

1.29 Jensen's inequality is:

$$f\left(\sum_{i=1}^M \lambda_i y_i\right) \leq \sum_{i=1}^M \lambda_i f(y_i)$$

Let $f = -\ln$ (negative logarithm is a convex function) and $y_i = \frac{1}{\lambda_i}$. This gives us:

$$-\ln\left(\sum_{i=1}^M \lambda_i \frac{1}{\lambda_i}\right) \leq \sum_{i=1}^M \lambda_i \left(-\ln\left(\frac{1}{\lambda_i}\right)\right)$$

$$\implies -\ln\left(\sum_{i=1}^M 1\right) \leq \sum_{i=1}^M \lambda_i \ln(\lambda_i)$$

$$\implies -\ln M \leq \sum_{i=1}^M \lambda_i \ln(\lambda_i)$$

Here, λ_i can represent probabilities as they sum up to one and range from 0 to 1. This can turn R.H.S into Entropy:

$$\implies \ln M \geq -\sum_{i=1}^M \lambda_i \ln(\lambda_i)$$

$$\implies \ln M \geq H[x]$$