

**2.51**

$$\begin{aligned} \exp(iA)\exp(-iA) &= 1 \\ \implies (\cos A + i \sin A)(\cos(-A) + i \sin(-A)) &= 1 \\ \implies (\cos A + i \sin A)(\cos A - i \sin A) &= 1 \\ \implies \cos^2 A - (i \sin A)^2 &= 1 \\ \implies \cos^2 A - i^2 \sin^2 A &= 1 \\ \implies \cos^2 A + \sin^2 A &= 1 \end{aligned}$$

$$\begin{aligned} \cos(A - B) &= \mathcal{R} \exp\{i(A - B)\} \\ &= \mathcal{R} \exp\{iA\} \exp\{-iB\} \\ &= \mathcal{R}(\cos A + i \sin A)(\cos(-B) + i \sin(-B)) \\ &= \mathcal{R}(\cos A + i \sin A)(\cos B - i \sin B) \\ &= \mathcal{R}(\cos A \cos B + i \sin A \cos B - i \cos A \sin B - i^2 \sin A \sin B) \\ &= \mathcal{R}(\cos A \cos B + i \sin A \cos B - i \cos A \sin B + \sin A \sin B) \\ &= \cos A \cos B + \sin A \sin B \end{aligned}$$

Using the above result,

$$\begin{aligned} \sin(A - B) &= \mathcal{I} \exp\{i(A - B)\} \\ &= \sin A \cos B - \cos A \sin B \end{aligned}$$