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UNIT - 5

Counting

* Principle of product and sum

They are fundamental principle of counting

→ Principle of product : Outcome of random exp. occur indipendently one after the other

→ Principle of sum : Outcome

→ sometimes these 2 prin. has to combine depending on the prob.

→ If one exp has m ways to perform and another exp can occur in n ways then there are $m \times n$ possible outcomes

→ If one exp has m possible outcom another has n then there are $m+n$ possible outcomes when one of this experiment takes place

Eg CS - 6 books two diff category books.
Math - 3 books
Chem - 2 book

CS & M + CS & Chem + M & chem
= 18 + 12 + 6
= 36 Ans.

* Permutation

If there are n objects then a sequence of 'r' diff. objects is called permutation of n object taken 'r' at a time

$$\boxed{{}^{n}P_{r} \cong P(n, r)}$$

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

* Combination :

* Important deduction :

$\longrightarrow$ $P(n, r) = \dfrac{n!}{(n-r)!}$

$\longrightarrow$ $P(n, r) = n!$ if $r = n$

$\longrightarrow$ $P(n, n-1) = n!$

$\longrightarrow$ $0! = 1$.

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→ Let A containing n object then the permutation of elements of a taken r at a time allowing repetaⁿ is n^r. Each such permutⁿ is also called a sequence of length r

→ The no. of permutⁿ of n object of which 'p' objects are of one type 'q' objects are of diff type and 'r' of 3rd type and remaining are of diff types. is given by

$$\boxed{\frac{n!}{p!\,q!\,r!}}$$

REP. ALLOWED

Qu Find out how many 5 digit no gr. than 30000 can be formed from 1, 2, 3, 4, 5

Ans

REP ALL.

$3 \times 5 \times 5 \times 5 \times 5$

$= 1875$

REP NOT → ${}^4P_4 = 4!$

$3 \times 4 \times 3 \times 2 \times 1.$

$= 72.$

$3P_1$

Qu Consider a set {a, b, c, d} in how many ways can we select two of this letter, rep not allowed.

→ $4P_2 = 12.$

Qu Find out the no. of permutⁿ from letter of word ENGINEERING

$$\frac{11!}{2!\,3!\,2!\,2!}$$

$P(n,r) = \frac{n!}{(n-r)!}$ → ORDER MATTERS

$C(n,r) = \frac{n!}{r!(n-r)!}$ ✗

Que How many permutn from letter of word:

MISSISSIPPI

i) taken all together.

ii) In how of these the vowels occupying the even places.

Ans $\frac{11!}{4!\,4!\,2!} = 34650$

ii) Vowels = I I I I

MISSISSIPPI
O E O E O E O E O E O

$^5C_4 \times \frac{7!}{4!\,2!}$ $\left(\frac{5!}{4!} \times \frac{7!}{4!\,2!}\right)$

5 - E
6 - O

Que2 No of permutation from EOS CONSTITUTION

(a) Vowels occur together

(b) Consonant and vowel occur alternatively

(d) N at the beginning and end.

(a) = $\frac{12!}{2!\,2!\,3!\,2!}$

7 + (OIUIO) Pack

(B) $\frac{7!}{2!\,3!} \times \frac{5!}{2!\,2!}$ even. ✗ $\frac{8!}{2!\,3!} \times \frac{5!}{2!\,2!}$

N T

(C) → GAP method.

→ Repetition is must.

(d) N _ _ _ _ _ _ _ _ _ _ N fixed

= $\frac{10!}{2!\,3!\,2!}$

Ans CO..N has 12 characters of which there are 2N, 3T, O-2, 1 U

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∴ Total no of permutations

$$= \frac{12!}{2!\,2!\,3!\,2!}$$

(b) vowels occur together

There are seven consonants

CNNSTTT

and 5 vowels

OIUIO

To find no of perm in which 5 v. occur tog. consider them as one pack

∴ There are 7 of const. and 1 of vowels total 8 for permutation of which there are 2N and 3T ∴ Total permut^n are

$$= \frac{8!}{2!\,3!}$$

but 5 ~~v~~ which was considered as a group contains 2O and 2I can be permuted themselve in $\frac{5!}{2!\,2!}$. Hence total no of perm^n are

$$\frac{5!}{2!\,2!} \times \frac{8!}{2!\,3!}$$

(c) $\frac{7!}{2!\,3!} \times \frac{5!}{2!\,2!}$

(d)

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Que In how many ways can the letter of the word
ALLAHABAD
can be arranged.
How many of these have
→ 2 L's occur together
→ 2 L's don't occur together.

Ans (a) No of words - 9
A - 4
L - 2

∴ Total no of permutation $= \frac{9!}{4!\,2!}$

(b) 2 L's occur together (consider as one grp)
$\Rightarrow \frac{9!}{4!}$

(c) ⇒ Total - 2L's together
$= \frac{9!}{4!\,2!} - \frac{9!}{2!}$

Combⁿ

Ques A candidate is req. to answer 6 of 10 questions which are divided into two groups containing 5 questions each and he is permited to attend not more than 4 questions from any group. In how many ways can he select the questions.

Ans No of choices

= 2	4	50
3	3	100
4	2	50
		100

COMBⁿ: DON'T MATTER IN WHICH SEQ. HE IS ANSG JUST ANS from 1 and 2.

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→ ORDER DOESN'T MATTER.

Ques In how many ways a cricket team of 11 players from 14 (a) How many of them will include a particular player
(b) Exclude a particular player.

Ans (o) = $\frac{14!}{11! \times 3!}$ ← $(C_{n,r})$

(a) $^{13}C_{10}$ (One is fixed)

(b) $^{13}C_{11}$ (Options reduced).

* Generating Permutation

We have learned $n!$ permut^n of $\{1, 2, 3, \ldots n\}$ it is important to generate a list of such perm^n. If we have complete list of perm^n of $\{1, 2 \ldots n-1\}$ then we can obtain complete list of permutation for $1 \to n$ by inserting n in n ways to each permutation of the list $(1, \ldots, n-1)$

for $n = 1$, the list is 1

$n = 2$ →	1 2		
	2 1		

inserting 2 in ||ly 3

$n = 3$ →	1 2 3	2 1 3
	1 3 2	2 3 1
	3 1 2	3 2 1
$n = 4$ … →	1 2 3 4	4 1 3 2
	1 2 4 3	1 4 3 2
	1 4 2 3	1 3 4 2
	4 1 2 3	1 3 2 4

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To generate comp list of permutation for the set $1 \rightarrow n$ we assign a dir^n to each integer $k \in \{1, 2 \ldots n\}$ by writing an arrow above it pointing in some dir^n

$\boxed{\overrightarrow{k} \text{ or } \overleftarrow{k}}$

We consider permut^n of $1 \rightarrow n$ in which each integer is given a dir^n is k/a directed perm^n

IMP → And integer k in directed permut^n is called mobile if its arrow points to a smaller integer adjecent to it.

Eg: $\overrightarrow{3}\ \overrightarrow{2}\ \overleftarrow{5}\ \overrightarrow{4}\ \overrightarrow{6}\ \overrightarrow{1}$

M X M X M

It follows that 1 digit can never be mobile since there is no integer in $1 \rightarrow n$ (i.e < 1)

The integer ~~is~~ n is mobile except two cases

→ n is the first integer and its arrow points to the left ($\overleftarrow{n} \ldots$)

→ n is the last ....

.... right ($\ldots\ldots \overrightarrow{n}$)

* Generating Permutan Algorithm : $\{1, 2 \ldots n\}$

step 0 → Begin with $\overleftarrow{1}, \overleftarrow{2} \ldots \overleftarrow{n}$

Step 1 → Find the largest mobile integer . Let 'm'

Step 2 → Switch m and adjecent integer its arrow points to

MOBILE INT.

Step 3 → Switch the dir^n for all the integer p>m

Step 4 → Write down the resulting permutation with dir^n and return to step 1

step 5 → Stop if there no mobile integer.

Eg : {1, 2}

n = 2

0 → $\overleftarrow{1}\ \overleftarrow{2}$

1 → m = 2

2 → $\overleftarrow{2}\ \overleftarrow{1}$

3 → Not valid

4 → $\overleftarrow{2}\ \overleftarrow{1}$

stop

= {1 2, 2, 1}

For n = 3

0 → $\overleftarrow{1}\ \overleftarrow{2}\ \overleftarrow{3}$

1 → m = 3

2 → ~~1 2 3~~ 2 → $\overleftarrow{1}\ \overleftarrow{3}\ \overleftarrow{2}$

3 → $\overleftarrow{1}\ \overleftarrow{3}\ \overleftarrow{2}$ (No change)

4 → ~~{1, 2, 3~~ 4 → {1 2 3, 1 3 2}

1 → m = 3 → ($\overleftarrow{1}\ \overleftarrow{3}\ \overleftarrow{2}$)

~~1~~ 2 → m = 3

2 ~~3~~ → $\overleftarrow{3}\ \overleftarrow{1}\ \overleftarrow{2}$ (No change in 3)

3 →

4 → ~~{1, 2, 3~~ 4 → {$\overleftarrow{1}\overleftarrow{2}\overleftarrow{3}$, 1 3 2, $\overleftarrow{3}\overleftarrow{1}\overleftarrow{2}$}

1 → m

Step 0 - $\overleftarrow{1}$ $\overleftarrow{2}$ $\overleftarrow{3}$

Step 1 - m = 3 (out of 2 and 3)

↳ MAKE IT BOLD

Step 2 - $\overleftarrow{1}$ $\overleftarrow{3}$ $\overleftarrow{2}$

Step 3 - $\overleftarrow{1}$ $\overleftarrow{3}$ $\overleftarrow{2}$ (p = 1 and 2 but !> m)

Step 4 - $\overleftarrow{1}$ $\overleftarrow{3}$ $\overleftarrow{2}$

Step 1. - m = 3

Step 2 - $\overleftarrow{3}$ $\overleftarrow{1}$ $\overleftarrow{2}$

Step 3 - $\overleftarrow{3}$ $\overleftarrow{1}$ $\overleftarrow{2}$

Step 4 - $\overleftarrow{3}$ $\overleftarrow{1}$ $\overleftarrow{2}$

Step 1 - m = 2

Step 2 - $\overleftrightarrow{3}$ $\overleftarrow{2}$ $\overleftarrow{1}$

Step 3 - $\overrightarrow{3}$ $\overleftarrow{2}$ $\overleftarrow{1}$

Step 4 - $\overrightarrow{3}$ $\overleftrightarrow{2}$ $\overleftarrow{1}$

Step 1 - m = 3

Step 2 - $\overleftarrow{2}$ $\overrightarrow{3}$ $\overleftarrow{1}$

Step 3 -

Step 4 -

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* Generating Combination Algorithm

Random Question

If $^nC_x = 56$ and
$^nP_x = 336$
find n and x

$$\frac{n!}{x!(n-x)!} = 56 \quad \text{— (i)} \qquad \frac{n!}{(n-x)!} = 336 \quad \text{— (ii)}$$

from (i) and (ii)

$$\frac{336}{x!} = 56 \Rightarrow x! = 6$$
$$\Rightarrow x = 3$$

$$\frac{n!}{(n-3)!} = 336$$

$\Rightarrow n(n-1)(n-2)\cancel{(n-3)!} = 336 \times \cancel{(n-3)!}$

$\Rightarrow n(n-1)(n-2) = 336$

$\Rightarrow n = 8$.

Step 0 : Begin with $a_{n-1} \dots a_1 a_0 = 0 \dots 00$

Step 1 : If $a_{n-1} \dots a_1 a_0 = 1 \dots 11$ then Stop

Step 2 : If $a_{n-1} \dots a_1 a_0 \neq 1 \dots 11$

~~Step 3 :~~ Then find the smallest integer j such that $a_j = 0$

Step 3 : Change $a_j, a_{j-1} \dots a_0$ (either from 0 to 1 or 1 to 0)

write down $a_{n-1} \dots a_1 a_0$ & return to Step 1

Eg: for $n = 3$

$a_2\, a_1\, a_0 = 000$ ← SMALLEST

$a_2\, a_1\, a_0 = 001$

CHANGE ALL 1 TO 0, 0 TO 1

010

011

100

101

110

111

ALL 1 ⇒ STOP

for $n = 4$

0000

0001

0010

0011

0100

0101

0110

0111

1000

1001

1010

1011

1100

1101

1110

1111 ← STOP

* Pigeonhole Principle. :-

aka : Dirichlet Drawer Principle

: Shoebox Principle

If n pigeon holes are occupied by n+1 or more pigeon then atleast one pigeon hole is occupied by more than 1 pigeon.

This principle tells us nothing about how to loc. the pigeonhole that contains two or more pigeon. It only asserts the existence of pigeonhole containing two or more pigeon.

Que In a grp of 13 children there must be atleast 2 children who were born in same month.

Pigeons are 13 children

PH are 12 months

Since 13 > 12 by PH Theo.

∵ 13 children are born in 12 months

∴ There must be 2 children born in same mon.

Que If 9 books are to be kept in 4 shelves will it be possible to avoid atleast 3 book on one shelve.

IMPOSSIBLE

* Generalized PH Principle

If n PH are occupied by $kn+1$ or more pigeons were k is +ve integer then atleast one PH is occupied by $k+1$ or more pigeon.

Que Find the min no. of students in a class to be sure that 4 out of them are born in same month.

$n = 12$

$k+1 = 4 \Rightarrow k = 3$

$\Rightarrow kn+1 \Rightarrow 3 \times 12 + 1 = 37$

Que ⑥ Prove that if 6 integers are selected from $\{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ there must be 2 integers whose sum is 15

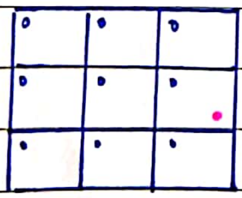
Que ⑦ Prove that if 10 pts are placed in 3x3cm square then two points must be less then or equal to $\sqrt{2}$ cm apart.

Selecting 6 Rand. no.

⑥

Box 1 :	3	12	$\boxed{3 \quad 12}$
Box 2 :	4	11	11 → 15
Box 3 :	5	10	10
Box 4 :	6	9	6
Box 5 :	7	8	8

⑦



Divide the square in 9 sq. that are 1cm x 1cm ∴
We have 10 pts to be placed in 9 squares. so there will be one square which contains two pts. Thus making max sep. of $\sqrt{2}$.

$P(E) = \frac{n(E)}{n(S)}$

Find Sample Space

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* Discrete Probabilities is

Qu There are 4R and 3B balls in bag. If one ball is taken out of this bag then represent the sample space and event of this ball being blue and find no. of elements in it.

Ans Sample Space size = 7 (R R R R B B B)

No of elements in this event = 3

Probab = 3/7.

Qu Two unbaised dice are thrown. What is the prob of

i) getting a sum of 6

ii) The no. showns are equal

iii) The diff of no. shown is 1

iv) The first die shows six

v) The total of no is greater than 1. (8) → 10/36.

Ans Sample Space size = $6^2 = 36$

i) Getting sum = 6

1:5, 2:4, 3:3; 4:2, 5:1

$= \frac{5}{36}$.

ii) Shows equal = 6/36 = 1/6

iii) Diff is 1 10/36

iv) 6/36 = 1/6

v) 36/36 = 1

Using P and C

Que A bag contains 7R and 4W balls. 2 balls are drawn at random. Find the prob that

i) Both balls are red

ii) One ball is R and other is W.

Ans Sample Space size $= {}^{11}C_2 = 55$

i) Both balls are red

$$\frac{{}^{7}C_2}{\text{Total}} = \frac{21}{55}$$

ii) One ball from each

$$\frac{{}^{7}C_1 \times {}^{4}C_1}{\text{Total}} = \frac{28}{55}$$

* Conditional Probabilities.

Let A and B be two events

$P(A)$ = prob of A

$P(B)$ = prob of B

then

$P(A/B)$ = Prob of event A w.r.t event B

2. knowing the sample space.

3. Prob. of event A to occur when event B was already occured.

$P(B/A)$ = Ulta of

$$= \boxed{\frac{P(A \cap B)}{P(A)}} \quad \text{if } P(A) \neq \phi$$

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Que FM and D are @ random lined up in queue. What is the prob that the daughter is at end when father is in the middle. ↳ FIRST OR LAST

Ans Sample space = {FMD, FDM, DFM, DMF, MFD, MDF}

SSS = 6.

Let A be the event when daughter is @ the end
B father is in middle.

$\therefore S_B = \{DFM, MFD\}$ $\therefore m(S_B) = 2$

$S_A = \{DFM, DMF, FMD, MFD\}$ $\therefore m(S_A) = 4$

$P(A/B) = 2/2 = 1$

Que A Dice when thrown twice it has been observed that the sum of it is 4 only if at least one of them on dice is 2

SSS = 36

A : Sum is 4

B : One is 2

$S_A = \{13, 22, 31\}$ $m(S_A) = 3$

$S_B = \{21, 22, 23, 24, 25, 26, 12, \cancel{22}, 32, 42, 52, 62\}$

$m(S_B) = 11$

$\therefore$ Ans $= 3/11$.

* Baye's Theoram:-

It is a formula to obtain how to update probability of hypothesis (incomplete defn) BEFORE AND AFTER EVEDENCES.

H : Hypothesis
E : Evedence.

BT states that the reln b/w prob of getting hypo before getting the evidence P(H) and prob of hypo. after getting the evidence is P(H/E) then Baye's Thm. is given by

$$BT = \boxed{P(H/E) = \frac{P(E/H) \times P(H)}{P(E)}}$$

POSTERIOR PROB — PRIOR PROBAB. — LIKELIHOOD RATIO.

Ques

Que A couple has two children the older of which is a boy.
i) What is the prob they haveaboy

A couple has two children one one of which is boy find the prob that that both are boys.?

Ans Let A: Both children are boy
B: The older children is a boy.
C: One of their children is a boy

S = { BB , BG , GB , GG }

i> B: .... (given)

$$P(A/B) = \frac{P(B/A)\,P(A)}{P(B)}$$

$P(A) = 1/4$ $\quad P(B) = 1/2$

$P(B/A) = 1$

$$\therefore P(A/B) = \frac{1 \times 1/4}{1/2} \Rightarrow 1/2 \text{ Ans.}$$

ii> C: ..... (given) : Evdnce.

$$P(A/c) = \frac{P(C/A) * P(A)}{P(C)}$$

$P(C) = \frac{3}{4}$ $\quad P(A) = 1/4$

$P(C/A) = 1$

$$\therefore P(A/C) = \frac{1 \times 1/4}{3/4} = 1/3 \text{ Ans}$$

Que An insurance company a 2000 scooter drivers 4000 car drivers and 6000 truck drivers.
The prob of accident involving scooter, car, truck individually having is given by 0.01 , 0.03 & 0.15 resp.
One of insured person met with an acc. what is the prob that he is scooter driver.
insurence.

Ans A : Scooter driver
B : Car
C : Truck
I : Person is insured.

$P(A) = 2000/12000 = 1/6$
$P(B) = 4000/12000 = 1/3$
$P(C) = 1/2$

MULTIPLE OCCURENCE

$P(I/A) = 0.01$
$P(I/B) = 0.03$
$P(I/C) = 0.15$

$P(A/I)$: He is a scooter driver

$$\boxed{P(A/I) = \frac{P(I/A) \cdot P(A)}{P(I/A) \cdot P(A) + P(I/B) \cdot P(B) + P(I/C) \cdot P(C)}}$$

* Random Variable :

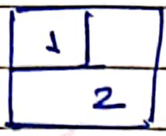
Let x be a RV.
↳ S of x is possible outcomes for the event.

i.e WHAT VALUE X CAN TAKE AS OUTPUT

1	•
•	2

X = 1 or 2 $P(x) = 1/4$ $(x = 1)$
$P(x) = 3/4$ $(x = 2)$

Eg 2.



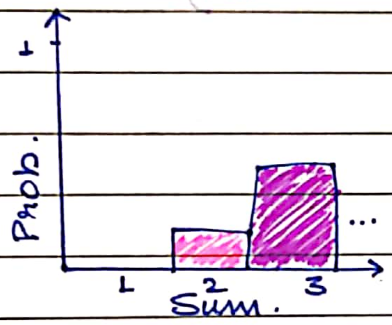
$P(1) = 0.25$
$P(2) = 0.75$

x Prob ~~of~~ sum of spin
The possible sum 2, 3, 4

$P(\text{sum } 2) = 0.25 \times 0.25$
$P(\quad 3) = 0.25 \times 0.75$
$P(\quad 4) = 0.75 \times 0.75$

sum of spin x	P(x)
2	....
⋮	

↓
PROB DIST. TABLE



→ GRAPHING

* Mean Median Mode
* Variance (σ^2)
* Standard Deviation $\sigma = \sqrt{\sigma^2}$
* Binomial Experiment
  - Each trial is independent
  - Only 2 poss. outcome
  - Prob of sucess should be same for each trials.

n = no of trial
$p = P(S)$ = Prob of success in trial
$q =$ failure
x = Random Variable

* Mean = np

Variance = $\sigma^2 = npq$

SD = $\sqrt{\sigma^2} = \sqrt{npq}$

Putting back ... type

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* Bi. Prob formula!

$$\boxed{\begin{aligned} P(x) &= {}^{n}C_{x}\, p^{x} q^{n-x} \\ &= {}^{n}C_{x}\, p^{(x)} (1-p)^{n-x} \end{aligned}}$$

* Poisson Distribution.

↳ Problems related with time.

$P(x) =$

* Gaussian

$P(x)$