3.3 Compute the distance between

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

using

a. $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^T \mathbf{y}$

Solution.

$$d(\mathbf{x}, \mathbf{y}) := \|\mathbf{x} - \mathbf{y}\| = \sqrt{\langle \mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{y} \rangle}$$

$$\mathbf{x} - \mathbf{y} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$$

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})^T (\mathbf{x} - \mathbf{y})}$$

$$= \sqrt{2 * 2 + 3 * 3 + 3 * 3}$$

$$= \sqrt{22}$$

b. $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^T \mathbf{A} \mathbf{y}, \qquad \mathbf{A} := \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

Solution.

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})^T \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} (\mathbf{x} - \mathbf{y})}$$

$$= \sqrt{\begin{bmatrix} 2 & 3 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}}$$

$$= \sqrt{\begin{bmatrix} 7 & 8 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}}$$

$$= \sqrt{14 + 24 + 9} = \sqrt{47}$$