4.15 The Hessian matrix given by 4.97 is:

$$\mathbf{H} = \mathbf{\Phi}^T \mathbf{R} \mathbf{\Phi}$$

where $\mathbf{R}$ is a diagonal matrix consisting of only positive values $R_{nn} = y_n(1 - y_n)$.

Let $\mathbf{S}$ be a diagonal matrix such that $S_{nn} = \sqrt{R_{nn}}$.

Then, for any vector $\mathbf{u}$,

$$\mathbf{u}^T \mathbf{H} \mathbf{u} = \mathbf{u}^T \mathbf{\Phi}^T \mathbf{R} \mathbf{\Phi} \mathbf{u}$$

$$= \mathbf{u}^T \mathbf{\Phi}^T \mathbf{S} \mathbf{S} \mathbf{\Phi} \mathbf{u}$$

$$= \mathbf{u}^T \mathbf{\Phi}^T \mathbf{S}^T \mathbf{S} \mathbf{\Phi} \mathbf{u}$$

$$= (\mathbf{S} \mathbf{\Phi} \mathbf{u})^T \mathbf{S} \mathbf{\Phi} \mathbf{u}$$

The resulting quantity is always positive, so $\mathbf{H}$ is positive definite.

From chapter 7 of MML Book, I know that if the Hessian is positive definite, then the function is convex. Therefore, the error function is convex.