

3.17 3.77 gives us the equation for the evidence function:

$$p(\mathbf{t}|\alpha, \beta) = \int p(\mathbf{t}|\mathbf{w}, \beta) p(\mathbf{w}|\alpha) d\mathbf{w}$$

From 3.11 and 3.12, we can see that:

$$\begin{aligned} p(\mathbf{t}|\mathbf{w}, \beta) &= \beta^{N/2} \left(\frac{1}{(2\pi)} \right)^{N/2} \exp\{-\beta E_D(\mathbf{w})\} \\ &= \left(\frac{\beta}{2\pi} \right)^{N/2} \exp\{-\beta E_D(\mathbf{w})\} \end{aligned}$$

From 3.52, we can see that:

$$\begin{aligned} p(\mathbf{w}|\alpha) d\mathbf{w} &= \mathcal{N}(\mathbf{w}|\mathbf{0}, \alpha^{-1}\mathbf{I}) \\ &= \frac{1}{(2\pi)^{D/2} |\alpha^{-1}\mathbf{I}|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{w} - \mathbf{0})^T (\alpha^{-1}\mathbf{I})^{-1} (\mathbf{w} - \mathbf{0}) \right\} \\ &= \frac{1}{(2\pi)^{D/2} (\alpha^{-D})^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{w} - \mathbf{0})^T (\alpha^{-1}\mathbf{I})^{-1} (\mathbf{w} - \mathbf{0}) \right\} \\ &= \frac{\alpha^{D/2}}{(2\pi)^{D/2}} \exp \left\{ -\frac{\alpha}{2} \mathbf{w}^T \mathbf{w} \right\} \\ &= \left(\frac{\alpha}{2\pi} \right)^{D/2} \exp \{-\alpha E_{\mathbf{w}}(\mathbf{w})\} \end{aligned}$$

Substituting in the equation for the evidence function, we get:

$$\begin{aligned} p(\mathbf{t}|\alpha, \beta) &= \int \left(\frac{\beta}{2\pi} \right)^{N/2} \exp\{-\beta E_D(\mathbf{w})\} \left(\frac{\alpha}{2\pi} \right)^{D/2} \exp\{-\alpha E_{\mathbf{w}}(\mathbf{w})\} d\mathbf{w} \\ &= \left(\frac{\beta}{2\pi} \right)^{N/2} \left(\frac{\alpha}{2\pi} \right)^{D/2} \int \exp\{-(\beta E_D(\mathbf{w}) + \alpha E_{\mathbf{w}}(\mathbf{w}))\} d\mathbf{w} \\ &= \left(\frac{\beta}{2\pi} \right)^{N/2} \left(\frac{\alpha}{2\pi} \right)^{D/2} \int \exp\{-E(\mathbf{w})\} d\mathbf{w} \end{aligned}$$

which is the same as 3.78.