4.12

$$\frac{d\sigma}{da} = \frac{d(1+\exp(-a))^{-1}}{da}$$

$$= \left(\frac{d(1+\exp(-a))^{-1}}{d(1+\exp(-a))}\right)\left(\frac{d(1+\exp(-a))}{d(-a)}(-1)\right)$$

$$= \left((-1)(1+\exp(-a))^{-2}\right)\exp(-a)(-1)$$

$$= (1+\exp(-a))^{-2}\exp(-a)$$

$$= (1+\exp(-a))^{-1}(1+\exp(-a))^{-1}\exp(-a)$$

$$= (1+\exp(-a))^{-1}\left(\frac{\exp(-a)}{1+\exp(-a)}\right)$$

$$= (1+\exp(-a))^{-1}\left(\frac{1+\exp(-a)-1}{1+\exp(-a)}\right)$$

$$= (1+\exp(-a))^{-1}\left(\frac{1+\exp(-a)}{1+\exp(-a)} - \frac{1}{1+\exp(-a)}\right)$$

$$= \sigma(a)(1-\sigma(a)).$$

which is the same as 4.88.