1.13 The variance of a Gaussian is estimated using the result (1.56) but with the maximum likelihood estimate μ_{ML} replaced with the true value μ of the mean:

$$\sigma^2_{ML} = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu_{ML})$$

$$\implies E[\sigma^2_{ML}] = E\left[\frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)^2\right]$$

$$= \frac{1}{N}\sum_{n=1}^{N} E[(x_n - \mu)^2]$$

$$= \frac{1}{N}\sum_{n=1}^{N} E[x_n^2 + \mu^2 - 2x_n\mu]$$

$$= \frac{1}{N}\sum_{n=1}^{N}(E[x_n^2] + E[\mu^2] - E[2x_n\mu])$$

$$= \frac{1}{N}\sum_{n=1}^{N}((\sigma^2 + \mu^2) + \mu^2 - 2E[x_n]\mu)$$

$$= \frac{1}{N}\sum_{n=1}^{N}(\sigma^2 + 2\mu^2 - 2\mu^2)$$

$$= \frac{1}{N}\sum_{n=1}^{N}\sigma^2$$

$$= \frac{1}{N}N\sigma^2$$

$$= \sigma^2$$