

2.2

$$\begin{aligned}
\sum_{x=-1,1} p(x|\mu) &= \sum_{x=-1,1} \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} \\
&= \left(\frac{1-\mu}{2}\right)^{(1-(-1))/2} \left(\frac{1+\mu}{2}\right)^{(1+(-1))/2} + \left(\frac{1-\mu}{2}\right)^{(1-1)/2} \left(\frac{1+\mu}{2}\right)^{(1+1)/2} \\
&= \left(\frac{1-\mu}{2}\right)^1 \left(\frac{1+\mu}{2}\right)^0 + \left(\frac{1-\mu}{2}\right)^0 \left(\frac{1+\mu}{2}\right)^1 \\
&= \left(\frac{1-\mu}{2}\right) + \left(\frac{1+\mu}{2}\right) \\
&= 1
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}[x] &= \sum_{x=-1,1} p(x)x \\
&= \sum_{x=-1,1} \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} x \\
&= \left(\frac{1-\mu}{2}\right)(-1) + \left(\frac{1+\mu}{2}\right)(1) \\
&= -\left(\frac{1-\mu}{2}\right) + \left(\frac{1+\mu}{2}\right) \\
&= \frac{-1+\mu+1+\mu}{2} \\
&= \mu
\end{aligned}$$

$$var[x] = E[x^2] - E[x]^2$$

$$E[x^2] = \sum_{x=-1,1} \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} x^2$$

$$\begin{aligned}
&= \left(\frac{1-\mu}{2}\right)(-1)^2 + \left(\frac{1+\mu}{2}\right)(1)^2 \\
&= \frac{(1-\mu)}{2} + \frac{(1+\mu)}{2} \\
&= 1
\end{aligned}$$

$$\implies \text{var}[x] = 1 - \mu^2$$

$$\begin{aligned}
H[x] &= - \sum_{x=-1,1} p(x) \ln p(x) \\
&= - \sum_{x=-1,1} \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} \ln \left(\left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} \right) \\
&= - \left(\frac{(1-\mu)}{2} \ln \left(\frac{1-\mu}{2} \right) + \frac{(1+\mu)}{2} \ln \left(\frac{1+\mu}{2} \right) \right) \\
&= - \left(\frac{1}{2} \ln \left(\frac{(1-\mu)(1+\mu)}{4} \right) + \frac{\mu}{2} \ln \left(\frac{1+\mu}{1-\mu} \right) \right) \\
&= - \frac{1}{2} \left(\ln \left(\frac{1-\mu^2}{4} \right) + \mu \ln \left(\frac{1+\mu}{1-\mu} \right) \right)
\end{aligned}$$