

### 5.34

$$E_n = -\ln \left\{ \sum_{l=1}^K \pi_l(\mathbf{x}_n, \mathbf{w}) \mathcal{N}(\mathbf{t}_n | \boldsymbol{\mu}_l(\mathbf{x}_n, \mathbf{w}), \sigma_l^2(\mathbf{x}_n, \mathbf{w})) \right\}$$

Since  $a_k^\pi$  affects  $E_n$  through every  $\pi_k$  (because of the denominator),

$$\frac{\partial E_n}{\partial a_k^\pi} = \sum_{l=1}^K \left( \frac{\partial E_n}{\partial \pi_l} \right) \left( \frac{\partial \pi_l}{\partial a_k^\pi} \right)$$

$$\begin{aligned} \frac{\partial E_n}{\partial \pi_l} &= - \frac{\mathcal{N}(\mathbf{t}_n | \boldsymbol{\mu}_l(\mathbf{x}_n, \mathbf{w}), \sigma_l^2(\mathbf{x}_n, \mathbf{w}))}{\left\{ \sum_{l=1}^K \pi_l(\mathbf{x}_n, \mathbf{w}) \mathcal{N}(\mathbf{t}_n | \boldsymbol{\mu}_l(\mathbf{x}_n, \mathbf{w}), \sigma_l^2(\mathbf{x}_n, \mathbf{w})) \right\}} \\ &= - \frac{\gamma_l}{\pi_l} \end{aligned}$$

Using 4.106,

$$\frac{\partial \pi_l}{\partial a_k^\pi} = \pi_l(I_{kl} - \pi_k)$$

Substituting, we get:

$$\begin{aligned} \frac{\partial E_n}{\partial a_k^\pi} &= \sum_{l=1}^K \left( -\frac{\gamma_l}{\pi_l} \right) (\pi_l(I_{kl} - \pi_k)) \\ &= - \sum_{l=1}^K \gamma_l(I_{kl} - \pi_k) \\ &= - \sum_{l=1}^K \gamma_l I_{kl} + \sum_{l=1}^K \gamma_l \pi_k \\ &= -\gamma_k + \pi_k \sum_{l=1}^K \gamma_l \\ &= \pi_k - \gamma_k \end{aligned}$$

which is the same as the result in 5.155.