1.16 $n(D,M)$ is the number of independent parameters of order M. So it just follows logically that the total number $N(D,M)$ of independent parameters in all of the terms up to and including the M^{th} order is given by:

$$N(D,M) = \sum_{m=0}^{M} n(D,m)$$

Now to prove 1.139.

First proving that the result holds for $M = 0$ and arbitrary $D \geq 1$:

$$L.H.S = N(D,0)$$

$$= \sum_{m=0}^{0} n(D,m)$$

$$= n(D,0)$$

Using 1.137, this becomes:

$$= \frac{(D+0-1)!}{(D-1)!0!}$$

$$= \frac{(D-1)!}{(D-1)!}$$

$$= 1$$

$$R.H.S = \frac{(D+0)!}{D!0!}$$

$$= \frac{D!}{D!}$$

$$= 1$$

Thus proved for $M = 0$.

Now assuming that 1.139 holds at order M, we need to show that it holds at order $M+1$.

$$L.H.S = N(D,(M+1))$$

$$= \sum_{m=0}^{M+1} n(D,m)$$

$$= \left(\sum_{m=0}^{M} n(D,m)\right) + n(D,(M+1))$$

$$= N(D,M) + n(D,(M+1))$$

Applying 1.137 and 1.139,

$$= \frac{(D+M)!}{D!\,M!} + \frac{(D+(M+1)-1)!}{(D-1)!\,(M+1)!}$$

$$= \frac{(D+M)!}{D!\,M!} + \frac{(D+M)!}{(D-1)!\,(M+1)!}$$

$$= \frac{(D+M)!}{(D-1)!\,M!}\left(\frac{1}{D} + \frac{1}{(M+1)}\right)$$

$$= \frac{(D+M)!}{(D-1)!\,M!}\left(\frac{(D+M+1)}{D(M+1)}\right)$$

$$= \frac{(D+M+1)!}{D!\,(M+1)!}$$

Thus proved that 1.139 holds for $M+1$.

Finally, applying Stirling's approximation for $D >> M$,

$$N(D,M) = \frac{(D+M)!}{D!\,M!}$$

$$\simeq \frac{(D+M)^{(D+M)}\,e^{-(D+M)}}{D^D\,e^{-D}M!}$$

$$= \frac{(D+M)^D(D+M)^M\,e^{-M}}{D^D\,M!}$$

$$= \frac{(1 + M/D)^D (D + M)^M \, e^{-M}}{M!}$$

Since $D >> M$, $(1 + M/D)^D \simeq 1$, and $(D + M)^M \simeq D^M$. So we can see that the quantity $N(D, M)$ grows like D^M.

Similarly, considering the case of $M >> D$,

$$N(D, M) = \frac{(D + M)!}{D! \, M!}$$

$$\simeq \frac{(D + M)^{(D+M)} \, e^{-(D+M)}}{D! \, M^M e^{-M}}$$

$$= \frac{(D + M)^D (D + M)^M \, e^{-D}}{D! \, M^M} \}$$

$$= \frac{(D + M)^D (D/M + 1)^M \, e^{-D}}{D!}$$

Since $M >> D$, $(D/M + 1)^M \simeq 1$, and $(D + M)^D \simeq M^D$. So we can see that the quantity $N(D, M)$ grows like M^D.

Assuming $M = 3$, Total number of independent parameters for

$$(i) D = 10 = N(10, 3) = \frac{(10 + 3)!}{10! \, 3!} = \frac{13!}{10! \, 3!} = \frac{13 * 12 * 11}{3 * 2 * 1} = 286$$

$$(ii) D = 100 = N(100, 3) = \frac{(100 + 3)!}{100! \, 3!} = \frac{103!}{100! \, 3!} = \frac{103 * 102 * 101}{3 * 2 * 1} = 176851$$