6.6 Prove the relationship in (6.44), which relates the standard definition of the variance to the raw-score expression for the variance.

Solution. To prove :

$$\mathbb{V}_X[x] = \mathbb{E}_X[x^2] - (\mathbb{E}_X[x])^2.$$

We know that $\mathbb{V}_X[x] \coloneqq \mathbb{E}_X[(x-\mu)^2]$, where $\mu = \mathbb{E}_X[x]$.

$\implies \mathbb{V}_X[x] = \mathbb{E}_X[x^2 + \mu^2 - 2x\mu]$

$\implies \mathbb{V}_X[x] = \mathbb{E}_X[x^2] + \mathbb{E}_X[\mu^2] - \mathbb{E}_X[2x\mu] \qquad$ by linearity of expectation

$\implies \mathbb{V}_X[x] = \mathbb{E}_X[x^2] + \mu^2 - 2\mu\mathbb{E}_X[x]$

$\implies \mathbb{V}_X[x] = \mathbb{E}_X[x^2] + \mu^2 - 2\mu\mu$

$\implies \mathbb{V}_X[x] = \mathbb{E}_X[x^2] + \mu^2 - 2\mu^2$

$\implies \mathbb{V}_X[x] = \mathbb{E}_X[x^2] - \mu^2$

$\implies \mathbb{V}_X[x] = \mathbb{E}_X[x^2] - (\mathbb{E}_X[x])^2$

Thus proved.