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$$\begin{aligned}
I[\mathbf{x}, \mathbf{y}] &= - \int \int p(\mathbf{x}, \mathbf{y}) \ln \left( \frac{p(\mathbf{x})p(\mathbf{y})}{p(\mathbf{x}, \mathbf{y})} \right) d\mathbf{x} d\mathbf{y} \\
&= - \int \int p(\mathbf{x}, \mathbf{y}) \ln \left( \frac{p(\mathbf{x})p(\mathbf{y})}{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})} \right) d\mathbf{x} d\mathbf{y} \\
&= - \int \int p(\mathbf{x}, \mathbf{y}) \ln \left( \frac{p(\mathbf{x})}{p(\mathbf{x}|\mathbf{y})} \right) d\mathbf{x} d\mathbf{y} \\
&= - \int \int p(\mathbf{x}, \mathbf{y}) \ln p(\mathbf{x}) d\mathbf{x} d\mathbf{y} + \int \int p(\mathbf{x}, \mathbf{y}) \ln p(\mathbf{x}|\mathbf{y}) d\mathbf{x} d\mathbf{y} \\
&= - \int \ln p(\mathbf{x}) \left( \int p(\mathbf{x}, \mathbf{y}) d\mathbf{y} \right) d\mathbf{x} + \int \int p(\mathbf{x}, \mathbf{y}) \ln p(\mathbf{x}|\mathbf{y}) d\mathbf{x} d\mathbf{y} \\
&= - \int \ln p(\mathbf{x}) p(\mathbf{x}) d\mathbf{x} + \int \int p(\mathbf{x}, \mathbf{y}) \ln p(\mathbf{x}|\mathbf{y}) d\mathbf{x} d\mathbf{y} \\
&= H[\mathbf{x}] - H[\mathbf{x}|\mathbf{y}]
\end{aligned}$$

Similarly,

$$\begin{aligned}
I[\mathbf{x}, \mathbf{y}] &= - \int \int p(\mathbf{x}, \mathbf{y}) \ln \left( \frac{p(\mathbf{x})p(\mathbf{y})}{p(\mathbf{x}, \mathbf{y})} \right) d\mathbf{x} d\mathbf{y} \\
&= - \int \int p(\mathbf{x}, \mathbf{y}) \ln \left( \frac{p(\mathbf{x})p(\mathbf{y})}{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})} \right) d\mathbf{x} d\mathbf{y} \\
&= - \int \int p(\mathbf{x}, \mathbf{y}) \ln \left( \frac{p(\mathbf{y})}{p(\mathbf{y}|\mathbf{x})} \right) d\mathbf{x} d\mathbf{y} \\
&= - \int \int p(\mathbf{x}, \mathbf{y}) \ln p(\mathbf{y}) d\mathbf{x} d\mathbf{y} + \int \int p(\mathbf{x}, \mathbf{y}) \ln p(\mathbf{y}|\mathbf{x}) d\mathbf{x} d\mathbf{y} \\
&= - \int \ln p(\mathbf{y}) \left( \int p(\mathbf{x}, \mathbf{y}) d\mathbf{x} \right) d\mathbf{y} + \int \int p(\mathbf{x}, \mathbf{y}) \ln p(\mathbf{y}|\mathbf{x}) d\mathbf{x} d\mathbf{y} \\
&= - \int \ln p(\mathbf{y}) p(\mathbf{y}) d\mathbf{y} + \int \int p(\mathbf{x}, \mathbf{y}) \ln p(\mathbf{y}|\mathbf{x}) d\mathbf{x} d\mathbf{y} \\
&= H[\mathbf{y}] - H[\mathbf{y}|\mathbf{x}]
\end{aligned}$$