

## Model-based testing Specifications – temporal logics

Abhik Roychoudhury  
<http://www.comp.nus.edu.sg/~abhik>

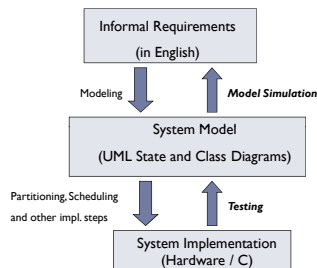
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## Flow of today's lecture

- ▶ Test generated from models
  - ▶ Run on implementation.
- ▶ How to find a “suitable” test case?
  - ▶ What is the purpose of testing?
- ▶ Finding a “suitable” test case guided by test specification
  - ▶ Given a test specification, we search the model to find a test?
- ▶ Two questions
  - ▶ How to describe test specifications – temporal logics.
  - ▶ How to search the system model – model checking.

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## Model-based system development



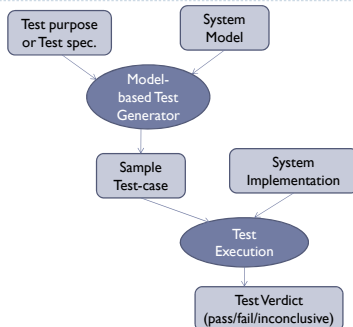
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## Model-based testing

- ▶ Generate test-cases from model, run them on the implementation.
- ▶ What are the criteria for generating test cases?
  - ▶ Generate a suite of test cases to ensure a structural coverage of the model
    - ▶ State coverage, Transition coverage for State Diagrams.
  - ▶ Generate test cases from the model based on some test specification
    - ▶ How to describe the test specification?
      - Temporal logic (discussed later)
    - ▶ How to find a test satisfying a test specification?
      - Model checking (discussed later)

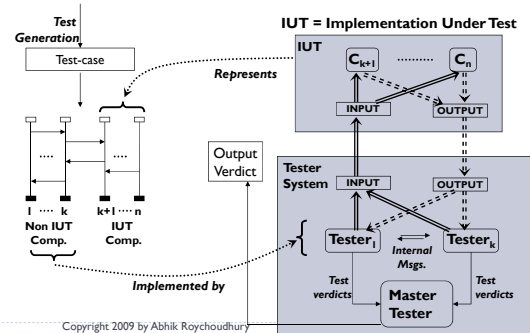
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## Test-purpose based test gen. & exec.



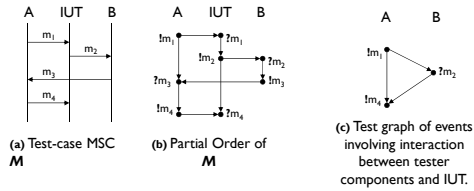
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## Test Execution Architecture



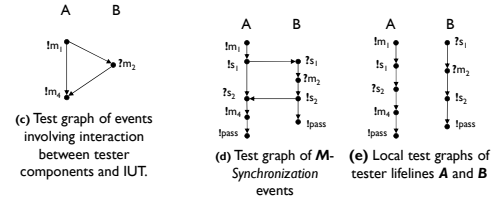
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### Test Execution – (1)



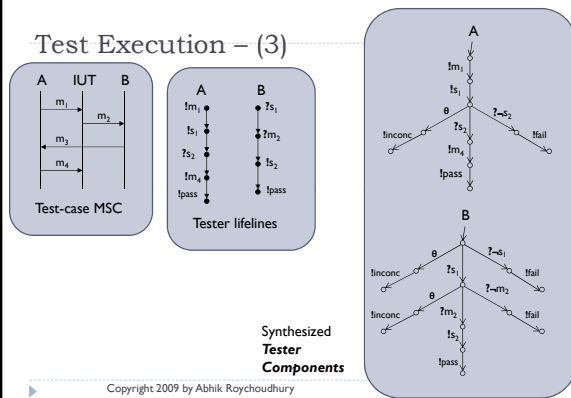
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### Test Execution – (2)



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### Test Execution – (3)



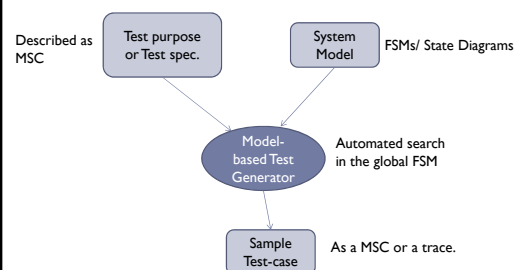
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### Test Verdicts

- ▶ **Pass**
  - ▶ All the tester components convey "Pass" to a Master tester.
- ▶ **Fail**
  - ▶ At least one tester component returns fail.
- ▶ **Inconclusive**
  - ▶ None of the tester components return fail, and
  - ▶ At least one tester component returns inconclusive.

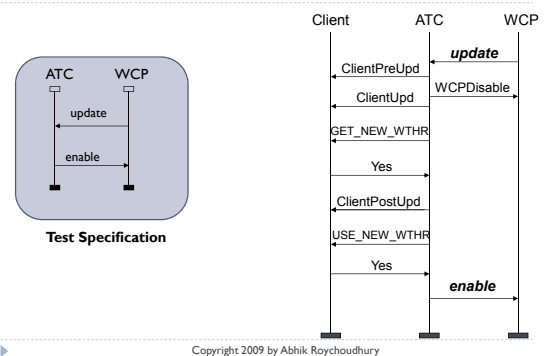
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### Test-purpose based test generation



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### Test spec. & Generated Test



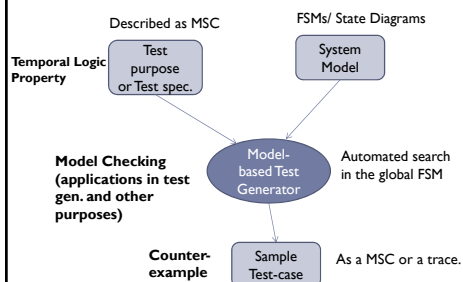
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## Test spec. & Generated test

- ▶ Test spec. is in the form of an MSC M.
- ▶ Def. 1
  - ▶ A trace  $\sigma$  satisfies a test specification M if  $\sigma$  contains at least one linearization of M as a **contiguous subsequence**.
- ▶ Def. 2
  - ▶ A trace  $\sigma$  satisfies a test specification M if  $\sigma$  contains at least one linearization of M as a **subsequence**.
- ▶ Which def. did we follow in the previous slide?

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## Test Generation



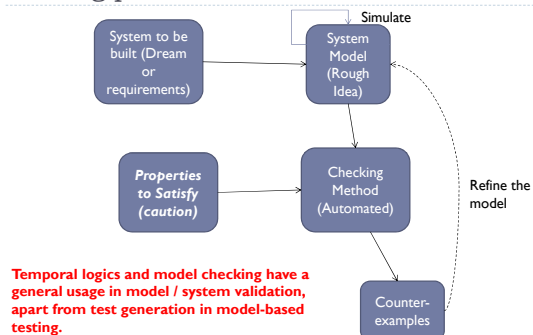
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## Organization

- ▶ So Far
  - ▶ What is a Model?
  - ▶ ATC – Running Example
  - ▶ How to model such requirements
  - ▶ How to validate the models
    - ▶ Simulations,
    - ▶ Model-based testing,
    - ▶ **Model Checking (discussed now)**
      - Temporal logics (the property specification)
      - Checking method
  - ▶ Also, model-based testing accomplished by model checking

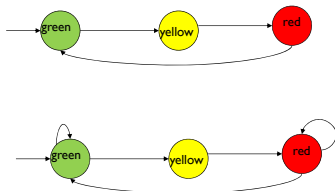
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## The big picture



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## Example System Model



Infinite length traces  
Possible to have infinitely many traces.

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## Temporal Logic

- ▶ On June 1 2007, I am teaching temporal logics which will be followed by teaching of model checking on June 8, 2007
- ▶ Teaching of temporal logics occurs 1 week before the teaching of model checking.
- ▶ Teaching of temporal logics is *always eventually* followed by the teaching of model checking.
- ▶ Teaching of temporal logics is *always immediately* followed by the teaching of model checking.

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### Example properties

- ▶ The light is *always* green.
- ▶ Whenever the light is red, it *eventually* becomes green.
- ▶ Whenever the light is green, it remains green *until* it becomes yellow.
- ▶ ...
- ▶ Are these properties true for the 2 example models in the previous slide?
  - ▶ Let us try the second property for example ...

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### When is a property satisfied?

- ▶ A property is **interpreted** on the traces of a system model.
  - ▶ Given a trace of the system model  $x$  and a property  $p$ , we can uniquely determine a yes/no answer to whether  $x$  satisfies  $p$ .
- ▶ A property  $p$  is satisfied by a system model  $M$ , if all traces of  $M$  satisfy  $p$ .
- ▶ So, given a system model what are its traces?

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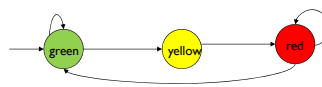
### Traces of a system model



- ▶ Only one trace, it has infinite length
    - ▶  $(\text{green}, \text{yellow}, \text{red})$  – repeated forever
- Written as  $(\text{green}, \text{yellow}, \text{red})^\omega$

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### Traces of a system model



- ▶ Infinitely many traces, each of infinite length
  - ▶  $(\text{green})^\omega$  - 1 trace
  - ▶  $(\text{green})^* \text{yellow} (\text{red})^\omega$  - many traces
  - ▶  $(\text{green})^* \text{yellow} (\text{red})^* (\text{green})^\omega$
  - ▶ ...
  - ▶  $(\text{green}, \text{yellow}, \text{red})^\omega$

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### Property Specification Language

- ▶ Properties in our property spec. language will be interpreted over **infinite length** traces.
  - ▶ Finite length traces can be converted into infinite length traces by putting a self-loop at last state.
- ▶ A property is satisfied by a system model if all execution traces satisfy the property.
  - ▶ In general, we cannot test the property on each exec. trace – infinitely many of them.
  - ▶ Model checking is smarter – we discuss it later!
- ▶ We formally describe the property spec. lang. or logic

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### Why study new logics ?

- ▶ Need a formalism to specify properties to be checked
- ▶ Our properties refer to dynamic system behaviors
  - ▶ **Eventually**, the system reaches a stable state
  - ▶ **Never** a deadlock can occur
- ▶ We want to maintain more than input-output properties (which are typical for transformational systems).
  - ▶ Input-output property: for input  $> 0$ , output should be  $> 0$
  - ▶ No notion of output or end-state in reactive systems.

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### Why study new logics ?

- ▶ Our properties express constraints on dynamic evolution of states.
- ▶ Propositional/first-order logics can only express properties of states, not properties of traces
- ▶ We study behaviors by looking at all execution traces of the system.
  - ▶ Linear-time Temporal Logic (LTL) is interpreted over execution traces of a system model.

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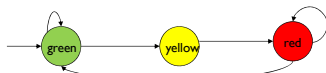
### Formally, system model is

- ▶ **Model for reactive systems**
  - ▶  $M = (S, I, \rightarrow, L)$
  - ▶  $S$  is the set of states
  - ▶  $S_0 \subseteq S$  is the set of initial states
  - ▶  $\rightarrow \subseteq S \times S$  is the transition relation
  - ▶ Set of (source-state, destination-state) pairs
  - ▶  $L$ : is the labeling function mapping  $S$  to  $2^{AP}$ 
    - ▶ Maps each state  $s$  to a subset of AP
    - ▶ These are the atomic prop. which are true in  $s$ .

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### Atomic Propositions

- ▶ All of our properties will contain atomic props.
  - ▶ These atomic props. will appear in the labeling function of the system model you verify.
  - ▶ The atomic props. represent some relationships among variables in the design that you verify.
  - ▶ Atomic props in the following example
    - ▶ **green, yellow, red (marked inside the states with obvious labeling function).**



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### Linear-time Temporal Logic

- ▶ The temporal logic that we study today build on a “static” logic like propositional logic.
  - ▶ Used to describe/constrain properties inside states.
- ▶ Temporal operators describe properties on execution traces.
  - ▶ Used to describe/constrain evolution of states.
- ▶ Time is **not** explicitly mentioned in the formulae
  - ▶ Properties describe how the system should evolve over time.

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### Linear-time Temporal Logic

- ▶ Does not capture exact timing of events, but rather the relative order of events
- ▶ We capture properties of the following form.
  - ▶ Whenever event  $e$  occurs, eventually event  $e'$  must occur.
- ▶ We do **not** capture properties of the following form.
  - ▶ At  $t=2$   $e$  occurs followed by  $e'$  occurring at  $t=4$ .

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### Notations and Conventions

- ▶ An LTL formula  $\varphi$  is interpreted over an infinite sequence of states  $\pi = s_0, s_1, \dots$ 
  - ▶ Use  $M, \pi \models \varphi$  to denote that formula  $\varphi$  holds in path  $\pi$  of system model  $M$ .
- ▶ Define semantics of LTL formulae w.r.t. a system model  $M$ .
  - ▶ **An LTL property  $\varphi$  is true of a system model iff all its traces satisfy  $\varphi$**
  - ▶  **$M \models \varphi$  iff  $M, \pi \models \varphi$  for all traces  $\pi$  in system model  $M$**

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## Notations and Conventions

- ▶  $M, \pi \models \varphi$ 
  - ▶ Path  $\pi = s_0, s_1, s_2, \dots$  in model  $M$  satisfies property  $\varphi$
- ▶  $M, \pi^k \models \varphi$ 
  - ▶ Path  $s_k, s_{k+1}, \dots$  in model  $M$  satisfies property  $\varphi$
- ▶ We now use these notations to define the syntax & semantics of LTL.

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## LTL - syntax

- ▶ Propositional Linear-time Temporal logic

$$\varphi = X\varphi \mid G\varphi \mid F\varphi \mid \varphi \cup \varphi \mid \varphi R \varphi \mid \neg\varphi \mid \varphi \wedge \varphi \mid \text{Prop}$$

- ▶ Prop is the set of atomic propositions
- ▶ Temporal operators
  - ▶ X (next - state)
  - ▶ F (eventually), G (globally)
  - ▶ U (until), R (release)

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## Semantics of propositional logic

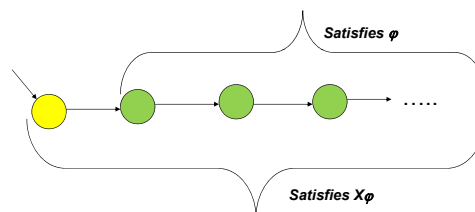
- ▶  $M, \pi \models p$  iff  $s_0 \models p$  i.e.  $p \in L(s_0)$  where  $L$  is the labeling function of Kripke Structure  $M$
- ▶  $M, \pi \models \neg \varphi$  iff  $\neg (M, \pi \models \varphi)$
- ▶  $M, \pi \models \varphi_1 \wedge \varphi_2$  iff  $M, \pi \models \varphi_1$  and  $M, \pi \models \varphi_2$

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## neXt-state operator of LTL

- ▶  $M, \pi \models X\varphi$  iff  $M, \pi^1 \models \varphi$ 
  - ▶ Path starting from **next state** satisfies  $\varphi$

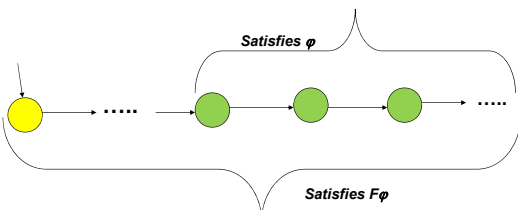


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## Finally operator of LTL

- ▶  $M, \pi \models F\varphi$  iff  $\exists k \geq 0, M, \pi^k \models \varphi$ 
  - ▶ Path starting from an **eventually** reached state satisfies  $\varphi$

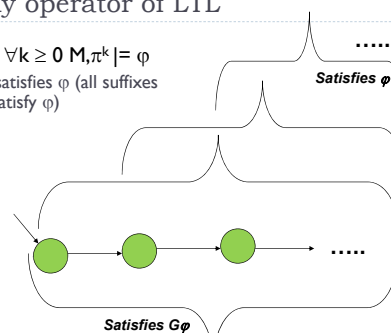


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## Globally operator of LTL

- ▶  $M, \pi \models G\varphi$  iff  $\forall k \geq 0, M, \pi^k \models \varphi$ 
  - ▶ Path **always** satisfies  $\varphi$  (all suffixes of the path satisfy  $\varphi$ )

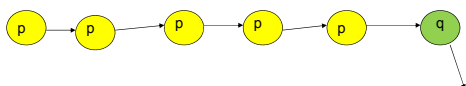


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### Until operator of LTL

- $M, \pi \models \varphi 1 \text{ U } \varphi 2$  iff  $\exists k \geq 0$  such that
  - $M, \pi^k \models \varphi 2$ , and
  - $\forall 0 \leq j < k \ M, \pi^j \models \varphi 1$

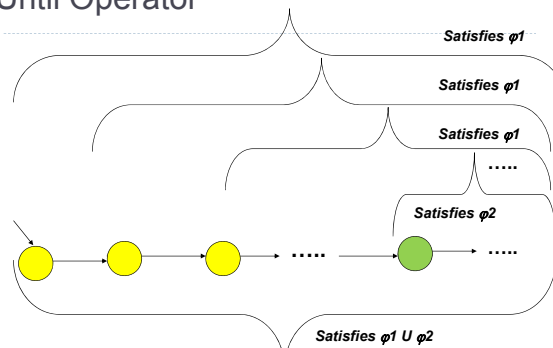


A trace satisfying  $p \text{ U } q$ , where  $p, q \in \text{Prop}$

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### Until Operator

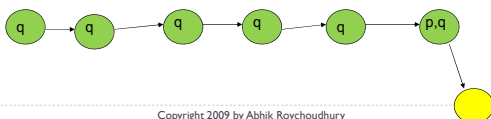


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### Release operator of LTL

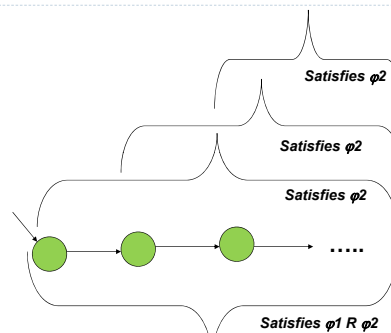
- $M, \pi \models \varphi 1 \text{ R } \varphi 2$  iff
  - Either  $\forall k \geq 0 \ M, \pi^k \models \varphi 2$
  - OR both of the following hold
    - $\exists k \geq 0 \ M, \pi^k \models \varphi 1$
    - $\forall 0 \leq j \leq k \ M, \pi^j \models \varphi 2$
- $\varphi 1$  releases the req. for  $\varphi 2$  to hold.



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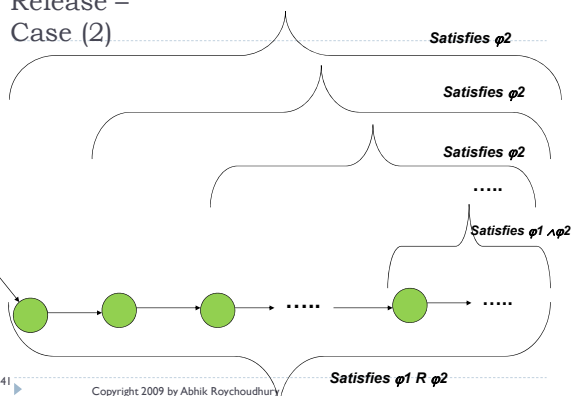
### Release – Case 1



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### Release – Case (2)



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### Exercise – (1)

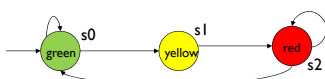
- ▶ The light is *always* green.
- ▶ Whenever the light is red, it eventually becomes green.
- ▶ Whenever the light is green, it remains green until it becomes yellow.
- ▶ Whenever the light is yellow, it becomes red *immediately* after.
- ▶ Encode these properties in LTL.

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## Exercise – (2)

- Check whether the four LTL properties in the previous slide are satisfied by our simple traffic light controller.



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## LTL Exercise – (3)

Consider a resource allocation protocol where  $n$  processes  $P_1, \dots, P_n$  are contending for exclusive access of a shared resource. Access to the shared resource is controlled by an arbiter process. The atomic proposition  $\text{req}_i$  is true only when  $P_i$  explicitly sends an access request to the arbiter. The atomic proposition  $\text{gnt}_i$  is true only when the arbiter grants access to  $P_i$ . Now suppose that the following LTL formula holds for our resource allocation protocol.

- $G(\text{req}_i \Rightarrow F \text{gnt}_i)$

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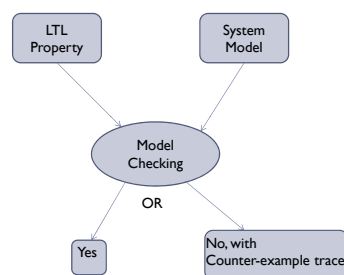
## LTL Exercise – (3)

- Explain in English what the property means.
- Is this a desirable property of the protocol?
- Suppose that the resource allocation protocol has a distributed implementation so that each process is implemented in a different site. Does the LTL property affect the communication overheads among the processes in any way?

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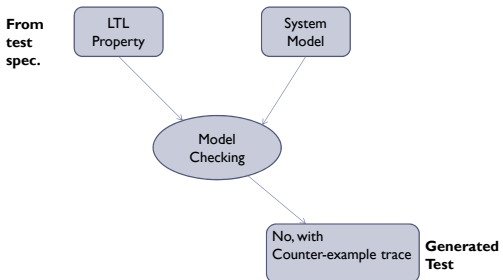
## Model Checking



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## Recap: Model Checking for model-based testing



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## Encoding test specifications

- Def. 1
  - A trace  $\sigma$  satisfies a test specification  $M$  if  $\sigma$  contains at least one linearization of  $M$  as a **contiguous subsequence**.
  - Given MSC  $M$ ,
    - define  $\text{Lin}(M)$  = set of linearizations of  $M$ .
    - For each linearization  $\sigma = e_1 e_2 \dots e_k$  define
      - Define  $\text{prop}_\sigma = F(e_1 \wedge X(e_2 \wedge X(\dots X(e_k) \dots)))$
      - Define property  $\phi_M$  corresponding to  $M$  as
        - $\phi_M = \neg (\vee_{\sigma \in \text{Lin}(M)} \text{prop}_\sigma)$
  - A counter-example to  $\phi_M$  is a test satisfying  $M$ .

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## Encoding test specifications

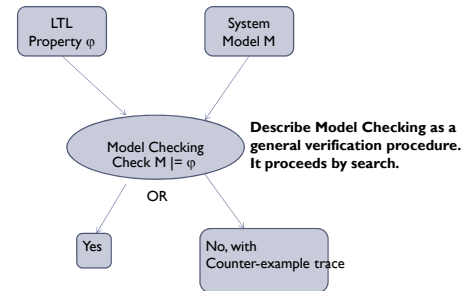
### ► Def. 2

- A trace  $\sigma$  satisfies a test specification  $M$  if  $\sigma$  contains at least one linearization of  $M$  as a **subsequence**.
- Given MSC  $M$ ,
  - define  $\text{Lin}(M)$  = set of linearizations of  $M$ .
  - For each linearization  $\sigma = e_1, e_2, \dots, e_k$  define
    - $n_\sigma = \neg(e_1 \vee e_2 \vee \dots \vee e_k)$
    - $\text{prop}_\sigma = (n_\sigma \text{ U } (e_1 \wedge \text{X}(n_\sigma \text{ U } (e_2 \wedge \text{X}(\dots \text{X}(n_\sigma \text{ U } e_k) \dots))))$
  - Define property  $\phi_M$  corresponding to  $M$  as
    - $\phi_M = \neg (\vee_{\sigma \in \text{Lin}(M)} \text{prop}_\sigma)$
- A counter-example to  $\phi_M$  is a test satisfying  $M$ .

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## Model Checking – Next class



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