

2.26 It is trivial to see that the L.H.S becomes the identity matrix after the multiplication.

Multiplying the R.H.S, we get:

$$\begin{aligned}
R.H.S &= (\mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D}\mathbf{A}^{-1})(\mathbf{A} + \mathbf{B}\mathbf{C}\mathbf{D}) \\
&= \mathbf{A}^{-1}(\mathbf{A} + \mathbf{B}\mathbf{C}\mathbf{D}) - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D}\mathbf{A}^{-1}(\mathbf{A} + \mathbf{B}\mathbf{C}\mathbf{D}) \\
&= \mathbf{A}^{-1}\mathbf{A} + \mathbf{A}^{-1}\mathbf{B}\mathbf{C}\mathbf{D} \\
&\quad - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D}\mathbf{A}^{-1}\mathbf{A} \\
&\quad - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D}\mathbf{A}^{-1}\mathbf{B}\mathbf{C}\mathbf{D} \\
&= \mathbf{I} + \mathbf{A}^{-1}\mathbf{B}\mathbf{C}\mathbf{D} \\
&\quad - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D} \\
&\quad - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D}\mathbf{A}^{-1}\mathbf{B}\mathbf{C}\mathbf{D} \\
&= \mathbf{I} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{C} - (\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1} - (\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D}\mathbf{A}^{-1}\mathbf{B}\mathbf{C})\mathbf{D} \\
&= \mathbf{I} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{C} - (\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B}^{-1})(\mathbf{I} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B}\mathbf{C}))\mathbf{D} \\
&= \mathbf{I} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{C} - (\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B}^{-1})(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})\mathbf{C})\mathbf{D} \\
&= \mathbf{I} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{C} - \mathbf{I}\mathbf{C})\mathbf{D} \\
&= \mathbf{I} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{C} - \mathbf{C})\mathbf{D} \\
&= \mathbf{I}
\end{aligned}$$

R.H.S also becomes the identity matrix. Thus proved.