

2.52 von Mises distribution is given by:

$$p(\theta|\theta_0, m) = \frac{1}{2\pi I_0(m)} \exp\{m \cos(\theta - \theta_0)\}$$

Applying Taylor expansion to the cosine function using 2.299:

$$= \frac{1}{2\pi I_0(m)} \exp \left\{ m \left(1 - \frac{(\theta - \theta_0)^2}{2} + O((\theta - \theta_0)^4) \right) \right\}$$

Ignoring the higher order terms,

$$\begin{aligned} &= \frac{1}{2\pi I_0(m)} \exp \left\{ m \left(1 - \frac{(\theta - \theta_0)^2}{2} \right) \right\} \\ &= \frac{1}{2\pi I_0(m)} \exp \left\{ m - \frac{m(\theta - \theta_0)^2}{2} \right\} \\ &= \frac{1}{2\pi I_0(m)} \exp(m) \exp \left\{ -\frac{(\theta - \theta_0)^2}{2m^{-1}} \right\} \end{aligned}$$

At large values of m , the Bessel function can be approximated using (this asymptotic form):

$$I_0(m) \approx \sqrt{\frac{2}{\pi m}} \cos(m - \frac{\pi}{4})$$

This gives us:

$$\begin{aligned} &= \frac{(\pi m)^{1/2}}{2\pi 2^{1/2} \cos(m - \frac{\pi}{4})} \exp(m) \exp \left\{ -\frac{(\theta - \theta_0)^2}{2m^{-1}} \right\} \\ &= \left(\frac{\exp(m)}{2 \cos(m - \frac{\pi}{4})} \right) \left(\frac{1}{(2\pi m^{-1})^{1/2}} \exp \left\{ -\frac{(\theta - \theta_0)^2}{2m^{-1}} \right\} \right) \end{aligned}$$

The left term is a constant and doesn't depend on θ . The term on the right resembles a normal distribution if $m^{-1} = \sigma^2$.