

**7.5** Express the following optimization problem as a standard linear program in matrix notation

$$\max_{\mathbf{x} \in \mathbb{R}^2, \xi \in \mathbb{R}} \mathbf{p}^T \mathbf{x} + \xi$$

subject to the constraints that  $\xi \geq 0$ ,  $x_0 \leq 0$  and  $x_1 \leq 3$ .

**Solution.**

$$\text{Let } \mathbf{p} = \begin{bmatrix} p_0 \\ p_1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} p_0 \\ p_1 \\ 1 \end{bmatrix} \text{ and let } \mathbf{y} = \begin{bmatrix} x_0 \\ x_1 \\ \xi \end{bmatrix}.$$

Then,  $\mathbf{p}^T \mathbf{x} + \xi = \mathbf{c}^T \mathbf{y}$ .

$$\text{Also, let } \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and let } \mathbf{b} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}.$$

Then, the constraints can be represented as  $\mathbf{A}\mathbf{y} \leq \mathbf{b}$ .

Therefore, the given optimization problem can be expressed as a linear program in the following way:

$$\begin{aligned} & \max_{\mathbf{y} \in \mathbb{R}^3} \mathbf{c}^T \mathbf{y} \\ & \text{subject to } \mathbf{A}\mathbf{y} \leq \mathbf{b}. \end{aligned}$$