2.8

$$\begin{aligned}
\mathbb{E}_y[\mathbb{E}_x[x|y]] &= \int_y \left(\int_x p(x|y)\, x\, dx \right)\, p(y)\, dy \\
&= \int_y \int_x p(x|y)\, p(y)\, x\, dx\, dy \\
&= \int_y \int_x p(x, y)\, x\, dx\, dy \\
&= \int_x \int_y p(x, y)\, x\, dy\, dx \\
&= \int_x x\ \int_y (p(x, y)\, dy)\, dx \\
&= \int_x x\, p(x)\, dx \\
&= \mathbb{E}[x]
\end{aligned}$$

$$\mathbb{E}_y[var_x[x|y]] + var_y[\mathbb{E}_x[x|y]]$$

$$= \mathbb{E}_y[\mathbb{E}_x[x^2|y] - \mathbb{E}_x[x|y]^2] + \left(\mathbb{E}_y[\mathbb{E}_x[x|y]^2] - \mathbb{E}_y[\mathbb{E}_x[x|y]]^2\right)$$

$$= \mathbb{E}_y[\mathbb{E}_x[x^2|y]] - \mathbb{E}_y[\mathbb{E}_x[x|y]^2] + \mathbb{E}_y[\mathbb{E}_x[x|y]^2] - \mathbb{E}_y[\mathbb{E}_x[x|y]]^2$$

$$= \mathbb{E}_y[\mathbb{E}_x[x^2|y]] - \mathbb{E}_y[\mathbb{E}_x[x|y]]^2$$

Evaluating $\mathbb{E}_y[\mathbb{E}_x[x^2|y]]$:

$$\begin{aligned}
\mathbb{E}_y[\mathbb{E}_x[x^2|y]] &= \int_y \left(\int_x p(x|y)\, x^2\, dx \right)\, p(y)\, dy \\
&= \int_y \int_x p(x|y)\, p(y)\, x^2\, dx\, dy \\
&= \int_y \int_x p(x, y)\, x^2\, dx\, dy \\
&= \int_x \int_y p(x, y)\, x^2\, dy\, dx
\end{aligned}$$

$$= \int_x x^2 \int_y (p(x, y)\, dy)\, dx$$

$$= \int_x x^2\, p(x)\, dx$$

$$= \mathbb{E}_x[x^2]$$

Substituting above, we get:

$$= \mathbb{E}_x[x^2] - \mathbb{E}_x[x]^2$$

$$= var[x]$$