4.25 We need to prove that the derivatives of $\sigma(a)$ and $\mathbf{\Phi}(\lambda a)$ are the same at $a = 0$ when $\lambda^2 = \pi/8$.

We already obtained the derivative of logistic sigmoid function in exercise 4.12, and it is given by:

$$\sigma(a)(1 - \sigma(a))$$

At $a = 0$, it becomes $\frac{1}{2}(1 - \frac{1}{2}) = \frac{1}{4}$.

Now to find the derivative of $\mathbf{\Phi}(\lambda a)$.

$$\mathbf{\Phi}(\lambda a) = \int_0^{\lambda a} \mathcal{N}(\theta|0, 1)\, d\theta$$

Taking derivative w.r.t a,

$$\frac{\partial \mathbf{\Phi}(\lambda a)}{\partial a} = \frac{\partial}{\partial a} \int_0^{\lambda a} \mathcal{N}(\theta|0, 1)\, d\theta$$

$$= \left(\frac{\partial \lambda a}{\partial a}\right) \frac{\partial}{\partial \lambda a} \int_0^{\lambda a} \mathcal{N}(\theta|0, 1)\, d\theta$$

Using this rule of derivatives, we get:

$$= \lambda \mathcal{N}(\lambda a|0, 1)$$

$$= \lambda \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(\lambda a)^2\right\}$$

At $a = 0$, this becomes:

$$= \frac{\lambda}{\sqrt{2\pi}}$$

Equating the derivatives of the 2 functions at $a = 0$, we get:

$$\frac{1}{4} = \frac{\lambda}{\sqrt{2\pi}}$$

$$\implies \frac{\sqrt{2\pi}}{4} = \lambda$$

$$\implies \lambda^2 = \frac{2\pi}{16}$$

$$\implies \lambda^2 = \frac{\pi}{8}$$