6.4 There are two bags. The first bag contains four mangos and two apples; the second bag contains four mangos and four apples.

We also have a biased coin, which shows "heads" with probability 0.6 and "tails" with probability 0.4. If the coin shows "heads", we pick a fruit at random from bag 1; otherwise we pick a fruit at random from bag 2.

Your friend flips the coin (you cannot see the result), picks a fruit at random from the corresponding bag, and present you a mango.

What is the probability that the mango was picked from bag 2?

Hint: Use Bayes' theorem.

Solution.

Let $X :=$ result of the coin flip and let $Y :=$ fruit that is picked.

Bag 2 implies that the result of the coin flip was "tails".

We need to find $P(X = tails|Y = mango)$.

Using Bayes' theorem:

$$P(X = tails|Y = mango) = \frac{P(Y = mango|X = tails) * P(X = tails)}{P(Y = mango)}$$

$$P(X = tails) = 0.4$$

$$P(Y = mango) = \Sigma_X P(X, Y = mango) = P(X = heads, Y = mango) + P(X = tails, Y = mango)$$

$$= P(Y = mango|X = heads) * P(X = heads) + P(Y = mango|X = tails) * P(X = tails)$$

$$= \frac{4}{6} * 0.6 + \frac{4}{8} * 0.4 = 0.4 + 0.2 = 0.6$$

$$P(Y = mango|X = tails) = \frac{4}{8} = 0.5$$

Substituting, we get

$$P(X = tails|Y = mango) = \frac{0.4 * 0.5}{0.6} = \frac{0.2}{0.6} = 0.33$$