

5.29 From 5.139, we have,

$$\begin{aligned}\tilde{E}(\mathbf{w}) &= E(\mathbf{w}) + \lambda \Omega(\mathbf{w}) \\ \implies \frac{\partial \tilde{E}}{\partial w_i} &= \frac{\partial E}{\partial w_i} + \frac{\partial \lambda \Omega(\mathbf{w})}{\partial w_i} \\ R.H.S &= \frac{\partial E}{\partial w_i} + \lambda \frac{\partial \Omega(\mathbf{w})}{\partial w_i}\end{aligned}$$

$$\frac{\partial \Omega(\mathbf{w})}{\partial w_i} = \frac{\partial}{\partial w_i} \left(- \sum_i \ln \left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2) \right) \right)$$

Since the partial derivative is only w.r.t one i , this becomes:

$$\begin{aligned}&= \frac{\partial}{\partial w_i} \left(- \ln \left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2) \right) \right) \\ &= - \frac{1}{\left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2) \right)} \frac{\partial}{\partial w_i} \left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2) \right) \\ &= - \frac{1}{\left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2) \right)} \frac{\partial}{\partial w_i} \left(\sum_{j=1}^M \frac{\pi_j}{(2\pi\sigma_j^2)} \exp \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \right) \\ &= - \frac{1}{\left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2) \right)} \left(\sum_{j=1}^M \frac{\pi_j}{(2\pi\sigma_j^2)} \exp \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \frac{\partial \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right)}{\partial w_i} \right) \\ &= - \frac{1}{\left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2) \right)} \left(\sum_{j=1}^M \frac{\pi_j}{(2\pi\sigma_j^2)} \exp \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \left(- \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right) \right) \\ &= \frac{1}{\left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2) \right)} \left(\sum_{j=1}^M \frac{\pi_j}{(2\pi\sigma_j^2)} \exp \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right)\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)} \left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right) \\
&= \sum_{j=1}^M \frac{\pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2)}{\left(\sum_{j=1}^M \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)} \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \\
&= \sum_{j=1}^M \gamma_j(w_i) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right)
\end{aligned}$$

Substituting this result in the partial derivative above, we get:

$$\frac{\partial \tilde{E}}{\partial w_i} = \frac{\partial E}{\partial w_i} + \lambda \sum_{j=1}^M \gamma_j(w_i) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right)$$

which is the same as the result in 5.141.