Van der Pol Oscillator

AE 425 Project

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Introduction

The Van der Pol oscillator is a non-conservative oscillator with non-linear damping. The oscillator is defined by the following second-order differential equation:

$$\frac{d^2x}{dt^2} - \mu(1 - x^2)\frac{dx}{dt} + x = 0 \tag{1}$$

where x is the position coordinate which is a function of the time t, and is a scalar parameter indicating the nonlinearity and the strength of the damping.

Two-dimensional form

Linard's theorem[2] is used to prove that the van der Pol Oscillator has a limit cycle. Applying the Linard transformation: $y = x - x^3/3 - \frac{\dot{x}}{\mu}$, the Van der Pol oscillator can be written in its two-dimensional form:

$$\dot{x} = \mu(x - \frac{1}{3}x^3 - y) \tag{2}$$

$$\dot{y} = \frac{1}{\mu}x \tag{3}$$

Another form which is used is based on the transformation $y = \dot{x}$:

$$\dot{x} = y \tag{4}$$

$$\dot{y} = \mu(1 - x^2)y - x \tag{5}$$

The system is not volume-preserving except when $\mu = 0$[1]. If $\mu = 0$, then the equation becomes $\ddot{x} + x = 0$, the simple harmonic oscillator, which is conservative. We formalize our analysis with some definitions.

Wesley Cao.
http://www.math.cornell.edu/.

Various.
www.wikipedia.com.