2.17 As per exercise 1.14, an arbitrary square matrix with elements w_{ij} can be written in the form $w_{ij} = w_{ij}^S + w_{ij}^A$ where w_{ij}^S and w_{ij}^A are symmetric and anti-symmetric matrices,respectively, satisfying

$$w_{ij}^S = w_{ji}^S$$

and

$$w_{ij}^A = -w_{ji}^A$$

for all i and j.

Let w_{ij} be the elements of $\mathbf{\Sigma}^{-1}$.

The term in the exponent of the Gaussian is:

$$-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})$$

Ignoring the $-\frac{1}{2}$, and letting $\mathbf{z} = (\mathbf{x}-\boldsymbol{\mu})$ we get:

$$\mathbf{z}^T\mathbf{\Sigma}^{-1}\mathbf{z}$$

$$= [z_1\, z_2\, \dots\, z_D] \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1D} \\ w_{21} & w_{22} & \dots & w_{2D} \\ \vdots & \vdots & \vdots & \vdots \\ w_{D1} & w_{D2} & \dots & w_{DD} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_D \end{bmatrix}$$

$$= \left[\sum_{i=1}^{D} z_i w_{i1} \; \sum_{i=1}^{D} z_i w_{i2} \; \dots \; \sum_{i=1}^{D} z_i w_{iD} \right] \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_D \end{bmatrix}$$

$$= \sum_{j=1}^{D} \sum_{i=1}^{D} z_i w_{ij} z_j$$

$$= \sum_{j=1}^{D}\sum_{i=1}^{D} z_i(w_{ij}^S + w_{ij}^A)z_j$$

$$= \sum_{j=1}^{D}\sum_{i=1}^{D} z_i w_{ij}^S z_j + \sum_{j=1}^{D}\sum_{i=1}^{D} z_i w_{ij}^A z_j$$

Considering the second term,

$$\sum_{j=1}^{D}\sum_{i=1}^{D} z_i w_{ij}^A z_j$$

Splitting into top right triangular, bottom left triangular, and diagonal values:

$$= \left(\sum_{i=1}^{D}\sum_{j=i+1}^{D} z_i w_{ij}^A z_j\right) + \left(\sum_{j=1}^{D}\sum_{i=j+1}^{D} z_i w_{ij}^A z_j\right) + \left(\sum_{i} z_i w_{ii}^A z_i\right)$$

The indices in the middle term can be exchanged without a problem, giving us:

$$\sum_{i=1}^{D}\sum_{j=i+1}^{D} z_j w_{ji}^A z_i$$

$$= \sum_{i=1}^{D}\sum_{j=i+1}^{D} z_i(-w_{ij}^A)z_j$$

Substituting above we get:

$$= \left(\sum_{i=1}^{D}\sum_{j=i+1}^{D} z_i w_{ij}^A z_j\right) + \left(\sum_{i=1}^{D}\sum_{j=i+1}^{D} z_i(-w_{ij}^A)z_j\right) + \left(\sum_{i} z_i w_{ii}^A z_i\right)$$

$$= \left(\sum_{i=1}^{D}\sum_{j=i+1}^{D} z_i(w_{ij}^A - w_{ij}^A)z_j\right) + \left(\sum_{i} z_i w_{ii}^A z_i\right)$$

Since the diagonal elements of an anti-symmetric matrix are 0, this becomes:

$$= 0 + 0$$

$$= 0$$

The terms in $\mathbf{z}^T \mathbf{\Sigma}^{-1} \mathbf{z}$ that include w_{ij}^A have summed up to 0, so only the symmetric terms remain.