2.4 The Binomial distribution is given by:

$$\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} = 1$$

Taking derivative w.r.t μ,

$$\frac{\partial}{\partial \mu} \sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} = 0$$

$$\sum_{m=0}^{N} \binom{N}{m} \frac{\partial \left(\mu^m (1-\mu)^{N-m}\right)}{\partial \mu} = 0$$

$$\sum_{m=0}^{N} \binom{N}{m} \left(m\mu^{m-1}(1-\mu)^{N-m} - (N-m)\mu^m (1-\mu)^{N-m-1}\right) = 0$$

$$\sum_{m=0}^{N} \binom{N}{m} \mu^{m-1}(1-\mu)^{N-m-1} \left(m(1-\mu) - (N-m)\mu\right) = 0$$

$$\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} \left(\frac{m}{\mu} - \frac{(N-m)}{(1-\mu)}\right) = 0$$

$$\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} \frac{(m - m\mu - N\mu + m\mu)}{\mu(1-\mu)} = 0$$

$$\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} \frac{(m - N\mu)}{\mu(1-\mu)} = 0$$

$$\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} (m - N\mu) = 0$$

$$\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} m = \sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} N\mu$$

$$\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} m = N\mu \sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m}$$

$$\sum_{m=0}^{N} \binom{N}{m} \mu^m (1-\mu)^{N-m} m = N\mu$$

$$\sum_{m=0}^{N} Bin(m|N,\mu)m = N\mu$$

$$\mathbb{E}[m] = N\mu$$

We can differentiate (2.264) twice with respect to μ by simply differentiating the result of the first derivative,

$$\frac{\partial}{\partial\mu}\sum_{m=0}^{N}\binom{N}{m}\mu^m(1-\mu)^{N-m}m = \frac{\partial}{\partial\mu}N\mu$$

We can get the derivative on the L.H.S using the result we got above:

$$\sum_{m=0}^{N}\binom{N}{m}\mu^m(1-\mu)^{N-m}\frac{(m-N\mu)}{\mu(1-\mu)}m = N$$

$$\sum_{m=0}^{N}\binom{N}{m}\mu^m(1-\mu)^{N-m}(m-N\mu)m = N\mu(1-\mu)$$

$$\sum_{m=0}^{N}\binom{N}{m}\mu^m(1-\mu)^{N-m}(m^2-Nm\mu) = N\mu - N\mu^2$$

$$\left(\sum_{m=0}^{N}\binom{N}{m}\mu^m(1-\mu)^{N-m}m^2\right)-\left(\sum_{m=0}^{N}\binom{N}{m}\mu^m(1-\mu)^{N-m}Nm\mu\right) = N\mu - N\mu^2$$

$$\left(\sum_{m=0}^{N} Bin(m|N,\mu)m^2\right) - \left(N\mu\sum_{m=0}^{N}\binom{N}{m}\mu^m(1-\mu)^{N-m}m\right) = N\mu - N\mu^2$$

$$\mathbb{E}[m^2] - N\mu\mathbb{E}[m] = N\mu - N\mu^2$$

$$\mathbb{E}[m^2] - \mathbb{E}[m]\mathbb{E}[m] = N\mu - N\mu^2$$

$$\mathbb{E}[m^2] - \mathbb{E}[m]^2 = N\mu(1-\mu)$$

$$var[m] = N\mu(1-\mu)$$