4.22 Applying 4.135 to 4.136:

$$p(\mathcal{D}) = \int p(\mathcal{D}|\boldsymbol{\theta})\, p(\boldsymbol{\theta})\, d\boldsymbol{\theta}$$

$$\simeq p(\mathcal{D}|\boldsymbol{\theta}_{MAP})\, p(\boldsymbol{\theta}_{MAP}) \int \exp\left\{-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}_{MAP})^T \mathbf{A}(\boldsymbol{\theta}-\boldsymbol{\theta}_{MAP})\right\} d\boldsymbol{\theta}$$

$$\simeq p(\mathcal{D}|\boldsymbol{\theta}_{MAP})\, p(\boldsymbol{\theta}_{MAP}) \frac{(2\pi)^{M/2}}{|\mathbf{A}|^{1/2}}$$

$$\Longrightarrow \ln p(\mathcal{D}) \simeq \ln p(\mathcal{D}|\boldsymbol{\theta}_{MAP}) + \ln p(\boldsymbol{\theta}_{MAP}) + \ln\left\{\frac{(2\pi)^{M/2}}{|\mathbf{A}|^{1/2}}\right\}$$

$$\Longrightarrow \ln p(\mathcal{D}) \simeq \ln p(\mathcal{D}|\boldsymbol{\theta}_{MAP}) + \ln p(\boldsymbol{\theta}_{MAP}) + \frac{M}{2}\ln(2\pi) - \frac{1}{2}\ln|\mathbf{A}|$$

which is the same as 4.137.