2.58 2.226 gives us the result of taking the gradient of both sides of (2.195):

$$-\nabla \ln g(\boldsymbol{\eta}) = g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\mathbf{u}(\mathbf{x})\mathrm{d}\mathbf{x}$$

Using definition 5.6 from MML, we get the derivative of a vector-valued function $g(\boldsymbol{\eta})\mathbf{u}(\mathbf{x})$ w.r.t the vector $\boldsymbol{\eta}$, giving us $\nabla g(\boldsymbol{\eta})\mathbf{u}(\mathbf{x})^T$ in the first term:

$$-\nabla\nabla \ln g(\boldsymbol{\eta}) = \nabla g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\mathbf{u}(\mathbf{x})^T\mathrm{d}\mathbf{x}$$

$$+g(\boldsymbol{\eta}) \int h(\mathbf{x}) \frac{\partial \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}}{\partial \boldsymbol{\eta}}\mathbf{u}(\mathbf{x})\mathrm{d}\mathbf{x}$$

$$= \nabla g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\mathbf{u}(\mathbf{x})^T\mathrm{d}\mathbf{x}$$

$$+g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\frac{\partial \boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})}{\partial \boldsymbol{\eta}}\mathbf{u}(\mathbf{x})\mathrm{d}\mathbf{x}$$

$$= \nabla g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\mathbf{u}(\mathbf{x})^T\mathrm{d}\mathbf{x}$$

$$+g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\frac{\partial \boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{x})}{\partial \boldsymbol{\eta}}\mathrm{d}\mathbf{x}$$

$$= \nabla g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\mathbf{u}(\mathbf{x})^T\mathrm{d}\mathbf{x}$$

$$+g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\frac{\partial \mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{x})^T \boldsymbol{\eta}}{\partial \boldsymbol{\eta}}\mathrm{d}\mathbf{x}$$

Using this result, we get:

$$= \nabla g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\mathbf{u}(\mathbf{x})^T\mathrm{d}\mathbf{x}$$

$$+g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\}\mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{x})^T\mathrm{d}\mathbf{x}$$

Using 2.224,

$$= -g(\boldsymbol{\eta})^2 \left(\int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\} \mathbf{u}(\mathbf{x}) \mathrm{d}\mathbf{x} \right) \left(\int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\} \mathbf{u}(\mathbf{x})^T \mathrm{d}\mathbf{x} \right)$$

$$+ \mathbb{E}[\mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{x})^T]$$

$$= - \left(g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\} \mathbf{u}(\mathbf{x}) \mathrm{d}\mathbf{x} \right) \left(g(\boldsymbol{\eta}) \int h(\mathbf{x}) \exp\{\boldsymbol{\eta}^T \mathbf{u}(\mathbf{x})\} \mathbf{u}(\mathbf{x})^T \mathrm{d}\mathbf{x} \right)$$

$$+ \mathbb{E}[\mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{x})^T]$$

$$= -\mathbb{E}[\mathbf{u}(\mathbf{x})]\mathbb{E}[\mathbf{u}(\mathbf{x})^T] + \mathbb{E}[\mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{x})^T]$$

$$= \mathbb{E}[\mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{x})^T] - \mathbb{E}[\mathbf{u}(\mathbf{x})]\mathbb{E}[\mathbf{u}(\mathbf{x})^T]$$

$$\implies -\nabla\nabla \ln g(\boldsymbol{\eta}) = cov[\mathbf{u}(\mathbf{x})]$$