

Model-based testing Specifications – temporal logics

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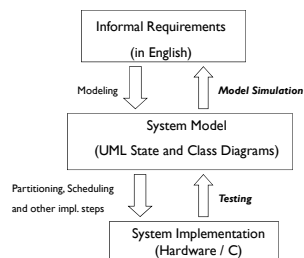
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Flow of today's lecture

- ▶ Test generated from models
 - ▶ Run on implementation.
- ▶ How to find a "suitable" test case?
 - ▶ What is the purpose of testing?
- ▶ Finding a "suitable" test case guided by test specification
 - ▶ Given a test specification, we search the model to find a test?
- ▶ Two questions
 - ▶ How to describe test specifications – temporal logics.
 - ▶ How to search the system model – model checking.

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Model-based system development



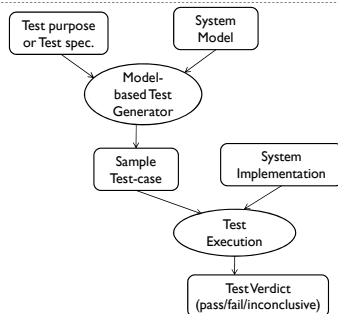
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Model-based testing

- ▶ Generate test-cases from model, run them on the implementation.
- ▶ What are the criteria for generating test cases?
 - ▶ Generate a suite of test cases to ensure a structural coverage of the model
 - ▶ State coverage, Transition coverage for State Diagrams.
- ▶ Generate test cases from the model based on some test specification
 - ▶ How to describe the test specification?
 - Temporal logic (discussed later)
 - ▶ How to find a test satisfying a test specification?
 - Model checking (discussed later)

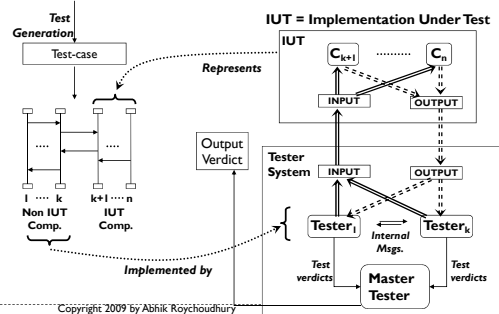
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Test-purpose based test gen. & exec.



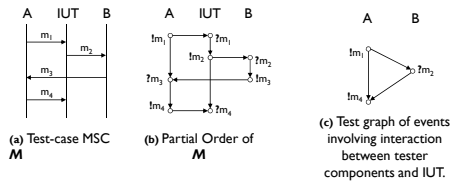
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Test Execution Architecture



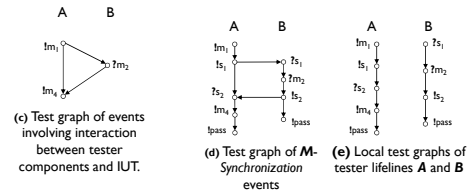
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Test Execution – (1)



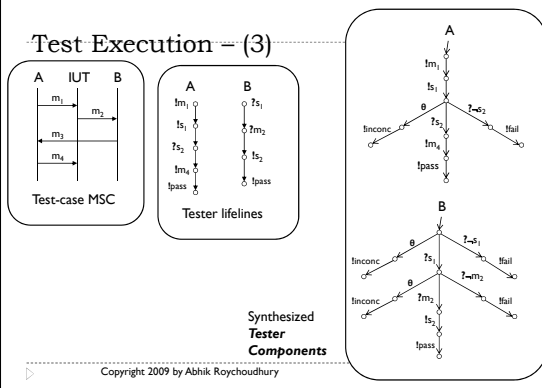
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Test Execution – (2)



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Test Execution – (3)



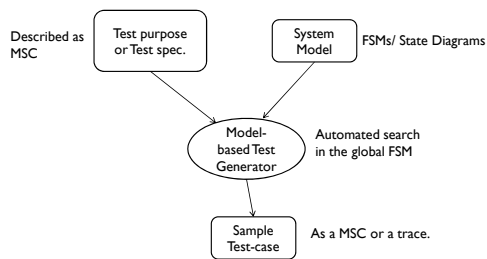
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Test Verdicts

- Pass
 - All the tester components convey "Pass" to a Master tester.
- Fail
 - At least one tester component returns fail.
- Inconclusive
 - None of the tester components return fail, and
 - At least one tester component returns inconclusive.

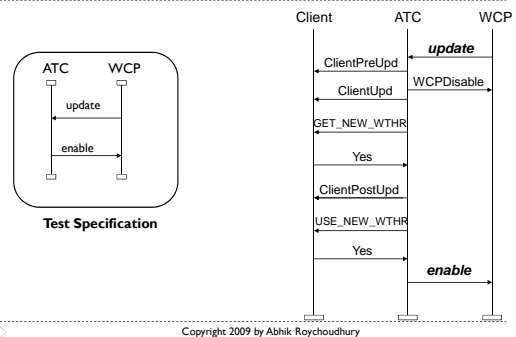
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Test-purpose based test generation



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Test spec. & Generated Test



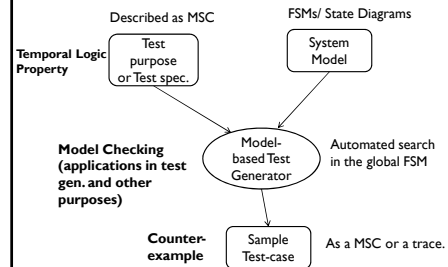
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Test spec. & Generated test

- ▶ Test spec. is in the form of an MSC M.
- ▶ Def. 1
 - ▶ A trace σ satisfies a test specification M if σ contains at least one linearization of M as a contiguous subsequence.
- ▶ Def. 2
 - ▶ A trace σ satisfies a test specification M if σ contains at least one linearization of M as a subsequence.
- ▶ Which def. did we follow in the previous slide?

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Test Generation



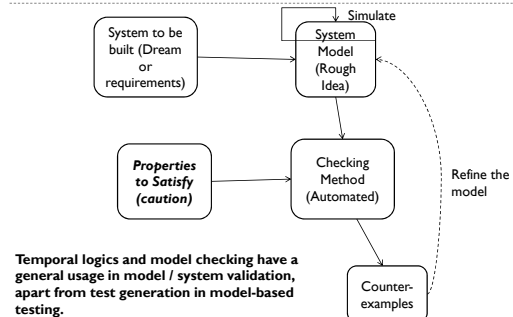
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Organization

- ▶ So Far
 - ▶ What is a Model?
 - ▶ ATC – Running Example
 - ▶ How to model such requirements
 - ▶ How to validate the models
 - ▶ Simulations,
 - ▶ Model-based testing,
 - ▶ Model Checking (discussed now)
 - Temporal logics (the property specification)
 - Checking method
 - ▶ Also, model-based testing accomplished by model checking

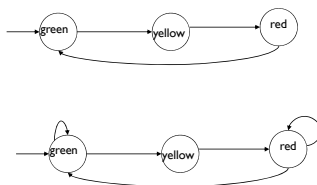
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The big picture



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Example System Model



Infinite length traces
Possible to have infinitely many traces.

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Temporal Logic

- ▶ On June ~~1~~ 2007, I am teaching temporal logics which will be followed by teaching of model checking on June ~~8~~ 2007
- ▶ Teaching of temporal logics occurs ~~1~~ week before the teaching of model checking.
- ▶ Teaching of temporal logics is *always eventually* followed by the teaching of model checking.
- ▶ Teaching of temporal logics is *always immediately* followed by the teaching of model checking.

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Example properties

- ▶ The light is *always* green.
- ▶ Whenever the light is red, it *eventually* becomes green.
- ▶ Whenever the light is green, it remains green *until* it becomes yellow.
- ▶ ...
- ▶ Are these properties true for the 2 example models in the previous slide?
 - ▶ Let us try the second property for example ...

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When is a property satisfied?

- ▶ A property is interpreted on the traces of a system model.
 - ▶ Given a trace of the system model x and a property p , we can uniquely determine a yes/no answer to whether x satisfies p .
- ▶ A property p is satisfied by a system model M , if all traces of M satisfy p .
- ▶ So, given a system model what are its traces?

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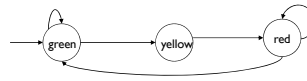
Traces of a system model



- ▶ Only one trace, it has infinite length
 - ▶ $(\text{green}, \text{yellow}, \text{red})^\omega$ – repeated forever
- Written as
 $(\text{green}, \text{yellow}, \text{red})^\omega$

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Traces of a system model



- ▶ Infinitely many traces, each of infinite length
 - ▶ $(\text{green})^\omega$ – 1 trace
 - ▶ $(\text{green})^* \text{yellow} (\text{red})^\omega$ – many traces
 - ▶ $(\text{green})^* \text{yellow} (\text{red})^* (\text{green})^\omega$
 - ▶ ...
 - ▶ $(\text{green}, \text{yellow}, \text{red})^\omega$

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Property Specification Language

- ▶ Properties in our property spec. language will be interpreted over infinite length traces.
 - ▶ Finite length traces can be converted into infinite length traces by putting a self-loop at last state.
- ▶ A property is satisfied by a system model if all execution traces satisfy the property.
 - ▶ In general, we cannot test the property on each exec. trace – infinitely many of them.
 - ▶ Model checking is smarter – we discuss it later!
- ▶ We formally describe the property spec. lang. or logic

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Why study new logics ?

- ▶ Need a formalism to specify properties to be checked
- ▶ Our properties refer to dynamic system behaviors
 - ▶ Eventually, the system reaches a stable state
 - ▶ Never a deadlock can occur
- ▶ We want to maintain more than input-output properties (which are typical for transformational systems).
 - ▶ Input-output property: for input > 0 , output should be > 0
 - ▶ No notion of output or end-state in reactive systems.

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Why study new logics ?

- ▶ Our properties express constraints on dynamic evolution of states.
- ▶ Propositional/first-order logics can only express properties of states, not properties of traces
- ▶ We study behaviors by looking at all execution traces of the system.
 - ▶ Linear-time Temporal Logic (LTL) is interpreted over execution traces of a system model.

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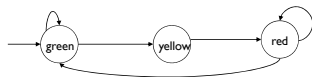
Formally, system model is

- ▶ Model for reactive systems
 - ▶ $M = (S, I, \rightarrow, L)$
 - ▶ S is the set of states
 - ▶ $S_0 \subseteq S$ is the set of initial states
 - ▶ $\rightarrow \subseteq S \times S$ is the transition relation
 - ▶ Set of (source-state, destination-state) pairs
 - ▶ L is the labeling function mapping S to 2^{AP}
 - ▶ Maps each state s to a subset of AP
 - ▶ These are the atomic prop. which are true in s .

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Atomic Propositions

- ▶ All of our properties will contain atomic props.
 - ▶ These atomic props. will appear in the labeling function of the system model you verify.
 - ▶ The atomic props. represent some relationships among variables in the design that you verify.
 - ▶ Atomic props in the following example
 - ▶ **green, yellow, red** (marked inside the states with obvious labeling function).



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Linear-time Temporal Logic

- ▶ The temporal logic that we study today build on a "static" logic like propositional logic.
 - ▶ Used to describe/constrain properties inside states.
- ▶ Temporal operators describe properties on execution traces.
 - ▶ Used to describe/constrain evolution of states.
- ▶ Time is not explicitly mentioned in the formulae
 - ▶ Properties describe how the system should evolve over time.

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Linear-time Temporal Logic

- ▶ Does not capture exact timing of events, but rather the relative order of events
- ▶ We capture properties of the following form.
 - ▶ Whenever event e occurs, eventually event e' must occur.
- ▶ We do not capture properties of the following form.
 - ▶ At $t=2$ e occurs followed by e' occurring at $t=4$.

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Notations and Conventions

- ▶ An LTL formula φ is interpreted over an infinite sequence of states $\pi = s_0, s_1, \dots$
 - ▶ Use $M, \pi \models \varphi$ to denote that formula φ holds in path π of system model M .
- ▶ Define semantics of LTL formulae w.r.t. a system model M .
 - ▶ **An LTL property φ is true of a system model iff all its traces satisfy φ**
 - ▶ **$M \models \varphi$ iff $M, \pi \models \varphi$ for all traces π in system model M**

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Notations and Conventions

- ▶ $M, \pi \models \varphi$
 - ▶ Path $\pi = s_0, s_1, s_2, \dots$ in model M satisfies property φ
- ▶ $M, \pi^k \models \varphi$
 - ▶ Path s_k, s_{k+1}, \dots in model M satisfies property φ
- ▶ We now use these notations to define the syntax & semantics of LTL.

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LTL - syntax

- ▶ Propositional Linear-time Temporal logic
- ▶ $\varphi = X\varphi \mid G\varphi \mid F\varphi \mid \varphi \cup \varphi \mid \varphi R \varphi \mid$
 $\neg\varphi \mid \varphi \wedge \varphi \mid \text{Prop}$
- ▶ Prop is the set of atomic propositions
- ▶ Temporal operators
 - ▶ X (next - state)
 - ▶ F (eventually), G (globally)
 - ▶ U (until), R (release)

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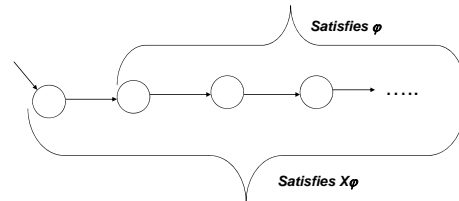
Semantics of propositional logic

- ▶ $M, \pi \models p$ iff $s_0 \models p$ i.e. $p \in L(s_0)$ where L is the labeling function of Kripke Structure M
- ▶ $M, \pi \models \neg\varphi$ iff $\neg (M, \pi \models \varphi)$
- ▶ $M, \pi \models \varphi_1 \wedge \varphi_2$ iff $M, \pi \models \varphi_1$ and $M, \pi \models \varphi_2$

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neXt-state operator of LTL

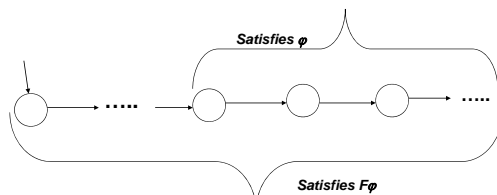
- ▶ $M, \pi \models X\varphi$ iff $M, \pi^1 \models \varphi$
 - ▶ Path starting from next state satisfies φ



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Finally operator of LTL

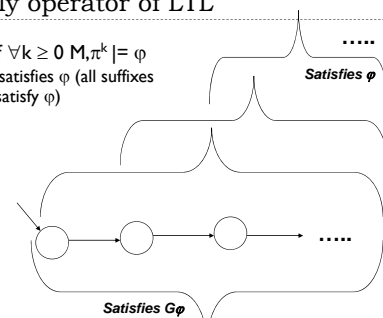
- ▶ $M, \pi \models F\varphi$ iff $\exists k \geq 0 M, \pi^k \models \varphi$
 - ▶ Path starting from an eventually reached state satisfies φ



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Globally operator of LTL

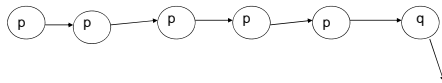
- ▶ $M, \pi \models G\varphi$ iff $\forall k \geq 0 M, \pi^k \models \varphi$
 - ▶ Path always satisfies φ (all suffixes of the path satisfy φ)



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Until operator of LTL

- $M, \pi \models \varphi_1 U \varphi_2$ iff $\exists k \geq 0$ such that
 - $M, \pi^k \models \varphi_2$, and
 - $\forall 0 \leq j < k, M, \pi^j \models \varphi_1$

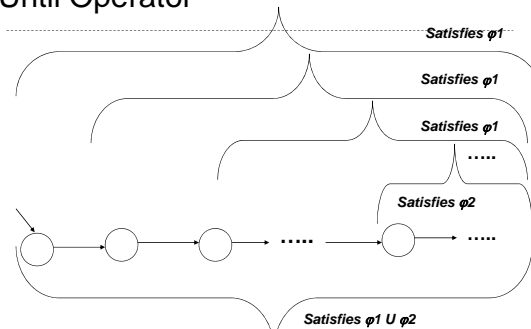


A trace satisfying $p U q$, where $p, q \in \text{Prop}$

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Until Operator

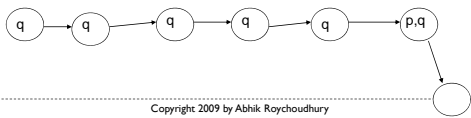


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Release operator of LTL

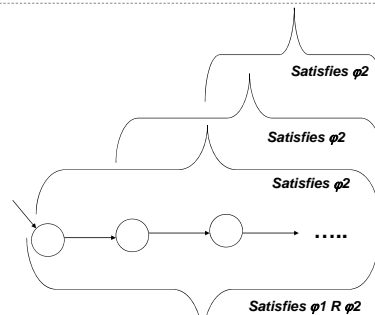
- $M, \pi \models \varphi_1 R \varphi_2$ iff
 - Either $\forall k \geq 0, M, \pi^k \models \varphi_2$
 - OR both of the following hold
 - $\exists k \geq 0, M, \pi^k \models \varphi_1$
 - $\forall 0 \leq j \leq k, M, \pi^j \models \varphi_2$
- φ_1 releases the req. for φ_2 to hold.



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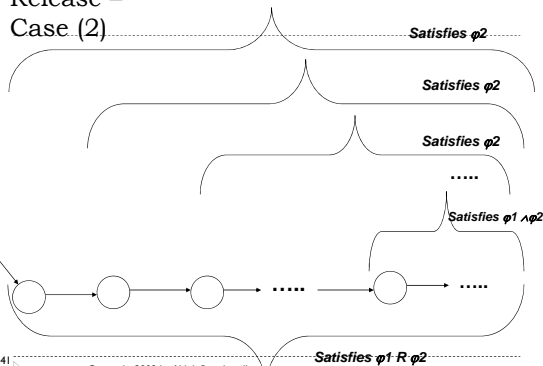
Release – Case 1



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Release – Case (2)



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Exercise – (1)

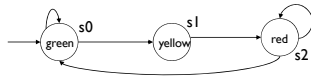
- ▶ The light is *always* green.
- ▶ Whenever the light is red, it *eventually* becomes green.
- ▶ Whenever the light is green, it remains green *until* it becomes yellow.
- ▶ Whenever the light is yellow, it becomes red *immediately* after.
- ▶ Encode these properties in LTL.

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Exercise – (2)

- Check whether the four LTL properties in the previous slide are satisfied by our simple traffic light controller.



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LTL Exercise – (3)

Consider a resource allocation protocol where n processes P_1, \dots, P_n are contending for exclusive access of a shared resource. Access to the shared resource is controlled by an arbiter process. The atomic proposition req_i is true only when P_i explicitly sends an access request to the arbiter. The atomic proposition gnt_i is true only when the arbiter grants access to P_i . Now suppose that the following LTL formula holds for our resource allocation protocol.

- $G (req_i \Rightarrow F gnt_i)$

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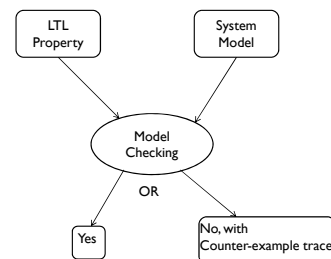
LTL Exercise – (3)

- Explain in English what the property means.
- Is this a desirable property of the protocol?
- Suppose that the resource allocation protocol has a distributed implementation so that each process is implemented in a different site. Does the LTL property affect the communication overheads among the processes in any way?

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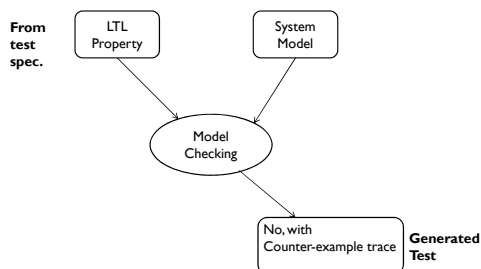
Model Checking



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Recap: Model Checking for model-based testing



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Encoding test specifications

- Def. 1
 - A trace σ satisfies a test specification M if σ contains at least one linearization of M as a **contiguous** subsequence.
- Given MSC M ,
 - define $\text{Lin}(M)$ = set of linearizations of M .
 - For each linearization $\sigma = e_1 e_2 \dots e_k$ define
 - Define $\text{prop}_\sigma = F(e_1 \wedge X(e_2 \wedge X(\dots X(e_k) \dots)))$
 - Define property ϕ_M corresponding to M as
 - $\phi_M = \neg (\bigvee_{\sigma \in \text{Lin}(M)} \text{prop}_\sigma)$
- A counter-example to ϕ_M is a test satisfying M .

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Encoding test specifications

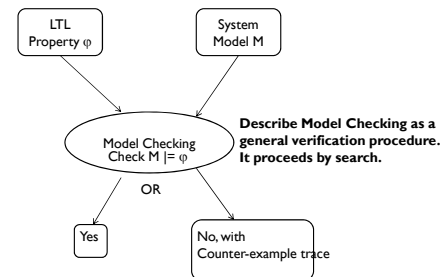
► Def. 2

- A trace σ satisfies a test specification M if σ contains at least one linearization of M as a subsequence.
- Given MSC M ,
 - define $\text{Lin}(M)$ = set of linearizations of M .
 - For each linearization $\sigma = e_1 e_2 \dots e_k$ define
 - $n_\sigma = \neg(e_1 \vee e_2 \vee \dots \vee e_k)$
 - $\text{prop}_\sigma = (n_\sigma \mathbf{U} (e_1 \wedge \mathbf{X}(n_\sigma \mathbf{U} (e_2 \wedge \mathbf{X}(\dots \mathbf{X}(n_\sigma \mathbf{U} e_k) \dots))))$
 - Define property ϕ_M corresponding to M as
 - $\phi_M = \neg (\vee_{\sigma \in \text{Lin}(M)} \text{prop}_\sigma)$
- A counter-example to ϕ_M is a test satisfying M .

► 49

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Model Checking – Next class



► 50

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