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UNIT - III

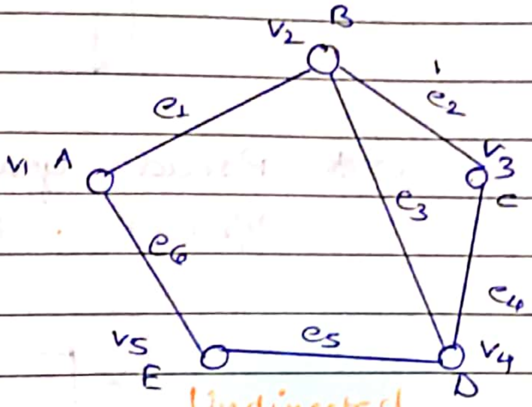
Graphs

Graph is denoted by G
→ set of vertices and Edeges

$G = (V, E)$

$V = \{v_1, v_2, v_3 \dots\}$

$E = \{e_1, e_2, e_3 \dots\}$



$= \{(A,B), (B,C) \ (B,D) \ (CD) \ (E,D), (A,E)\}$

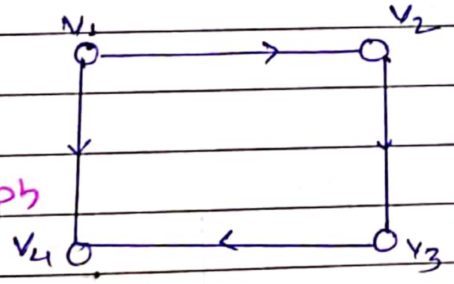
Undirected → can change order

can change

Not in directed graph $(A \to B)$ eg.

Diagraph

Directed graph



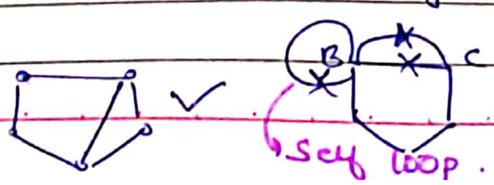
$\{(v_1, v_2) (v_2, v_3) (v_3, v_4) (v_4, v_1)$

Graph Loop + Parallel Edge.

Undirected	Directed.
→ Simple Graph	→ Simple directed G
→ Multi	→ Directed multigraph
→ Pseudo	

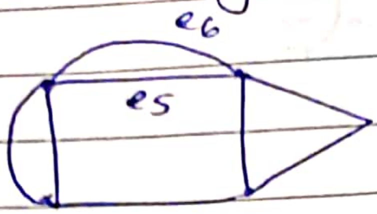
* Simple graph. ✗ ✗

A graph in which each edge connects two 2 diff vertices and where no 2 edges connects same pair of vertices



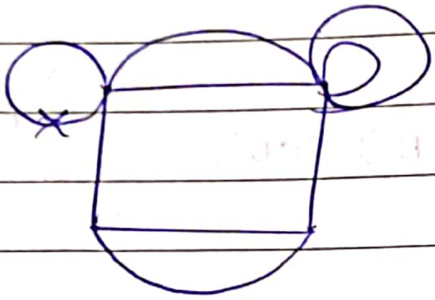
* Multi-graph : ×

Graphs having multiple edges (not loop)



* Pseudo graph : ✓

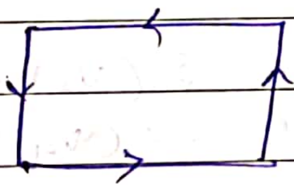
Muliple edges and loop also.



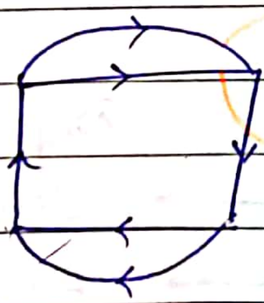
Note: We may even have more than 1 loop at a single vertex

* Simple Directed Graph × ×

simple having dirn.

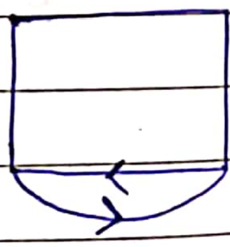


* Directed Multigraph



parallel edges (Same dirn).

Even if dirn not given.



Not parallel edges.

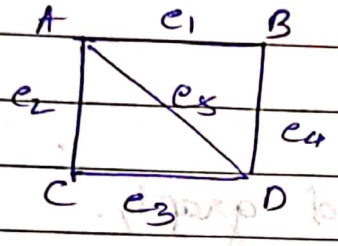
* **Mixed Graph.**

In mixed graph some are directed and some are not directed.

Terms

✓* **Adj vertex :**

If 2 vertices are joined by same edge they are called AV.



for A → B, C, D

✓* **Adj Edge**

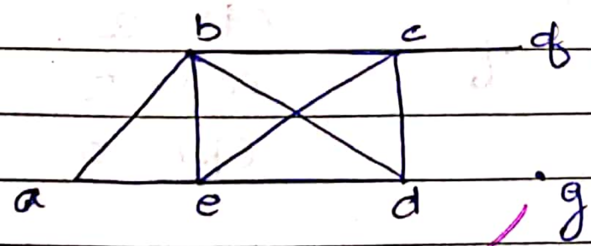
If 2 edges are incident on same vertex.

Eg for A e_1, e_2, e_5 are adj.

for C e_2 and e_3

✓* **Degree of an undirected graph.**

The deg of vertex of an undirted graph is the no of edges incident to it. except that loop at a vertex contribute twice for deg of vertex



deg (a) = 2
(b) = 4
(c) = 4
(d) = 3
(e) = 4
(f) = 1.
(g) = 0.

Should have set of vertices, set of edges not compulsary

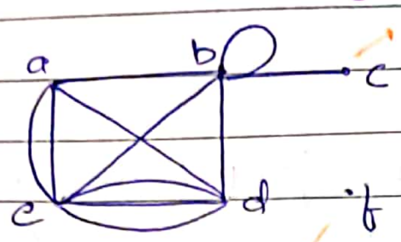
Single but valid. Graph

Degree of graph not possible.

Note : A vertex of deg = 0 is called isolated and vertex of deg = 1 is called pendant.

Eg

pendent



isolated

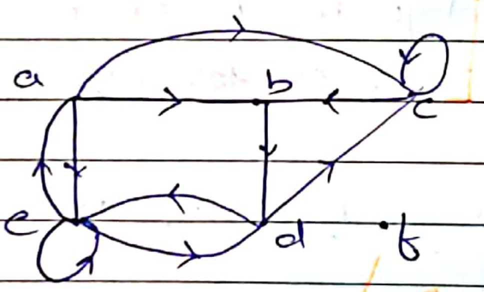
deg(a) = 4
deg(b) = 6 (2 for 1 lp)
(c) = 1 (pendant)
(d) = 5
(e) = 6
(f) = 0 (isolated)

* Degree in a directed graph.

There are two types of degrees
Coming ⟶ ⟶ Indegree denoted by deg(−)
Going ⟶ ⟶ Out-degree deg(+)

Note: Loop at vertex contributes 1 to both ID and OD of a vertex.

pendent



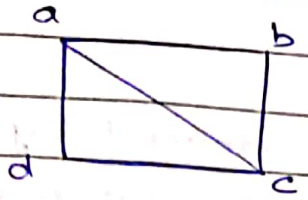
↳ isolated vertex

In-	Out-
deg(a) = 1	deg(a) = 3
(b) = 2	(b) = 1
(c) = 3	(c) = 2
(d) = 2	(d) = 2
(e) = 3	(e) = 3
(f) = 0	(f) = 0

Handshaking Thm. :
Let G=(V,E) be an undir graph with m edges then

$$\boxed{2m = \sum_{v \in V} deg(v)}$$

Valid   Valid.



$$\left.\begin{aligned}(a) &= 3\\ (b) &= 2\\ (c) &= 3\\ (d) &= 2\end{aligned}\right\} = 10$$

$m = 5$

$2m = 10$ Same.

Q4 A simple graph G has 24 edges and degree of each vertex is 4 find total no of vertices

$$2 \times 24 = x \times 4$$
$$x = 12.$$

Ques A grap : 21 edges, 3 vertices of degree 4, other with degree 2. find total no. of vertices

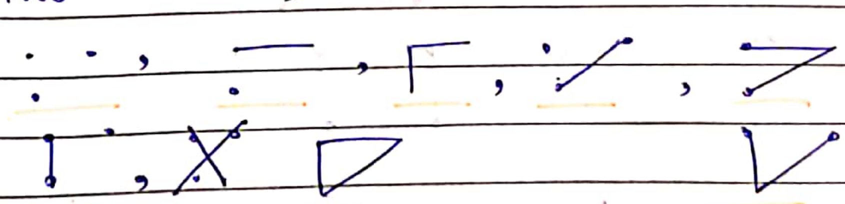
$$2 \times 21 = 3 \times 4 + x \times 2.$$
$$42 = 12 + 2x$$
$$x = 15$$
$$\therefore \text{Total} = 15 + 3 = 18.$$

* With n vertex(s) no. of simple graph

→ 1 : One.

→ 2 : Two • •, •—•

→ 3. i)



$= \cancel{6}\ \cancel{7}$ 8

→ n $\boxed{2^{\frac{n(n-1)}{2}}}$ Only Simple Graphs

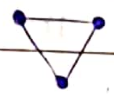
Complete Graph

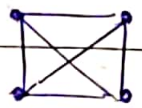
A CG of n vertices is denoted by K_n

→ It is a simple graph that contains exactly 1 edge b/w each pair of distinct vertices

for $n=1$: (K_1)

$n=2$: 

$n=3$:  (every with other)

$n=4$:  ( )

IIIly for $n = 5, 6, 7, 8 \dots$

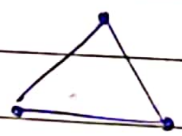
If only / even single is not connected to all it is not complete

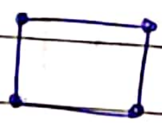
Cycles:

$C_n \quad \forall\ n \geq 3$

Consist of n vertices $v_1 \dots v_n$ and edges $\{(v_1, v_2), (v_2, v_3) \dots (v_{n-1}, v_n)$ and $(v_n, v_1)\}$

⇒ All in sequence.

$n = 3$: C_n = 

$n = 4$: C_n : 

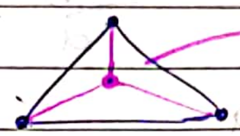
IIIly for $n = 5, 6, 7 \dots$

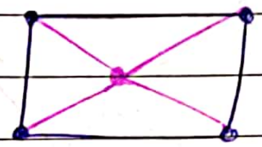
Contains single cycle.

or Some no of vertices connected in close chain form

Wheels:

We obtain the wheel W_n when we add an add^n al vertex to a cycle and connect this new vertex to each of the n vertices of C_n

Eg W_3 =  Take add^n al v.

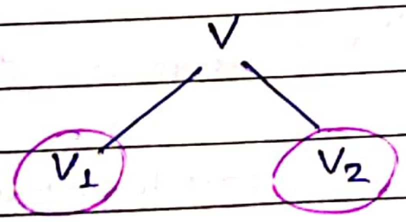
W_4 = 

Single virtex is connected ~~by~~ with all vetices of a cycle having $n-1$ vertices

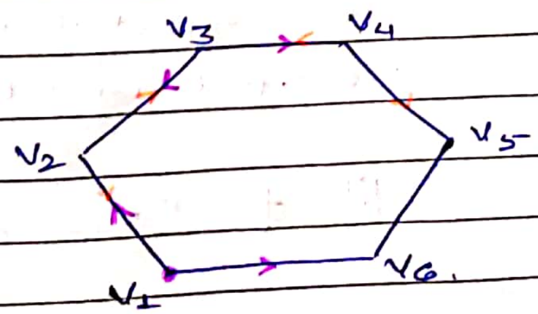
Bipartite Graph.

A simple graph is called BG if its vertex set V can be partitioned into 2 disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and vertex in V_2 (so that no edge in G connects either 2 vertices in V_1 or vertices in V_2).

When this cond^n holds we call the pair V_1, V_2 a ~~bat~~ bipartition of V.



V set 1 and 2

Eg: 

No two connected directly

← C_6

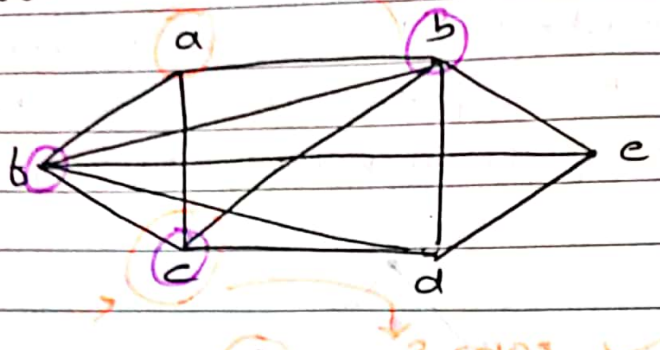
$V = \{V_1, V_2, V_3, V_4, V_5\}$

$V_1 : V_1, V_3, V_5$

$V_2 : V_2, V_6, V_4$

H.W Where to use biparted graph → ?
→ Convert mathematically

Ques Check:

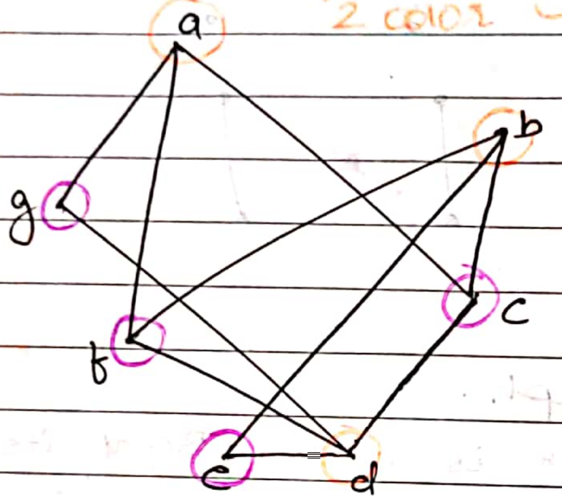


V_a

V_f V_c V_b

No

Ques Check.



a b, d

g f c e

Yes

M2: Give 2 colors to divide each vertex so

If no 2 v has same color ⇒ bip.

→ Check all adj should have diff color.

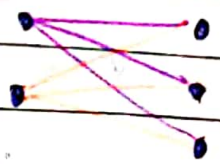
* Complete bipartite graph:

Eg: $K_{2,3}$ → Complete

→ 2 element

→ 3 elements (in other set)

Each element of set 1 is connected with each element in set 2



Illy for $K_{3,5}$, $K_{4,7}$ ... etc.

* Theoram - 1

If G(V, E) is a directed graph then the sum of out degree of vertices of a diograph equals the sum of in degrees of the V which equals the number of edges in the diograph

$$\sum_{i=1}^{n} indeg(v_i) = \sum_{i=1}^{n} outdeg(v_i) = n(e)$$

* Th. - 2.

If G(V, E) is undirected graph with E edges then the sum of degree of vertices in an undir. grap is even and exactly the twice the no. of edges.

$$\sum_{i=1}^{n} deg(v_i) = 2 \cdot |E|$$

Que P.T the no of vertices of odd degree in a graph is always even (By default undir).

$$\sum_{i=1}^{n} deg(v_i) = \sum_{even} deg(v_i) + \sum_{odd} deg(v_i)$$

↳ even ↳ must be even.

$$= 2|E|$$

↳ even

Que A graph has 24 edges and degree of each vert. is 4 find no of vertices

$n \times 4 = 24 \times 2$ Here n = no of vertices.

$n = 24/4 \times 2 = 6 \times 2$ $(n \times 4 = 4_1 + 4_2 + 4_3 \ldots 4_n)$

$= 12$ An

Que A Graph consist of 21 edges and 3 vertices of degree 4 all other vert. of degree 2 Find total no of vertices

Ans: $3 \times 4 + n \times 2 = 2 \times 21$ (no of elements)

$\hookrightarrow |E|$

Que A Graph has 24 edges and degree of each vertex is K then which of the following is possible no of vertices

(a) 20 (b) 15 (c) 10 (d) 8 ✓

$8 \times 3 = 24.$

* Operations on Graphs:

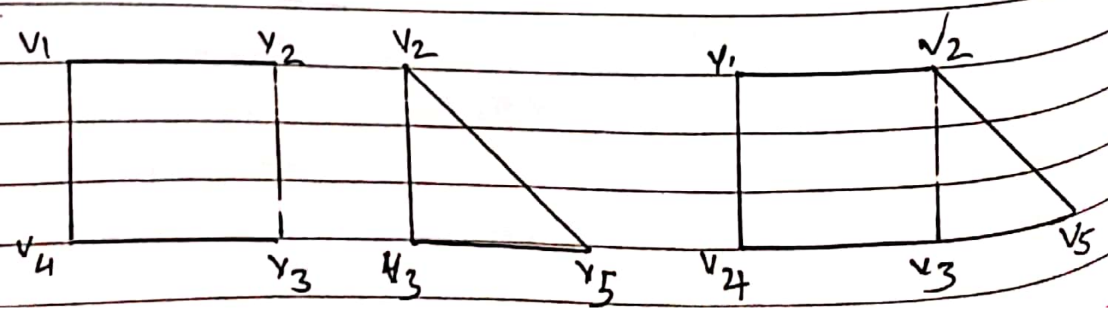
→ Union → Intersection → Ring sum
→ Decomposition → Deletion → Fusion

→ Union

$G_1(V_1, E_1)$ and $G_2(V_2, E_2)$

$\downarrow$

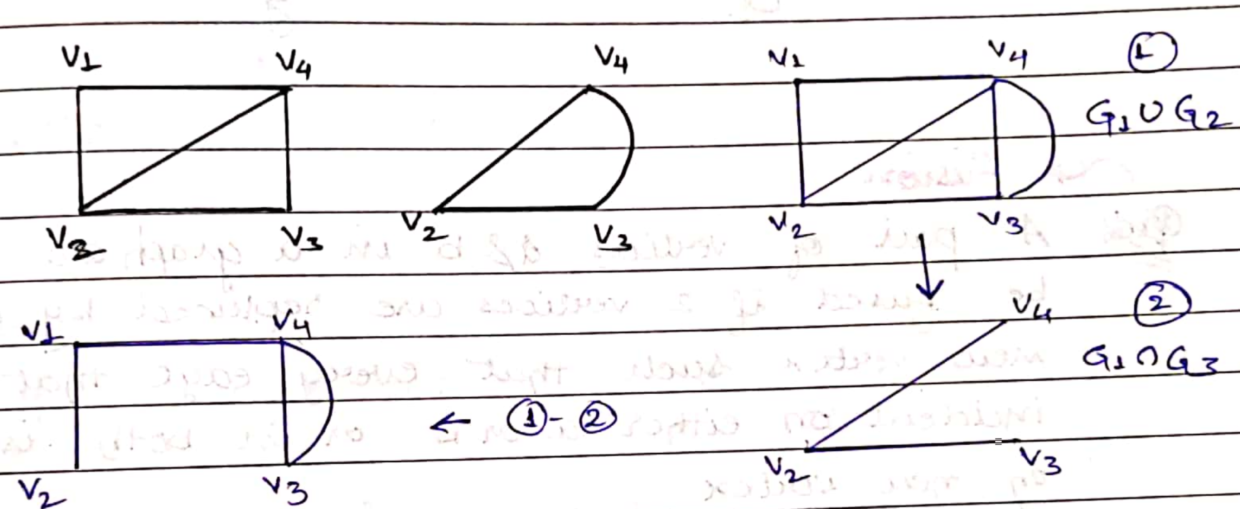
$G_3(V_3, E_3)$



Ring Sum :

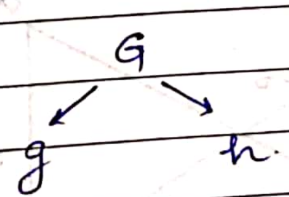
Ring sum is represented by ⊕

It consist of the vertex set $V_1 \cup V_2$ and edges that are either in G_1 and G_2 but not in both.



Decomposition :

A Graph G is said to have been decomposed into two sub graph 'g' and 'h' if $g \cup h = G$ and $g \cap h = \phi$

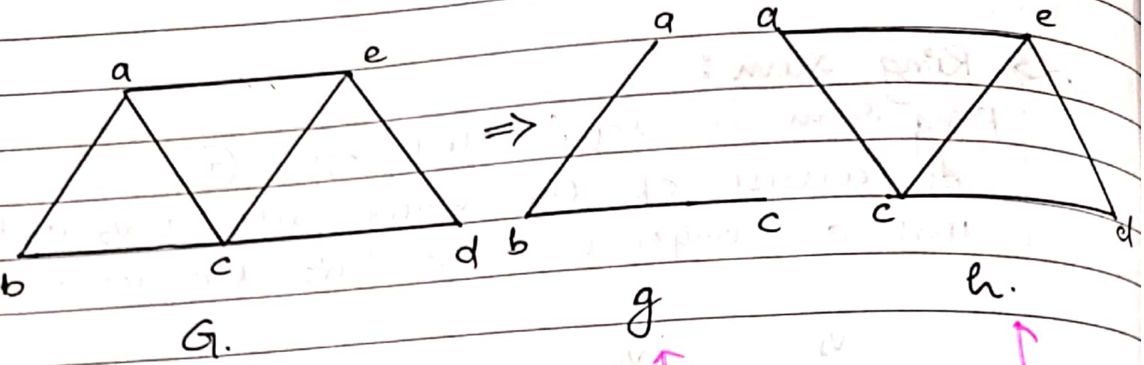


Each edge of G either occur in g or h but not in both but some vertices may occur in both.

⇒ In decomposition isolated vertices are not considered

→ Deleting vertex (Like b) edges from it also deleted

→ Deleting edge don't delete vertex



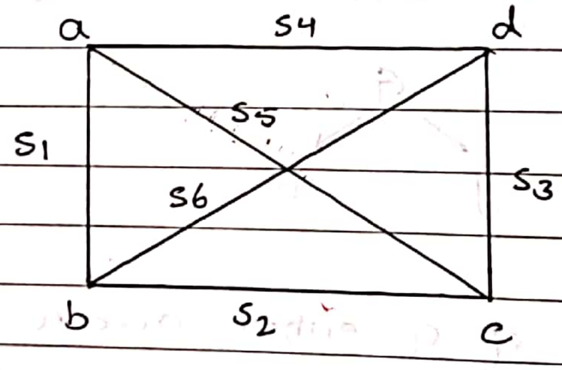
Subgraphs

Fusion

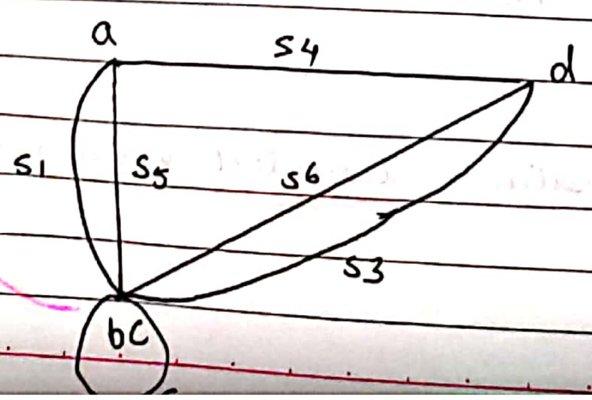
Def A pair of vertices a & b in a graph are said to be fused if 2 vertices are replaced by a single new vertex such that every edge that was incident on either a or b or in both is incident on new vertex

→ Fusion of 2 vertices does not alter the no of edges and reduces no. of vertices by one (1).

↳ self loop



Fusion
→ Only 2 V.



* Types of Graphs :-

~ NULL Graph:

Graph having n nos. of vertices but having no edges. i.e all vertices are isolated.

$$G(V, E)$$

$$E = \phi \qquad V \neq \phi.$$

~ Complete Graph:

Loop free undirected graph where $\forall$ all vertices $v_i, v_j \in V$ but $v_i \neq v_j$. There exist an edge e $(\exists e(v_i, v_j))$

It is denoted by K_n

↳ no of vertices

same vertex

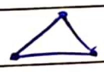
⇒ Loop   No loop.

having $\frac{n(n-1)}{2}$ edges.

K_1 is not possible

K_1

K_2 

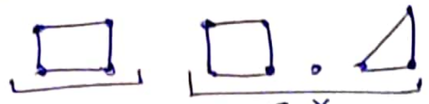
K_3  ...

~ Regular Graph:

A graph in which every vertex has a same degree is called a regular graph

If every vertex as r degree then the graph is regular graph of r degree

If a graph G has n vertices of r degree then this regular graph of r degree will have $\boxed{\frac{1}{2} rn}$ edges



Connectd CX

there exist connection b/w vertices (direct or indirect)

Que Draw connected regular graph of degree 0, 1 & 2.
(b) ~~Draw 2-3~~ Draw two 3-regular graphs with six vertices.

↳ regular graph of 3 degree.

Ans (a)

Def^n A graph is said to be connected if there exist atleast one path to reach any vertex to any other vertex through edges is k/a C. graph.

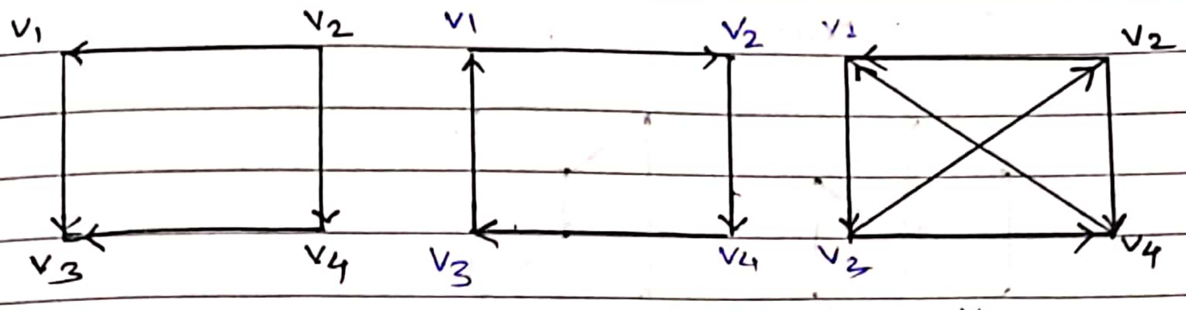
A disconnected graph is union of 2 or more connected sub graphs each pair of each which has not vertex in common

* Connected directed graph:
Let G be a directed graph the connectedness in a d. grap is denoted as

(a) connected or weakly connected if it is connected as an undirected graph where each directed edge is considered as undirected edge.

(b) directed graph G is called unilateral connected graph if for any pair of vertices of the graph at least one of them reachable from other vertex.

(c) Strongly connected if for any p of v. both of these are reachable from other vertex



$V_4 \to V_2$ X (a)

All $\to$ Pos.

(c) X

(b)

Unilateral

$V_4 \to V_2$ ~~X~~ ✓

↳ $V_4 \to V_1$

V_3 V_2

~~(b)~~

(c)

Strongly

⇒ (follow some particular dirⁿ (↻ or ↺))

Ans cond... (a)

Representⁿ 0-regular

$\frac{1}{2}\, r \times m = 0$ edges.

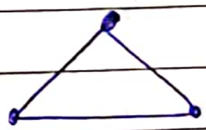
↱ 1

↳ 0

K_1

(Complete graph).

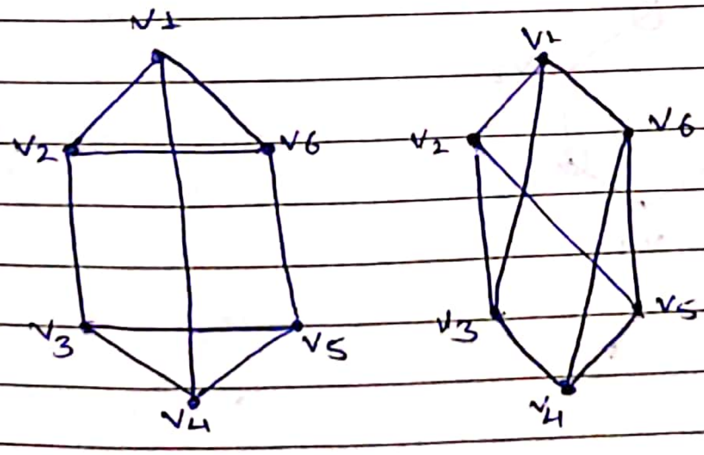
1 - regular



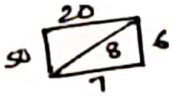
2- regular.

□ ⬠ ⬡ ....

(b)

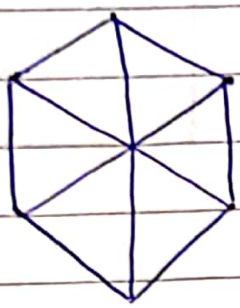


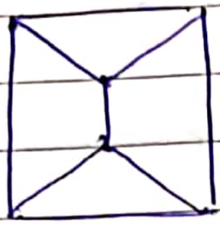
Until and unless specified draw normal graph
No multi
No pseudo.

* Weighted grap: 

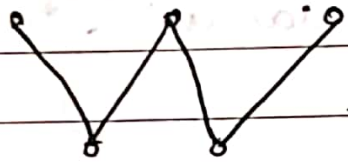
dir or undir.

Sir's ans.

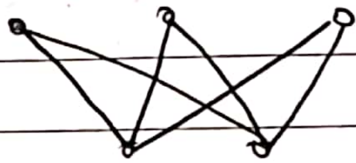




Bi partite

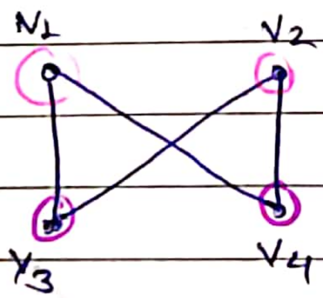


X



✓

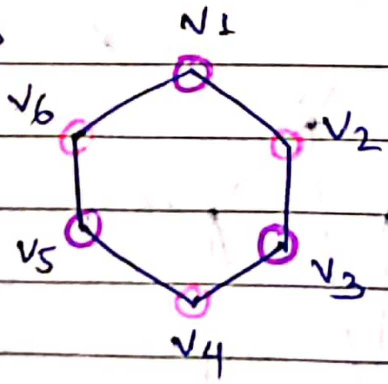
Construct a graph which is both regular and bipartite



v_1 v_2

v_3 v_4

· God's



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* New Terms

1. WALK : A sequence of V and E of a graph if we traverse a graph then we get a WALK
   (V and E can be repeated)

2. TRAIL : A walk in which no edges are repeated.
   → V can.

3. CIRCUIT : Traversing a graph s.t no edges are repeated but V can but it is close (i.e initial and final vertice is same

4. PATH : It is a trial in which neither vertices nor edges are repeated

5. CYCLE : No V Rep no E rep but it is close

WALK. — E R ✓, V R ✓

OPEN WALK
- TRAIL — E not R, V may R
- PATH — E not R, V not R

CLOSED WALK
- CIRCUIT — E not R, V may R.
- CYCLE — E not R, V not R

○ → Visited

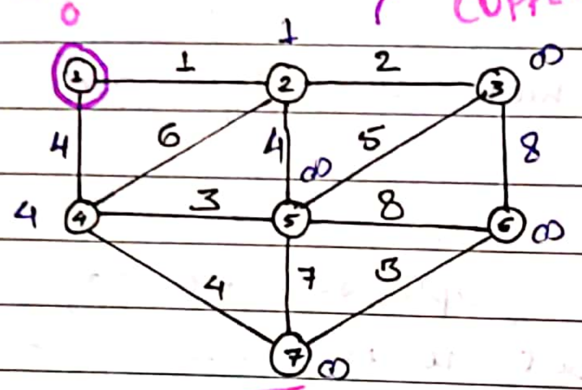
* Dijkstra's Algorithms

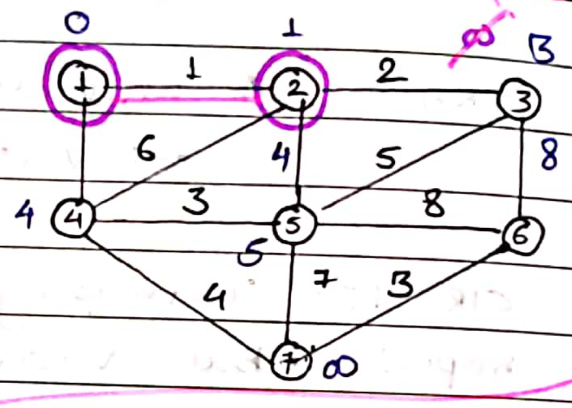
→ Shortest distance b/w 2 nodes.

Eg:-

SHORTEST PATH (UPPER VALUE)

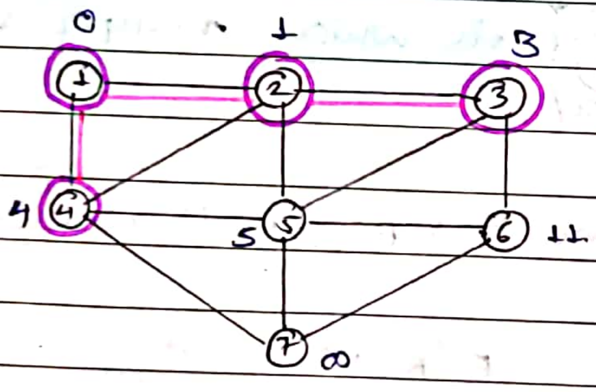
FROM 1 (1+2)

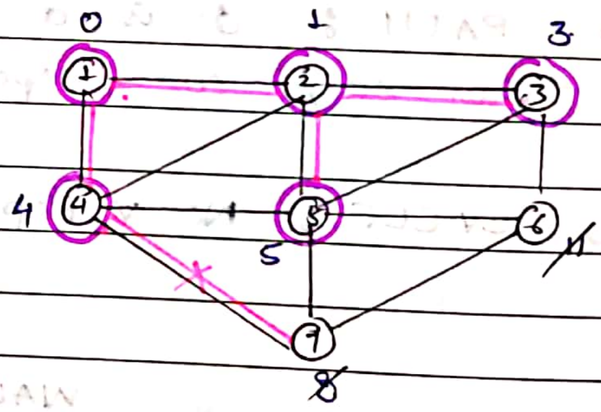


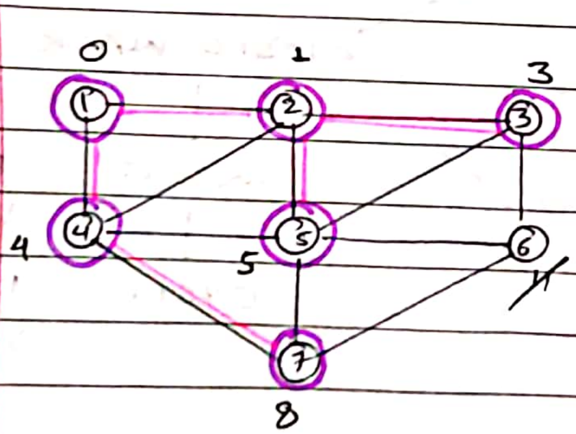


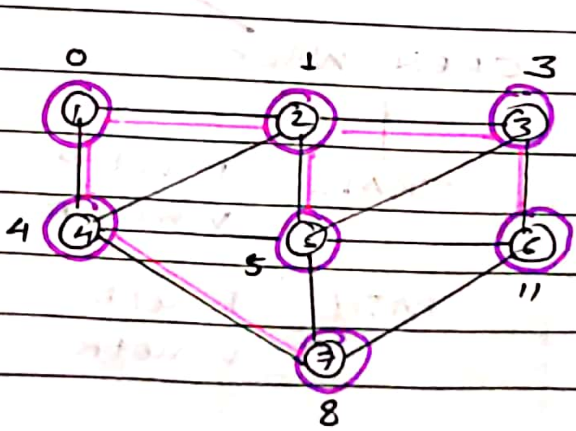
CONSIDER IF NOT DIRECTLY CONNECTED

Change it if it is less then previous value.



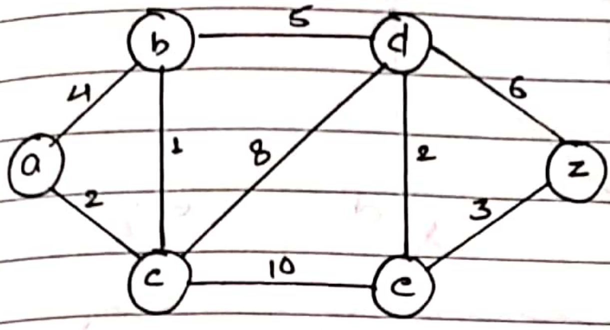






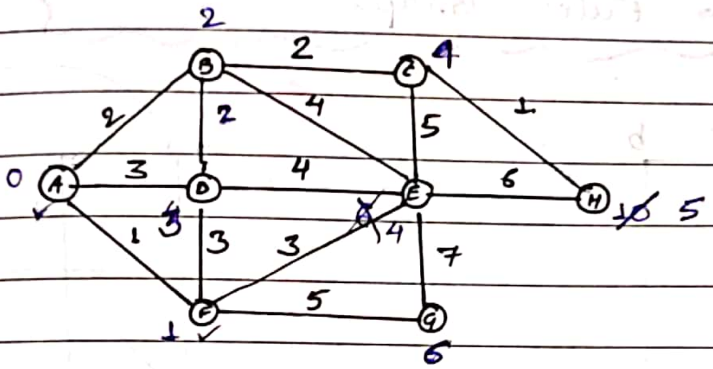
$a \to z = 13.$

Eg2:



FOLLOW THIS

Eg2



A then F

then B (move to node having min. upper value)

A → H is 5

(A → B → C → H).

* Euler Graph and Hamilton Graph

START = END ← Imp.

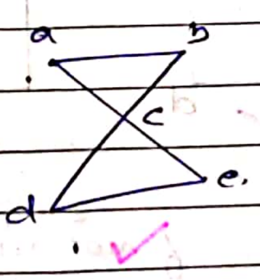
An Euler circuit is the graph G is a simple circuit containing every edge of G.

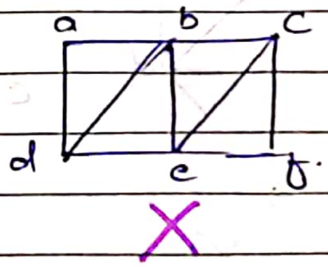
→ Edge not repeated.

→ Vertex can :)

Note: Try moving all nodes without r. edges.

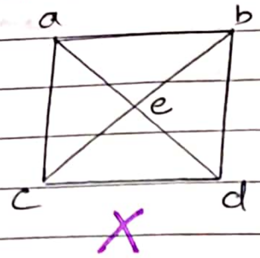
Eg:

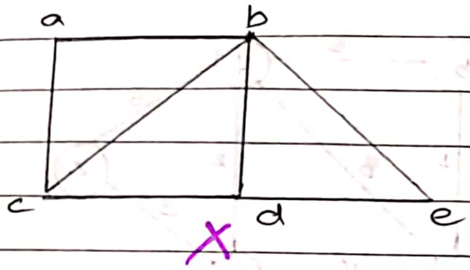




a – b – e – c – d – e – a

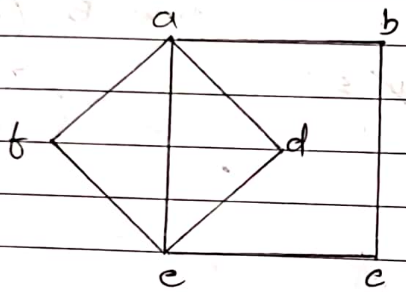
→ Start and End must be same.

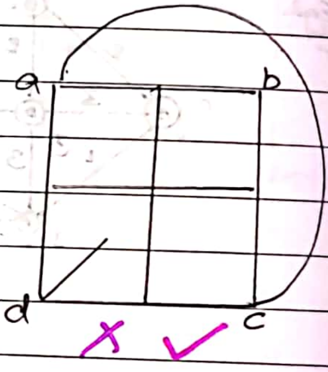




X X

→ Degree of all vertices should be even → Euler Graph.

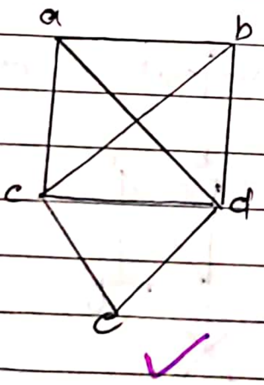


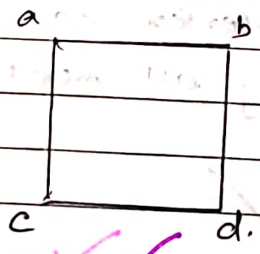


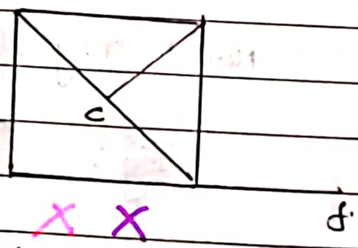
START = END

* A simple circuit in a graph G that passes through every vertex exactly once is k/a Hamilton circuit.

→ Visit all vertices, edges not compulsory to visit.



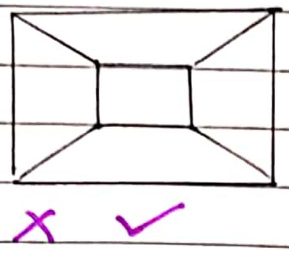




f not in circuit (pendent is present)

c has 3 also f. has 1 deg. (P is P)

→ Graph having pendent is neither EC nor HC.



* Isomorphic Graph:

⇝ If 2 graphs given $G1(V_1, E_1)$ and $G2(V_2, E_2)$ are isomorphic when they are same in structure,

⇝ If 2 graphs are iso'c then

→ G1 and G2 must have same no. of vertices

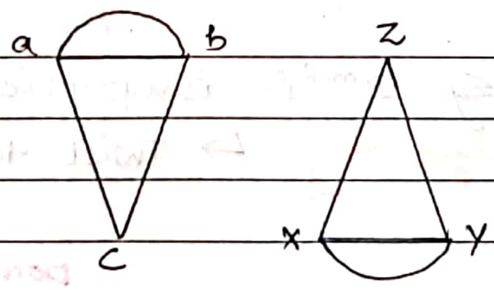
→ same edges

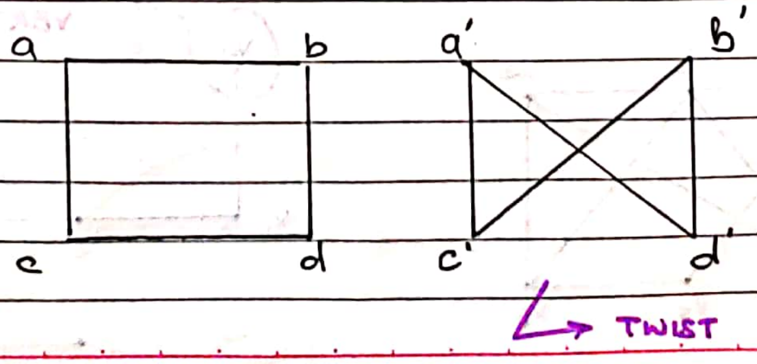
→ equal no vertices with same degree ~~eo~~

→ equal loops

⇝ If u and v are adjecent in G1 then correspondg vertices in G2 must also be adjecent.

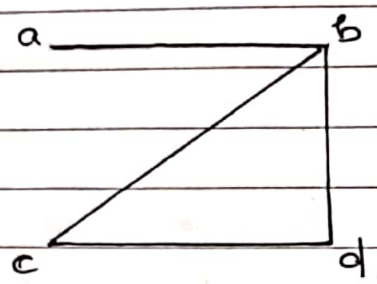
Eg:





H ⊁ Sub Graph Symbol.
↳ deleting vertices or edges

Ques Consider the Graph Shown



(a) $V' = \{a, b, f\}$; $E' = \{(a, b), (a, f)\}$ ✗
(b) $V' = \{a, b, d\}$; $E' = \{(a, b), (a, d)\}$ ✗
(c) $V' = \{a, b, c\}$; $E' = \{(a, b), (b, c), (b, d)\}$ ✗
(d) $V' = \{b, c, d\}$; $E' = \{(b, c) (b, d)\}$ ✓

✗ = NOT A SUBGRAPH

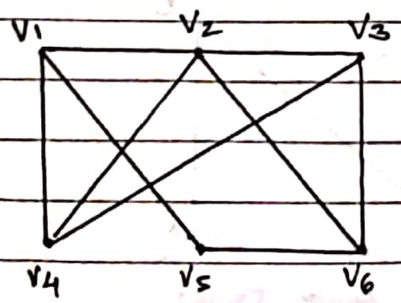
* Planar Graph :-

A graph or a multigraph is said to be Planar if it can be drawn in a plane with edges do not ~~etc~~ cross or intersect each other except at vertices.

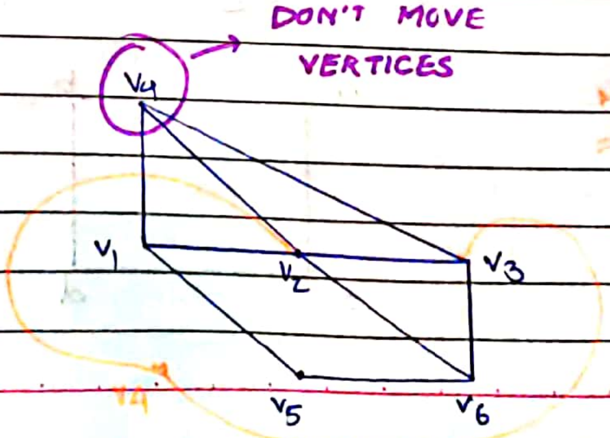
Eg: 2 nos in isomorphic. → NAMING CAN CHANGE

→ By moving edges only ↳ Twist to make planar

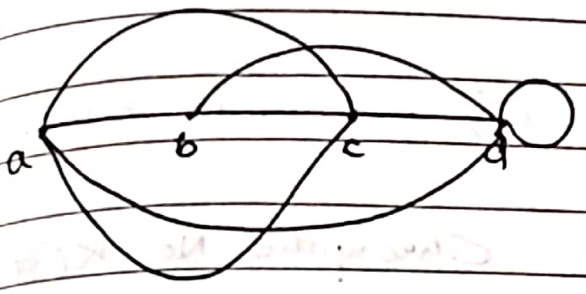
Ques
i)

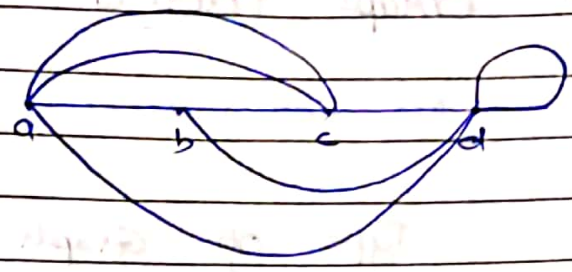


DON'T MOVE VERTICES

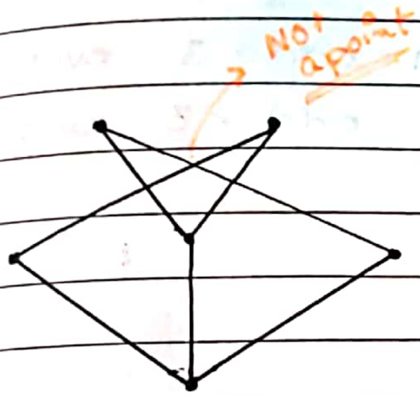


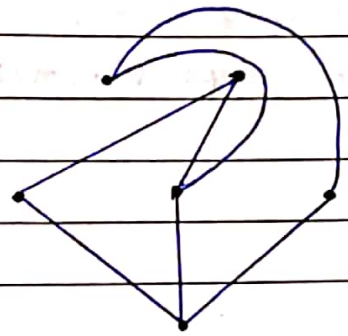
2)



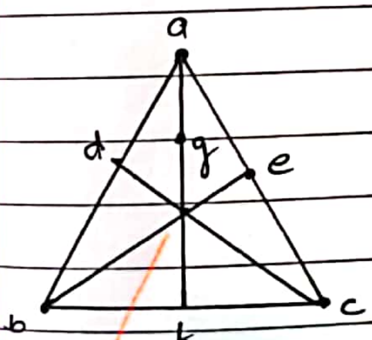


3)





4)



Non planner.

NOT A POINT

* Graph Coloring

MIN NO COLORS TO BE USED

- No two vertices (adjecent) have same color.

N NODES

1 Have degree N-1

Rest N-1 have degree 1

Type of Graph	Chromatic No K(G)
Complete graph K_n	n
Star graph C_n, $n>1$	2
Cycle graph C_n, $n>1$	n = odd ⇒ 3 even ⇒ 2
Wheel graph W_n, $n \geq 2$	odd ⇒ 3 even ⇒ 4