

5.7 5.24 is the error function in multi-class classification.

$$\begin{aligned}
\frac{\partial E}{\partial a_k} &= \frac{\partial}{\partial a_k} \left(- \sum_{j=1}^K \{t_{nj} \ln y_{nj}\} \right) \\
&= - \sum_{j=1}^K \frac{\partial \{t_{nj} \ln y_{nj}\}}{\partial a_k} \\
&= - \sum_{j=1}^K t_{nj} \left(\frac{\partial \{\ln y_{nj}\}}{\partial a_k} \right) \\
&= - \sum_{j=1}^K t_{nj} \left(\frac{1}{y_{nj}} \right) \left(\frac{\partial y_{nj}}{\partial a_k} \right)
\end{aligned}$$

Here, y_{nk} is given by the softmax function. We calculated it's derivative w.r.t a_k in exercise 4.17, which gave us $\frac{\partial y_j}{\partial a_k} = y_j(I_{jk} - y_k)$. Applying this result here, we get:

$$\begin{aligned}
&= - \sum_{j=1}^K t_{nj} \left(\frac{1}{y_{nj}} \right) (y_{nj}(I_{jk} - y_{nk})) \\
&= - \sum_{j=1}^K t_{nj}(I_{jk} - y_{nk}) \\
&= - \sum_{j=1}^K t_{nj}I_{jk} + \sum_{j=1}^K t_{nj}y_{nk} \\
&= -t_{nk} + y_{nk} \sum_{j=1}^K t_{nj}
\end{aligned}$$

t_{nj} will be 1 for one j , and 0 for the rest. So the expression becomes:

$$\begin{aligned}
&= -t_{nk} + y_{nk} \\
&= y_{nk} - t_{nk}
\end{aligned}$$

which satisfies 5.18.