Uncertainty

Cause of uncertainty

- Most intelligent system may have some degree of uncertainty associated with them
- Reason of uncertainty may be
  - Missing or unavailable data
  - Unreliable or ambiguous data
  - Presence of conflicting measurement techniques
  - knowledge representation which captures knowledge from expert's guess or observation, may not be accurate or correct
- It is not always possible to represent data in precise and consistent manner
- Hence a means to handle uncertainty in the intelligent system is desired

Uncertainty

- While implementing uncertainty scheme, three issues should be considered
  - Representation of uncertain data
  - Combination of two or more pieces of uncertain data
  - Drawing inference using uncertain data
- Methods available for handling uncertain data is
  - Probability theory
  - Fuzzy logic
  - Temporal logic

Probability-basic concept

- Probability is likelihood of occurrence of an event or chance that an event will occur.
- S be sample space representing all the possible outcome of an experiment or problem
- Probability of A is defined as
- = $$P(A) = \frac{(number\_of\_outcomes\_favourable\_to\_A)}{Total\_outcomes}$$
- = $$\frac{n(A)}{n(S)}$$

Example

- Consider the experiment of tossing three fair coins, sample sapce is given by
- S={HHH, HTH, HHT, HTT, TTT, THH, THT, TTH}
- E= Event that at least two head appear

  ={HHH, HTH, HHT, THH}
- F=event that first coin show tail

  ={TTT, TTH, THT, THH}

  n(E∩F)={THH}
- P(E)=n(E)/n(S)= 4/8=1/2
- P(F)=n(E)/n(S)=4/8=1/2
- P(E∩F)= n(E∩F)/n(S)=1/8

Conditional probability

- Probability of an event E is called conditional probability of E given that event F has already occurred denoted by P(E|F)
- Elements of F which favor E are common elements of E and F ---- sample points $E \cap F$

$$P(E \mid F) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{\frac{n(E \cap F)}{n(S)}}{\frac{n(F)}{n(S)}}$$

$$= \frac{P(E \cap F)}{P(F)}$$ ----------------------(1)

Conditional prob

- Example Given that probability of a person chosen at random being literate is 0.40 and probability of any person chosen at random having age > 60 is 0.005. Finf Probability that person chosen at random of age > 60 years if literate
- E= event that person age > 60
- F= event that person is literate
- Find $P(F|E)$
- $P(E)=0.005$ $P(F)= 0.40$
- $P(E \text{ and } F) = P(E) * P(F)= 0.005 * 0.40= 0.002$
- $P(F|E)= P(E \text{ and } F)/ P(E)=0.002/0.005=2/5$

Joint probability

- Joint probability is defined as the probability of occurrence of two independent events in conjunction
- Probability of both events occurring together
- Joint probability of A and B is written as
  - P(E∩ F) when E and F are dependent

$$P(E \bigcap F) = P(E \mid F) P(F) \quad \text{-------(2)}$$

$$P(E \bigcap F) = P(E) * P(F) \quad \text{when E and F Are independent}$$

P(EUF)

- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
- if E and F have some sample points in common
- Otherwise
- $P(E \cup F) = P(E) + P(F)$

Multiplication theorem of probability

$$P(E \cap F) = P(E \mid F)P(F)$$ (From equation (2))

Similarly

$$P(E \cap F) = P(F \mid E)P(E)$$ ------------- (3)

Using equation 2 and 3

$$P(E \mid F) * P(F) = P(F \mid E) * P(E)$$

$$P(E \mid F) = \frac{P(F \mid E) * P(E)}{P(F)}$$ ---------- (4)

This is said to be **Bayes theorem**

Example

Example : An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement . What is the probability that both drawn balls are black

E= First ball drawn is black
F= second ball drawn is black
Find P(E∩ F)

P(E)= 10/15

P(F|E)=9/14

P(E∩ F)= (9/14)*(10/15) [Eq(3)]

=3/7 Ans

Multiplication theorem of probability

- This rule can be extended to more than two events. If E, F and G are the events then
- $P(E \cap F \cap G)$

$$= P(E) * P(F|E) * P(G|E \cap F)$$

$$= P(E) * P(F|E) * P(G|EF)$$

Joint Distribution

- k random variables $X_1, \ldots, X_k$
- The joint distribution of these variables is a table in which each entry gives the probability of one combination of values of $X_1, \ldots, X_k$
- Example:

	Toothache	¬Toothache
Cavity	0.04	0.06
¬Cavity	0.01	0.89

$P(\neg Cavity \wedge Toothache)$

$P(Cavity \wedge \neg Toothache)$

Joint Distribution Says It All

	Toothache	¬Toothache
Cavity	0.04	0.06
¬Cavity	0.01	0.89

- $P(\text{Toothache}) = P((\text{Toothache} \wedge \text{Cavity}) \vee (\text{Toothache} \wedge \neg\text{Cavity}))$

  $= P(\text{Toothache} \wedge \text{Cavity}) + P(\text{Toothache} \wedge \neg\text{Cavity})$

  $= 0.04 + 0.01 = 0.05$

- $P(\text{Toothache} \vee \text{Cavity})$

  $= P((\text{Toothache} \wedge \text{Cavity}) \vee (\text{Toothache} \wedge \neg\text{Cavity})$

  $\vee (\neg\text{Toothache} \wedge \text{Cavity}))$

  $= 0.04 + 0.01 + 0.06 = 0.11$

Joint Distribution Says It All

	Toothache	¬Toothache
Cavity	0.04	0.06
¬Cavity	0.01	0.89

- P(Toothache) = ??

- P(Toothache v Cavity) = ??

Conditional Probability

- Definition:
  $P(A|B) = P(A \wedge B) / P(B)$
- Read $P(A|B)$: probability of A given B

- can also write this as:
  $P(A \wedge B) = P(A|B)\, P(B)$
- called the **product rule**

Example

	Toothache	¬Toothache
Cavity	0.04	0.06
¬Cavity	0.01	0.89

- $P(Cavity|Toothache) = P(Cavity \wedge Toothache) / P(Toothache)$
  - $P(Cavity \wedge Toothache) = ?$
  - $P(Toothache) = ?$
  - $P(Cavity|Toothache) = 0.04/0.05 = 0.8$

Bayes theorem

- There are two bags I and II
- Bag I contains 2 White and 3 Red balls
- Bag II contains 4 White and 5 Red balls
- We can find
  - Prob of selecting a bag
  - Prob of drawing a ball of a particular color , given the bag
- Can we find
  - Prob that ball is drawn from a particular bag if the color of ball is known?
- This is the case of finding reverse probability using conditional probability
- Formulae developed was given by mathematician Bayes and hence known as bayes theorem

Theorem of total probability

- Set of events $E_1, E_2, \ldots\ldots E_n$ represents the partition of the sample space S if
  (a) $E_1 \cap E_2 \cap \ldots\ldots E_n = \emptyset$
  (b) $E_1 \cup E_2 \cup \ldots\ldots \cup E_n = S$
  (c) $P(E_i) > 0$

Theorem : Let $\{E_1, E_2, \ldots\ldots E_n\}$ be a partition of the sample space S and suppose that each of the events $E_1, E_2, \ldots\ldots E_n$ has non zero probabaility. Let A be any event associated with S then

$P(A) = P(E_1)\ P(A|E_1) + P(E_2)\ P(A|E_2), \ldots\ldots + P(E_n)\ P(A|E_n)$

$$= \sum_{j=1}^{n} P(E_i) P(A \mid E_i)$$

Bayes theorem

- If $E_1, E_2, \ldots\ldots E_n$ are non empty events which constitute a partition of sample space S such that $E_1 \cup E_2 \cup \ldots\ldots \cup E_n = S$ and E_i ‘s are disjoint and A is any event of non zero probability, then

$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)P(A \mid E_j)}$$