in

$n = 2$ → 1 2, 2 1 — Illy 3

$n = 3$ →

| | | |
|---|---|---|
| 1 2 3 | | 2 1 3 |
| 1 3 2 | | 2 3 1 |
| 3 1 2 | | 3 2 1 |

$n = 4$ ... →

| | | |
|---|---|---|
| 1 2 3 4 | | 4 1 3 2 |
| 1 2 4 3 | | 1 4 3 2 |
| 1 4 2 3 | | 1 3 4 2 |
| 4 1 2 3 | | 1 3 2 4 |

To generate comp list of permutation for the set $1 \rightarrow n$ we assign a dir$^n$ to each integer $k \in \{1, 2 \ldots n\}$ by writing an arrow above it pointing in some dir$^n$

$$\boxed{\overrightarrow{k} \text{ or } \overleftarrow{k}}$$

We consider permut$^n$ of $1 \rightarrow n$ in which each integer is given a dir$^n$ is k/a directed perm$^n$

IMP → And integer k in directed permut$^n$ is called mobile if its arrow points to a smaller integer adjecent to it.

Eg: $\overleftarrow{3}\ \overrightarrow{2}\ \overleftarrow{5}\ \overrightarrow{4}\ \overrightarrow{6}\ \overrightarrow{1}$

M X M X M

It follows that 1 digit can never be mobile since there is no integer in $1 \rightarrow n$ (i.e < 1)

The integer ~~n~~ n is mobile except two cases

→ n is the first integer and its arrow points to the left ($\overleftarrow{n}\ldots$)

→ n is the last ....

.... right ($\ldots\overrightarrow{n}$)

* Generating Permuta$^n$ Algorithm : $\{1, 2 \ldots n\}$

Step 0 → Begin with $\overleftarrow{1}, \overleftarrow{2} \ldots \overleftarrow{n}$

Step 1 → Find the largest mobile integer. Let 'm'

Step 2 → Switch m and adjecent integer its arrow points to

MOBILE INT.

Step 3 → Switch the dirⁿ for all the integer p>m

Step 4 → Write down the resulting permutation with dirⁿ and return to step 1

Step 5 → Stop if there no mobile integer.

Eg : {1, 2}

n = 2

0 → $\overleftarrow{1}\ \overleftarrow{2}$

1 → m = 2

2 → $\overleftarrow{2}\ \overleftarrow{1}$

3 → Not valid

4 → $\overleftarrow{2}\ \overleftarrow{1}$

stop

{1 2, 2, 1}

For n = 3

0 → $\overleftarrow{1}\ \overleftarrow{2}\ \overleftarrow{3}$

1 → m = 3

2 → 1 2 3 ✗ 2 → $\overleftarrow{1}\ \overleftarrow{3}\ \overleftarrow{2}$

3 → $\overleftarrow{1}\ \overleftarrow{3}\ \overleftarrow{2}$ (No change)

4 → {1, 2, 3 ✗ 4 → {1 2 3, 1 3 2}

1 → n = 3 ($\overleftarrow{1}\ \overleftarrow{3}\ \overleftarrow{2}$)

~~1~~ 2 → m = 3

2 ~~3~~ → $\overleftarrow{3}\ \overleftarrow{1}\ \overleftarrow{2}$ (No change in 3)

3 →

4 → {1, 2, 3 ✗ 4 → {$\overleftarrow{1}\overleftarrow{2}\overleftarrow{3}$, $\overleftarrow{1}\overleftarrow{3}\overleftarrow{2}$, $\overleftarrow{3}\overleftarrow{1}\overleftarrow{2}$}

1 → m

Scanned by CamScanner

Step 0 - $\overleftarrow{1}$ $\overleftarrow{2}$ $\overleftarrow{3}$

Step 1 - m = 3 ( out of 2 and 3)

↳ MAKE IT BOLD

Step 2 - $\overleftarrow{1}$ $\overleftarrow{3}$ $\overleftarrow{2}$

Step 3 - $\overleftarrow{1}$ $\overleftarrow{3}$ $\overleftarrow{2}$ (p = 1 and 2 but ! > m)

Step 4 - $\overleftarrow{1}$ $\overleftarrow{3}$ $\overleftarrow{2}$

Step 1. - m = 3

Step 2 - $\overleftarrow{3}$ $\overleftarrow{1}$ $\overleftarrow{2}$

Step 3 - $\overleftarrow{3}$ $\overleftarrow{1}$ $\overleftarrow{2}$

Step 4 - $\overleftarrow{3}$ $\overleftarrow{1}$ $\overleftarrow{2}$

Step 1 - m = 2

Step 2 - $\overleftrightarrow{3}$ $\overleftarrow{2}$ $\overleftarrow{1}$

Step 3 - $\overrightarrow{3}$ $\overleftarrow{2}$ $\overleftarrow{1}$

Step 4 - $\overrightarrow{3}$ $\overleftarrow{2}$ $\overleftarrow{1}$

Step 1 - m = 3

Step 2 - $\overleftarrow{2}$ $\overrightarrow{3}$ $\overleftarrow{1}$

Step 3 -

Step 4 -

⋮

# * Generating Combination Algorithm

## # Random Question

If ${}^nC_x = 56$ and
${}^nP_x = 336$
find $n$ and $x$

$$\frac{n!}{x!(n-x)!} = 56 \quad \text{— (i)} \qquad \frac{n!}{(n-x)!} = 336 \quad \text{— (ii)}$$

from (i) and (ii)

$$\frac{336}{x!} = 56 \Rightarrow x! = 6$$
$$\Rightarrow x = 3$$

$$\frac{n!}{(n-3)!} = 336$$

$\Rightarrow n(n-1)(n-2)\cancel{(n-3)!} = 336 \times \cancel{(n-3)!}$

$\Rightarrow n(n-1)(n-2) = 336$

$\Rightarrow n = 8$.

▶

Step 0 : Begin with $a_{n-1} \ldots a_1 a_0 = 0 \ldots 00$

Step 1 : If $a_{n-1} \ldots a_1 a_0 = 1 \ldots 11$ then Stop

Step 2 : If $a_{n-1} \ldots a_1 a_0 \neq 1 \ldots 11$

~~Step 3 :~~ Then find the smallest integer $j$ such that $a_j = 0$

Step 3 : Change $a_j, a_{j-1} \ldots a_0$ (either from 0 to 1 or 1 to 0)

write down $a_{n-1} \ldots a_1 a_0$ & return to step 1

Eg: for $n = 3$

$a_2 a_1 a_0 = 000$ ← SMALLEST

$a_2 a_1 a_0 = 001$ → CHANGE ALL 1 TO 0, 0 TO 1

010

011

100

101

110

111 → ALL 1 ⇒ STOP

for $n = 4$

0000

0001

0010

0011

0100

0101

0110

0111

1000

1001

1010

1011

1100

1101

1110

1111 ← STOP

* Pigeonhole Principle. :-

aka : Dirichlet Drawer Principle

: Shoebox Principle

If $n$ pigeon holes are occupied by $n+1$ or more pigeon then atleast one pigeon hole is occupied by more than 1 pigeon.

This principle tells us nothing about how to loc. the pigeonhole that contains two or more pigeon. It only asserts the existence of pigeonhole containing two or more pigeon.

Que In a grp of 13 children there must be atleast 2 children who were born in same month.

Pigeons are 13 children

PH are 12 months

Since $13 > 12$ by PH Theo.

$\because$ 13 children are born in 12 months

$\therefore$ There must be 2 children born in same mo

Que If 9 books are to be kept in 4 shelves will it be possible to avoid atleast 3 book on one shelve.

IMPOSSIBLE

* Generalized PH Principle

If $n$ PH are occupied by $kn+1$ or more pigeons were $k$ is +ve integer then atleast one PH is occupied by $k+1$ or more pigeon.

Que Find the min no of students in a class to be sure that 4 out of them are born in same month.

$n = 12$

$k+1 = 4 \Rightarrow k = 3$

$\Rightarrow kn+1 \Rightarrow 3\times12+1 = 37$

Que ⑥ Prove that if 6 integers are selected from {3, 4, 5, 6, 7, 8, 9, 10, 11, 12} there must be 2 integers whose sum is 15

Que ⑦ Prove that if 10 pts are placed in 3x3 cm square then two points must be less then or equal to $\sqrt{2}$ cm apart.

⑥

| | | |
|---|---|---|
| Box 1 : | 3 | 12 |
| Box 2 : | 4 | 11 |
| Box 3 : | 5 | 10 |
| Box 4 : | 6 | 9 |
| Box 5 : | 7 | 8 |

Selecting 6 Rand. no.

$\boxed{3 \quad 12}$ → 15

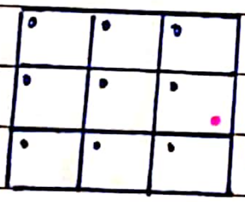
11

10

6

8

⑦



Divide the square in 9 sq. that are 1cm x 1cm.

We have 10 pts to be placed in 9 squares. so there will be one square which contains two pts. Thus making max sep. of $\sqrt{2}$.

$P(E) = \frac{n(E)}{n(S)}$

Find Sample Space



* Discrete Probabilities :-

Que There are 4R and 3B balls in bag. If one ball is taken out of this bag then represent the sample space and event of this ball being blue and find no. of elements in it.

Ans Sample Space size = 7 (R R R R B B B)

No of elements in this event = 3

Probab = 3/7.

Que Two unbaised dice are thrown. What is the prob of

i) getting a sum of 6

ii) The no. showns are equal

iii) The diff of no. shown is 1

iv) The first die shows six

v) The total of no is greater than 1. (8) → 10/36.

Ans Sample Space size = $6^2 = 36$

i) Getting sum = 6

1:5 , 2:4 , 3:3 ; 4:2 , 5:1

= 5/36.

ii) Shows equal = 6/36 = 1/6

iii) Diff is 1. 10/36

iv) 6/36 = 1/6

v) 36/36 = 1

Using P and C

Que A bag contains 7R and 4W balls . 2 balls are drawn at random . Find the prob that

i> Both balls are red

ii> One ball is R and other is W.

Ans Sample Space size = $^{11}C_2$ = 55

i> Both balls are red

$$\frac{^7C_2}{\text{Total}} = \frac{21}{55}$$

ii> One ball from each

$$\frac{^7C_1 \times {}^4C_1}{\text{Total}} = \frac{28}{55}$$

* Conditional Probabilities.

Let A and B be two events

P(A) = prob of A

P(B) = prob of B

then

P(A/B) = Prob of event A w.r.t event B

2. knowing the sample space.

3. Prob. of event A to occur when event B was already occured.

P(B/A) = Vita of

$$= \boxed{\frac{P(A \cap B)}{P(A)}} \quad \text{if } P(A) \neq \phi$$

Camlin Page
Date / /

Ques F M and D are @ random lined up in queue. What is the prob that the daughter is at end when father is in the middle. ↳ FIRST OR LAST

Ans Sample space = { FMD, FDM, DFM, DMF, MFD, MDF }

SSS = 6.

Let A be the event when daughter is @ the end
B father is in middle.

$S_B$ = { DFM, MFD } ∴ $n(S_B) = 2$

$S_A$ = { DFM, DMF, FMD, MFD } ∴ $n(S_A) = 4$

$P(A/B) = 2/2 = 1$

Ques A Dice when thrown twice it has been observed that the sum of it is 4 only if at least one of them on dice is 2

SSS = 36

A : Sum is 4
B : One is 2

$S_A$ = { 13, 22, 31 }  $n(S_A) = 3$

$S_B$ = { 21, 22, 23, 24, 25, 26, 12, ~~22~~, 32, 42, 52, 62 }

$n(S_B) = 11$

∴ Ans = 3/11.

Scanned by CamScanner

* Baye's Theoram :-

It is a formula to obtain how to update probability of hypothesis (I incomplete def$^n$)

H : Hypothesis
E : Evedence.

BT states that the rel$^n$ b/w prob of ~~getting~~ hypo before getting the evidence P(H) and prob of hypo. after getting the evidence is P(H/E) then Baye's Thm. is given by

$$BT = \boxed{P(H/E) = \frac{P(E/H) \times P(H)}{P(E)}}$$