2.47

$$\frac{\Gamma(\nu/2+1/2)}{\Gamma(\nu/2)}\left(\frac{\lambda}{\pi\nu}\right)^{1/2}\left(\frac{\lambda}{\nu}(x-\mu)^2+1\right)^{-(\nu/2+1/2)}$$

$$=\frac{\Gamma(\nu/2+1/2)}{\Gamma(\nu/2)}\left(\frac{\lambda}{\pi\nu}\right)^{1/2}\left(\left(\frac{\lambda}{\nu}(x-\mu)^2+1\right)^{\frac{\nu}{\lambda(x-\mu)^2}}\right)^{\frac{-(\nu/2+1/2)\lambda(x-\mu)^2}{\nu}}$$

$$=\frac{\Gamma(\nu/2+1/2)}{\Gamma(\nu/2)}\left(\frac{\lambda}{\pi\nu}\right)^{1/2}\left(\left(\frac{\lambda}{\nu}(x-\mu)^2+1\right)^{\frac{\nu}{\lambda(x-\mu)^2}}\right)^{-(\lambda(x-\mu)^2/2+\lambda(x-\mu)^2/2\nu)}$$

Applying an approximation to Gamma function as $\nu \to +\infty$ as per this formula where $\Gamma(x+\alpha)=\Gamma(x)x^{\alpha}$ as $x \to +\infty$:

$$=\frac{\Gamma(\nu/2)(\nu/2)^{1/2}}{\Gamma(\nu/2)}\left(\frac{\lambda}{\pi\nu}\right)^{1/2}\left(\left(\frac{\lambda}{\nu}(x-\mu)^2+1\right)^{\frac{\nu}{\lambda(x-\mu)^2}}\right)^{-(\lambda(x-\mu)^2/2+\lambda(x-\mu)^2/2\nu)}$$

Using the limit $lim_{x\to+\infty}(1+1/x)^x = e$ from link, when we apply $\nu \to +\infty$, we get:

$$=\left(\frac{1}{2}\right)^{1/2}\left(\frac{\lambda}{\pi}\right)^{1/2}\exp\left\{-\lambda(x-\mu)^2/2\right\}$$

$$=\frac{1}{(2\pi\lambda^{-1})^{1/2}}\,exp\left\{-\frac{\lambda}{2}(x-\mu)^2\right\}$$

which gives us a normal distribution where $\lambda^{-1}=\sigma^2$.