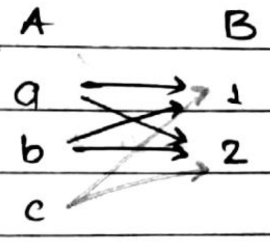
UNIT- 2.

Relations

subset of cartesian product of the n sets.

$R_1 = \{(a,1), (1,2)\}$

$R_2 = \{(b,1)(a,1)\}$



Cartesian product

$A = \{a, b, c\}$

$B = \{1, 2\}$

$A \times B = \{(a,1)(a,2)(b,1)(b,2)(c,1)(c,2)\}$

* Representing Relⁿ

→ Matrix

→ Digraph.

Eg: $A = \{1, 2, 3\}$ $B = \{1, 2\}$

$R = \{(2,1)(3,1)(3,2)\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \end{array}$$

If ordered pair = 1.
Not = 0.

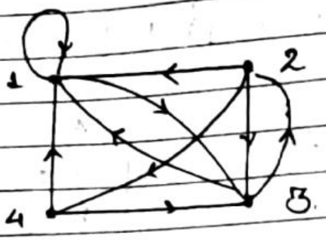
Ques $A = \{a_1, a_2, a_3\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

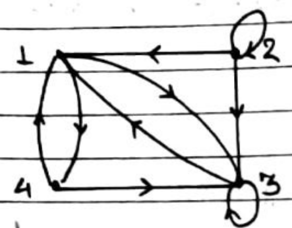
$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccccc} b_1 & b_2 & b_3 & b_4 & b_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(a_1,b_2)(a_2,b_2)(a_2,b_3)$
$\ldots\}$

* $R = \{ (1,1)(1,3)(2,1)(2,3)(2,4)(3,1)(3,2)(4,1)(4,3) \}$



Qus Write relⁿ R and Matrix representation.



$R = \{ (1,3),(1,4),(2,2),(2,1),(2,3),(3,1)(4,1)(4,3) \}$

→ From A to A relⁿ.

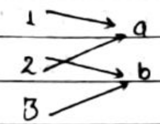
Matrix :

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

* Representing Relⁿ :-

Matrix | Diagraph.

$$\begin{array}{c|c} A \times B & \begin{matrix} a & b \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$



$R_1 = \{ (1,a)(2,b)(2,a)(3,b) \}$

* Types of Relⁿs :-

→ Reflexive :-

A relⁿ R on set A is called reflexive if $(a,a) \in R$ for every element $a \in A$

Eg: $A = \{1,2,3\}$

$R_1 = \{ (1,1)(2,2)(3,3) \}$

Que $A = \{a, b, c\}$

i) $R_1 = \{(a,a)(b,b)(c,c)\}$

ii) $R_2 = A \times A$

iii) $R_3 = \phi$ ← Everything missing.

iv) $R_4 = \{ (a,a)(b,b)(c,c)(a,b)(b,c) \}$

v) $R_5 = \{ (a,b)(b,a)(a,a)(b,b) \}$

↳ (c,c) missing

→ Irreflexive :-

A relⁿ R on set A is said to be irreflexive $\forall a \in R$ $(a,a) \notin R$.

Ques $A = \{a, b, c\}$

i) $R_1 = \phi$ ← Everything present

ii) $R_2 = A \times A$

iii) $R_3 = \{(a,a)\ (b,b)\ (c,c)\}$

iv) $R_4 = \{(a,b)\ (b,a)\ (a,a)\ (b,b)\}$

v) $R_5 = \{(a,b)\ (b,a)\ (b,c)\ (a,c)\}$

→ Symmetric :

A relⁿ R on set is said to be symmetric if $\forall a,b \in R$

if $(a,b) \in R$ then $(b,a) \in R$

Ques.

i) $\{(a,b)\ (b,a)\}$

ii) $\{(b,c)\ (c,b)\ (b,b)\ (c,c)\}$

iii) $\{(a,a)\ (b,b)\ (c,c)\}$

iv) $A \times A$

v) ϕ ← Nothing to compare

vi) $\{(a,b)\ (b,c)\ (a,c)\}$ ×

vii) $\{(a,b)\ (b,a)\ (a,c)\}$ ×

→ Antisymmetric :

A relⁿ R on set A is said to be antisymmetric if $\forall a,b \in R$. if $(a,b) \in R$ and $(b,a) \in R$ then $a = b$

↳ $(b,a) \notin R$ if $a \neq b$.

* Sym. pairs not allowed diagonal pairs allowed.

(a,b) (b,a) (a,a)

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & & \\ b & & 1 & \\ c & & & 1 \end{array}$$

Ques :

i) $\{(a,b)\ (b,c)\ (a,c)\}$ ✓

ii) $\{(a,b)\ (a,a)\ (b,b)\}$ ×

iii) $\{(a,a)\ (b,b)\ (c,c)\}$ ×

iv) $\{(a,b)\ (b,a)\ (b,c)\ (c,c)\}$ ×

v) $A \times A$ ← Gives all pairs ×

vi) $\{(a,b)\ (b,c)\ (a,c)\ (c,a)\ (a,a)\ (c,c)\}$ ×

vii) ϕ ✓

→ Asymmetric :

A relⁿ R on set A is asymmetric if $\forall a,b \in R$

if $(a,b) \in R$ then $(b,a) \notin R$.

→ Transitive

A relⁿ R on set A is said to be transitive if $\forall a,b \in A$, $(a,b) \in R$ & $(b,c) \in R$ then $(a,c) \in R$.

if ↑ and ↑ then must ↑

⇝ if only (a,b) is there but (b,c) is not there then still it is trans.

Ques

i) $R_1 = \{(a,b)\ (b,a)\ (a,a)\ (b,b)\}$

$a \to b \to a$ ✓

then

$a \to b \to b$

ii) $\{(a,a)\ (b,b)\ (c,c)\}$

iii) $R_3 = \{(a,b)\}$ ← No relⁿ starting with b

iv) $R_4 = \{(a,b), (c,b)\}$

Ques $A = \{1, 2, 3, 4, 5\}$ → ~~No~~ ulta.

i) $\{(2,2)(2,3)(2,4)(3,2)(3,3)(3,4)\}$

Not reflexive ~ Not Antisymmetric

Not Symmetric — Not Asymmetric

Not irreflexive — ~~Not~~ Transitive.

ii) $\{(1,1)(1,2)(2,1)(2,2)(3,3)(4,4)\}$

Reflexive.

Symmetric

Transitive.

iii) $\{(2,4)(4,2)\}$

Symmetric, Irr. Not transitive (2,2) not present

iv) $\{(1,2)(2,3)(3,4)\}$

Irreflexive, Anti, Asy.

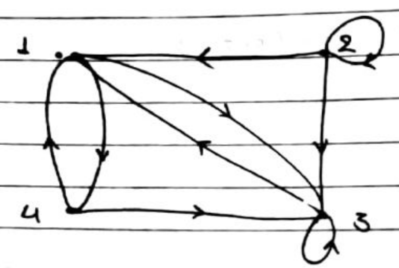
v) $\{(1,1)(2,2)(3,3)(4,4)\}$

Reflexive, Symmetric, Antisymmetric, Transitive.

vi) $\{(1,3)(1,4)(2,3)(2,4)(3,1)(3,4)\}$

Irreflexive.

Ques



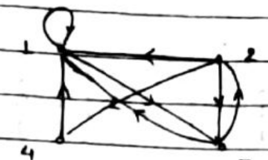
$A = \{1, 2, 3, 4\}$

$R = \{(1,3)(1,4)(2,1)(2,3)(2,2)(3,3)(3,1)(4,1)$
$(4,3)\}$

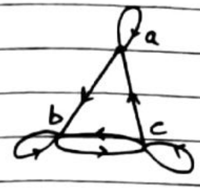
Ques Draw:

$R = \{(1,1)(1,3)(2,1)(2,3)(2,4)(3,1)(3,2)(4,1)\}$

Set $\{1,2,3,4\}$



Ques



$R = \{(a,a)(a,b)(b,b)(b,c)(c,a)(c,b)$
$(c,c)\}$

Symmetric ✗

Reflexive

	a	b	c
a	1	1	0
b	0	1	1
c	1	1	1

Matrix representⁿ.

* Partial Order Relⁿ :-

A relⁿ R on set A is POR if it is Reflexive, Antisymmetric and Transitive

Ques $A = \{1, 2, 3\}$

i) $R_1 = \phi$ Not ref.

ii) $R_2 = \{(1,1)(2,2)(3,3)\}$

iii) $R_3 = \{(1,1)(2,2)(3,3)(1,3)(2,3)\}$

iv) $R_4 = A \times A$

* Equivalence Rel^m:

A rel^m R on A is said to be equivalence if it is reflexive, symmetric and Transitive.

Ques

$$\begin{array}{c|c} & a \ b \ c \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} , \quad \begin{array}{c|c} & a \ b \ c \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array} , \quad \begin{array}{c|c} & a \ b \ c \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Transitive ~~Difficult~~

R = {(a,a) (a,c) (b,b) (c,a) (c,c)}	R = {(a,b)}	Not Ref	POR = ?
R ✓ An ✓	Not Ref	POR ✗.	
T ✓	POR ✗	Eq ✗.	
POR ✓	Eq ✗		

Ques

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Not Ref	Not Sym.	Not Ref
POR ✗	POR ✗	POR ✗
Eq ✗	Eq ✗.	Eq ✗

Ques A = {1, 2, 3, 4}

R_1 = {(1,2) (1,3) (1,4) (2,3) (2,4) (3,1) (3,2) (3,4) (4,1) (4,2) (4,3)}

R_2 = {(1,2) (1,3) (1,4) (2,3) (2,4) (3,4)}

R_3 = {(1,1)

* Equivalence Class and Partitions.

To find the equ. class and partition relm must be equ. relm

The equ. class of x is denoted by |x| or [x]

where |x| = {y | y ∈ A and (x,y) ∈ R}

Eg: A = {1, 2, 3, 4, 5}

R = {(1,1) (2,2) (3,3) (4,4) (5,5) (1,2) (2,1) (4,5) (5,4)}

|1| = {1, 2} One is with One and Two

|2| = {1, 2}

|3| = {3}

|4| = {4, 5}

|5| = {4, 5}

P_1	P_2	P_3
{1,2}	{3}	{4,5}.

$$\boxed{\begin{array}{l} P_1 \cup P_2 \cup P_3 = A \\ P_1 \cap P_2 \cap P_3 = \phi. \end{array}}$$

* Closure :-

* Reflexive closure: The RC of R (Rel^m) is R together with all pairs (a,a)

Ex: A = {0, 1, 2, 3}

R = {(0,1) (1,1) (1,2) (2,0) (2,2) (3,0)}

R^* = {$\underbrace{(0,0)\ (3,3)}_{\text{Added}}$ (0,1) (1,1) (1,2) (2,0) (2,2) (3,0)}

Symmetric Closure

The SC of R is R together with all pairs (b,a) such (a,b) is in R.

Eg: $A = \{0,1,2,3\}$

$R = \{(0,1)(1,1)(1,2)(2,0)(2,2)(3,0)\}$

$R^* = \{(0,1)(1,0)(1,1)(1,2)(2,1)(2,0)(0,2)(2,2)(3,0)(0,3)\}$

↳ Added.

Transitive Closure:

⟶ Warshall's Algorithm

Ex: $A = \{1,2,3,4\}$

$R = \{(1,2)(2,1)(2,3)(3,4)\}$

$$M_R = \begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\end{array} = W_0$$

Will go till W_4

4 elements.

Column = 2, Row = 2] Where we are getting 1 first.

$C \times R = (2,2)$

put 1 at (2,2)

$$W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Column = 1, 2

Rows = 1, 2, 3

= (1,1)(1,2)(1,3)(2,1)(2,2)(2,3)

$$W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Column = 1, 2

Row = 4

(1,4)(2,4)

$$W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Column = 1, 2, 3

Row = ϕ

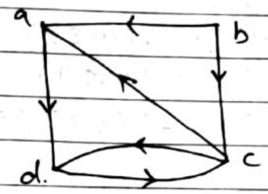
$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4 elements in A.

$R^* = \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)(2,4)(3,4)\}$.

↳ This is Transitive.

Que $A = \{a,b,c,d\}$



$R = \{(a,d)(b,a)(b,c)(c,a)(c,d)(d,c)\}$

$$M_R = \begin{array}{c|c} & \begin{matrix}a & b & c & d\end{matrix} \\ \begin{matrix}a\\b\\c\\d\end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\end{array} = W_0$$

Column = b, c

Row = d

$C \times R = (b,d)(c,d)$.

$$W_1 = \begin{array}{c|c} & \begin{matrix}a & b & c & d\end{matrix} \\ \begin{matrix}a\\b\\c\\d\end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\end{array}$$

Column = ϕ

Rows = a, c, d

$$W_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Column = b, d
Rows = a, d
· (b,a)(b,d)(d,a)(d,d)

$$W_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Columns = a, b, c, d
Rows = a, c, d
(a,a)(a,c)(a,d)
(b,a)(b,c)(b,d)
(c,a)(c,c)(c,d)
(d,a)(d,c)(d,d)

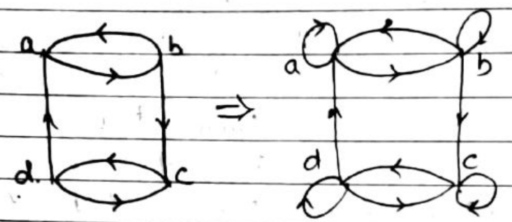
$$W_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1. \end{bmatrix}$$

Ques Use WA and find TC

$A = \{1, 2, 3, 4\}$

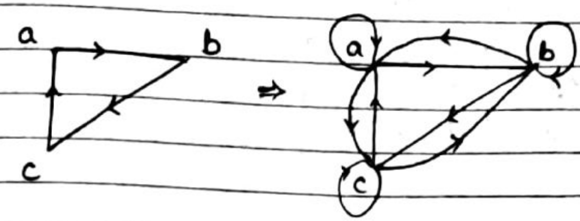
(a) {(1,2) (2,1) (2,3) (3,4) (4,1)}
(b) {(2,1) (2,3) (3,1) (3,4) (4,1) (4,3)}
(c) {(1,2) (1,3) (1,4) (2,3) (2,4) (3,4)}
(d) {(1,1) (1,4) (2,1) (2,3) (3,1) (3,2) (3,4) (4,2)}

Ques



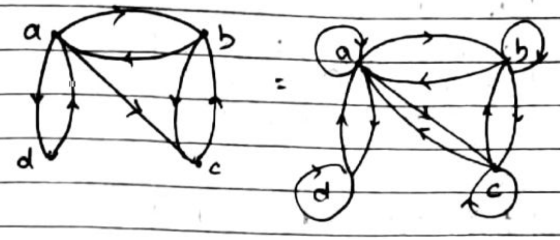
(Reflexive Closure Diagraph.)

Ques



Reflexive and Symmetric C.D.

(b)



* Transitive ~~Cot~~ Closure Th^m :-

Let M_R be the 0,1 matrix of rel^n R on a set with n elements then 0,1 matrix of Transitive Closure is

$$M_R^{*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \text{ - - - - } M_R^{[n]}$$

M_R closure

[r^{th} boolean product of A is denoted by $A^{[r]}$]

$$\boxed{\begin{aligned} A^{[2]} &= A \odot A \\ A^{[3]} &= A^{[2]} \odot A \\ A^{[4]} &= A^{[3]} \odot A. \end{aligned}}$$

$M_R^{[2]} = M_R \odot M_R$
$M_R^{[3]} = M_R^{[2]} \odot M_R$
$M_R^{[4]} = M_R^{[3]} \odot M_R$

Qu4 Find TC using Tran Thm.

Ex: $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

M_R

$M_R^{(2)} = M_R \odot M_R$

$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 \vee 0 \vee 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$M_R^{(3)} = M_R^{(2)} \odot M_R$

$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$M_R^* = M_R \vee M_R^{(2)} \vee M_R^{(3)}$

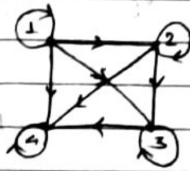
$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

* Hasse Diagram | POSET Diagram

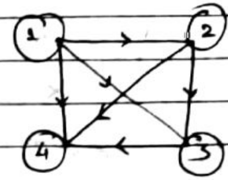
→ If a relⁿ is partial order relⁿ then only we can draw the HD of it.

$A = \{1, 2, 3, 4\}$

$R = \{(1,1)(1,2)(1,3)(1,4)(2,2)(2,3)(2,4)(3,3)(3,4)(4,4)\}$.



Remove Reflexive. Pairs (i.e remove loops)

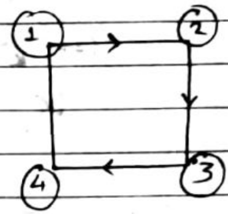


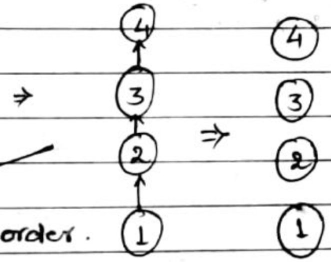
Remove Trans. Pairs

1 - 3 - 4 R (1,4)

1 - 2 - 3

2 - 3 - 4





In ↓ing order.

Read: 1 is related to 1
— do — 2
3
4

2 is R to 2
3
4

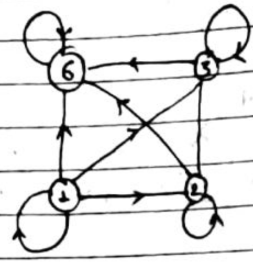
3 is R to 3
4

4 is R to 4

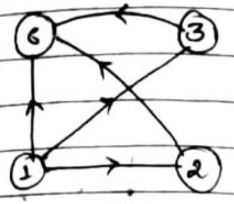
Refer page +3

Ques $A = \{1, 2, 3, 6\}$

$R = \{(1,1)(1,2)(1,3)(1,6)(2,2)(2,6)$
$(3,3)(3,6)(6,6)\}$



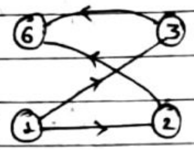
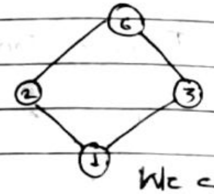
Remove Reflexive Pairs



1 - 2 - 6

1 - 3 - 6

Remove from this
not from this
i.e not from ref. pairs

=  ⇒ 

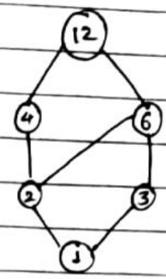
We can draw this directly with LOGIC.

Ques. (A) $[(1, 2, 3, 4, 5), \leq]$

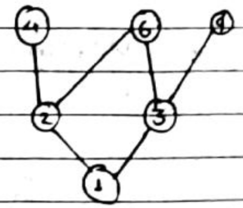
$1 \leq 1$ $2 \leq 2$ Illy for
$1 \leq 2$ 3   3, 4, 5.
3   4
4   5
5

Relⁿ only ↑ dirⁿ
Refer page +2

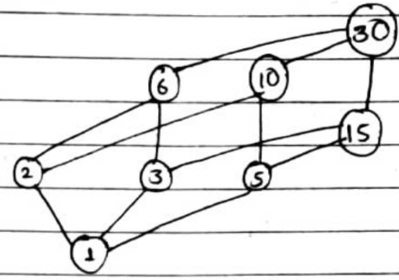
Ques $[\{1, 2, 3, 4, 6, 12\}, /]$



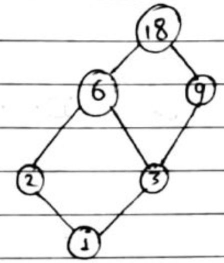
$[\{1, 2, 3, 4, 6, 9\}, /]$



Ques $[\{1, 2, 3, 5, 6, 10, 15, 30\}, /]$



Ques $[\{1, 2, 3, 6, 9, 18\}, /]$



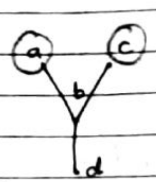
Maximal Minimal
Maximum Minimum - from.

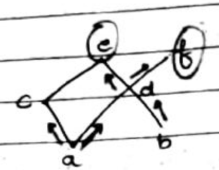
Top element.

* **Maximal Element:**
If in POSET an element is not related to any other element.

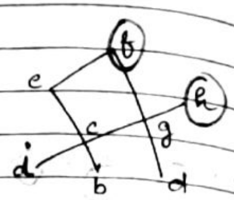
* **Minimal Element:**
No element is related to it.

Eg
Eg:

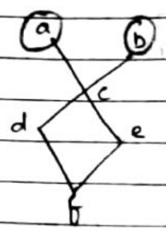




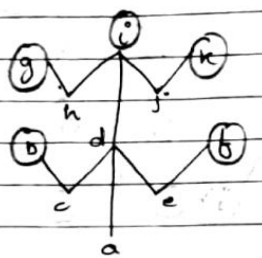
No M & m



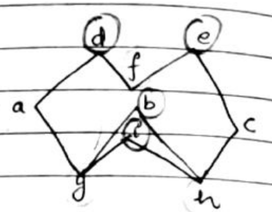
$MAX^m := \phi$
$MIN^m = d$ → Must be unique.



No max
Min^m: f



No M & m.



No M & m.

* Upper Bound and Lower Bound.

* UB :-
If [A, R] is a POSET $a \in A$ is called upper bound of subset B $(B \subseteq A)$ if xRa, $\forall x \in B$

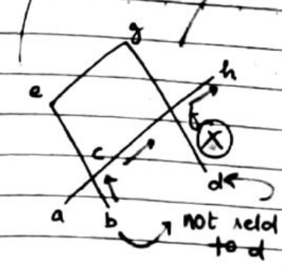
Represent^n of POSET

C is reld to e and c

Move up only

In ques.

Eg:

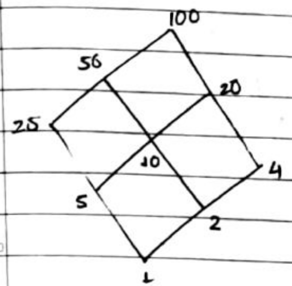


A = {a, b, c, d, e, f, g, h}
B = {e, c}

e,c e,c
UB = {e, g}
Must be reld to both.
LB = {a, b, c}

ii) B = {c, f, d}
UB = {g, h, f}
LB = {a, b} X ? = {a, b, c}
LB = ϕ.

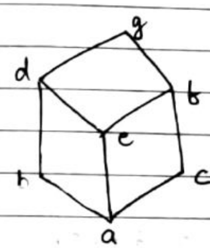
Ex:



B = {5, 10, 2, 4}
LB = {1}
UB = {20, 100}.

B = {5, 10}
LB = {1, 5}
UB = {10, 20, 50, 100}

Ex

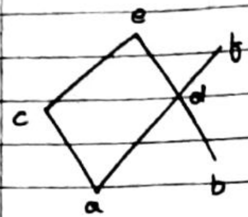


B = {d, g} LB = {d, b, a, e}
UB = {g}

B = {e, f} - LB = {a, e}
UB = {f, g}

* Least Upper Bound (LUB) → Min element in UB
  → / Join / Supreme
  → $\vee$ → Denoted
Greater Lower Bound (GLB) → Max element in LB
  ↳ / Meet / Infimum ↳ $\wedge$

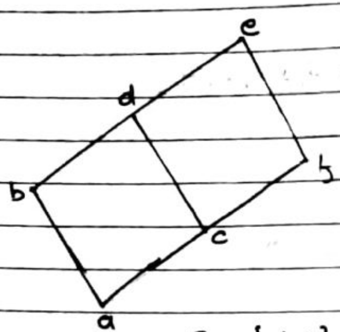
Que



B = {c, d}
UB = {e}
LUB = ~~{e}~~
LB = {a}
GLB = ~~{a}~~

B = {e, f}
UB = ϕ
LUB = ϕ
LB = {d}
GLB = ~~{d}~~

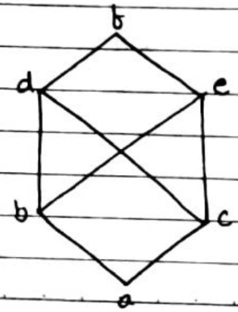
Que



B = {a, c, f}
UB = {f, e}
LUB = f
LB = {a}
GLB = a.

B = {d, c}
UB = {e, d}
LUB = d
LB = a, c
GLB = c.

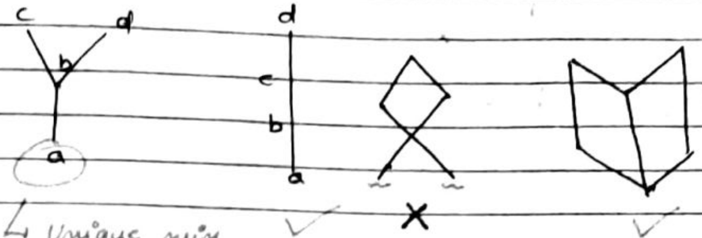
Que



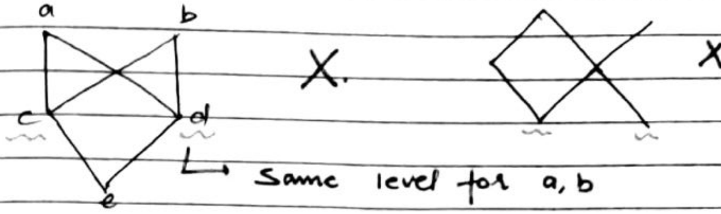
B = {d, e}
UB = {f}
LUB = f
LB = {a, b, c}
GLB = ϕ
Same level

B = {b, c} Same level
UB = {d, e, f}
LUB = ϕ
LB = a
GLB = a

* Meet - Semi Lattice
In a POSET if GLB exist for every pair of element then POSET is called meet semi lattice.

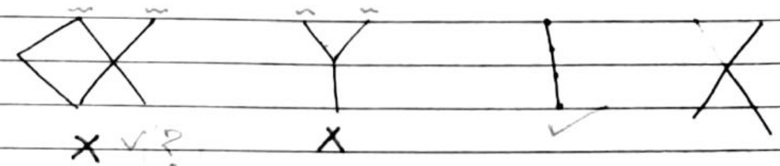


↳ Unique min element.

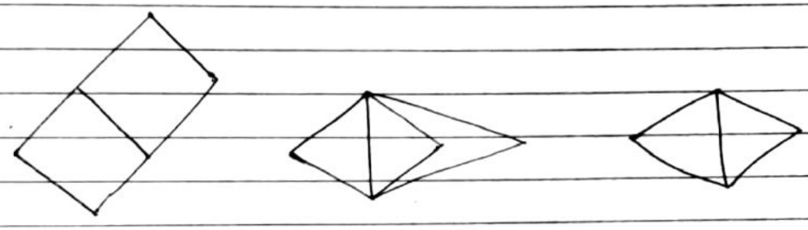


↳ Same level for a, b

* Join Semi Lattice
LUB exist ...



Que



Ques: $\{D_{110}, \mid\}$

↳ Convert to Hasse then Conclude.

$\{ \ldots\ldots\ldots \}$

$\{ \ldots\ldots\ldots \}$

UNIT - 3

Functions

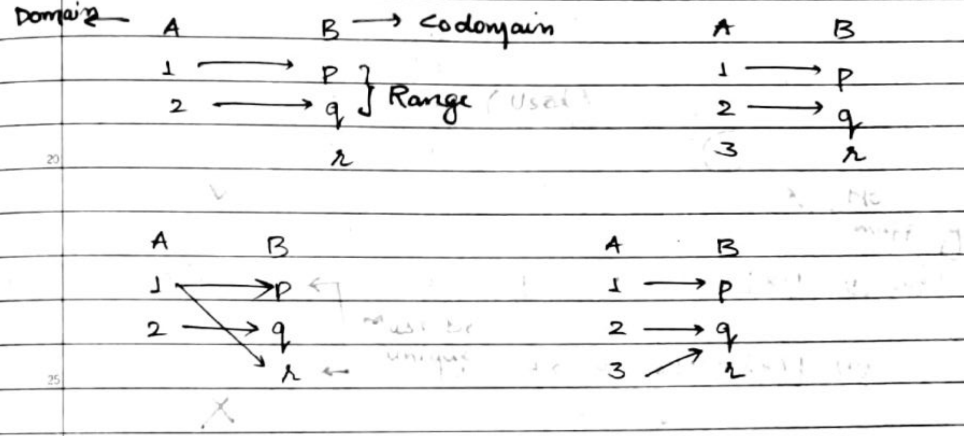
→ It is a special type of relⁿ

→ A funcⁿ F which maps from A to B ($F : A \to B$) — domain, co-domain
when every element of domain must have a mapping

→ domain is also called as pre-image

→ For every Pre-image we should get unique pre-image (In relⁿ we can avoid some elements of A but in funcⁿ we should use all elements of set A)

→ Range is subset of co-domain
Range is nothing but elements of funcⁿ
↳ also called as image.



Ques $f(x) = x^2$ $\quad R \to R$ ✓ (Real nos.)

$f(x) = \sqrt{x}$ $\quad R \to R$ ✗ { For -ve elements } no mapping.

* Types of functions :

→ One to One (Injective)

→ Onto (Surjective)

→ One to One and Onto (Bijective).

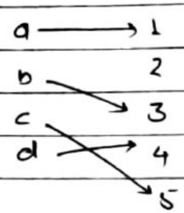
* One to One :

A func^n F is said to be one to one if

$f(a) = f(b) \Rightarrow a = b \ \forall a, b$ in domain of F.

Distinct element of A → Distinct element of B.

Eg: {a, b, c, d} to {1, 2, 3, 4, 5}



$f(a) = 1$

$f(b) = 3$

$f(c) = 5$

$f(d) = 4$.

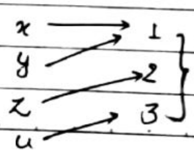
Qus (i) $f(x) = x^2$ × for 1 and −1 (int → int)

(ii) $f(x) = x+1$ $R \to R$ ✓

* Onto : $F: X \to Y$ → Capital.

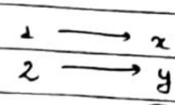
Every image in Y must have a pre image in X

Eg:



} → Every element has Yes primage.

Eg



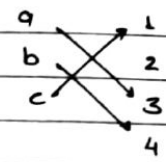
Not onto.

(z) → No pre image.

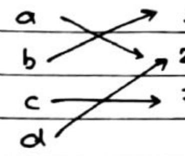
* Bijective :

If it is one to one and onto.

Ques



I



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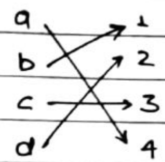
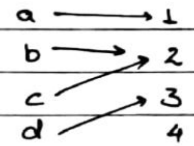
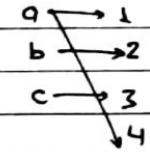


I O B



× Nothing.



Not a func^n.

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* Composition of Func^n

$g : A \longrightarrow B$

$f : B \rightarrow C$

Composition of f^n f and g
all $a \in A$ by
$f o g$

Ex: $f(x) = 2x + 3$
$g(x) = 3x + 2$

$(fog)x = f(g(x))$
$= f(3x+2) = 2(3x+2) + 3$
$= 6x + 7$.

$(gof)x = g(2x+3)$
$= 3(2x+3) + 2$
$= 6x + 11$.

$\boxed{(gof)x \longrightarrow g(f(x))}$

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* Inverse of ~~of~~ a func^n:

Given fun^n $f : A \rightarrow B$ if rel^n $f^{-1} : B \rightarrow A$ is a fun^n then it is called inverse of a func^n

Ex:

A	B		B	A
a	→ 1		1 →	a
b	→ 2		2 →	b
c	→ 3		3 →	c
	4		4	~~d~~

Not a func^n

$\therefore$ B is not ~~a~~ inverse of A.

Ex:

A	B		B	A
a	→ 1		1 →	a
b	→ 1		1 →	b
c	→ 2		2 →	c

~~B should be~~

If you want to get inverse then func^n should by bijective.

Eg: $A = \{a, b, c\}$ $\quad f(a) = 2$

$B = \{1, 2, 3\}$ $\quad f(b) = 3$

$f(c) = 1$

* Discrete Numeric fun^n:-

Let N be the set of natural nos. and R be the set of Real nos. then fun^n $f : N \rightarrow R$ is called DNf.

In practice we write DNf as a sequences. like $(f(0), f(1), f(2) \dots)$

The sequences can be written like this.

$a = a_0, a_1, a_2 \dots a_r \dots$

Ex: Let a be DNf.

$a_r = 7r^2 + 1$, $r \geq 0$

$a_0 = 7(0)+1 = 81$

$a_1 = 7(1)^2+1 = 8$

$a_2 = 7(2)^2 = 29$

* DNf can be defined by using diff. formulas

$$a_r = \begin{cases} 2+r & 0 \leq r \leq 5 \\ 2-r & r > 5 \text{ and } r \text{ is odd.} \\ r/2 & r > 5 \text{ and } r \text{ is even} \end{cases}$$

* Generating fun^n:

A Gf is a way of encoding an infinite seq. of nos. (a_n) by treating them as a coeff. of power series

Def^n : Let $a_0, a_1 \dots a_\infty$ be a series of real nos. denoted by $\{a_n\}$ then a series in power of x such that

$$g(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots \infty$$

or $\boxed{g(x) = \sum_{n=0}^{\infty} a_n x^n}$

It is used to solve the counting problems and recurrence rel^n.

Qu Find Gf for following series.

1, -1, 1, -1 ...

$g(x) = a_0 + a_1 x + a_2 x^2 + \dots$

$a_0 = 1$

$a_1 = -1$

$a_2 = 1$

$g(x) = 1 - x + x^2 - x^3 \dots$

$\Rightarrow g(x) = \sum_{n=0}^{\infty} (-1)^n (x)^n$.

ii) $2^0, 2^1, 2^3, 2^4 \dots$

$= \cancel{2^0 x + 2^1 x}$

$= 2^0 + 2^1 x + 2^2 x^2 \dots$

$= \sum_{n=0}^{\infty} 2^n x^n$.

* Recurrence Rel^n

→ It is a recursively defined sequence

→ A Reqe^n is an eq^n that recr.ly defined seq. where next term depends on prev. term.

Ex: Fibonacci series.

Qu Find first 4 terms.

i) $a_k = 2a_{k-1} + k$, $a_1 = 1$

$a_2 = 2a_{(2-1)} + 2$

$= 2a_1 + 2$

$= 4$

$a_3 = 2a_2 + 3$

$= 8 + 3$

$= 11$

$a_4 = 2a_3 + 4$

$= 2(11) + 4$

$= 26$.

ii) $a_k = a_{k-1} + 2$, $a_0 = 1$

$a_1 = a_0 + 2 = 3$

$a_2 = a_1 + 2 = 5$

$a_3 = a_2 + 2 = 7$.

* First Order Linear Recurrence Re^n:

Rec Rel^n : $a_n = C_1 a_{n-1} + C_2 a_{n_2} + \dots C_k a_{n-n}$

↓ r format

$r^n = C_1 r^{n-1} + C_2 r^{n-2} + \dots C_k r^{n-k}$

If it is a homogenous eqn then equate it to zero

$r^n - C_1 r^{n-1} - C_2 r^{n-2} \dots - C_k \boxed{r^{n-k}} = 0$

Divide by smallest r^{th} term

$\boxed{r^k - C_1 r^{k-1} + C_2 r^{k-2} \dots C_k = 0}$.

Characteristic eqn.

Ex: $a_{n+1} = 2a_n$, $a_0 = 1$

→ Convert eqn to r format

→ Divide it by lowest value of r

→ Find the value of r

→ Find values of constant (here A is constant)

→ Substitute the values of r and A in

$$\boxed{a_n = A(r^n)}$$

① $r^{n+1} = 2r^n$

② Divide by r^n ⇒ $r = 2$

③ $a_n = A(r^n)$

$a_0 = A(r^0) \leftarrow a_0 = 1.$

$1 = A(2^0)$, $\qquad A = 1.$

④ $a_n = 1(2^n)$.

* 2nd Order Linear Homogenous Rec. Reln.

→ If we have

$Ba_{n+2} + Ca_{n+1} + Da_n = 0$

→ Convert to r^n format

→ Divide it by lowest r^n term

→ Find the roots Eg $r^2 + r + 1$

r_1 and r_2

→ To Solve $a_n = A(r_1)^n + B(r_2)^n$

→ Find A and B and put in

Ex $a_n = 5a_{n-1} - 6a_{n-2}$, $a_0 = 1$, $a_1 = 1$.

① $a_n - 5a_{n-1} + 6a_{n-2} = 0$

② $r^n - 5r^{n-1} + 6r^{n-2} = 0$

③ $r^2 - 5r + 6 = 0$

$r_1 = -3 \quad r_2 = -2 \qquad r_1 = 3 \quad r_2 = 2$

④ $a_n = A.(3)^n + B(2)^n$

at $n = 0$

$a_0 = A(3)^0 + B(2)^0$

$1 = A + B$ — (i)

at $n = 1$

$a_1 = A(3)^1 + B(2)^1$

$1 = 3A + 2B$

from (i)

$1 = 3(1-B) + 2B$

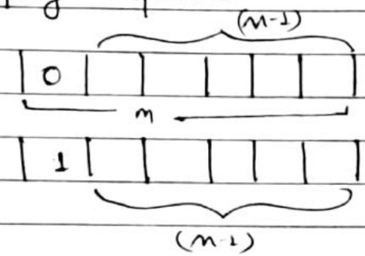
$1 = 3 - 3B + 2B$

⇒ $B = 2$.

∴ $A = -1$.

Final eqn = $a_n = -1(3)^n + 2.(2)^n$.

* Binary Sequence



$T_n = T(n-1) + T(n-1)$

$T(n) = 2T(n-1)$

$T(1) = 2$ ways

(0 or 1)

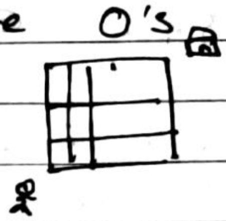
∴ When $T(2) = 2T(1)$

$= 4$ ways.

Ques 1: Binary seq without consecutive 0's

Ques 2:

Ques 3:



only R and ↑

Take only 1 or 2 steps.