

1.14 For every i, j , if $w_{ij} = w_{ij}^S + w_{ij}^A$, then $w_{ji} = w_{ij}^S - w_{ij}^A$.

This is feasible since we have 2 variables for 2 equations.

$$\begin{aligned}
2 \sum_{i=1}^D \sum_{j=1}^D w_{ij} x_i x_j &= \sum_{i=1}^D \sum_{j=1}^D w_{ij} x_i x_j + \sum_{i=1}^D \sum_{j=1}^D w_{ji} x_j x_i \\
&= \sum_{i=1}^D \sum_{j=1}^D (w_{ij}^S + w_{ij}^A) x_i x_j + \sum_{i=1}^D \sum_{j=1}^D (w_{ij}^S - w_{ij}^A) x_j x_i \\
&= \sum_{i=1}^D \sum_{j=1}^D ((w_{ij}^S + w_{ij}^A) x_i x_j + (w_{ij}^S - w_{ij}^A) x_j x_i) \\
&= \sum_{i=1}^D \sum_{j=1}^D 2w_{ij}^S x_i x_j \\
&\implies 2 \sum_{i=1}^D \sum_{j=1}^D w_{ij} x_i x_j = \sum_{i=1}^D \sum_{j=1}^D 2w_{ij}^S x_i x_j \\
&\implies \sum_{i=1}^D \sum_{j=1}^D w_{ij} x_i x_j = \sum_{i=1}^D \sum_{j=1}^D w_{ij}^S x_i x_j
\end{aligned}$$

The number of independent parameters in the matrix w_{ij}^S is given by the sum of the number of elements above (or below, either works) the diagonal, given by $\left(\frac{1}{2}(D^2 - D)\right)$, plus the number of elements in the diagonal, given by D .

$$\begin{aligned}
&= \frac{1}{2}(D^2 - D) + D \\
&= \frac{D^2}{2} - \frac{D}{2} + D \\
&= \frac{D^2}{2} + \frac{D}{2} \\
&= \frac{D(D+1)}{2}
\end{aligned}$$