1.11 The log-likelihood function is:

$$\ln\, p(\mathbf{x}|\mu, \sigma^2) = -\frac{1}{2\sigma^2} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2} \ln(\sigma^2) - \frac{N}{2} \ln(2\pi)$$

Setting the derivative w.r.t μ,, we get:

$$\begin{aligned}
\frac{d}{d\mu} \ln\, p(\mathbf{x}|\mu, \sigma^2) = \frac{d}{d\mu} \left(-\frac{1}{2\sigma^2} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2} \ln(\sigma^2) - \frac{N}{2} \ln(2\pi) \right) \\
= -\frac{1}{2\sigma^2} \sum_{n=1}^{N} \frac{d}{d\mu} (x_n - \mu) - 0 - 0 \\
= -\frac{1}{2\sigma^2} \sum_{n=1}^{N} 2(x_n - \mu)(-1) \\
= \frac{1}{\sigma^2} \sum_{n=1}^{N} (x_n - \mu) \\
= \frac{1}{\sigma^2} \sum_{n=1}^{N} x_n - \frac{1}{\sigma^2} N\mu
\end{aligned}$$

Setting it to 0, we get:

$$\begin{aligned}
0 = \frac{1}{\sigma^2} \sum_{n=1}^{N} x_n - \frac{1}{\sigma^2} N\mu_{ML} \\
\implies \sum_{n=1}^{N} x_n = N\mu_{ML} \\
\implies \mu_{ML} = \frac{1}{N} \sum_{n=1}^{N} x_n
\end{aligned}$$

Setting the derivative w.r.t σ^2, we get:

$$\frac{d}{d\sigma^2} \ln\, p(\mathbf{x}|\mu, \sigma^2) = \frac{d}{d\sigma^2} \left(-\frac{1}{2\sigma^2} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2} \ln(\sigma^2) - \frac{N}{2} \ln(2\pi) \right)$$

$$= \frac{1}{2(\sigma^2)^2} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2} \frac{1}{\sigma^2} - 0$$

$$= \frac{1}{2(\sigma^2)^2} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2} \frac{1}{\sigma^2}$$

Setting it to 0, we get:

$$0 = \frac{1}{2(\sigma_{ML}^2)^2} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2} \frac{1}{\sigma_{ML}^2}$$

$$\Longrightarrow \frac{1}{2(\sigma_{ML}^2)^2} \sum_{n=1}^{N} (x_n - \mu)^2 = \frac{N}{2} \frac{1}{\sigma_{ML}^2}$$

$$\Longrightarrow \frac{1}{\sigma_{ML}^2} \sum_{n=1}^{N} (x_n - \mu)^2 = N$$

$$\Longrightarrow \sigma_{ML}^2 = \frac{1}{N} \sum_{n=1}^{N} (x_n - \mu)^2$$

Since μ is also unknown, we substitute μ_{ML}, and we get:

$$\sigma_{ML}^2 = \frac{1}{N} \sum_{n=1}^{N} (x_n - \mu_{ML})^2$$