2.2

$$\sum_{x=-1,1} p(x|\mu) = \sum_{x=-1,1} \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2}$$

$$= \left(\frac{1-\mu}{2}\right)^{(1-(-1))/2} \left(\frac{1+\mu}{2}\right)^{(1+(-1))/2} + \left(\frac{1-\mu}{2}\right)^{(1-1)/2} \left(\frac{1+\mu}{2}\right)^{(1+1)/2}$$

$$= \left(\frac{1-\mu}{2}\right)^{1} \left(\frac{1+\mu}{2}\right)^{0} + \left(\frac{1-\mu}{2}\right)^{0} \left(\frac{1+\mu}{2}\right)^{1}$$

$$= \left(\frac{1-\mu}{2}\right) + \left(\frac{1+\mu}{2}\right)$$

$$= 1$$

$$\mathbb{E}[x] = \sum_{x=-1,1} p(x)x$$

$$= \sum_{x=-1,1} \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} x$$

$$= \left(\frac{1-\mu}{2}\right)(-1) + \left(\frac{1+\mu}{2}\right)(1)$$

$$= -\left(\frac{1-\mu}{2}\right) + \left(\frac{1+\mu}{2}\right)$$

$$= \frac{-1+\mu+1+\mu}{2}$$

$$= \mu$$

$$var[x] = E[x^2] - E[x]^2$$

$$E[x^2] = \sum_{x=-1,1} \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} x^2$$

$$= \left(\frac{1-\mu}{2}\right)(-1)^2 + \left(\frac{1+\mu}{2}\right)(1)^2$$

$$= \frac{(1-\mu)}{2} + \frac{(1+\mu)}{2}$$

$$= 1$$

$$\implies var[x] = 1 - \mu^2$$

$$H[x] = -\sum_{x=-1,1} p(x) \ln p(x)$$

$$= -\sum_{x=-1,1} \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} \ln\left(\left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2}\right)$$

$$= -\left(\frac{(1-\mu)}{2} \ln\left(\frac{1-\mu}{2}\right) + \frac{(1+\mu)}{2} \ln\left(\frac{1+\mu}{2}\right)\right)$$

$$= -\left(\frac{1}{2} \ln\left(\frac{(1-\mu)(1+\mu)}{4}\right) + \frac{\mu}{2} \ln\left(\frac{1+\mu}{1-\mu}\right)\right)$$

$$= -\frac{1}{2}\left(\ln\left(\frac{1-\mu^2}{4}\right) + \mu \ln\left(\frac{1+\mu}{1-\mu}\right)\right)$$