

**2.50** The multivariate Student's t-distribution is given by 2.162:

$$St(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}, \nu) = \frac{\Gamma(\nu/2 + D/2)}{\Gamma(\nu/2)} \frac{|\boldsymbol{\Lambda}|^{1/2}}{(\pi\nu)^{D/2}} \left(1 + \frac{\Delta^2}{\nu}\right)^{-D/2-\nu/2}$$

Applying an approximation to Gamma function as  $\nu \rightarrow +\infty$  as per this formula where  $\Gamma(x + \alpha) = \Gamma(x)x^\alpha$  as  $x \rightarrow +\infty$ :

$$\begin{aligned} &= \left(\frac{\nu}{2}\right)^{D/2} \frac{|\boldsymbol{\Lambda}|^{1/2}}{(\pi\nu)^{D/2}} \left(1 + \frac{\Delta^2}{\nu}\right)^{-D/2-\nu/2} \\ &= \left(\frac{1}{2\pi}\right)^{D/2} \frac{1}{|\boldsymbol{\Lambda}|^{-1/2}} \left(1 + \frac{\Delta^2}{\nu}\right)^{-D/2-\nu/2} \\ &= \left(\frac{1}{2\pi}\right)^{D/2} \frac{1}{|\boldsymbol{\Lambda}|^{-1/2}} \left(1 + \frac{\Delta^2}{\nu}\right)^{-D/2} \left(1 + \frac{\Delta^2}{\nu}\right)^{-\nu/2} \end{aligned}$$

Using the limit  $\lim_{x \rightarrow +\infty} (1 + 1/x)^x = e$  from link, when we apply  $\nu \rightarrow +\infty$ , we get:

$$\begin{aligned} &= \left(\frac{1}{2\pi}\right)^{D/2} \frac{1}{|\boldsymbol{\Lambda}|^{-1/2}} \left(1 + \frac{\Delta^2}{\nu}\right)^{-D/2} \left(\left(1 + \frac{\Delta^2}{\nu}\right)^{\nu/\Delta^2}\right)^{-\Delta^2/2} \\ &= \left(\frac{1}{2\pi}\right)^{D/2} \frac{1}{|\boldsymbol{\Lambda}|^{-1/2}} (1)^{-D/2} e^{-\Delta^2/2} \\ &= \left(\frac{1}{2\pi}\right)^{D/2} \frac{1}{|\boldsymbol{\Lambda}|^{-1/2}} \exp\{-\Delta^2/2\} \\ &= \frac{1}{(2\pi)^{D/2}} \frac{1}{|\boldsymbol{\Lambda}^{-1}|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Lambda}(\mathbf{x} - \boldsymbol{\mu})\right\} \end{aligned}$$

which is a Gaussian distribution comparable to equation 2.43.