Constraint Satisfaction Problem (CSP)

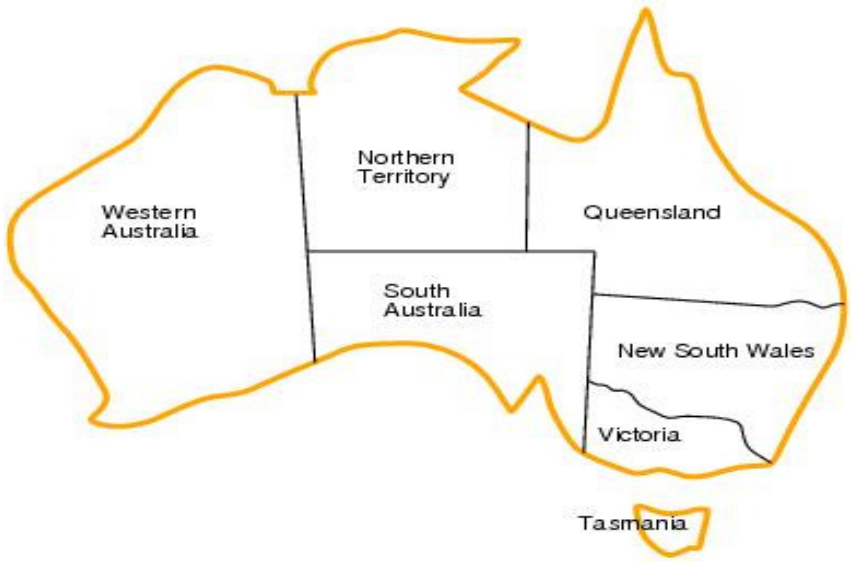
Constraint Satisfaction Problems

- What is a CSP?
  - Finite set of variables $V_1, V_2, \ldots, V_n$
    - Nonempty domain of possible values for each variable $D_{V1}, D_{V2}, \ldots D_{Vn}$
  - Finite set of constraints $C_1, C_2, \ldots, C_m$
    - Each constraint C_i limits the values that variables can take,
      - e.g., $V_1 \neq V_2$
- A *state* is defined as an *assignment* of values to some or all variables.
- *Consistent assignment*
  - assignment does not violate the constraints
- CSP benefits
  - Standard representation pattern
  - Generic goal and successor functions
  - Generic heuristics (no domain specific expertise).

CSPs (continued)

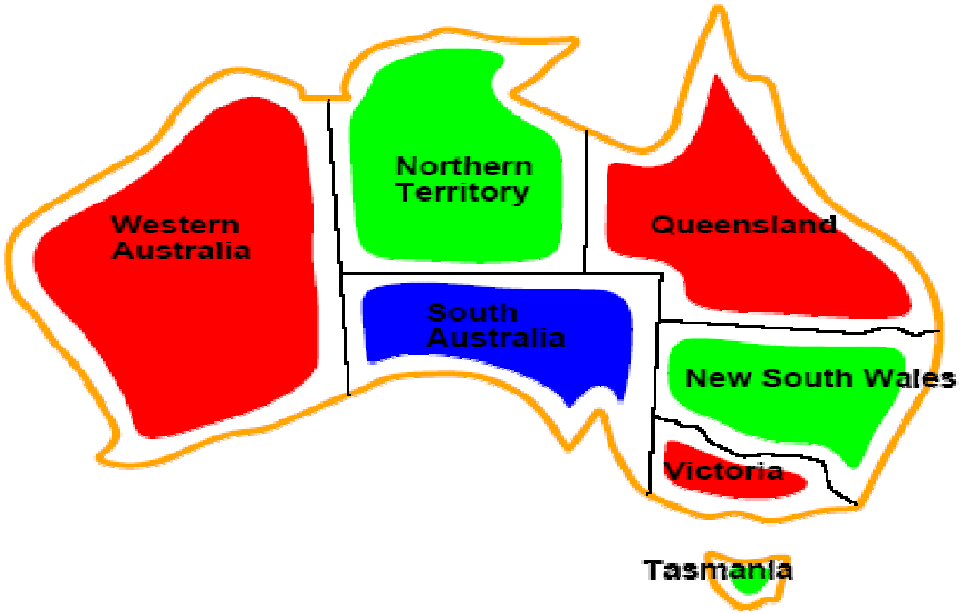
- An assignment is *complete* when every variable is mentioned.
- A *solution* to a CSP is a complete assignment that satisfies all constraints.
- Some CSPs require a solution that maximizes an *objective function*.
- Examples of Applications:
  - Scheduling the time of observations on the Hubble Space Telescope
  - Airline schedules
  - Cryptography
  - Computer vision -> image interpretation
  - Scheduling your MS or PhD thesis exam ☺

CSP example: map coloring



- Variables: *WA, NT, Q, NSW, V, SA, T*
- Domains: D_i=*{red,green,blue}*
- Constraints:adjacent regions must have different colors.
  - E.g. *WA* ≠ *NT*

CSP example: map coloring



- Solutions are assignments satisfying all constraints, e.g.

  {WA=red,NT=green,Q=red,NSW=green,V=red,SA=blue,T=green}

Constraint Satisfaction

- Many AI problem can be viewed as problems of constraint satisfaction in which goal is to solve problem state that satisfies given set of constraint instead of finding optimal path to the solution.
- Examples:
  - crypt-airthmetic /cryptography BASE+BALL=GAMES
  - n-Queen problem: condition is that on same row, column or diagonal no two queens attack each other http://www.hbmeyer.de/backtrack/achtdamen/eight.htm#up
  - Mapcoloring: Given a map color the regions of map using three colors blue,red and green such that no two neighboring countries have same color.
  - Crossword puzzle
- Design tasks can be viewed as constraint satisfaction problems in which a design must be created within fixed limits on time, cost, and materials.

Constraint Satisfaction

- Define CSP as follows
  - A set of variables (x1,x2….xn) with
  - set of possible values (n1,n2…..nk)for each xi
  - A set of constraints that are assumed to hold between the values of variables
  - Problem is to find all the values of xi such that all constraints are satisfied
- Constraint satisfaction is a search procedure
  - that operates in a space of constraint sets.
  - The initial state
    - contains the constraints that are originally given in the problem description.
    - Set of unassigned variables
  - A goal state
    - Is any state which satisfies all the constraints
    - All the variables are assigned values that satisfies all constraints
  - Operator
    - Assign value to unassigned variable , provided that it does not conflict with previously assigned variables

Constraint Satisfaction

- Constraint Satisfaction is a two step process:
  - First constraints are discovered and propagated as far as possible throughout the system.
  - Then if there still not a solution, search begins.
  - A guess about something is made and added as a new constraint
  - Propagation can then occur with this new constraint
- First step propagation occurs from fact that there are
  - dependencies amongst constraint
  - These dependencies occur because
  - many constraints involve more than one object
  - and many object participate in more than one constraint
- Constraint propagation also occurs due to presence of inference rules that allow additional constraints to be added which are inferred form the given ones.

Algorithm: Constraint Satisfaction

1. Propagate available constraints.
   a. Set OPEN to set of all objects that must have values assigned to them in a complete solution.
   b. Then do until an inconsistency is detected or until OPEN is empty:
      i. Select an object OB from OPEN. Strengthen as much as possible the set of constraints that apply to OB.
      ii. If this set is different from the set that was assigned the last time OB was examined or if this is the first time OB has been examined, then add to OPEN all objects that share any constraints with OB.
      iii. Remove OB from OPEN.
2. If the union of the constraints discovered above defines a solution, then quit and report the solution.

Algorithm: Constraint Satisfaction

3. If the union of the constraints discovered above defines a contradiction, then return the failure.

4. If neither of the above occurs, then make a guess at in order to proceed.

5. Loop until a solution is found or all possible solutions have been eliminated:

    a. Select an object whose value is not yet determined
    b. select a way of strengthening the constraints on that object.
    c. Recursively invoke constraint satisfaction with the current set of constraints augmented by strengthening constraint just selected.

Constraint Satisfaction: Example

- Crypt arithmetic Problem:

```
  SEND
+MORE
-----
MONEY
```

Initial State:

- No two letters have the same value
- The sums of the digits must be as shown in the problem

Goal State:

- All letters have been assigned a digit in such a way that all the initial constraints are satisfied.

Cryptasithmetic Problem: Constraint Satisfaction

The solution process proceeds in cycles. At each cycle, two significant things are done:

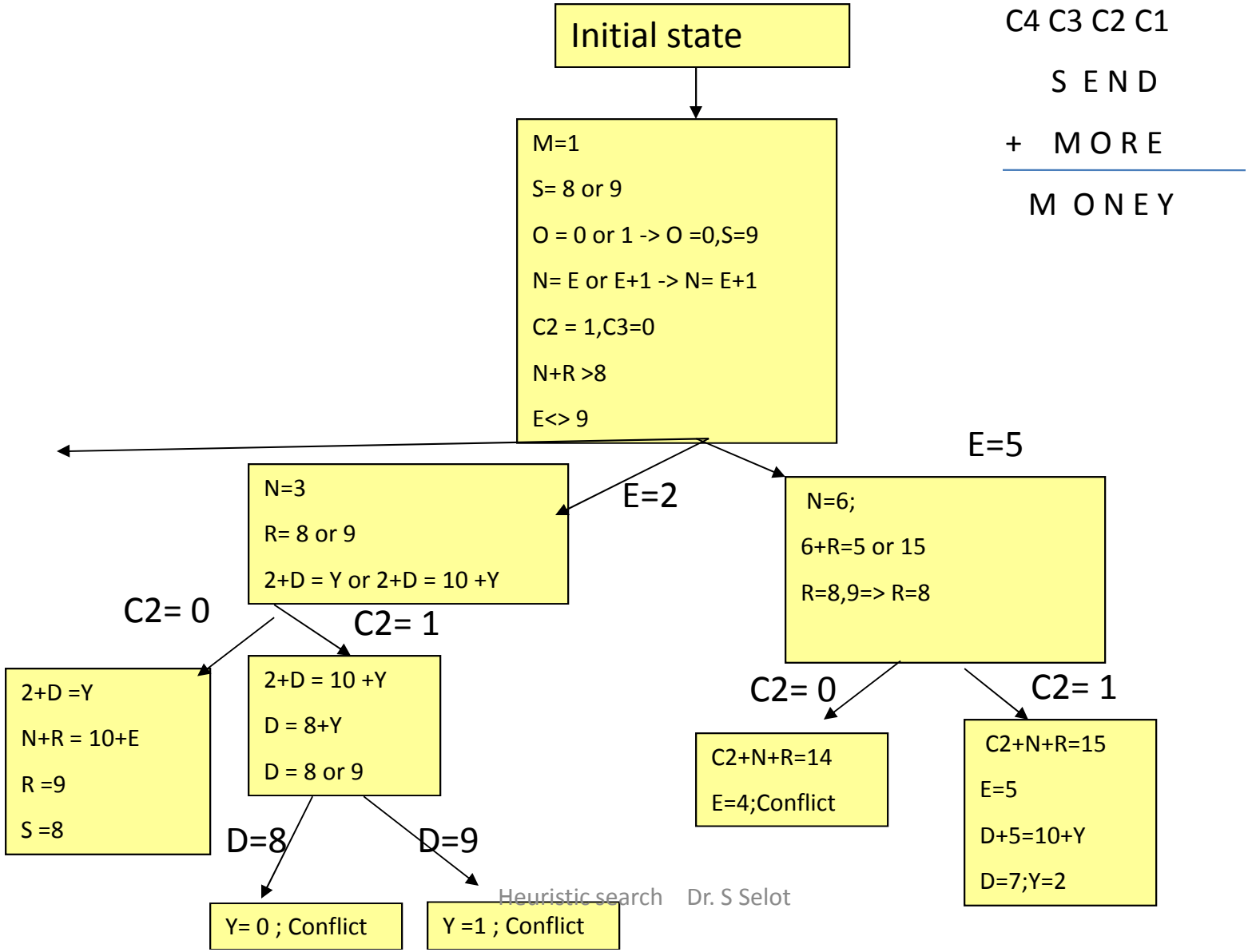
1. Constraints are **propagated** by using rules that correspond to the properties of arithmetic.
2. A value is **guessed** for some letter whose value is not yet determined.

A few **Heuristics** can help to select the best guess to try first:

- If there is a letter that has only two possible values and other with six possible values, there is a better chance of guessing right on the first than on the second.

- Another useful Heuristic is that if there is a letter that participates in many constraints then it is a good idea to prefer it to a letter that participates in a few.

Solving a Cryptarithmetic Problem



More puzzle

- TAKE+THAT=SHEET
- US+AS=ALL
- BASE+BALL=GAMES
  CROSS+ROADS=DANGER
- GREEN+ORANGE=COLORS