5.34

$$E_n = -\ln\left\{\sum_{l=1}^{K} \pi_l(\mathbf{x}_n, \mathbf{w}) \mathcal{N}\left(\mathbf{t}_n | \boldsymbol{\mu}_l(\mathbf{x}_n, \mathbf{w}), \sigma_l^2(\mathbf{x}_n, \mathbf{w})\mathbf{I}\right)\right\}$$

Since a_k^π affects E_n through every π_k (because of the denominator),

$$\frac{\partial E_n}{\partial a_k^\pi} = \sum_{l=1}^{K} \left(\frac{\partial E_n}{\partial \pi_l}\right) \left(\frac{\partial \pi_l}{\partial a_k^\pi}\right)$$

$$\begin{aligned}
\frac{\partial E_n}{\partial \pi_l} &= -\frac{\mathcal{N}_{nl}}{\left\{\sum_{l=1}^{K} \pi_l \mathcal{N}_{nl}\right\}} \\
&= -\frac{\gamma_{nl}}{\pi_l}
\end{aligned}$$

Using 4.106,

$$\frac{\partial \pi_l}{\partial a_k^\pi} = \pi_l(I_{kl} - \pi_k)$$

Substituting, we get:

$$\begin{aligned}
\frac{\partial E_n}{\partial a_k^\pi} &= \sum_{l=1}^{K} \left(-\frac{\gamma_l}{\pi_l}\right) (\pi_l(I_{kl} - \pi_k)) \\
&= -\sum_{l=1}^{K} \gamma_l (I_{kl} - \pi_k) \\
&= -\sum_{l=1}^{K} \gamma_l I_{kl} + \sum_{l=1}^{K} \gamma_l \pi_k \\
&= -\gamma_k + \pi_k \sum_{l=1}^{K} \gamma_l \\
&= \pi_k - \gamma_k
\end{aligned}$$

which is the same as the result in 5.155.