

1.1 The error function given by 1.2 is:

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\}^2$$

$$\text{where } y(x, \mathbf{w}) = w_0 + w_1x + w_2x^2 + \dots + w_Mx^M = \sum_{j=0}^M w_jx^j$$

To minimize the error function w.r.t \mathbf{w} , we take its derivative w.r.t \mathbf{w} and set it to 0.

Here, $E(\mathbf{w})$ is a scalar and \mathbf{w} is a vector.

The derivative of a scalar valued function with respect to a vector of variables then is a row vector. This row vector has one column for each variable we want to differentiate by.

Therefore,

$$\begin{aligned} \frac{dE(\mathbf{w})}{d\mathbf{w}} &= \frac{d}{d\mathbf{w}} \left(\frac{1}{2} \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\}^2 \right) = \frac{1}{2} \sum_{n=1}^N \frac{d}{d\mathbf{w}} \{y(x_n, \mathbf{w}) - t_n\}^2 \\ &= \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\} \left[\frac{d\{y(x_n, \mathbf{w}) - t_n\}}{dw_0} \quad \frac{d\{y(x_n, \mathbf{w}) - t_n\}}{dw_1} \quad \dots \quad \frac{d\{y(x_n, \mathbf{w}) - t_n\}}{dw_M} \right] \\ &= \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\} \begin{bmatrix} 1 & x_n & \dots & (x_n)^M \end{bmatrix} \end{aligned}$$

Setting the derivative to $\mathbf{0}$ row vector, we get the following set of equations:

$$\begin{aligned} \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\} (x_n)^j &= 0, \quad \forall j = 0..M \\ \implies \sum_{n=1}^N \left\{ \left(\sum_{i=0}^M w_i (x_n)^i \right) - t_n \right\} (x_n)^j &= 0 \\ \implies \sum_{n=1}^N \left(\sum_{i=0}^M w_i (x_n)^i \right) (x_n)^j &= \sum_{n=1}^N t_n (x_n)^j \\ \implies \sum_{n=1}^N \sum_{i=0}^M w_i (x_n)^i (x_n)^j &= \sum_{n=1}^N t_n (x_n)^j \end{aligned}$$

Here we just multiplied $(x_n)^j$ to every term inside the summation

$$\begin{aligned}
&\Rightarrow \sum_{n=1}^N \sum_{i=1}^M w_i (x_n)^{i+j} = \sum_{n=1}^N t_n (x_n)^j \\
&\Rightarrow \sum_{i=1}^M \sum_{n=1}^N w_i (x_n)^{i+j} = \sum_{n=1}^N t_n (x_n)^j \\
&\Rightarrow \sum_{i=1}^M w_i \sum_{n=1}^N (x_n)^{i+j} = \sum_{n=1}^N t_n (x_n)^j \\
&\Rightarrow \sum_{i=1}^M w_i A_{ij} = T_j \quad \text{by applying 1.123}
\end{aligned}$$

Note: i and j can be switched with no effect, as they both are indices going from 0 to M.