2.3 First we prove 2.262:

$$\binom{N}{m} + \binom{N}{m-1} = \frac{N!}{m!(N-m)!} + \frac{N!}{(m-1)!(N-m+1)!}$$

$$= \frac{N!}{(m-1)!(N-m)!}\left(\frac{1}{m} + \frac{1}{N-m+1}\right)$$

$$= \frac{N!}{(m-1)!(N-m)!}\left(\frac{N-m+1+m}{m(N-m+1)}\right)$$

$$= \frac{N!}{(m-1)!(N-m)!}\left(\frac{(N+1)}{m(N-m+1)}\right)$$

$$= \frac{(N+1)!}{(m)!(N-m+1)!}$$

Now we prove 2.263:

For $N = 1$,

$$\sum_{m=0}^{N} \binom{N}{m} x^m$$

$$= \sum_{m=0}^{1} \binom{1}{m} x^m$$

$$= \binom{1}{0} x^0 + \binom{1}{1} x^1$$

$$= 1 + x$$

$$= (1+x)^N$$

Now let's assume the equality holds for $N-1$ and prove it for N.

This gives us:

$$(1+x)^{N-1} = \sum_{m=0}^{N-1} \binom{N-1}{m} x^m$$

Multiplying both sides by $(1+x)$:

$$(1+x)^N = (1+x)\sum_{m=0}^{N-1}\binom{N-1}{m}x^m$$
$$= \sum_{m=0}^{N-1}\binom{N-1}{m}x^m + \sum_{m=0}^{N-1}\binom{N-1}{m}x^{m+1}$$

$$= \begin{array}{cccccccccc} \tbinom{N-1}{0}x^0 & + & \tbinom{N-1}{1}x^1 & + & \dots & + & \tbinom{N-1}{N-1}x^{N-1} & + & & \\ \dots & + & \tbinom{N-1}{0}x^1 & + & \dots & + & \tbinom{N-1}{N-2}x^{N-1} & + & \tbinom{N-1}{N-1}x^N \end{array}$$

$$\begin{gathered} = \binom{N-1}{0}x^0 \\ + \left(\binom{N-1}{1} + \binom{N-1}{0}\right)x^1 \\ + \left(\binom{N-1}{2} + \binom{N-1}{1}\right)x^2 \\ + \dots \\ + \left(\binom{N-1}{N-1} + \binom{N-1}{N-2}\right)x^{N-1} \\ + \binom{N-1}{N-1}x^N \end{gathered}$$

$$\begin{gathered} = \binom{N-1}{0}x^0 \\ + \binom{N}{1}x^1 \\ + \binom{N}{2}x^2 \\ + \dots \\ + \binom{N}{N-1}x^{N-1} \\ + \binom{N-1}{N-1}x^N \end{gathered}$$

$$\binom{N-1}{0} = 1 = \binom{N}{0} \text{ and } \binom{N-1}{N-1} = 1 = \binom{N}{N}$$

Thus, we get:

$$(1+x)^N = \binom{N}{0}x^0 + \binom{N}{1}x^1 + \binom{N}{2}x^2 + \ldots + \binom{N}{N-1}x^{N-1} + \binom{N}{N}x^N$$

$$\implies (1+x)^N = \sum_{m=0}^{N} \binom{N}{m} x^m$$

Finally, we prove 2.264:

$$\begin{aligned}
&\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} \\
&= \sum_{m=0}^{N} \binom{N}{m} \mu^m \frac{(1-\mu)^N}{(1-\mu)^m} \\
&= (1-\mu)^N \sum_{m=0}^{N} \binom{N}{m} \left(\frac{\mu}{1-\mu}\right)^m \\
&= (1-\mu)^N \left(1 + \left(\frac{\mu}{1-\mu}\right)\right)^N \\
&= (1-\mu)^N \left(\frac{1-\mu+\mu}{1-\mu}\right)^N \\
&= (1-\mu)^N \left(\frac{1}{1-\mu}\right)^N \\
&= 1
\end{aligned}$$

Thus, we can see that the binomial distribution is normalized.