3.18 From 3.79, we have:

$$E(\mathbf{w}) = \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi w}||^2 + \frac{\alpha}{2}\mathbf{w}^T\mathbf{w}$$

$$= \frac{\beta}{2}(\mathbf{t} - \mathbf{\Phi w})^T(\mathbf{t} - \mathbf{\Phi w}) + \frac{\alpha}{2}\mathbf{w}^T\mathbf{w}$$

$$= \frac{\beta}{2}\mathbf{t}^T\mathbf{t} + \frac{\beta}{2}(\mathbf{\Phi w})^T(\mathbf{\Phi w}) - \beta\mathbf{t}^T\mathbf{\Phi w} + \frac{\alpha}{2}\mathbf{w}^T\mathbf{w}$$

Adding and subtracting $\left(\frac{\beta}{2}(\mathbf{\Phi m}_N)^T(\mathbf{\Phi m}_N) - \beta\mathbf{t}^T\mathbf{\Phi m}_N\right)$, we get:

$$= \frac{\beta}{2}\mathbf{t}^T\mathbf{t} + \frac{\beta}{2}(\mathbf{\Phi m}_N)^T(\mathbf{\Phi m}_N) - \beta\mathbf{t}^T\mathbf{\Phi m}_N$$
$$-\frac{\beta}{2}(\mathbf{\Phi m}_N)^T(\mathbf{\Phi m}_N) + \beta\mathbf{t}^T\mathbf{\Phi m}_N$$
$$+\frac{\beta}{2}(\mathbf{\Phi w})^T(\mathbf{\Phi w}) - \beta\mathbf{t}^T\mathbf{\Phi w} + \frac{\alpha}{2}\mathbf{w}^T\mathbf{w}$$

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi m}_N||^2$$
$$-\frac{\beta}{2}(\mathbf{\Phi m}_N)^T(\mathbf{\Phi m}_N) + \beta\mathbf{t}^T\mathbf{\Phi m}_N$$
$$+\frac{\beta}{2}(\mathbf{\Phi w})^T(\mathbf{\Phi w}) - \beta\mathbf{t}^T\mathbf{\Phi w} + \frac{\alpha}{2}\mathbf{w}^T\mathbf{w}$$

Adding and subtracting $\frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$, we get:

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi m}_N||^2 + \frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$
$$-\frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N - \frac{\beta}{2}(\mathbf{\Phi m}_N)^T(\mathbf{\Phi m}_N) + \beta\mathbf{t}^T\mathbf{\Phi m}_N$$
$$+\frac{\beta}{2}(\mathbf{\Phi w})^T(\mathbf{\Phi w}) - \beta\mathbf{t}^T\mathbf{\Phi w} + \frac{\alpha}{2}\mathbf{w}^T\mathbf{w}$$

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi m}_N||^2 + \frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$

$$+\frac{1}{2}\left(-\alpha\mathbf{m}_N^T\mathbf{m}_N - \beta(\mathbf{\Phi}\mathbf{m}_N)^T(\mathbf{\Phi}\mathbf{m}_N) + 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{m}_N + \beta(\mathbf{\Phi}\mathbf{w})^T(\mathbf{\Phi}\mathbf{w}) - 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{w} + \alpha\mathbf{w}^T\mathbf{w}\right)$$

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi}\mathbf{m}_N||^2 + \frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$

$$+\frac{1}{2}\left(\alpha\mathbf{w}^T\mathbf{w} - \alpha\mathbf{m}_N^T\mathbf{m}_N + \beta(\mathbf{\Phi}\mathbf{w})^T(\mathbf{\Phi}\mathbf{w}) - \beta(\mathbf{\Phi}\mathbf{m}_N)^T(\mathbf{\Phi}\mathbf{m}_N) + 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{m}_N - 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{w}\right)$$

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi}\mathbf{m}_N||^2 + \frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$

$$+\frac{1}{2}\left(\alpha\mathbf{w}^T\mathbf{w} + \beta\mathbf{w}^T\mathbf{\Phi}^T\mathbf{\Phi}\mathbf{w} - \alpha\mathbf{m}_N^T\mathbf{m}_N - \beta\mathbf{m}_N^T\mathbf{\Phi}^T\mathbf{\Phi}\mathbf{m}_N + 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{m}_N - 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{w}\right)$$

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi}\mathbf{m}_N||^2 + \frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$

$$+\frac{1}{2}\left(\mathbf{w}^T(\alpha\mathbf{I} + \beta\mathbf{\Phi}^T\mathbf{\Phi})\mathbf{w} - \mathbf{m}_N^T(\alpha\mathbf{I} + \beta\mathbf{\Phi}^T\mathbf{\Phi})\mathbf{m}_N + 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{m}_N - 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{w}\right)$$

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi}\mathbf{m}_N||^2 + \frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$

$$+\frac{1}{2}\left(\mathbf{w}^T\mathbf{A}\mathbf{w} - \mathbf{m}_N^T\mathbf{A}\mathbf{m}_N + 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{m}_N - 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{w}\right)$$

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi}\mathbf{m}_N||^2 + \frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$

$$+\frac{1}{2}\left(\mathbf{w}^T\mathbf{A}\mathbf{w} + \mathbf{m}_N^T\mathbf{A}\mathbf{m}_N - 2\mathbf{m}_N^T\mathbf{A}\mathbf{m}_N + 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{m}_N - 2\beta\mathbf{t}^T\mathbf{\Phi}\mathbf{w}\right)$$

Applying the definition from 3.84, where

$$\beta\mathbf{t}^T\mathbf{\Phi} = (\mathbf{A}\mathbf{m}_N)^T = \mathbf{m}_N^T\mathbf{A}^T = \mathbf{m}_N^T\mathbf{A}$$

, we get:

$$= \frac{\beta}{2}||\mathbf{t} - \mathbf{\Phi}\mathbf{m}_N||^2 + \frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$

$$+\frac{1}{2}\left(\mathbf{w}^T\mathbf{A}\mathbf{w}+\mathbf{m}_N^T\mathbf{A}\mathbf{m}_N-2\mathbf{m}_N^T\mathbf{A}\mathbf{m}_N+2\mathbf{m}_N^T\mathbf{A}\mathbf{m}_N-2\beta\mathbf{t}^T\boldsymbol{\Phi}\mathbf{w}\right)$$

$$=\frac{\beta}{2}||\mathbf{t}-\boldsymbol{\Phi}\mathbf{m}_N||^2+\frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$
$$+\frac{1}{2}\left(\mathbf{w}^T\mathbf{A}\mathbf{w}+\mathbf{m}_N^T\mathbf{A}\mathbf{m}_N-2\beta\mathbf{t}^T\boldsymbol{\Phi}\mathbf{w}\right)$$

$$=\frac{\beta}{2}||\mathbf{t}-\boldsymbol{\Phi}\mathbf{m}_N||^2+\frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$
$$+\frac{1}{2}\left(\mathbf{w}^T\mathbf{A}\mathbf{w}+\mathbf{m}_N^T\mathbf{A}\mathbf{m}_N-2\mathbf{m}_N^T\mathbf{A}\mathbf{w}\right)$$

$$=\frac{\beta}{2}||\mathbf{t}-\boldsymbol{\Phi}\mathbf{m}_N||^2+\frac{\alpha}{2}\mathbf{m}_N^T\mathbf{m}_N$$
$$+\frac{1}{2}\left((\mathbf{w}-\mathbf{m}_N)^T\mathbf{A}(\mathbf{w}-\mathbf{m}_N)\right)$$

$$=E(\mathbf{m}_N)+\frac{1}{2}(\mathbf{w}-\mathbf{m}_N)^T\mathbf{A}(\mathbf{w}-\mathbf{m}_N)$$

which is the same as 3.80.