

**6.1** Consider the following bivariate distribution  $p(x, y)$  of two discrete random variables  $X$  and  $Y$ .

$Y$	$y_1$	0.01	0.02	0.03	0.1	0.1
	$y_2$	0.05	0.1	0.05	0.07	0.2
	$y_3$	0.1	0.05	0.03	0.05	0.04
		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
$X$						

Compute:

- a. The marginal distributions  $p(x)$  and  $p(y)$ .

**Solution.**

$$p(x)$$

$$p(X = x_1) = \sum_{t=1}^3 p(X = x_1, Y = y_t) = 0.01 + 0.05 + 0.1 = 0.16.$$

$$p(X = x_2) = \sum_{t=1}^3 p(X = x_2, Y = y_t) = 0.02 + 0.1 + 0.05 = 0.17.$$

$$p(X = x_3) = \sum_{t=1}^3 p(X = x_3, Y = y_t) = 0.03 + 0.05 + 0.03 = 0.11.$$

$$p(X = x_4) = \sum_{t=1}^3 p(X = x_4, Y = y_t) = 0.1 + 0.07 + 0.05 = 0.22.$$

$$p(X = x_5) = \sum_{t=1}^3 p(X = x_5, Y = y_t) = 0.1 + 0.2 + 0.04 = 0.34.$$

$p(x)$	0.16	0.17	0.11	0.22	0.34
	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$

$$p(y)$$

$$p(Y = y_1) = \sum_{t=1}^5 p(X = x_t, Y = y_1) = 0.01 + 0.02 + 0.03 + 0.1 + 0.1 = 0.26.$$

$$p(Y = y_2) = \sum_{t=1}^5 p(X = x_t, Y = y_2) = 0.05 + 0.1 + 0.05 + 0.07 + 0.2 = 0.47.$$

$$p(Y = y_3) = \sum_{t=1}^5 p(X = x_t, Y = y_3) = 0.1 + 0.05 + 0.03 + 0.05 + 0.04 = 0.27.$$

$p(y)$	0.26	0.47	0.27
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$y_1 \quad y_2 \quad y_3$

b. The conditional distributions  $p(x|Y = y_1)$  and  $p(y|X = x_3)$ .

**Solution.**

$$p(X = x_1|Y = y_1) = p(X = x_1, Y = y_1)/p(Y = y_1) = 0.01/0.26 = 0.038$$

$$p(X = x_2|Y = y_1) = p(X = x_2, Y = y_1)/p(Y = y_1) = 0.02/0.26 = 0.077$$

$$p(X = x_3|Y = y_1) = p(X = x_3, Y = y_1)/p(Y = y_1) = 0.03/0.26 = 0.115$$

$$p(X = x_4|Y = y_1) = p(X = x_4, Y = y_1)/p(Y = y_1) = 0.1/0.26 = 0.384$$

$$p(X = x_5|Y = y_1) = p(X = x_5, Y = y_1)/p(Y = y_1) = 0.1/0.26 = 0.384$$

$p(x Y = y_1)$	0.038	0.077	0.115	0.384	0.384
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$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

$$p(Y = y_1|X = x_3) = p(Y = y_1, X = x_3)/p(X = x_3) = 0.03/0.11 = 0.272$$

$$p(Y = y_2|X = x_3) = p(Y = y_2, X = x_3)/p(X = x_3) = 0.05/0.11 = 0.455$$

$$p(Y = y_3|X = x_3) = p(Y = y_3, X = x_3)/p(X = x_3) = 0.03/0.11 = 0.272$$

$p(y X = x_3)$	0.272	0.455	0.272
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$y_1 \quad y_2 \quad y_3$