4.1 Compute the determinant using Laplace expansion (using the first row) and the Sarrus rule for

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 2 & 4 \end{bmatrix}$$

Solution.

If we use Laplace expansion along row 1,

$$\det(\mathbf{A}) = \Sigma_{k=1}^{n}(-1)^{k+1}a_{1k}\det(\mathbf{A}_{1,k})$$

$$= (-1)^{1+1}a_{11} \cdot \begin{vmatrix} 4 & 6 \\ 2 & 4 \end{vmatrix} + (-1)^{2+1}a_{12} \cdot \begin{vmatrix} 2 & 6 \\ 0 & 4 \end{vmatrix} + (-1)^{3+1}a_{13} \cdot \begin{vmatrix} 2 & 4 \\ 0 & 2 \end{vmatrix}$$

$= (-1)^2 * 1 * (4 * 4 - 6 * 2) + (-1)^3 * 3 * (2 * 4 - 6 * 0) + (-1)^4 * 5 * (2 * 2 - 4 * 0)$

$= 1 * 1 * (16 - 12) + (-1) * 3 * (8 - 0) + 1 * 5 * (2 * 2 - 4 * 0)$

$= 4 - 3 * 8 + 5 * 4$

$= 4 - 24 + 20$

$= 0$

Using Sarrus rule,

$\det(\mathbf{A}) = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{31}a_{22}a_{13} - a_{11}a_{32}a_{23} - a_{21}a_{12}a_{33}$

$= 1 * 4 * 4 + 2 * 2 * 5 + 0 * 3 * 6 - 0 * 4 * 5 - 1 * 2 * 6 - 2 * 3 * 4$

$= 16 + 20 + 0 - 0 - 12 - 24$

$= 36 - 36$

$= 0$