1.21

$$a \leq b$$

$$\implies a^{1/2} \leq b^{1/2}$$

$$\implies a^{1/2}a^{1/2} \leq a^{1/2}b^{1/2}$$

$$\implies a \leq (ab)^{1/2}$$

$$\{p(\mathbf{x}, C_1)p(\mathbf{x}, C_2)\}^{1/2}$$

$$= \{p(C_1|\mathbf{x})p(\mathbf{x})p(C_2|\mathbf{x})p(\mathbf{x})\}^{1/2}$$

$$= \{p(C_1|\mathbf{x})p(C_2|\mathbf{x})\}^{1/2}p(\mathbf{x})$$

When $\mathbf{x} \in R_1$,

$$p(C_1|\mathbf{x}) \geq p(C_2|\mathbf{x})$$

$$\implies \{p(C_1|\mathbf{x})p(C_2|\mathbf{x})\}^{1/2}p(\mathbf{x}) \geq p(C_2|\mathbf{x})p(\mathbf{x})$$

Similarly, when $\mathbf{x} \in R_2$,

$$p(C_2|\mathbf{x}) \geq p(C_1|\mathbf{x})$$

$$\implies \{p(C_1|\mathbf{x})p(C_2|\mathbf{x})\}^{1/2}p(\mathbf{x}) \geq p(C_1|\mathbf{x})p(\mathbf{x})$$

$$\implies \int_{R_1} \{p(C_1|\mathbf{x})p(C_2|\mathbf{x})\}^{1/2}p(\mathbf{x})d\mathbf{x} + \int_{R_2} \{p(C_1|\mathbf{x})p(C_2|\mathbf{x})\}^{1/2}p(\mathbf{x})d\mathbf{x}$$

$$\geq$$

$$\int_{R_1} p(C_2|\mathbf{x})p(\mathbf{x})d\mathbf{x} + \int_{R_2} p(C_1|\mathbf{x})p(\mathbf{x})d\mathbf{x}$$

$$\implies \int \{p(C_1|\mathbf{x})p(C_2|\mathbf{x})\}^{1/2} p(\mathbf{x}) d\mathbf{x} \geq \int_{R_1} p(C_2|\mathbf{x})p(\mathbf{x}) d\mathbf{x} + \int_{R_2} p(C_1|\mathbf{x})p(\mathbf{x}) d\mathbf{x}$$

$$\implies \int \{p(C_1, \mathbf{x})p(C_2, \mathbf{x})\}^{1/2} d\mathbf{x} \geq \int_{R_1} p(C_2, \mathbf{x}) d\mathbf{x} + \int_{R_2} p(C_1, \mathbf{x}) d\mathbf{x}$$

Using equation 1.78,

$$\implies \int \{p(C_1, \mathbf{x})p(C_2, \mathbf{x})\}^{1/2} d\mathbf{x} \geq p(mistake)$$