

4.7 From 4.59, we have:

$$\begin{aligned}
\sigma(-a) &= \frac{1}{1 + \exp(-(-a))} \\
&= \frac{1}{1 + \exp(a)} \\
&= \frac{1}{\exp(a) (\exp(-a) + 1)} \\
&= \frac{\exp(-a)}{1 + \exp(-a)} \\
&= \frac{1 - 1 + \exp(-a)}{1 + \exp(-a)} \\
&= \frac{1 + \exp(-a)}{1 + \exp(-a)} - \frac{1}{1 + \exp(-a)} \\
&= 1 - \frac{1}{1 + \exp(-a)} \\
&\implies \sigma(-a) = 1 - \sigma(a)
\end{aligned}$$

To prove the inverse of logistic sigmoid function, we prove the following:

$$\begin{aligned}
&\sigma\left(\ln\left\{\frac{y}{1-y}\right\}\right) = y \\
L.H.S &= \frac{1}{1 + \exp\left(-\left(\ln\left\{\frac{y}{1-y}\right\}\right)\right)} \\
&= \frac{1}{1 + \exp\left(\ln\left\{\frac{1-y}{y}\right\}\right)} \\
&= \frac{1}{1 + \left\{\frac{1-y}{y}\right\}} \\
&= \frac{1}{\left\{\frac{y+1-y}{y}\right\}} \\
&= \frac{1}{\left\{\frac{1}{y}\right\}} \\
&= y
\end{aligned}$$

Thus proved both.