

1.40 \ln is a concave function, so $-\ln$ is a convex function.

Applying $-\ln$ to the formula for Jensen's inequality, we get:

$$\begin{aligned} -\ln\left(\sum_{i=1}^M \lambda_i x_i\right) &\leq \sum_{i=1}^M \lambda_i (-\ln(x_i)) \\ \implies \ln\left(\sum_{i=1}^M \lambda_i x_i\right) &\geq \sum_{i=1}^M \lambda_i \ln(x_i) \end{aligned}$$

Let $\lambda_i = (1/M) \forall i$

$$\begin{aligned} \implies \ln\left(\sum_{i=1}^M \frac{1}{M} x_i\right) &\geq \sum_{i=1}^M \frac{1}{M} \ln(x_i) \\ \implies \ln\left(\sum_{i=1}^M \frac{1}{M} x_i\right) &\geq \frac{1}{M} \sum_{i=1}^M \ln(x_i) \\ \implies \ln\left(\sum_{i=1}^M \frac{1}{M} x_i\right) &\geq \frac{1}{M} \ln\left(\prod_{i=1}^M x_i\right) \\ \implies \ln\left(\sum_{i=1}^M \frac{1}{M} x_i\right) &\geq \ln\left(\left(\prod_{i=1}^M x_i\right)^{1/M}\right) \\ \implies \sum_{i=1}^M \frac{1}{M} x_i &\geq \left(\prod_{i=1}^M x_i\right)^{1/M} \\ \implies \frac{1}{M} \sum_{i=1}^M x_i &\geq \left(\prod_{i=1}^M x_i\right)^{1/M} \\ \implies \text{Arithmetic mean} &\geq \text{Geometric mean} \end{aligned}$$