2.54 Computing first derivative of the von Mises distribution:

$$\frac{\partial}{\partial\theta}\left(\frac{1}{2\pi I_0(m)}\exp\{m\ \cos(\theta-\theta_0)\}\right)$$

$$= \frac{1}{2\pi I_0(m)}\frac{\partial \exp\{m\ \cos(\theta-\theta_0)\}}{\partial\theta}$$

$$= \frac{1}{2\pi I_0(m)}\frac{\partial \exp\{m\ \cos(\theta-\theta_0)\}}{\partial m\ \cos(\theta-\theta_0)}\frac{\partial m\ \cos(\theta-\theta_0)}{\partial\theta}$$

$$= \frac{1}{2\pi I_0(m)}\exp\{m\ \cos(\theta-\theta_0)\}(-m\ \sin(\theta-\theta_0))$$

If we set this first derivative to 0, we get $\sin(\theta-\theta_0)=0, \implies \theta=\theta_0$ or $\theta=\theta_0+\pi n$ where n is an integer.

Computing first derivative of the von Mises distribution:

$$\frac{\partial}{\partial\theta}\left(\frac{1}{2\pi I_0(m)}\exp\{m\ \cos(\theta-\theta_0)\}(-m\ \sin(\theta-\theta_0))\right)$$

$$= \frac{1}{2\pi I_0(m)}\left(\exp\{m\ \cos(\theta-\theta_0)\}\frac{\partial(-m\ \sin(\theta-\theta_0))}{\partial\theta} + (-m\ \sin(\theta-\theta_0))\frac{\partial \exp\{m\ \cos(\theta-\theta_0)}{\partial\theta}\right)$$

$$= \frac{1}{2\pi I_0(m)}\left(-\exp\{m\ \cos(\theta-\theta_0)\}m\ \cos(\theta-\theta_0)) + (m\ \sin(\theta-\theta_0))^2\exp\{m\ \cos(\theta-\theta_0)\right)$$

At $\theta=\theta_0+\pi n$, $\sin(\theta-\theta_0)$ becomes 0. So we get:

$$= \frac{1}{2\pi I_0(m)}\left(-m\exp\{m\ \cos(\theta_0+\pi n-\theta_0)\}\ \cos(\theta_0+\pi n-\theta_0))\right)$$

$$= \frac{1}{2\pi I_0(m)}(-m\exp\{m\ \cos(\pi n)\}\ \cos(\pi n))$$

At $\theta=\theta_0$, $\cos(\pi n)=1$, and the second derivative becomes negative, giving us a maxima.

At $\theta=\theta_0+\pi(\mod 2\pi)$, $\cos(\pi n)=-1$, and the second derivative becomes positive, giving us a minima.