5.2 Compute the derivative $f'(x)$ of the logistic sigmoid

$$f(x) = \frac{1}{1 + exp(-x)}$$

Solution.

$$f(x) = \frac{1}{1 + exp(-x)} = (1 + exp(-x))^{-1}$$

Using Chain rule,

$$f'(x) = \frac{d(1 + exp(-x))^{-1}}{dx} = \frac{d(1 + exp(-x))^{-1}}{d(1 + exp(-x)}*\frac{d(1 + exp(-x))}{d(exp(-x))}*\frac{d(exp(-x))}{d(-x)}*\frac{d(-x)}{x}$$

$$f'(x) = (-(1 + exp(-x))^{-2}) * (0 + 1) * (exp(-x)) * (-1)$$

$$f'(x) = (-(1 + exp(-x))^{-2}) * (1) * (exp(-x)) * (-1)$$

$$f'(x) = ((1 + exp(-x))^{-2}) * (exp(-x))$$

$$f'(x) = \frac{exp(-x)}{(1 + exp(-x))^2}$$