

Keywords

1. Combinatorics
2. Factorial
3. Set
4. Permutation
5. Variation
6. Combination
7. Symmetry of combination

Factorial

product of first n natural numbers

$$n! = 1*2*3*...*(n-1)*n$$

$$0! = 1$$

Set

Group of distinct elements

$s = \{a,b,c\}$ – Set

$s = [a,b,c,a]$ – Not set (a is repeated)

All possible combinations for **$S = \{a,b,c,d,e\}$** in a group of 3 elements

1. abc
2. abd
3. abe
4. acd
5. ace
6. ade
7. bcd

8. bce
9. bde
10. cde

All possible permutation for the $S = \{a, b, c, d, e\}$ in a group of 3 elements

1. abc
 - 1.1. abc
 - 1.2. acb
 - 1.3. bac
 - 1.4. bca
 - 1.5. cab
 - 1.6. cba
2. abd – this has again 6 permutations
 - .
 - .
 - .
10. cde – this again has again 6 permutations

for the above example $n=5 = |S|$, $k=3$

For each combination we have $k!$ permutations

Ordering the k -combinations of S in all possible ways produces the k -permutations of S

permutation formula = $P(n, k) = n! / (n - k)! = n * (n - 1) * (n - 2) * \dots * (n - k + 1)$

combination formula = $C(n, k) = n! / ((n - k)! k!) = P(n, k) / k!$

Permutation means pick and order all the elements in the set

k-permutation/variation means pick and order some of the elements in the set ($k \leq n$)

combination means selecting some of the elements in the set (if you select all the elements in the set then there is only one possible way to do it)

Permutation with repetition

Ordered arrangements of k elements of a set S , where repetition is allowed = $|S|^k = n^k$

Symmetry of combination

$$C(n, k) = C(n, n-k)$$

$$C(n, 0) = 1 = C(n, n)$$

$$C(n, k) = 0, \text{ when } k > n$$

$$P(n, k) = 0, \text{ when } k > n$$

$$P(n, k) = C(n, k) * k! = n! / (n-k)!$$

$$\begin{array}{|c|c|c|} \hline & & n! \\ \hline C(n, k) & = & \frac{n!}{(n-k)! * k!} \\ \hline \end{array} = \frac{P(n, k)}{k!} = C(n, n-k)$$

$$n=5$$

$$c(5, 0) = 1$$

$$c(5, 1) = 5$$

$$c(5, 2) = 10$$

$$c(5, 3) = 10$$

$$c(5, 4) = 5$$

$$c(5, 5) = 1$$

$$c(6, 0) = 1$$

$$c(6, 6) = 1$$

$$c(n, 0) = 1 = c(n, n)$$

$$\begin{array}{|c|c|c|} \hline & & n! \\ \hline P(n, k) & = & \frac{n!}{(n-k)!} \\ \hline \end{array} = C(n, k) * k!$$