

5.8

$$\begin{aligned}\frac{d \tanh(a)}{da} &= \frac{d}{da} \left(\frac{e^a - e^{-a}}{e^a + e^{-a}} \right) \\&= (e^a - e^{-a}) \frac{d(e^a + e^{-a})^{-1}}{da} + (e^a + e^{-a})^{-1} \frac{d(e^a - e^{-a})}{da} \\&= (e^a - e^{-a})(-1)(e^a + e^{-a})^{-2} \left(\frac{d(e^a + e^{-a})}{da} \right) + (e^a + e^{-a})^{-1}(e^a - e^{-a}) \\&= -\frac{(e^a - e^{-a})}{(e^a + e^{-a})^2} (e^a - e^{-a}) + (e^a + e^{-a})^{-1}(e^a - e^{-a}) \\&= -\frac{(e^a - e^{-a})^2}{(e^a + e^{-a})^2} + \frac{(e^a - e^{-a})}{(e^a + e^{-a})} \\&= -\frac{(e^a - e^{-a})^2}{(e^a + e^{-a})^2} + 1 \\&= 1 - \left(\frac{e^a - e^{-a}}{e^a + e^{-a}} \right)^2 \\&= 1 - \tanh(a)^2\end{aligned}$$