

3.1 First we show how tanh and logistic sigmoid function are related:

$$\begin{aligned}
\sigma(2a) &= \frac{1}{1 + \exp(-2a)} \\
\implies 2\sigma(2a) - 1 &= \frac{2}{1 + \exp(-2a)} - 1 \\
&= \frac{2 - (1 + \exp(-2a))}{1 + \exp(-2a)} = \frac{1 - \exp(-2a)}{1 + \exp(-2a)} = \frac{1 - \frac{1}{\exp(2a)}}{1 + \frac{1}{\exp(2a)}} \\
&= \frac{\frac{\exp(2a) - 1}{\exp(2a)}}{\frac{\exp(2a) + 1}{\exp(2a)}} = \frac{\exp(2a) - 1}{\exp(2a) + 1} = \tanh(a).
\end{aligned}$$

From the above result, we can see that $\sigma(a) = \frac{1}{2}(\tanh(\frac{a}{2}) + 1)$.

Applying that to the general linear combination of logistic sigmoid functions of this form:

$$w_0 + \sum_{j=1}^M w_j \sigma\left(\frac{x - \mu_j}{s}\right)$$

we get :

$$\begin{aligned}
&= w_0 + \sum_{j=1}^M w_j \left(\frac{1}{2} \left(\tanh\left(\frac{x - \mu_j}{2s}\right) + 1 \right) \right) \\
&= w_0 + \sum_{j=1}^M \frac{w_j}{2} \tanh\left(\frac{x - \mu_j}{2s}\right) + \sum_{j=1}^M \frac{w_j}{2} \\
&\implies u_0 = w_0 + \sum_{j=1}^M \frac{w_j}{2}
\end{aligned}$$

and

$$\implies u_j = \frac{w_j}{2}, \quad \forall j \neq 0.$$

Note: typo in (3.102), the denominator should be 2s not s.