

**3.10** Rotate the vectors

$$\mathbf{x}_1 := \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{x}_2 := \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

by  $30^\circ$ .

**Solution.**

$$\text{Rotation matrix } \mathbf{R} = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\text{Rotated vector } \mathbf{x}_{1v} = \mathbf{R}\mathbf{x}_1 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \sqrt{3} - \frac{3}{2} \\ 1 + \frac{3\sqrt{3}}{2} \end{bmatrix}$$

$$\text{Rotated vector } \mathbf{x}_{2v} = \mathbf{R}\mathbf{x}_2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix}$$