

3.10 The predictive distribution is given by:

$$\begin{aligned}
p(t|\mathbf{t}, \alpha, \beta) &= \int p(t|\mathbf{w}, \beta) p(\mathbf{w}|\mathbf{t}, \alpha, \beta) d\mathbf{w} \\
&= \int \mathcal{N}(t|y(\mathbf{x}, \mathbf{w}), \beta^{-1}) \mathcal{N}(\mathbf{w}|\mathbf{m}_N, \mathbf{S}_N) d\mathbf{w} \\
&= \int \mathcal{N}(t|\phi(\mathbf{x})^T \mathbf{w}, \beta^{-1}) \mathcal{N}(\mathbf{w}|\mathbf{m}_N, \mathbf{S}_N) d\mathbf{w}
\end{aligned}$$

Comparing this to 2.113 and 2.114, we get:

$$\begin{aligned}
\mathbf{x} &= \mathbf{w} \\
\boldsymbol{\mu} &= \mathbf{m}_N \\
\boldsymbol{\Lambda} &= \mathbf{S}_N^{-1} \\
\mathbf{y} &= t \\
\mathbf{A} &= \phi(\mathbf{x})^T \\
\mathbf{b} &= 0 \\
\mathbf{L} &= \beta
\end{aligned}$$

Applying 2.115, we get,

$$\begin{aligned}
p(t|\mathbf{t}, \alpha, \beta) &= \mathcal{N}(t|\phi(\mathbf{x})^T \mathbf{m}_N, \beta^{-1} + \phi(\mathbf{x})^T \mathbf{S}_N \phi(\mathbf{x})) \\
&= \mathcal{N}(t|\phi(\mathbf{x})^T \mathbf{m}_N, \sigma_N^2(\mathbf{x}))
\end{aligned}$$

where $\sigma_N^2(\mathbf{x}) = \frac{1}{\beta} + \phi(\mathbf{x})^T \mathbf{S}_N \phi(\mathbf{x})$.