1.10 First, we prove that $E[x+z] = E[x]+E[z]$ when x and z are statistically independent.

$$E[x+z] = \int_z \int_x (x+z)\, p(x,z)\, dx\, dz$$

Since x and z are independent, $p(x,z) = p(x)p(z)$. This gives us:

$$= \int_z \int_x (x+z)\, p(x)\, p(z)\, dx\, dz$$

$$= \int_z \int_x (x\, p(x)\, p(z) \; + z\, p(x)\, p(z))\, dx\, dz$$

$$= \int_z \int_x x\, p(x)\, p(z)\, dx\, dz \; + \int_z \int_x z\, p(x)\, p(z)\, dx\, dz$$

$$= \int_z \int_x x\, p(x)\, p(z)\, dx\, dz \; + \int_x \int_z z\, p(x)\, p(z)\, dx\, dz$$

$$= \int_z p(z) \left(\int_x x\, p(x)\, dx \right) dz \; + \int_x p(x) \left(\int_z z\, p(z)\, dz \right) dx$$

$$= \int_z p(z)\, E[x]\, dz \; + \int_x p(x)\, E[z]\, dx$$

$$= E[x] \int_z p(z)\, dz \; + E[z] \int_x p(x)\, dx$$

$$= E[x]\,(1) \; + E[z]\,(1)$$

$$= E[x] + E[z].$$

Now we prove the following:

$$var[x+z] = var[x] + var[z]$$

From 1.39, we know that $var[x] = E[x^2] - E[x]^2$.

Applying that, we get:

$$var[x+z] = E[(x+z)^2] - E[x+z]^2$$

$$= E[x^2 + z^2 + 2xz] - (E[x] + E[z])^2$$

$$= E[x^2 + z^2 + 2xz] - (E[x]^2 + E[z]^2 + 2E[x]E[z])$$

The first term is simplified as:

$$E[x^2 + z^2 + 2xz] = \int_x \int_z (x^2 + z^2 + 2xz)\, p(x, z)\, dx\, dz$$

$$= \int_x \int_z (x^2 + z^2 + 2xz)\, p(x)\, p(z)\, dx\, dz$$

$$= \int_x \int_z x^2\, p(x)\, p(z)\, dx\, dz + \int_x \int_z z^2\, p(x)\, p(z)\, dx\, dz + \int_x \int_z 2xz\, p(x)\, p(z)\, dx\, dz$$

$$= \int_z p(z) \left(\int_x x^2\, p(x)\, dx \right) dz + \int_x p(x) \left(\int_z z^2\, p(z)\, dz \right) dx + 2 \int_x x\, p(x) \left(\int_z z\, p(z)\, dz \right) dx$$

$$= \int_z p(z)\, E[x^2]\, dz + \int_x p(x)\, E[z^2]\, dx + 2 \int_x x\, p(x)\, E[z]\, dx$$

$$= E[x^2] \int_z p(z)\, dz + E[z^2] \int_x p(x)\, dx + 2\, E[z] \int_x x\, p(x)\, dx$$

$$= E[x^2]\, (1) + E[z^2]\, (1) \; + 2E[z]E[x]$$

$$= E[x^2] + E[z^2] \; + 2E[z]E[x]$$

Substituting this result in the expression for $var[x + z]$, we get:

$$var[x + z] = E[x^2] + E[z^2] \; + 2E[z]E[x] - (E[x]^2 + E[z]^2 + 2E[x]E[z])$$

$$= E[x^2] + E[z^2] \; + 2E[z]E[x] - E[x]^2 - E[z]^2 - 2E[x]E[z])$$

$$= (E[x^2] - E[x]^2) + (E[z^2] - E[z]^2)$$

$$= var[x] + var[z].$$

Thus proved.