**6.20** Given the covariance matrix in equation 6.65:

$$\mathbf{C}_{N+1} = \begin{pmatrix} \mathbf{C}_N & \mathbf{k} \\ \mathbf{k}^T & c \end{pmatrix}$$

Comparing this to equation 2.78,

$$\begin{pmatrix} \mathbf{\Sigma}_{aa} & \mathbf{\Sigma}_{ab} \\ \mathbf{\Sigma}_{ba} & \mathbf{\Sigma}_{bb} \end{pmatrix} = \begin{pmatrix} \mathbf{C}_N & \mathbf{k} \\ \mathbf{k}^T & c \end{pmatrix}$$

Exchange a and b in equation 2.81 for our convenience here, we get:

$$\mu_{b|a} = \mu_b + \mathbf{\Sigma}_{ba}\mathbf{\Sigma}_{aa}^{-1}(\mathbf{x}_a - \mu_a)$$

$$\Longrightarrow m(\mathbf{x}_{N+1}) = \mu_{t_{N+1}|\mathbf{t}_N} = \mu_{t_{N+1}} + \mathbf{k}^T\mathbf{C}_N^{-1}(\mathbf{t} - \boldsymbol{\mu}_{\mathbf{t}})$$

$$= 0 + \mathbf{k}^T\mathbf{C}_N^{-1}(\mathbf{t} - \mathbf{0})$$

$$= \mathbf{k}^T\mathbf{C}_N^{-1}\mathbf{t}$$

Similarly, comparing to equation 2.82,

$$\Sigma_{b|a} = \Sigma_{bb} - \mathbf{\Sigma}_{ba}\mathbf{\Sigma}_{aa}^{-1}\mathbf{\Sigma}_{ab}$$

$$\Longrightarrow \sigma^2(\mathbf{x}_{N+1}) = \Sigma_{t_{N+1}|\mathbf{t}} = c - \mathbf{k}^T\mathbf{C}_N^{-1}\mathbf{k}$$