**2.55** From 2.184, we have:

$$\theta_0^{ML} = \tan^{-1}\left\{\frac{\sum_n \sin\theta_n}{\sum_n \cos\theta_n}\right\}$$

and from 2.168, we have:

$$\bar{r}\cos(\bar{\theta}) = \frac{1}{N}\sum_n^N \cos\theta_n \qquad \bar{r}\sin(\bar{\theta}) = \frac{1}{N}\sum_n^N \sin\theta_n$$

Substituting this result in 2.184, we get:

$$\begin{aligned}
\theta_0^{ML} &= \tan^{-1}\left\{\frac{N\bar{r}\sin(\bar{\theta})}{N\bar{r}\cos(\bar{\theta})}\right\} \\
&= \tan^{-1}\left\{\frac{\sin(\bar{\theta})}{\cos(\bar{\theta})}\right\} \\
&= \tan^{-1}\tan(\bar{\theta}) \\
&\implies \theta_0^{ML} = \bar{\theta}
\end{aligned}$$

Substituting this result in 2.185, we get:

$$A(m_{ML}) = \frac{1}{N}\sum_{n=1}^N \cos(\theta_n - \bar{\theta})$$

$$= \frac{1}{N}\sum_{n=1}^N (\cos(\theta_n)\cos(\bar{\theta}) + \sin(\theta_n)\sin(\bar{\theta})) \qquad \text{using 2.178}$$

$$= \frac{1}{N}\left(\cos(\bar{\theta})\sum_{n=1}^N \cos(\theta_n) + \sin(\bar{\theta})\sum_{n=1}^N \sin(\theta_n)\right)$$

Again applying 2.168, we get:

$$\begin{aligned}
&= \frac{1}{N}\big(\cos(\bar{\theta})(N\bar{r}\cos(\bar{\theta})) + \sin(\bar{\theta})(N\bar{r}\sin(\bar{\theta}))\big) \\
&= \frac{1}{N}N\bar{r}(\cos^2(\bar{\theta}) + \sin^2(\bar{\theta})) \\
&= \bar{r}(1) \\
&\implies A(m_{ML}) = \bar{r}
\end{aligned}$$