2.61 Let us consider a case of a finite set of points that are relatively close to each other and not far from the origin.

For all $\mathbf{x}$ that are far away from the given finite set of points, the volume V increases but the fraction $1/V$ never truly goes to 0. Let these points belong to a set $\mathcal{FARSET}$.

For these far points, V is the volume of a D-ball (a hypersphere in D dimensions) centered at $\mathbf{x}$ with radius r that is approximately equal to the distance from the given set of finite points.

$$V \simeq \frac{\pi^{D/2}\, r^D}{\Gamma(D/2+1)}$$

We can approximately treat the given set of points as a single points since the distances to $\mathbf{x}$ are much greater than the distances between the points.

K/N is a constant. Given all these assumptions, the integral

$$\frac{K}{N}\int_{\mathbf{x}\in\mathcal{FARSET}} \frac{1}{V}\, d\mathbf{x}$$

can be approximated as

$$\begin{aligned}
&\simeq \frac{K}{N}\int_{\boldsymbol{\theta}}\int_{r\in\mathcal{FARSET}} \frac{1}{\left(\frac{\pi^{D/2}\, r^D}{\Gamma(D/2+1)}\right)}\, dr\, d\boldsymbol{\theta} \\
&= \left(\frac{K}{N}\right)\left(\frac{\Gamma(D/2+1)}{\pi^{D/2}}\right)\int_{\boldsymbol{\theta}}\int_{r\in\mathcal{FARSET}} r^{-D}\, dr\, d\boldsymbol{\theta} \\
&= \left(\frac{K}{N}\right)\left(\frac{\Gamma(D/2+1)}{\pi^{D/2}}\right)\int_{\boldsymbol{\theta}} \left.\frac{r^{-D+1}}{-D}\right|_{r\in\mathcal{FARSET}} d\boldsymbol{\theta}
\end{aligned}$$

$\left.\frac{r^{-D+1}}{-D}\right|_{r\in\mathcal{FARSET}}$ diverges. Therefore, K-nearest-neighbour density model defines an improper distribution whose integral over all space is divergent.