6.11 Equation 6.25 gives us:

$$k(\mathbf{x},\mathbf{x'}) = \exp\left(-\mathbf{x}^T\mathbf{x}/2\sigma^2\right)\exp\left(\mathbf{x}^T\mathbf{x'}/\sigma^2\right)\exp\left(-(\mathbf{x'})^T\mathbf{x'}/2\sigma^2\right)$$

The Maclaurin expansion of exponential function is:

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Applying this to the middle term, we get:

$$\exp\left(\mathbf{x}^T\mathbf{x'}/\sigma^2\right) = \sum_{n=0}^{\infty}\frac{1}{n!}(\mathbf{x}^T\mathbf{x'}/\sigma^2)^n$$

$$= \sum_{n=0}^{\infty}\frac{1}{\sigma^{2n}\,n!}\left(\sum_{i=1}^{N}x_i\,x_i^{'}\right)^n$$

Using Multinomial Theorem, we have this expansion:

$$(x^Tx')^n = \left(\sum_{i=1}^{d}x_ix_i'\right)^n = \sum_{\substack{k_1+k_2+\cdots+k_N=n\\k_1,k_2,\ldots,k_N\geq 0}}\frac{n!}{k_1!k_2!\cdots k_N!}\prod_{i=1}^{d}(x_ix_i')^{k_i}$$

Substituting this back, we get:

$$= \sum_{n=0}^{\infty}\frac{1}{\sigma^{2n}\,n!}\left(\sum_{\substack{k_1+k_2+\cdots+k_N=n\\k_1,k_2,\ldots,k_N\geq 0}}\frac{n!}{k_1!k_2!\cdots k_N!}\prod_{i=1}^{N}(x_ix_i')^{k_i}\right)$$

$$= \sum_{n=0}^{\infty}\left(\sum_{\substack{k_1+k_2+\cdots+k_N=n\\k_1,k_2,\ldots,k_N\geq 0}}\left(\left(\left(\frac{1}{\sigma^{2n}\,n!}\right)\left(\frac{n!}{k_1!k_2!\cdots k_N!}\right)\right)^{1/2}\prod_{i=1}^{N}(x_i)^{k_i}\right)\right.$$

$$\left.\left(\left(\left(\frac{1}{\sigma^{2n}\,n!}\right)\left(\frac{n!}{k_1!k_2!\cdots k_N!}\right)\right)^{1/2}\prod_{i=1}^{N}(x_i^{'})^{k_i}\right)\right)$$

If we define:

$$\mathbf{k} = (k_1, k_2, \ldots, k_N)$$

and

$$\phi_{\mathbf{k},n}(\mathbf{x}) = \left(\left(\left(\frac{1}{\sigma^{2n}\, n!} \right) \left(\frac{n!}{k_1! k_2! \cdots k_N!} \right) \right)^{1/2} \prod_{i=1}^{N} (x_i)^{k_i} \right)$$

Then,

$$\boldsymbol{\phi}(\mathbf{x}) = [\quad \ldots \quad \phi_{\mathbf{k},n}(\mathbf{x}) \quad \ldots \quad]^T$$

and the expansion of the middle term becomes this inner product:

$$\exp\left(\mathbf{x}^T \mathbf{x'}/\sigma^2\right) = \boldsymbol{\phi}(\mathbf{x})^T \boldsymbol{\phi}(\mathbf{x'})$$