Revision of Temporal Logics via Exercises

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Warm-up LTL Exercises

- 1. Can you establish that
- $\varphi \mathrm{R} \Psi = \neg(\neg\varphi \ \mathrm{U} \neg\Psi)$
- 2. Express the following in LTL
  - Along any path, a state satisfying p occurs at most once.

LTL Exercises (1)

- Express each of the following properties (stated in English) as an LTL formula. Assume that p, q and r are atomic propositions.
  - If p occurs, q never occurs in the future.
  - Always if p occurs, then eventually q occurs followed immediately by r.
  - Any occurrence of p is followed eventually by an occurrence of q. Furthermore, r never occurs between p and q.

LTL Exercises (2)

- Specify the following in LTL.
  - The light is always green.
  - Whenever the light is red, it eventually becomes green.
  - Whenever the light is yellow, it becomes red immediately after.
- Use the atomic propositions
  - {red, yellow, green}

LTL Exercises (3)

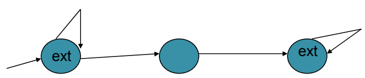
Consider a resource allocation protocol where n processes $P_1,\ldots,P_n$ are contending for exclusive access of a shared resource. Access to the shared resource is controlled by an arbiter process. The atomic proposition req_i is true only when P_i explicitly sends an access request to the arbiter. The atomic proposition gnt_i is true only when the arbiter grants access to P_i. Now suppose that the following LTL formula holds for our resource allocation protocol.

- $\mathrm{G}\ (req_i \Rightarrow \mathrm{F}\ gnt_i)$

LTL Exercise (continued)

- Explain in English what the property means.
- Is this a desirable property of the protocol ?
- Suppose that the resource allocation protocol has a distributed implementation so that each process is implemented in a different site. Does the LTL property affect the communication overheads among the processes in any way ?

Relationship of CTL and LTL



Satisfies the LTL formula FG ext

What about the CTL formula AFAG ext ?

Relationship of CTL and LTL

- As shown
  - FG p and AFAG p are not equivalent.
    - p is an atomic proposition.
- Are the following equivalent formulae?
  - GF p
  - AGAF p
    - i.e. any Kripke structure satisfying GFp satisfies AGAFp and vice-versa?

LTL & CTL Exercises

- Consider the LTL formula GFp and the CTL formula AGEFp where p is an atomic proposition. Give an example of a Kripke Structure which satisfies AGEFp but does not satisfy GFp. You may assume that p is the only atomic proposition for constructing the labeling function.

CTL Exercise

Construct a CTL formula which is satisfied by the initial state of the system model in Figure (a), but not satisfied by the initial state of the system model of Figure (b). Since I have shown only the valuation for the atomic proposition q in the different states, it will be the only atomic proposition appearing in your formula. (We can assume that there are other atomic propositions as well, but their valuations are not shown).

Hint: Try to use the next-state (X) operator

Figure (a)

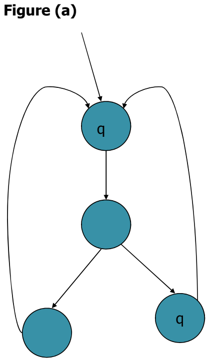
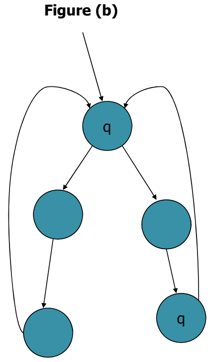


Figure (b)



CTL Exercise

- Show that all the ten CTL operators
  - AF,EF,AG,EG,AU,EU,AR,ER

can be expressed in terms of EX,EG,EU and the propositional logic operators.