

5.14 Equation 5.69 gives us:

$$\frac{\partial E_n}{\partial w_{ji}} = \frac{E_n(w_{ji} + \epsilon) - E_n(w_{ji} - \epsilon)}{2\epsilon} + O(\epsilon^2)$$

Applying Taylor expansion to E_n , evaluated at w_{ji} , we get:

$$E_n(x) = E_n(w_{ji}) + \frac{E'_n(w_{ji})}{1!}(x - w_{ji}) + O(x - w_{ji})^2$$

Evaluating the expression at $w_{ji} + \epsilon$, we get

$$\begin{aligned} E_n(w_{ji} + \epsilon) &= E_n(w_{ji}) + \frac{E'_n(w_{ji})}{1!}(w_{ji} + \epsilon - w_{ji}) \\ &\quad + \frac{E''_n(w_{ji})}{2!}(w_{ji} + \epsilon - w_{ji})^2 + O(w_{ji} + \epsilon - w_{ji})^3 \\ &= E_n(w_{ji}) + \epsilon E'_n(w_{ji}) + \frac{E''_n(w_{ji})}{2}(\epsilon)^2 + O(\epsilon)^3 \end{aligned}$$

Evaluating the expression at $w_{ji} - \epsilon$, we get

$$\begin{aligned} E_n(w_{ji} - \epsilon) &= E_n(w_{ji}) + \frac{E'_n(w_{ji})}{1!}(w_{ji} - \epsilon - w_{ji}) \\ &\quad + \frac{E''_n(w_{ji})}{2!}(w_{ji} - \epsilon - w_{ji})^2 + O(w_{ji} - \epsilon - w_{ji})^3 \\ &= E_n(w_{ji}) - \epsilon E'_n(w_{ji}) + \frac{E''_n(w_{ji})}{2}(\epsilon)^2 - O(\epsilon)^3 \end{aligned}$$

Substituting in 5.69, we get

$$\begin{aligned} \frac{\partial E_n}{\partial w_{ji}} &= \frac{1}{2\epsilon} \left(E_n(w_{ji}) + \epsilon E'_n(w_{ji}) + \frac{E''_n(w_{ji})}{2}(\epsilon)^2 + O(\epsilon)^3 \right. \\ &\quad \left. - E_n(w_{ji}) - \epsilon E'_n(w_{ji}) + \frac{E''_n(w_{ji})}{2}(\epsilon)^2 - O(\epsilon)^3 \right) + O(\epsilon^2) \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2\epsilon} (2\epsilon E'_n(w_{ji}) + 2O(\epsilon)^3) + O(\epsilon^2) \\
&= E'_n(w_{ji}) + O(\epsilon)^2
\end{aligned}$$

(by folding all $O(\epsilon)^2$ terms into one term).

We can see that the terms that are $O(\epsilon)$ cancel on the R.H.S.