

**4.4** Maximization of the class separation criterion given by (4.23), along with the constraint is:

$$\begin{aligned} &\text{maximize } \mathbf{w}^T \mathbf{m}_k \\ &\text{subject to } \mathbf{w}^T \mathbf{w} = 1 \end{aligned}$$

where  $\mathbf{m}_k = \mathbf{m}_2 - \mathbf{m}_1$ .

Using a Lagrange multiplier  $\lambda$ , we can turn this into a Lagrangian function:

$$\text{maximize } \mathbf{w}^T \mathbf{m}_k + \lambda(\mathbf{w}^T \mathbf{w} - 1)$$

Taking derivative w.r.t  $\mathbf{w}$  and setting it to  $\mathbf{0}$ , we get:

$$\mathbf{0} = \mathbf{m}_k + 2\lambda\mathbf{w}$$

$$\implies \mathbf{w} = \frac{1}{2\lambda} \mathbf{m}_k$$

$$\implies \mathbf{w} \propto \mathbf{m}_k$$

$$\implies \mathbf{w} \propto \mathbf{m}_2 - \mathbf{m}_1$$