5.30 From 5.139, we have,

$$\tilde{E}(\mathbf{w}) = E(\mathbf{w}) + \lambda \mathbf{\Omega}(\mathbf{w})$$

$$\implies \frac{\partial \tilde{E}}{\partial \mu_j} = \frac{\partial E}{\partial \mu_j} + \frac{\partial \lambda \mathbf{\Omega}(\mathbf{w})}{\partial \mu_j}$$

$$= 0 + \lambda \frac{\partial \mathbf{\Omega}(\mathbf{w})}{\partial \mu_j}$$

$$= \lambda \frac{\partial \mathbf{\Omega}(\mathbf{w})}{\partial \mu_j}$$

$$\frac{\partial \mathbf{\Omega}(\mathbf{w})}{\partial \mu_j} = \frac{\partial}{\partial \mu_j}\left(-\sum_i \ln\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)\right)$$

$$= -\sum_i \frac{\partial}{\partial \mu_j} \ln\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)$$

$$= -\sum_i \frac{1}{\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)} \frac{\partial}{\partial \mu_j}\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)$$

Since this partial derivative is w.r.t a specific j, it becomes:

$$= -\sum_i \frac{1}{\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)} \left(\pi_j \, \frac{\partial \mathcal{N}(w_i|\mu_j, \sigma_j^2)}{\partial \mu_j}\right)$$

Simplifying the derivative inside the braces:

$$\frac{\partial \mathcal{N}(w_i|\mu_j, \sigma_j^2)}{\partial \mu_j}$$

$$= \frac{\partial}{\partial \mu_j}\left(\frac{1}{(2\pi\sigma_j^2)^{(1/2)}} \exp\left(-\frac{1}{2}\left(\frac{w_i - \mu_j}{\sigma_j}\right)^2\right)\right)$$

$$= \frac{1}{(2\pi\sigma_j^2)^{(1/2)}} \left(\frac{\partial}{\partial \mu_j} \exp\left(-\frac{1}{2}\left(\frac{w_i - \mu_j}{\sigma_j}\right)^2\right)\right)$$

$$= \frac{1}{(2\pi\sigma_j^2)^{(1/2)}} \left(\exp\left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \frac{\partial}{\partial \mu_j} \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \right)$$

$$= \frac{1}{(2\pi\sigma_j^2)^{(1/2)}} \left(\exp\left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right)$$

$$= \left(\frac{1}{(2\pi\sigma_j^2)^{(1/2)}} \exp\left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \right) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right)$$

$$= \mathcal{N}(w_i|\mu_j, \sigma_j^2) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right)$$

Substituting this result in the partial derivative above, we get:

$$\frac{\partial \tilde{E}}{\partial \mu_j} = -\lambda \sum_i \frac{1}{\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right)} \left(\pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right)$$

$$= -\lambda \sum_i \left(\frac{\pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)}{\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right)} \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right)$$

$$= -\lambda \sum_i \left(\gamma_j(w_i) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right)$$

$$= \lambda \sum_i \gamma_j(w_i) \, \frac{(\mu_j - w_i)}{\sigma_j^2}$$

which is the same as the result in 5.142.