

5.9 The conditional distribution of targets becomes:

$$\begin{aligned} p(t|\mathbf{x}, \mathbf{w}) &= y(\mathbf{x}, \mathbf{w})^{(t+1)/2} \{1 - y(\mathbf{x}, \mathbf{w})\}^{1-(t+1)/2} \\ &= y(\mathbf{x}, \mathbf{w})^{(t+1)/2} \{1 - y(\mathbf{x}, \mathbf{w})\}^{(1-t)/2} \end{aligned}$$

It can be easily seen that when $t = 1$, $(t + 1)/2 = 1$ and $(1 - t)/2 = 0$, and when $t = -1$, $(t + 1)/2 = 0$ and $(1 - t)/2 = 1$. Since y is modeling t , it will also have the same range and will have to be transformed similarly.

The error function becomes:

$$\begin{aligned} E(\mathbf{w}) &= - \sum_{n=1}^N \left\{ \frac{(t_n + 1)}{2} \ln \left(\frac{y_n + 1}{2} \right) + \frac{(1 - t_n)}{2} \ln \left(\frac{1 - y_n}{2} \right) \right\} \\ &= - \frac{1}{2} \sum_{n=1}^N \left\{ (t_n + 1) \ln \left(\frac{y_n + 1}{2} \right) + (1 - t_n) \ln \left(\frac{1 - y_n}{2} \right) \right\} \\ &= - \frac{1}{2} \sum_{n=1}^N \{ (t_n + 1) \ln(y_n + 1) - (t_n + 1) \ln 2 + (1 - t_n) \ln(1 - y_n) - (1 - t_n) \ln 2 \} \\ &= - \frac{1}{2} \sum_{n=1}^N \{ (t_n + 1) \ln(y_n + 1) + (1 - t_n) \ln(1 - y_n) - (t_n + 1 + 1 - t_n) \ln 2 \} \\ &= - \frac{1}{2} \sum_{n=1}^N \{ (t_n + 1) \ln(y_n + 1) + (1 - t_n) \ln(1 - y_n) - 2 \ln 2 \} \\ &= - \frac{1}{2} \sum_{n=1}^N \{ (t_n + 1) \ln(y_n + 1) + (1 - t_n) \ln(1 - y_n) \} + N \ln 2 \end{aligned}$$

We can find an appropriate activation function, by performing the inverse of the above transformation on the logistic sigmoid function.

$$h(a) = 2\sigma(a) - 1$$

$$= \frac{2}{1 + e^{-a}} - 1$$

$$\begin{aligned}
&= \frac{2 - (1 + e^{-a})}{1 + e^{-a}} \\
&= \frac{1 - e^{-a}}{1 + e^{-a}}
\end{aligned}$$