**4.18**

$$E(\mathbf{w}_1, \ldots, \mathbf{w}_K) = -\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\ln y_{nk}$$

$$\implies \frac{\partial E(\mathbf{w}_1, \ldots, \mathbf{w}_K)}{\partial \mathbf{w}_j} = \frac{\partial}{\partial \mathbf{w}_j}\left(-\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\ln y_{nk}\right)$$

$$= \frac{\partial}{\partial \mathbf{w}_j}\left(-\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\ln y_{nk}\right)$$

$$= -\sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\partial\left(t_{nk}\ln y_{nk}\right)}{\partial \mathbf{w}_j}$$

$$= -\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\frac{\partial \ln y_{nk}}{\partial \mathbf{w}_j}$$

$$= -\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\left(\frac{\partial \ln y_{nk}}{\partial y_{nk}}\right)\left(\frac{\partial y_{nk}}{\partial a_{nj}}\right)\left(\frac{\partial a_{nj}}{\partial \mathbf{w}_j}\right)$$

$$= -\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\left(\frac{\partial \ln y_{nk}}{\partial y_{nk}}\right)\left(\frac{\partial y_k(\boldsymbol{\phi}_n)}{\partial a_{nj}}\right)\left(\frac{\partial \mathbf{w}_j^T\boldsymbol{\phi}_n}{\partial \mathbf{w}_j}\right)$$

$$= -\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\left(\frac{1}{y_{nk}}\right)\left(y_{kn}(I_{jk} - y_{nj})\right)\left(\boldsymbol{\phi}_n\right)$$

$$= -\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}(I_{jk} - y_{nj})\left(\boldsymbol{\phi}_n\right)$$

$$= -\sum_{n=1}^{N}\left(\sum_{k=1}^{K} t_{nk}I_{jk} - \sum_{k=1}^{K} t_{nk}y_{nj}\right)\left(\boldsymbol{\phi}_n\right)$$

$$= -\sum_{n=1}^{N}\left(t_{nj} - y_{nj}\sum_{k=1}^{K} t_{nk}\right)\left(\boldsymbol{\phi}_n\right)$$

$$= -\sum_{n=1}^{N}\left(t_{nj} - y_{nj}(1)\right)\left(\boldsymbol{\phi}_n\right)$$

$$= -\sum_{n=1}^{N}(y_{nj} - t_{nj})(\boldsymbol{\phi}_n)$$

which is the same as 4.109.