Batched Related Key Oblivious Psuedo Random Function

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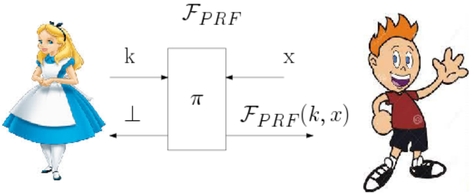
Table of contents

Introduction



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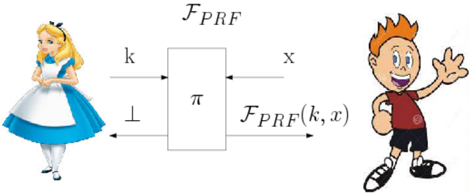

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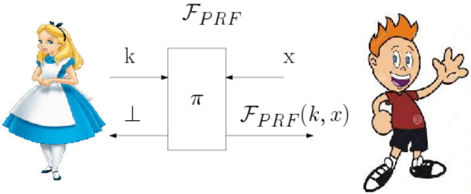

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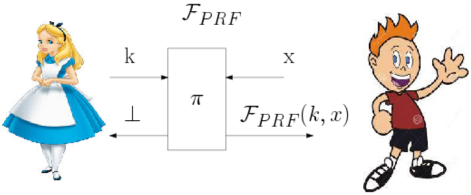

- In this talk we will talk about a protocol for evaluating a large batch OPRF instances.
- We will consider semi-honest settings.
- The above protocol is due to [KKRT16] called Batched Related Key OPRF(BaRK-OPRF).

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- OPRFs were introduced by [FIPR05] but their protocol uses expensive public key operations. Also they use number of OTs proportional to bit length of PRF input. Thus not useful if we have to carry out large number of OPRF instances.☹
- The protocol of [CNS07] constructs an OPRF from unique blind signature schemes.

Notations

- We denote vectors in bold for eg. $\mathbf{a}$.
- OT_l^m means m instances of OTs of l-bit strings.
- We denote Matrices in capital say M.
- For matrix M
  - M_i means i^{th} row of M
  - M^j means j^{th} coloumn of M.
- For vector $\mathbf{a}$ and bit b, $\mathbf{a} \cdot b$ means $\mathbf{a}$ if $b = 1$ else $\mathbf{0}$.
- For vectors $\mathbf{a}$ and bit $\mathbf{b}$, $\mathbf{a} \cdot \mathbf{b}$ means bitwise AND.

Correlation Robustness

Definition (1)

*Let H be a hash function. Then H is a d-**Hamming correlation robust** if for any strings* $\mathbf{z_1} \ldots . \mathbf{z_m} \in \{0,1\}^*$ *and* $\mathbf{a_1}, \ldots . \mathbf{a_m}, \mathbf{b_1}, \ldots . \mathbf{b_m} \in \{0,1\}^n$ *with* $\|\mathbf{b_i}\|_H \geq d$*, the following distribution, induced by random sampling of* $\mathbf{s} \longleftarrow \{0,1\}^n$*, is psuedo-random :*

$$H(\mathbf{z_1}||\mathbf{a_1} \oplus [\mathbf{b_1} \cdot \mathbf{s}]), \ldots .., H(\mathbf{z_m}||\mathbf{a_m} \oplus [\mathbf{b_m} \cdot \mathbf{s}])$$

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Psuedo-Random Codes

Definition (2)

Let $\mathcal{C}$ be a family of functions. We say that $\mathcal{C}$ is a (d, ϵ) **psuedorandom code(PRC)** *if for all strings $x \neq x'$,*

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Lemma (1)

Suppose $G : \{0,1\}^\kappa \times \{0,1\}^* \to \{0,1\}^n$ is a psuedorandom function. Define $\mathcal{C} = \{G(s, \cdot) \mid s \in \{0,1\}^\kappa\}$. Then $\mathcal{C}$ is a (d, ϵ)-psuedorandom code where :

$$2^{-\epsilon} = 2^{-n} \sum_{i=0}^{d-1} \binom{n}{i} + \vartheta(\kappa)$$

High Level Idea

- We see 1-out-of-∞ OT as OPRF. The intuition begind this is the fact that Sender has ability to evaluate the function at any point but remains oblivious to Receiver's choice. Also Receiver can't evaluate function at any other point as it doesn't have access to key.

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- However in the construction as we shall see, more information is leaked than required. Hence we will meet the definition of relaxed OPRF as defined in [FIPR05].

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Definition (4)

Let F be a relaxed PRF with output length v, for which we can write the key $= (k^*, k)$*. Then F has* $\mathbf{m-related-key-PRF(m-RK-PRF)security}$ *if the advantage of any PPT adversary in the following game is negligible :*

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1. The adversary chooses a large no. of strings $x_1, ...x_n$ which it will query to $\widetilde{F}$. $x_1, ...x_n \in \{0, 1\}^*$.

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Remark

Pair (K^*, K_i) will be used as *key* to F for input x_i for $i \in [n]$. We assume that $p_1, ...p_m \in [n]$. This is because $p_i's$ are basically meant to determine the key (k^*, k_{p_i}) that will be used when input to F is $y_i's$ for $y \in [m]$.

Remark

$y_i \neq x_{p_i}$ for $i \in [m]$. If this is not true then adversary will win trivially, as it means that adversary is choosing is a string which has been queried before with has been queried before with same key hence it can easily distinguish from random.

Game continued...

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Remark

Intuitively the *PRF* is instantiated with n related keys(sharing k^* value). The adversary learns the relaxed output on one chosen input for each key. Then any m additional *PRF* outputs are indistinguishable from random.

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Lemma (2)

Let $\mathcal{C}$ be a $(d, \epsilon + \log m) - PRC$ let H be $d -$ hamming correlation robust hash function. Let us define following relaxed PRF for $C \leftarrow_{randomly} \mathcal{C}$:

$$F\Big(((C,s),(q_j,j)),r\Big) = H(j||q_j \oplus [C(r)\cdot s])$$
$$\widetilde{F}\Big(((C,s),(q_j,j)),r\Big) = (j, C, q_j \oplus [C(r)\cdot s])$$

Then F has $\mathbf{m-RK-PRF}$ security .

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Input of R : m selection strings $\mathbf{r} = (\mathbf{r}_1, ...\mathbf{r}_m), r_i \in \{0,1\}^*, i \in [m]$.

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    - From the output S receives, S forms $m \times k$ matrix Q such that

$$\mathbf{Q}^i = \mathbf{T}^i_{s_i}, \ \ \textit{for } i \in [k]$$
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From relaxed-*PRF*'s output, R can calculate PRF's output.

Thank You ☺

Appendix

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where $\vartheta(\kappa)$ is negligible else we can build a distinguisher $\mathcal{D}$ that distinguishes output of PRF from Random function by calculating hamming weights of inputs. □

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The [IKNP03] construction does that ☺.
This protocol reduces reduces OT_l^m to OT_m^k which is ultimately reduced to OT_k^k $(m \gg k)$. Overhead is few symmetric key operations.

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We can easily verify that $\mathbf{z}_j = \mathbf{x}_{j,r_j}$

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- Now instead of choice bit r_i receiver has choice string $\mathbf{r_i}$ of length l.And let C be an error correcting code of codeword length k.
- The receiver will prepare matrices T and U such that $\mathbf{T}_j \oplus \mathbf{U}_j = C(\mathbf{r_j})$.
- Equation 2 will now become :

$$\mathbf{Q}_j = [C(\mathbf{r_j}) \cdot \mathbf{s}] \oplus \mathbf{T}_j \tag{3}$$

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6. It is easy to verify that R can't learn any other value and is able to learn value corresponding to his choice integer only.

Proof of lemma 2

Proof.

In the $m - RK - PRF$ game with the defined PRF, we can write adversary's(controlling R) view as :

$$\left(C, \{\mathbf{T}_{0,j}\}_{j\in[m]}, \{H(p_i||\mathbf{T}_{0,p_i} \oplus [(C(\mathbf{x}_{p_i}) \oplus C(\mathbf{y}_i)) \cdot \mathbf{s}])\}_{i\in[m]}\right) \tag{4}$$

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$$\Pr\left[A_1^\complement \cap A_2^\complement ..... \cap A_m^\complement\right] = 1 - \Pr\left[(A_1^\complement \cap A_2^\complement ..... \cap A_m^\complement)^\complement\right]$$
$$= 1 - \Pr\left[A_1 \cup A_2 ..... \cup A_m\right]$$

Proof Continued.

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Conditioning on this event apply $d-$ hamming correlation robust property of H to conclude that H outputs are indistinguishable from random. $\square$

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