2.41

$$Gam(\lambda|a,b) = \frac{1}{\Gamma(a)} b^a \lambda^{a-1} exp(-b\lambda)$$

λ ranges from 0 to ∞. So we need to prove that

$$\int_0^\infty Gam(\lambda|a,b)\, d\lambda = \int_0^\infty \frac{1}{\Gamma(a)} b^a \lambda^{a-1} exp(-b\lambda)\, d\lambda = 1$$

$$\frac{1}{\Gamma(a)} \int_0^\infty b^a \lambda^{a-1} exp(-b\lambda)\, d\lambda$$

$$= \frac{1}{\Gamma(a)} \int_0^\infty (b\lambda)^{a-1} exp(-b\lambda)\, d(b\lambda)$$

Let $u = b\lambda$. The limits don't change since they are positive and negative infinity and $b\lambda$ will also range from negative infinity to positive infinity. So we get:

$$= \frac{1}{\Gamma(a)} \int_0^\infty (u)^{a-1} exp(-u)\, d(u)$$

$$= \frac{1}{\Gamma(a)} \Gamma(a)$$

$$= 1.$$