1.35

$$H[x] = -\int_{-\infty}^{\infty} \mathcal{N}(\mu,\sigma^2)\ \ln(\mathcal{N}(\mu,\sigma^2))\,dx$$

$$= -\int_{-\infty}^{\infty} \mathcal{N}(\mu,\sigma^2)\ \ln\left(\frac{1}{(2\pi\sigma^2)^{1/2}} exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}\right)\ dx$$

$$= -\int_{-\infty}^{\infty} \mathcal{N}(\mu,\sigma^2)\ \left(-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x-\mu)^2}{2\sigma^2}\right)\ dx$$

$$= \int_{-\infty}^{\infty} \mathcal{N}(\mu,\sigma^2)\ \left(\frac{1}{2}\ln(2\pi\sigma^2) + \frac{(x-\mu)^2}{2\sigma^2}\right)\ dx$$

$$= \frac{1}{2}\ln(2\pi\sigma^2)\int_{-\infty}^{\infty} \mathcal{N}(\mu,\sigma^2)\,dx + \int_{-\infty}^{\infty} \mathcal{N}(\mu,\sigma^2)\frac{(x-\mu)^2}{2\sigma^2}\,dx$$

$$= \frac{1}{2}\ln(2\pi\sigma^2)\int_{-\infty}^{\infty} \mathcal{N}(\mu,\sigma^2)\,dx + \frac{1}{2\sigma^2}\int_{-\infty}^{\infty} \mathcal{N}(\mu,\sigma^2)(x-\mu)^2\,dx$$

$$= \frac{1}{2}\ln(2\pi\sigma^2)(1) + \frac{1}{2\sigma^2}\sigma^2$$

$$= \frac{1}{2}(\ln(2\pi\sigma^2) + 1)$$