

1.26 Adding and subtracting $E_t[\mathbf{t}|\mathbf{x}]$, we get:

$$\begin{aligned}
\|\mathbf{y}(\mathbf{x}) - \mathbf{t}\|^2 &= \|\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}] + E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}\|^2 \\
&= ((\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]) + (E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}))^T ((\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]) + (E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t})) \\
&= (\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}])^T (\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]) \\
&\quad + (E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t})^T (\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]) \\
&\quad + (\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}])^T (E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}) \\
&\quad + (E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t})^T (E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}) \\
&= \|\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]\|^2 + 2(E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t})^T (\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]) + \|E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}\|^2
\end{aligned}$$

The middle integral becomes 0, as we can see here:

$$\begin{aligned}
&\int 2(E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t})^T (\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]) p(\mathbf{x}, \mathbf{t}) d\mathbf{t} \\
&= \int 2(\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}])^T (E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}) p(\mathbf{x}, \mathbf{t}) d\mathbf{t} \\
&= 2(\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}])^T \int (E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}) p(\mathbf{x}, \mathbf{t}) d\mathbf{t} \\
&= 2(\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}])^T (E_t[\mathbf{t}|\mathbf{x}] - E_t[\mathbf{t}|\mathbf{x}]) p(\mathbf{x}) \\
&= 0
\end{aligned}$$

Therefore, we get:

$$\begin{aligned}
E[L(\mathbf{t}, \mathbf{y}(\mathbf{x}))] &= \int \int (\|\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]\|^2 + \|E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}\|^2) p(\mathbf{x}, \mathbf{t}) d\mathbf{x} d\mathbf{t} \\
&= \int \int \|\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]\|^2 p(\mathbf{x}, \mathbf{t}) d\mathbf{x} d\mathbf{t} + \int \int \|E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}\|^2 p(\mathbf{x}, \mathbf{t}) d\mathbf{x} d\mathbf{t}
\end{aligned}$$

$$\begin{aligned}
&= \int \int ||\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]||^2 p(\mathbf{x}, \mathbf{t}) d\mathbf{x} d\mathbf{t} + \int \int ||E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}||^2 p(\mathbf{t}|\mathbf{x}) p(\mathbf{x}) d\mathbf{t} d\mathbf{x} \\
&= \int \int ||\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]||^2 p(\mathbf{x}, \mathbf{t}) d\mathbf{x} d\mathbf{t} + \int \left(\int ||E_t[\mathbf{t}|\mathbf{x}] - \mathbf{t}||^2 p(\mathbf{t}|\mathbf{x}) d\mathbf{t} \right) p(\mathbf{x}) d\mathbf{x} \\
&= \int ||\mathbf{y}(\mathbf{x}) - E_t[\mathbf{t}|\mathbf{x}]||^2 p(\mathbf{x}) d\mathbf{x} + \int var[\mathbf{t}|\mathbf{x}] p(\mathbf{x}) d\mathbf{x}
\end{aligned}$$