1.15 1.133 gives us:

$$\sum_{i_1=1}^{D} \sum_{i_2=1}^{D} \cdots \sum_{i_M=1}^{D} w_{i_1 i_2 \ldots i_M} x_{i_1} x_{i_2} \ldots x_{i_M}$$

In this expression, the interchange symmetries can be seen when, for example, for some $1 \leq k < l \leq M$, $i_1 = k$ and $i_2 = l$, and $i_1 = l$ and $i_2 = k$ give us the same result.

But in 1.134:

$$\sum_{i_1=1}^{D} \sum_{i_2=1}^{i_1} \cdots \sum_{i_M=1}^{i_{M-1}} w_{i_1 i_2 \ldots i_M} x_{i_1} x_{i_2} \ldots x_{i_M}$$

we can see that $i_1 \geq i_2 \geq \ldots \geq i_{M-1}$. So, for $1 \leq k < l \leq M$, $i_1 = l$ and $i_2 = k$ is possible but $i_1 = k$ and $i_2 = l$ is not. So the symmetries disappear.

To find the the number of independent parameters, we can see that for a fixed i_1, the number of independent parameters is $n(i_1, M-1)$. The $(M-1)$ is there because the fixed i_1 makes first summation irrelevant.

Therefore, the total number of independent parameters becomes:

$$n(D, M) = \sum_{i=1}^{D} n(i, M-1)$$

Now to prove 1.136 by induction. First we prove the result for D = 1 and arbitrary M:

$$L.H.S = \sum_{i=1}^{D} \frac{(i+M-2)!}{(i-1)!(M-1)!}$$

$$= \sum_{i=1}^{1} \frac{(i+M-2)!}{(i-1)!(M-1)!}$$

$$= \frac{(1+M-2)!}{(1-1)!(M-1)!}$$

$$= \frac{(M-1)!}{0!(M-1)!}$$

$$= 1$$

$$R.H.S = \frac{(D+M-1)!}{(D-1)!M!}$$

$$= \frac{(1+M-1)!}{(1-1)!M!}$$

$$= \frac{M!}{0!M!}$$

$$= 1$$

$L.H.S = R.H.S$, thus proved for $D = 1$.

Assuming it is correct for dimension D and verifying that it is correct for dimension D + 1:

$$L.H.S = \sum_{i=1}^{D+1} \frac{(i+M-2)!}{(i-1)!(M-1)!}$$

$$= \left(\sum_{i=1}^{D} \frac{(i+M-2)!}{(i-1)!(M-1)!} \right) + \frac{((D+1)+M-2)!}{((D+1)-1)!(M-1)!}$$

$$= \frac{(D+M-1)!}{(D-1)!M!} + \frac{(D+M-1)!}{D!(M-1)!}$$

$$R.H.S = \frac{((D+1)+M-1)!}{((D+1)-1)!M!}$$

$$= \frac{((D+1)+M-1)\,(D+M-1)!}{((D+1)-1)\,(D-1)!M!}$$

$$= \left(\frac{((D+1)+M-1)}{((D+1)-1)}\right)\left(\frac{(D+M-1)!}{(D-1)!M!}\right)$$

$$= \left(\frac{(D+M)}{D}\right)\left(\frac{(D+M-1)!}{(D-1)!M!}\right)$$

$$= \left(1+\frac{M}{D}\right)\left(\frac{(D+M-1)!}{(D-1)!M!}\right)$$

$$= \frac{(D+M-1)!}{(D-1)!M!} + \frac{M}{D}\frac{(D+M-1)!}{(D-1)!M!}$$

$$= \frac{(D+M-1)!}{(D-1)!M!} + \frac{(D+M-1)!}{D!(M-1)!}$$

Since L.H.S = R.H.S, 1.136 is verified.

Now to prove 1.137 by induction. First we prove it for M = 2.

Using 1.135 to simplify the L.H.S of 1.137:

$$n(D,2) = \sum_{i=1}^{D} n(i,1) = \sum_{i=1}^{D} i = \frac{D(D+1)}{2}$$

Simplifying the R.H.S of 1.137:

$$\frac{(D+M-1)!}{(D-1)!M!} = \frac{(D+2-1)!}{(D-1)!2!}$$

$$= \frac{(D+1)!}{(D-1)!2!}$$

$$= \frac{(D+1)D}{2}$$

Thus, the result is true for M = 2.

Assuming the result holds for M-1, we check if it holds for M:

$$L.H.S = n(D, M)$$

$$= \sum_{i=1}^{D} n(i, M-1)$$

$$= n(D, M-1) + \sum_{i=1}^{D-1} n(i, M-1)$$

$$= n(D, M-1) + n(D-1, M)$$

$$R.H.S = \frac{(D+M-1)!}{(D-1)!M!}$$

$$= \frac{(D+(M-1)+1-1)!}{(D-1)!(M-1)!M}$$

$$= \frac{(D+M-1)\,(D+(M-1)-1)!}{M\,(D-1)!(M-1)!}$$

$$= \left(\frac{(D+M-1)}{M}\right)\left(\frac{(D+(M-1)-1)!}{(D-1)!(M-1)!}\right)$$

$$= \left(\frac{(D-1)}{M}+1\right)\left(\frac{(D+(M-1)-1)!}{(D-1)!(M-1)!}\right)$$

$$= \left(\frac{(D-1)}{M}\right)\left(\frac{(D+(M-1)-1)!}{(D-1)!(M-1)!}\right) + \left(\frac{(D+(M-1)-1)!}{(D-1)!(M-1)!}\right)$$

$$= \left(\frac{(D-1)}{M}\right)\left(\frac{(D+(M-1)-1)!}{(D-1)!(M-1)!}\right) + n(D, M-1)$$

$$\left(\frac{(D-1)}{M}\right)\left(\frac{(D+(M-1)-1)!}{(D-1)!(M-1)!}\right)$$

$$= \left(\frac{(D+(M-1)-1)!}{(D-2)!M!}\right)$$

$$= \left(\frac{((D-1)+M-1)!}{((D-1)-1)!M!}\right)$$

$$= n(D-1, M)$$

$$\implies R.H.S = n(D, M-1) + n(D-1, M) = L.H.S$$

Thus proved 1.137.