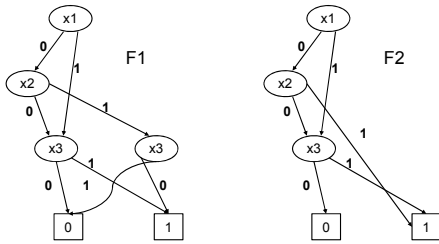


## Steps for Computing $F1 \wedge F2$



Variable ordering:  $x1 < x2 < x3$

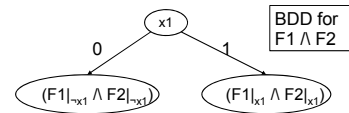
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Recall:  $F1 \wedge F2 =$

$$(x1 \wedge (F1|_{x1} \wedge F2|_{x1})) \vee (\neg x1 \wedge (F1|_{\neg x1} \wedge F2|_{\neg x1}))$$

Due to variable ordering, we start from  $x1$  and compute  $F1 \wedge F2$  recursively. Thus, we proceed as...

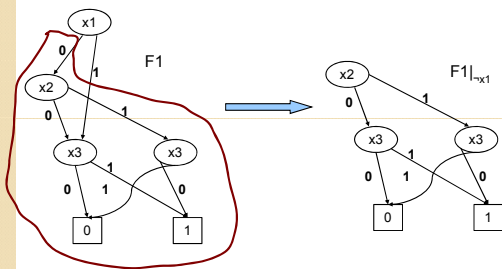


Next, we compute the left subtree first, for  $x1 = 0$ .

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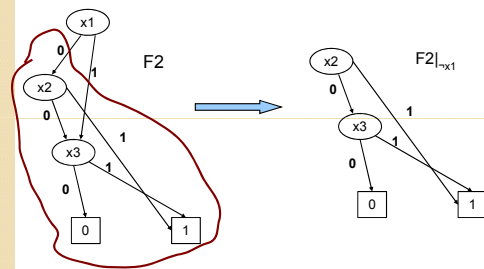
We determine the BDD's representing  $F1|_{\neg x1}$  and  $F2|_{\neg x1}$



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We determine the BDD's representing  $F1|_{\neg x1}$  and  $F2|_{\neg x1}$

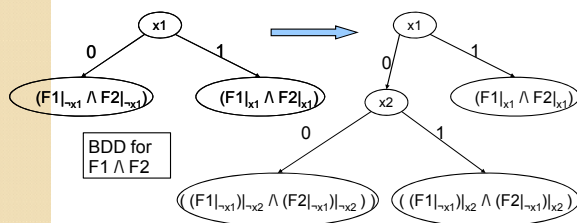


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Then, using BDDs for  $F1|_{\neg x1}$  and  $F2|_{\neg x1}$ , we will recursively compute  $F1|_{\neg x1} \wedge F2|_{\neg x1} =$

$$(x2 \wedge ((F1|_{\neg x1})|_{x2} \wedge (F2|_{\neg x1})|_{x2})) \vee (\neg x2 \vee ((F1|_{\neg x1})|_{\neg x2} \wedge (F2|_{\neg x1})|_{\neg x2}))$$

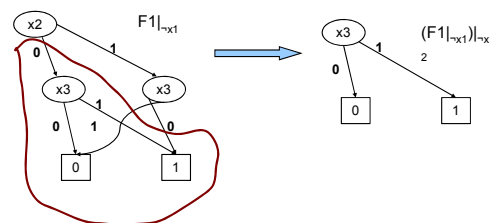


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Next, we consider the leftmost subtree corresponding to  $x1=0, x2=0$  in the BDD of  $F1 \wedge F2$ .

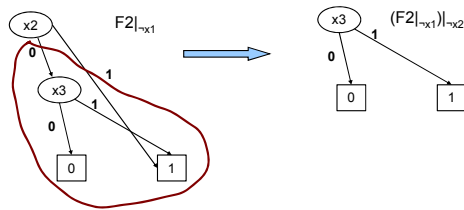
We determine the BDD's for  $(F1|_{\neg x1})|_{\neg x2}$  and  $(F2|_{\neg x1})|_{\neg x2}$ .



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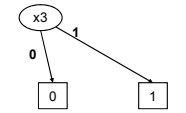
Next, we consider the leftmost subtree corresponding to  $x_1=0, x_2=0$  in the BDD of  $F_1 \wedge F_2$ . We determine the BDD's for  $(F_1|_{\neg x_1})|_{\neg x_2}$  and  $(F_2|_{\neg x_1})|_{\neg x_2}$ .



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From the last two slides it is easy to see that both  $(F_1|_{\neg x_1})|_{\neg x_2}$  and  $(F_2|_{\neg x_1})|_{\neg x_2}$  simply correspond to  $x_3$ . Therefore, BDD for  $(F_1|_{\neg x_1})|_{\neg x_2} \wedge (F_2|_{\neg x_1})|_{\neg x_2}$  is the following...

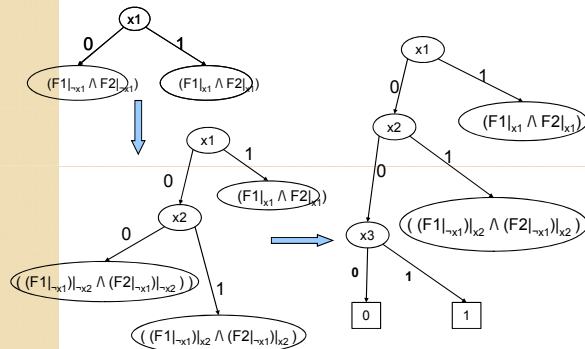


BDD for  $(F_1|_{\neg x_1})|_{\neg x_2} \wedge (F_2|_{\neg x_1})|_{\neg x_2}$

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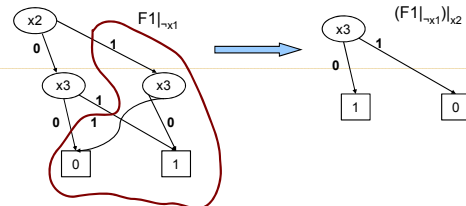
Picture so far: BDD for  $F_1 \wedge F_2$



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Next, we compute the BDD for subtree corresponding to  $x_1=0, x_2=1$  in the BDD of  $F_1 \wedge F_2$ , i.e. for the expression  $((F_1|_{\neg x_1})|_{x_2} \wedge (F_2|_{\neg x_1})|_{x_2})$ . We determine BDDs for  $(F_1|_{\neg x_1})|_{x_2}$  and  $(F_2|_{\neg x_1})|_{x_2}$ .

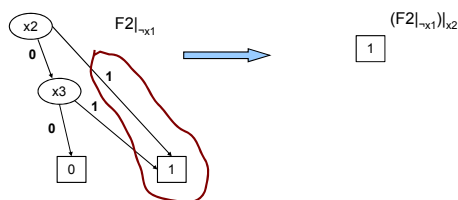


From BDD of  $F_1$ , see slide 7

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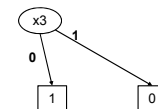
Next, we compute the BDD for subtree corresponding to  $x_1=0, x_2=1$  in the BDD of  $F_1 \wedge F_2$ , i.e. for the expression  $((F_1|_{\neg x_1})|_{x_2} \wedge (F_2|_{\neg x_1})|_{x_2})$ . We determine BDDs for  $(F_1|_{\neg x_1})|_{x_2}$  and  $(F_2|_{\neg x_1})|_{x_2}$ .



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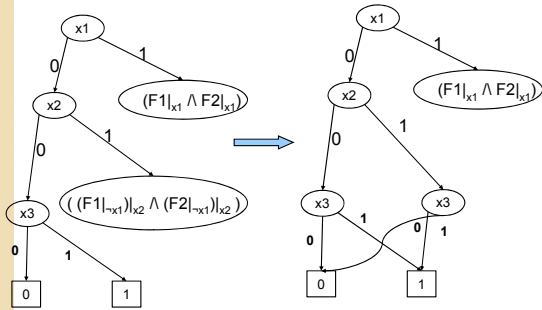
Since BDD for  $(F_2|_{\neg x_1})|_{x_2}$  is simply *true* (represented by BDD leaf node 1), BDD for the expression  $((F_1|_{\neg x_1})|_{x_2} \wedge (F_2|_{\neg x_1})|_{x_2})$  is given by...



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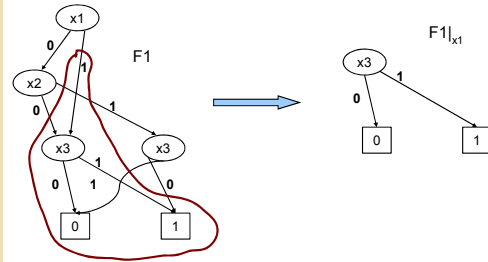
Hence, the BDD for  $F1 \wedge F2$  so far...



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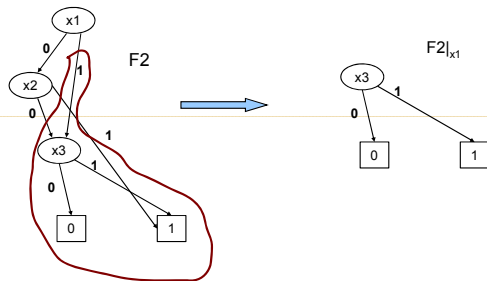
We continue in the similar manner to determine the BDD for the remaining case (for  $x=1$ ), i.e. BDD for the expression  $F1|_{x1} \wedge F2|_{x1}$ .



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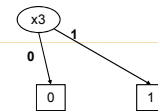
We continue in the similar manner to determine the BDD for the remaining case (for  $x=1$ ), i.e. BDD for the expression  $F1|_{x1} \wedge F2|_{x1}$ .



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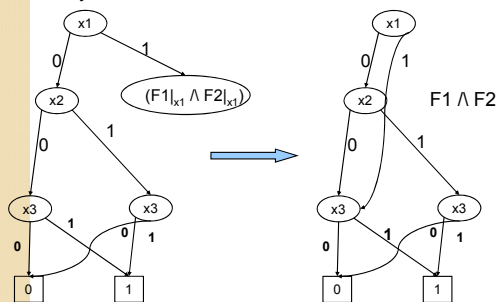
Note that the BDDs for  $F1|_{x1}$  and  $F2|_{x1}$  are identical to those for  $(F1|_{x1})|_{x2}$  and  $(F2|_{x1})|_{x2}$  (see slides 6, 7). Hence, we can simply reuse that result (see slide 8), and so the BDD for  $F1|_{x1} \wedge F2|_{x1}$  is...



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Finally, BDD for  $F1 \wedge F2$  is...



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