2.7 Posterior mean value of μ is given by (2.19) and (2.20) since posterior distribution will be given by (2.18):

$$\mathbb{E}[\mu] = \frac{m+a}{m+a+l+b}$$

Prior mean is :

$$\mu_{prior} = \frac{a}{a+b}$$

Maximum likelihood estimate for μ is given by (2.8):

$$\mu_{MLE} = \frac{m}{N} = \frac{m}{m+l}$$

Posterior mean can be written as λ times the prior mean plus $(1-\lambda)$ times the maximum likelihood estimate, where $0 \leq \lambda \leq 1$:

$$\begin{aligned}
\mathbb{E}[\mu] &= \frac{m+a}{m+a+l+b} \\
&= \frac{m}{m+a+l+b} + \frac{a}{m+a+l+b} \\
&= \frac{m}{(m+l)} \frac{(m+l)}{(m+a+l+b)} + \frac{a}{(a+b)} \frac{(a+b)}{(m+a+l+b)} \\
&= \mu_{MLE} \left(\frac{m+l}{m+a+l+b} \right) + \mu_{prior} \left(\frac{a+b}{m+a+l+b} \right)
\end{aligned}$$

Let $\lambda = \left(\frac{m+l}{m+a+l+b} \right)$. Then, $(1-\lambda) = \left(\frac{a+b}{m+a+l+b} \right)$.

It can be easily seen that $0 \leq \lambda \leq 1$ since a, b, m, l are all ≥ 0.

$$\implies \mathbb{E}[\mu] = \lambda \mu_{MLE} + (1-\lambda)\mu_{prior}$$