2.27

$$\mathbb{E}[\mathbf{y}] = \mathbb{E}[\mathbf{x} + \mathbf{z}]$$

$$= \mathbb{E}[\mathbf{x}] + \mathbb{E}[\mathbf{z}]$$

simply using linearity of expectation.

Using 2.63 to define covariance,

$$\Sigma_{\mathbf{yy}} = \mathbb{E}[(\mathbf{y} - \mathbb{E}[\mathbf{y}])(\mathbf{y} - \mathbb{E}[\mathbf{y}])^T]$$

$$= \mathbb{E}[((\mathbf{x} + \mathbf{z}) - (\mathbb{E}[\mathbf{x}] + \mathbb{E}[\mathbf{z}]))((\mathbf{x} + \mathbf{z}) - (\mathbb{E}[\mathbf{x}] + \mathbb{E}[\mathbf{z}]))^T]$$

$$= \mathbb{E}[((\mathbf{x} - \mathbb{E}[\mathbf{x}]) + (\mathbf{z} - \mathbb{E}[\mathbf{z}]))((\mathbf{x} - \mathbb{E}[\mathbf{x}]) + (\mathbf{z} - \mathbb{E}[\mathbf{z}]))^T]$$

$$= \mathbb{E}[(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T + (\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{z} - \mathbb{E}[\mathbf{z}])^T + (\mathbf{z} - \mathbb{E}[\mathbf{z}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T + (\mathbf{z} - \mathbb{E}[\mathbf{z}])(\mathbf{z} - \mathbb{E}[\mathbf{z}])^T]$$

$$\begin{gathered} = \mathbb{E}[(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T] \\ + \mathbb{E}[(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{z} - \mathbb{E}[\mathbf{z}])^T] \\ + \mathbb{E}[(\mathbf{z} - \mathbb{E}[\mathbf{z}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T] \\ + \mathbb{E}[(\mathbf{z} - \mathbb{E}[\mathbf{z}])(\mathbf{z} - \mathbb{E}[\mathbf{z}])^T] \end{gathered}$$

$$= \Sigma_{\mathbf{xx}} + \Sigma_{\mathbf{xz}} + \Sigma_{\mathbf{zx}} + \Sigma_{\mathbf{zz}}$$

Since $\mathbf{x}$ and $\mathbf{z}$ are independent, their covariance is $\mathbf{0}$, giving us:

$$= \Sigma_{\mathbf{xx}} + \mathbf{0} + \mathbf{0} + \Sigma_{\mathbf{zz}}$$

$$= \Sigma_{\mathbf{xx}} + \Sigma_{\mathbf{zz}}$$