6.2 Consider a mixture of two Gaussian distributions (illustrated in Figure 6.4),

$$0.4\mathcal{N}\left(\begin{bmatrix}10\\2\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right) + 0.6\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}8.4 & 2.0\\2.0 & 1.7\end{bmatrix}\right)$$

a. Compute the marginal distributions for each dimension.

Solution.

In the marginal distributions, we simply drop the irrelevant variables.

$$\implies p(x_1) = 0.4\mathcal{N}(10, 1) + 0.6\mathcal{N}(0, 8.4)$$

$$\text{and } p(x_2) = 0.4\mathcal{N}(2, 1) + 0.6\mathcal{N}(0, 1.7)$$

b. Compute the mean, mode and median for each marginal distribution.

Solution.

Using 6.81 (which is applicable since 0.4+0.6=1),

$$\mathbb{E}[x_1] = 0.4 * 10 + 0.6 * 0 = 4$$

$$\mathbb{E}[x_2] = 0.4 * 2 + 0.6 * 0 = 0.8$$

Each distribution has a peak of its own, so $p(x_1)$ has 2 modes at 10 and 0, and $p(x_2)$ has 2 modes at 2 and 0.

Median is given by the point where *cdf* is 0.5.

So, median of x_1 is at a where $\int_{-\infty}^{a} 0.4\mathcal{N}(10,1) + 0.6\mathcal{N}(0,8.4) = 0.5$.

Similarly, median of x_2 is at b where $\int_{-\infty}^{b} 0.4\mathcal{N}(2,1) + 0.6\mathcal{N}(0,1.7) = 0.5$.

c. Compute the mean and mode for the two-dimensional distribution.

Solution.

$$\text{Mean} = \begin{bmatrix} \mathbb{E}[x_1] \\ \mathbb{E}[x_2] \end{bmatrix} = \begin{bmatrix} 4 \\ 0.8 \end{bmatrix}$$

There are 2 modes, at the peaks of the individual gaussian distributions:

$$\begin{bmatrix} 10 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$