2.12 Verifying that the uniform distribution is normalized:

$$\int_a^b U(x|a,b)\,dx = \int_a^b \frac{1}{b-a}\,dx$$

$$= \frac{x}{b-a}\Big|_a^b$$

$$= \frac{b}{b-a} - \frac{a}{b-a}$$

$$= \frac{b-a}{b-a}$$

$$= 1$$

Finding mean:

$$\mathbb{E}[x] = \int_a^b U(x|a,b)\,x\,dx$$

$$= \int_a^b \frac{1}{b-a}\,x\,dx$$

$$= \frac{x^2}{2(b-a)}\Big|_a^b$$

$$= \frac{b^2}{2(b-a)} - \frac{a^2}{2(b-a)}$$

$$= \frac{(b-a)(b+a)}{2(b-a)}$$

$$= \frac{a+b}{2}$$

Finding Variance:

$$var[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2$$

$$\mathbb{E}[x^2] = \int_a^b U(x|a,b)\,x^2\,dx$$

$$= \int_a^b \frac{1}{b-a}\,x^2\,dx$$

$$= \frac{x^3}{3(b-a)}\Big|_a^b$$

$$= \frac{b^3}{3(b-a)} - \frac{a^3}{3(b-a)}$$

$$= \frac{(b-a)(b^2+a^2+ba)}{3(b-a)}$$

$$= \frac{(b^2+a^2+ba)}{3}$$

$$\implies var[x] = \frac{(b^2+a^2+ba)}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{(b^2+a^2+ba)}{3} - \frac{(a^2+b^2+2ab)}{4}$$

$$= \frac{4(b^2+a^2+ba) - 3(a^2+b^2+2ab)}{12}$$

$$= \frac{4b^2+4a^2+4ba-3a^2-3b^2-6ab}{12}$$

$$= \frac{b^2+a^2-2ab}{12}$$

$$= \frac{(b-a)^2}{12}$$