

**4.14** If a dataset is linearly separable, then the decision boundary can perfectly separate the classes. This means that for  $y_n = 1$ ,  $(\mathbf{w}^T \phi_n) > 0$  and for  $y_n = 0$ ,  $(\mathbf{w}^T \phi_n) < 0$ .

$$\text{For } t_n = 1, \text{ the error} = -\ln y_n = -\ln \left( \frac{1}{1 + \exp(-\mathbf{w}^T \phi_n)} \right) = \ln (1 + \exp(-\mathbf{w}^T \phi_n)).$$

This error can be minimized if  $\exp(-\mathbf{w}^T \phi_n) \rightarrow -\infty, \implies \mathbf{w}^T \phi_n \rightarrow \infty$ .

$$\text{For } t_n = 0, \text{ the error} = -(1 - \ln y_n) = -\left(1 - \ln \left( \frac{1}{1 + \exp(-\mathbf{w}^T \phi_n)} \right)\right) = -1 - \ln (1 + \exp(-\mathbf{w}^T \phi_n)).$$

This error can be minimized if  $\exp(-\mathbf{w}^T \phi_n) \rightarrow \infty, \implies \mathbf{w}^T \phi_n \rightarrow -\infty$ .