2.45

$$p(\mathbf{\Lambda}|\mathbf{X}) \propto p(\mathbf{X}|\mathbf{\Lambda})p(\mathbf{\Lambda})$$

$$R.H.S = \prod_{n=1}^{N} \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}, \mathbf{\Lambda}^{-1})\, \mathcal{W}(\mathbf{\Lambda}|\mathbf{W}, \nu)$$

$$R.H.S = \mathcal{W}(\mathbf{\Lambda}|\mathbf{W}, \nu)\, \prod_{n=1}^{N} \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}, \mathbf{\Lambda}^{-1})$$

$$= B|\mathbf{\Lambda}|^{(\nu-D-1)/2} exp\left(-\frac{1}{2}Tr(\mathbf{W}^{-1}\mathbf{\Lambda})\right) \prod_{n=1}^{N} \frac{1}{(2\pi)^{D/2}|\mathbf{\Lambda}^{-1}|^{1/2}} exp\left(-\frac{1}{2}(\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda}(\mathbf{x}_n - \boldsymbol{\mu})\right)$$

$$= B|\mathbf{\Lambda}|^{(\nu-D-1)/2} exp\left(-\frac{1}{2}Tr(\mathbf{W}^{-1}\mathbf{\Lambda})\right) \frac{1}{(2\pi)^{ND/2}|\mathbf{\Lambda}^{-1}|^{N/2}} exp\left(\sum_{n=1}^{N} -\frac{1}{2}(\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda}(\mathbf{x}_n - \boldsymbol{\mu})\right)$$

$$= \frac{B|\mathbf{\Lambda}|^{(\nu-D-1)/2}}{(2\pi)^{ND/2}|\mathbf{\Lambda}|^{-N/2}} exp\left(-\frac{1}{2}Tr(\mathbf{W}^{-1}\mathbf{\Lambda})\right) exp\left(-\frac{1}{2}\sum_{n=1}^{N}(\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda}(\mathbf{x}_n - \boldsymbol{\mu})\right)$$

$$= \frac{B|\mathbf{\Lambda}|^{(\nu-D-1+N)/2}}{(2\pi)^{ND/2}} exp\left(-\frac{1}{2}Tr(\mathbf{W}^{-1}\mathbf{\Lambda}) - \frac{1}{2}\sum_{n=1}^{N}(\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda}(\mathbf{x}_n - \boldsymbol{\mu})\right)$$

Since $\mathbf{x}^T\mathbf{y} = Tr(\mathbf{x}^T\mathbf{y}) = Tr(\mathbf{y}\mathbf{x}^T)$, we can say that $(\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda}(\mathbf{x}_n - \boldsymbol{\mu}) = Tr((\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda}(\mathbf{x}_n - \boldsymbol{\mu})) = Tr((\mathbf{x}_n - \boldsymbol{\mu})(\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda})$, giving us:

$$= \frac{B|\mathbf{\Lambda}|^{(\nu-D-1+N)/2}}{(2\pi)^{ND/2}} exp\left(-\frac{1}{2}Tr(\mathbf{W}^{-1}\mathbf{\Lambda}) - \frac{1}{2}Tr\left(\sum_{n=1}^{N}(\mathbf{x}_n - \boldsymbol{\mu})(\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda}\right)\right)$$

$$= \frac{B|\mathbf{\Lambda}|^{(\nu-D-1+N)/2}}{(2\pi)^{ND/2}} exp\left(-\frac{1}{2}\,Tr\left(\mathbf{W}^{-1}\mathbf{\Lambda} + \sum_{n=1}^{N}(\mathbf{x}_n - \boldsymbol{\mu})(\mathbf{x}_n - \boldsymbol{\mu})^T\mathbf{\Lambda}\right)\right)$$

$$= \frac{B|\mathbf{\Lambda}|^{(\nu-D-1+N)/2}}{(2\pi)^{ND/2}} exp\left(-\frac{1}{2}\,Tr\left(\left(\mathbf{W}^{-1} + \sum_{n=1}^{N}(\mathbf{x}_n - \boldsymbol{\mu})(\mathbf{x}_n - \boldsymbol{\mu})^T\right)\mathbf{\Lambda}\right)\right)$$

which has the same form as a $Wishart$ distribution.