3.10 The predictive distribution is given by:

$$p(t|\mathbf{t}, \beta) = \int p(t|\mathbf{w}, \beta)\, p(\mathbf{w}|\mathbf{t})\, d\mathbf{w}$$

$$= \int \mathcal{N}(t|y(\mathbf{x}, \mathbf{w}), \beta^{-1})\, \mathcal{N}(\mathbf{w}|\mathbf{m}_N, \mathbf{S}_N)\, d\mathbf{w}$$

$$= \int \mathcal{N}(t|\phi(\mathbf{x})^T\mathbf{w}, \beta^{-1})\, \mathcal{N}(\mathbf{w}|\mathbf{m}_N, \mathbf{S}_N)\, d\mathbf{w}$$

Comparing this to 2.113 and 2.114, we get:

$$\mathbf{x} = \mathbf{w}$$

$$\boldsymbol{\mu} = \mathbf{m}_N$$

$$\boldsymbol{\Lambda} = \mathbf{S}_N^{-1}$$

$$\mathbf{y} = t$$

$$\mathbf{A} = \phi(\mathbf{x})^T$$

$$\mathbf{b} = 0$$

$$\mathbf{L} = \beta$$

Applying 2.115, we get,

$$p(t|\mathbf{t}, \beta) = \mathcal{N}(t|\phi(\mathbf{x})^T\mathbf{m}_N,\ \beta^{-1} + \phi(\mathbf{x})^T\mathbf{S}_N\phi(\mathbf{x}))$$

$$= \mathcal{N}(t|\phi(\mathbf{x})^T\mathbf{m}_N, \sigma_N^2(\mathbf{x}))$$

where $\sigma_N^2(\mathbf{x}) = \frac{1}{\beta} + \phi(\mathbf{x})^T\mathbf{S}_N\phi(\mathbf{x})$.