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UNIT - III

Graphs.

Graph is denoted by G

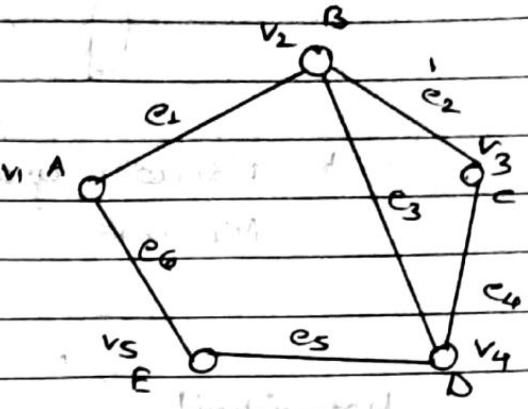
→ Set of Vertices and Edges

$G = (V, E)$

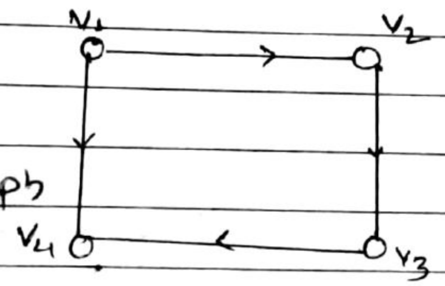
$V = \{v_1, v_2, v_3 \dots\}$

$E = \{e_1, e_2, e_3 \dots\}$

$= \{(A,B), (B,C), (B,D), (CD), (E,D), (A,E)\}$



Not in directed graph $(A \to B)$ eg.



Diagraph

Directed graph

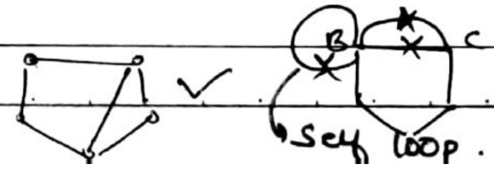
$\{(v_1, v_2) (v_2, v_3) (v_3, v_4) (v_4, v_1)\}$

Graph Loop + Parallel Edge.

Undirected	Directed.
→ Simple Graph	→ Simple directed G
→ Multi	→ Directed multigraph
→ Pseudo	

* Simple graph. ×

A graph in which each edge connects two 2 diff vertices and where no 2 edges connects same pair of vertices



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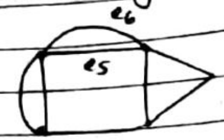
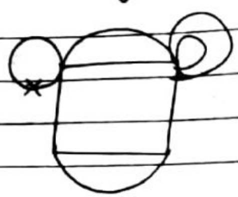
* Multi-graph : ✗
Graphs having multiple edges (not loop)

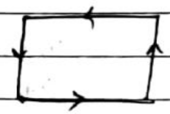


* Pseudo graph : ✓
Muliple edges and loop also.

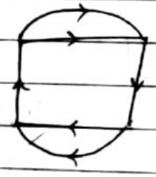


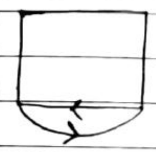
Note: We may even have more than 1 loop at a single vertex

* Simple Directed Graph ✗
simple having dirⁿ.



* Directed Multigraph



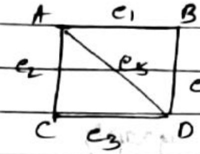


Not parallel edges.

* Mixed Graph.
In mixed graph some are directed and some are not directed.

Terms

* Adj vertex :
If 2 vertices are joined by same edge they are called AV.

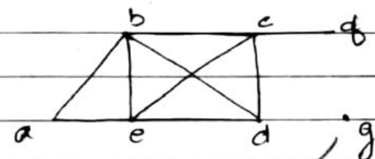


for A → B, C, D

* Adj Edge
If 2 edges are incident on same vertex.

Eg for A e_1, e_2, e_5 are adj.
for C e_2 and e_3

* Degree of an undirected graph.
The deg of vertex of an undirted graph is the no of edges incident to it. except that loop at a vertex contribute twice for deg of vertex



deg (a) = 2
(b) = 4
(c) = 4
(d) = 3
(e) = 4
(f) = 1.
(g) = 0.

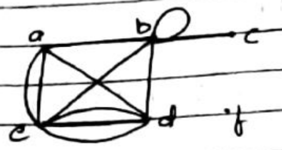
Single but valid. Grap.

Should have set of vertices
set of edges not compulsary

Degree of graph not possible.

Note : A vertex of deg = 0 is called isolated and vertex of deg = 1 is called pendent.

Eg

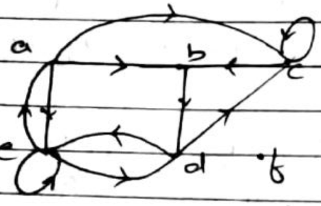


deg(a) = 4
deg(b) = 6 (2 for 1 loop)
(c) = 1 (Pendant)
(d) = 5
(e) = 6
(f) = 0 (isolated)

* Degree in a directed graph.

There are two types of degrees
→ Indegree denoted by deg(−)
→ Out-degree deg(+)

Note: Loop at vertex contributes 1 to both ID and OD of a vertex.

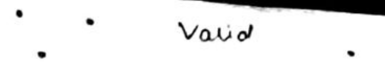


In-	Out-
deg(a) = 1	deg(a) = 3
(b) = 2	(b) = 1
(c) = 3	(c) = 2
(d) = 2	(d) = 2
(e) = 3	(e) = 3
(f) = 0	(f) = 0

* Handshaking Thm. :

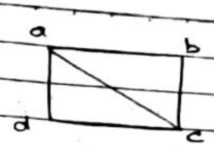
Let G=(V,E) be an undir graph with m edges then

$$\boxed{2m = \sum_{v \in V} \deg(v)}$$



Valid

Valid



(a) = 3
(b) = 2
(c) = 3
(d) = 2
} = 10

m = 5
2m = 10

Same

Qu A simple graph G has 24 edges and degree of each vertex is 4 find total no of vertices

$2 \times 24 = x \times 4$
$x = 12$.

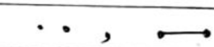
Ques A grap : 21 edges, 3 vertices of degree 4, other with degree 2 find total no. of vertices

$2 \times 21 = 3 \times 4 + x \times 2$.
$42 = 12 + 2x$
$x = 15$

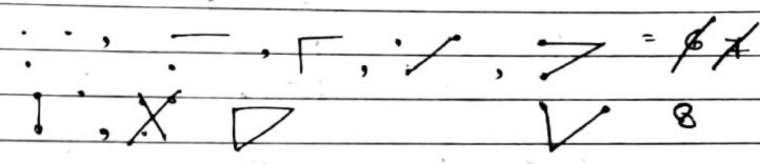
∴ Total = 15 + 3 = 18.

* With x vertex(s) no. of simple graph

→ 1 : One.
→ 2 : Two



→ 3 :



= 8

→ n

$$\boxed{2^{\frac{n(n-1)}{2}}}$$

Only Simple Graphs

* Complete Graph

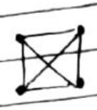
A CG of n vertices is denoted by K_n

→ It is a simple graph that contains exactly 1 edge b/w each pair of distinct vertices

for $n = 1$: (K_1)

$n = 2$: 

$n = 3$:  (every with other)

$n = 4$:  ( )

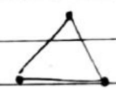
|||ly for $n = 5, 6, 7, 8 \ldots$

* Cycles :

$C_n \quad \forall\, n \geq 3$

Consist of n vertices $v_1 \ldots v_n$ and edges $\{(v_1, v_2)$, (v_2, v_3), $\ldots (v_{n-1}, v_n)$ and $(v_n, v_1)\}$

⇒ All in sequence.

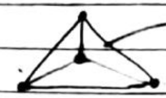
$n = 3$: C_n = 

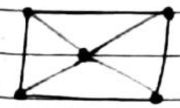
$n = 4$: C_n : 

|||ly for $n = 5, 6, 7 \ldots$

* Wheels :

We obtain the wheel W_n when we add an add^nal vertex to a cycle and connect this new vertex to each of the n vertices of C_n

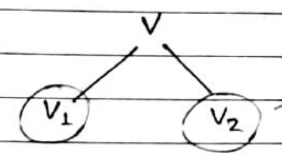
Eg W_3 =  Take add^nal v.

W_4 : 

* Bipartite Graph.

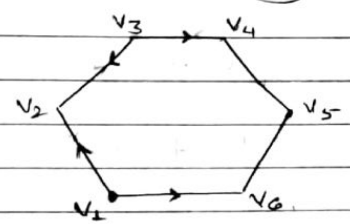
A simple graph is called BG if its vertex set V can be partitioned into 2 disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and vertex in V_2 (so that no edge in G connects either 2 vertices in V_1 or vertices in V_2)

When this cond^n holds we call the pair V_1, V_2 a ~~bat~~ bipartition of V.



V set 1 and 2

Eg :



← C_6

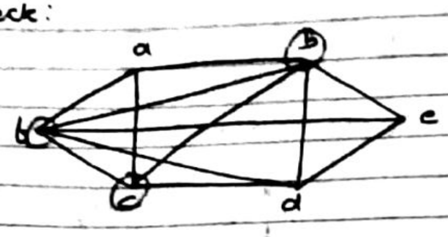
$V = \{v_1, v_2, v_3, v_4, v_5\}$

$V_1 : v_1, v_3, v_5$

$V_2 : v_2, v_6, v_4$

Where to use biparted graph → ?
→ convert mathematically

Que Check:

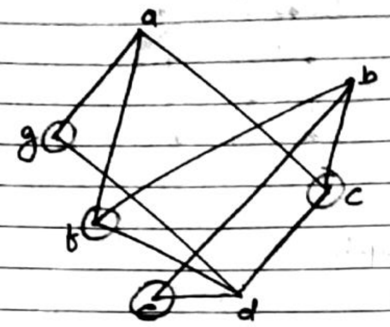


V_a

V_f V_c V_b

No

Que Check.



a b, d

g f c e

Yes

M2: Give 2 colors to each vertex & divide

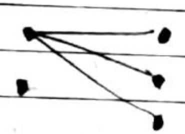
- If no 2 V has same color ⇒ bip.
- → Check all adj should have diff colors.

* Complete bipartite graph.:

Eg: $K_{2,3}$

Complete → 2 element, 3 elements (Another set)

Each element of set 1 is connected with each element in set 2



Illy for $K_{3,5}$, $K_{4,7}$ ... etc.

* Theorem - 1

If $G(V,E)$ is a directed graph then the sum of out degree of vertices of a diagraph equals the sum of in degrees of the V which equals the number of edges in the diagraph

$$\sum_{i=1}^{m} indeg(v_i) = \sum_{i=1}^{?} outdeg(v_i) = n(e)$$

* Th. - 2.

If $G(V,E)$ is undirected graph with E edges then the sum of degree of vertices in an undir. grap is even and exactly the twice the no. of edges.

$$\sum_{i=1}^{m} deg(v_i) = 2|E|$$

Que P.T the no of vertices of odd degree in a graph is always even

$$\sum_{i=1}^{m} deg(v_i) = \sum_{even} deg(v_i^e) + \sum_{odd} deg(v_i^o)$$

→ even, → must be even.

$= 2|E|$

→ even

Que A graph has 24 edges and degree of each vert. is 4 find no of vertices

$n \times 4 = 24 \times 2$ Here n = no of vertices.

$n = 24/4 = 6 \times 2$ $(n \times 4 = 4_1 + 4_2 + 4_3 \ldots 4_n)$

$= 12$ Ans

Ques A Graph consist of 21 edges and 3 vertices of degree 4 all other vert. of degree 2 Find total no of vertices

Ans: $3 \times 4 + n \times 2 = 2 \times 21$

Ques A Graph has 24 edges and degree of each vertex is K then which of the following is possible no of vertices

(a) 20 (b) 15 (c) 10 (d) 8

$8 \times 3 = 24.$

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* Operations on Graphs:

→ Union → Intersection → Ring Sum

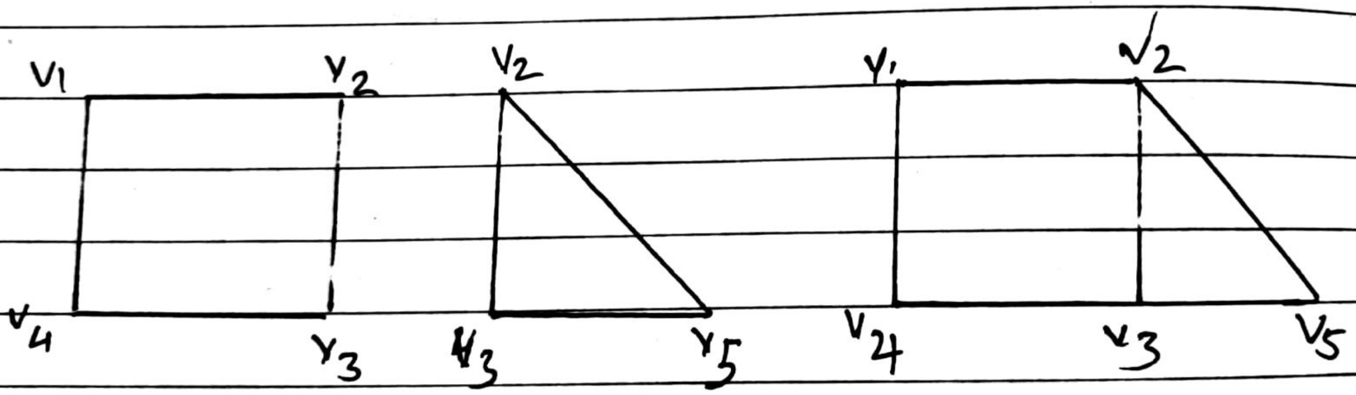
→ Decomposition → Deletion → Fusion

→ Union

$G_1 (V_1, E_1)$ and $G_2 (V_2, E_2)$

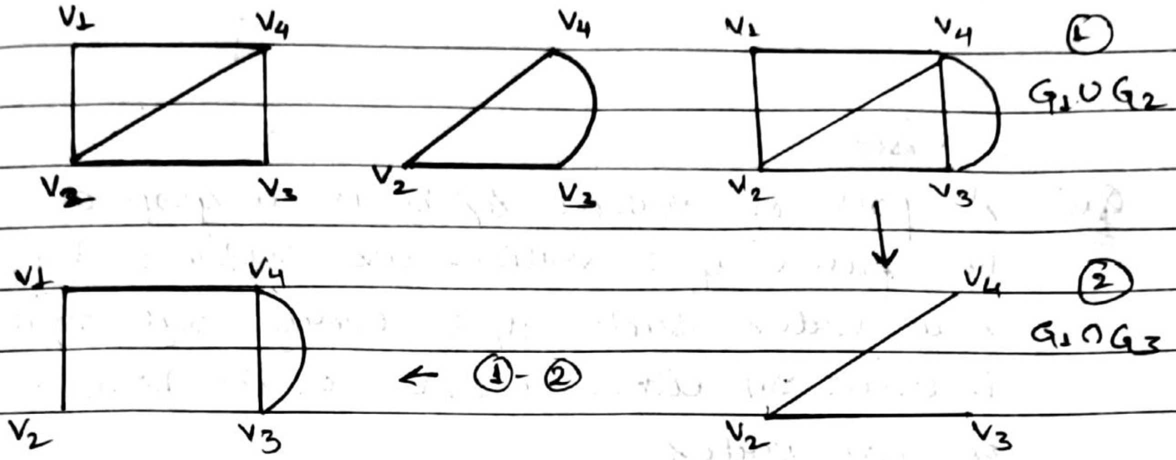
↓

$G_3 (V_3, E_3)$



Ring Sum :

Ring sum is represented by $\oplus$
It consist of the Vertex set $V_1 \cup V_2$ and edges that are either in G_1 and G_2 but not in both.



Decomposition :-

A Graph G is said to have been decomposed into two sub graph 'g' and 'h' if $g \cup h = G$ and $g \cap h = \phi$

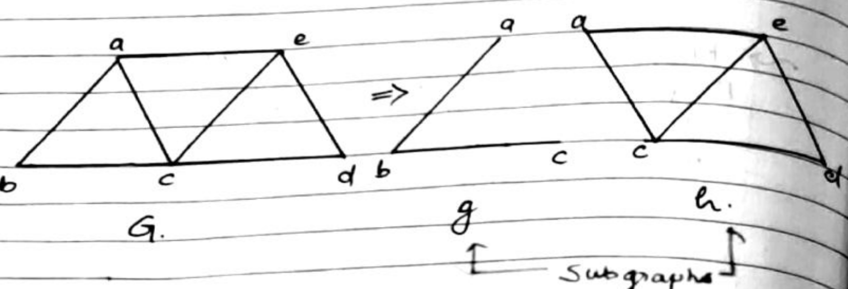
G
↙ ↘
g h.

Each edge of G either occur in g or h but not in both but some vertices may occur in both.

⇒ In decomposition isolated vertices are not consider

→ Deleting vertex (Like b) edges from it also deleted

→ Deleting edge don't delete vertex

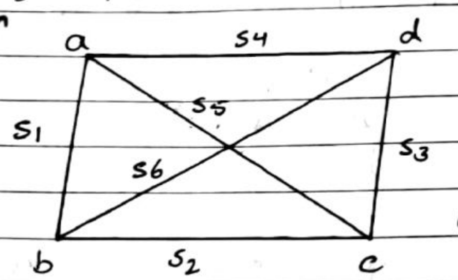


↝ Fusion

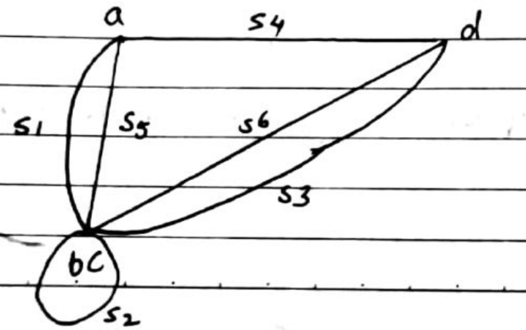
A pair of vertices a & b in a grap are said to be fused if 2 vertices are replaced by a single new vertex such that every edge that was incident on either a or b or in both is incident on new vertex

→ Fusion of 2 vertices does not alter the no of edges and reduces no. of vertices by one (1).

↳ self loop



Fusion
→ Only 2 V.



* Types of Graphs :-

↝ NULL Graph:

Graph having n nos of vertices but having no edges. i.e all vertices are isolated.

$G(V, E)$

$E = \phi \qquad V \neq \phi$.

↝ Complete Graph:

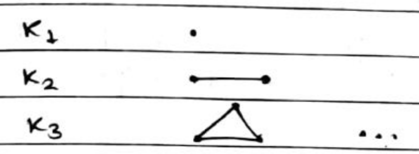
Loop free undirected graph, where $\forall$ all vertices $v_i, v_j \in V$ but $v_i \neq v_j$. There exist an edge e $(\exists e(v_i, v_j))$

same vertex → Loop   No loop.

It is denoted by K_n ↳ no of vertices

having $\frac{n(n-1)}{2}$ edges.

K_1 is not possible

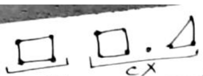


↝ Regular Graph:

A graph in which every vertex has a same degree is called a regular graph

If every vertex as r degree then the graph is regular graph of r degree

If a graph G has n vertices of r degree then this regular graph of r degree will have $\boxed{\frac{1}{2} rn}$ edges



Connected   CX

→ There exist connection b/w vertices (direct or indirect)

Ques Draw connected regular graph of degree 0, 1 & 2.
(b) ~~Draw 2-3~~ Draw two 3-regular graphs with six vertices
↳ regular graph of 3 degree.

Ans (a)

def[n] A graph is said to be connected if there exist atleast one path to reach any vertex to any other vertex through edges is k/a C. graph.

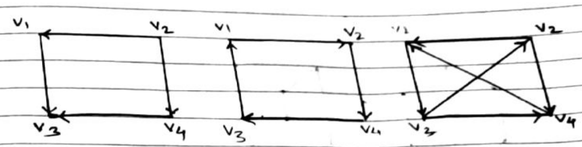
A disconnected graph is union of 2 or more connected sub graphs each pair of ~~each~~ which has not vertex in common

* Connected directed graph:
Let G be a directed graph the connectedness in a d.grap is denoted as ⇓

(a) connected or weakly connected if it is connected as an undirected graph where each directed edge is considered as undirected edge.

(b) directed graph G is called unilateral ⇓ connected graph if for any pair of vertices of the graph at least one of them reachable from other vertex.

(c) Strongly ⇓ connected if for any P of V. both of these are reachable from other vertex



$V_4 \to V_2$ X   All → Poss. $V_4 \to V_2$ ✓
↳ $V_4 \to V_1$ V_3 V_2

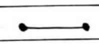
(a)   ~~(c)~~ X (b) Unilateral   ~~(b)~~ (c) Strongly

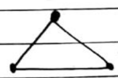
⇒ (follow some particular dir[n] (↻ or ↺))

Ans cond... (a)
Represent[n] 0-regular

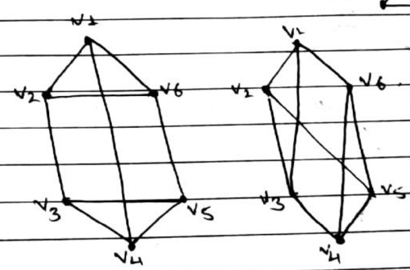
• $\frac{1}{2} \, \overset{\to 1}{r} \times m = 0$ edges.
K_1 ↲   ↳ 0

(Complete graph).

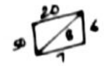

1-regular


2-regular. □ ⌂ ⬠ ....

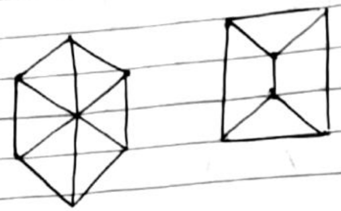
(b)



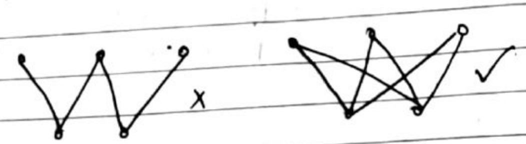
Until and unless specified draw normal graph
No multi
No pseudo.

* Weighted grap:  dir or undire

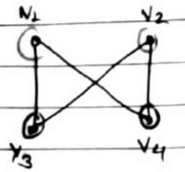
Sir's ans.



* Bi partite



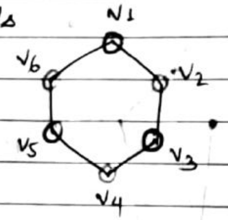
Q4 Construct a graph which is both regular and bipartite



V_1 V_2

V_3 V_4

God's



* New Terms

1. WALK: A sequence of V and E of a graph if we traverse a graph then we get a WALK (V and E can be repeated)

2. TRAIL: A walk in which no edges are repeated → V can.

3. CIRCUIT: Traversing a graph s.t no edges are repeated but V can but it is close (i.e initial and final vertice is same

4. PATH: It is a trial in which neither vertices nor edges are repeated

5. CYCLE: NO V Rep no E rep but it is close

E R ✓
V R ✓

WALK.

OPEN WALK

- TRAIL: E not R, V may R
  - PATH: E not R, V not R

CLOSED WALK

- CIRCUIT: E not R, V may R.
  - CYCLE: E not R, V not R