5.35

$$E_n = -\ln\left\{\sum_{k=1}^{K}\pi_k(\mathbf{x}_n,\mathbf{w})\,\mathcal{N}\left(\mathbf{t}|\boldsymbol{\mu}_k(\mathbf{x}_n,\mathbf{w}),\left(\sigma_k(\mathbf{x}_n,\mathbf{w})^2\mathbf{I}\right)\right)\right\}$$

Since $a^{\mu}_{kl} = \mu_{kl}$,

$$\frac{\partial E_n}{\partial a^{\mu}_{kl}} = -\frac{1}{\left\{\sum_{k=1}^{K}\pi_k\,\mathcal{N}_{nk}\right\}}\left(\pi_k\frac{\partial\mathcal{N}_{nk}}{\partial a^{\mu}_{kl}}\right)$$

$$\frac{\partial\mathcal{N}_{nk}}{\partial a^{\mu}_{kl}} = \frac{\partial\mathcal{N}_{nk}}{\partial\mu_{kl}}$$

$$= \frac{\partial}{\partial\mu_{kl}}\left(\frac{1}{(2\pi)^{L/2}|\,(\sigma_k^2\mathbf{I})\,|^{(1/2)}}\exp\left\{-\frac{1}{2}(\mathbf{t}-\boldsymbol{\mu}_k)^T\left(\sigma_k^2\mathbf{I}\right)^{-1}(\mathbf{t}-\boldsymbol{\mu}_k)\right\}\right)$$

$$= \frac{1}{(2\pi)^{L/2}|\,(\sigma_k^2\mathbf{I})\,|^{(1/2)}}\left(\frac{\partial}{\partial\mu_{kl}}\exp\left\{-\frac{1}{2\sigma_k^2}(\mathbf{t}-\boldsymbol{\mu}_k)^T(\mathbf{t}-\boldsymbol{\mu}_k)\right\}\right)$$

$$= \frac{1}{(2\pi)^{L/2}|\,(\sigma_k^2\mathbf{I})\,|^{(1/2)}}\exp\left\{-\frac{1}{2\sigma_k^2}(\mathbf{t}-\boldsymbol{\mu}_k)^T\left(\sigma_k^2\mathbf{I}\right)^{-1}(\mathbf{t}-\boldsymbol{\mu}_k)\right\}$$
$$\left(\frac{\partial}{\partial\mu_{kl}}\left(-\frac{1}{2\sigma_k^2}(\mathbf{t}-\boldsymbol{\mu}_k)^T(\mathbf{t}-\boldsymbol{\mu}_k)\right)\right)$$

$$= \frac{1}{(2\pi)^{L/2}|\,(\sigma_k^2\mathbf{I})\,|^{(1/2)}}\exp\left\{-\frac{1}{2}(\mathbf{t}-\boldsymbol{\mu}_k)^T\left(\sigma_k^2\mathbf{I}\right)^{-1}(\mathbf{t}-\boldsymbol{\mu}_k)\right\}$$
$$\left(-\frac{1}{2\sigma_k^2}\left(\frac{\partial}{\partial\mu_{kl}}\left(\mathbf{t}^T\mathbf{t}-2\boldsymbol{\mu}_k^T\mathbf{t}+\boldsymbol{\mu}_k^T\boldsymbol{\mu}_k\right)\right)\right)$$

Calculating the partial derivatives inside the braces:

$$\frac{\partial\mathbf{t}^T\mathbf{t}}{\partial\mu_{kl}} = 0$$

$$\frac{\partial(-2\boldsymbol{\mu}_k^T\mathbf{t})}{\partial\mu_{kl}} = -2t_l$$

$$\frac{\partial \boldsymbol{\mu}_k^T \boldsymbol{\mu}_k}{\partial \mu_{kl}} = (2\boldsymbol{\mu}_k) \frac{\partial \boldsymbol{\mu}_k}{\partial \mu_{kl}} = 2\mu_{kl}$$

Substituting these results back, we get:

$$\begin{aligned}
\frac{\partial E_n}{\partial a_{kl}^{\mu}} &= -\frac{1}{\left\{\sum_{k=1}^{K} \pi_k \, \mathcal{N}_{nk}\right\}} \left(\pi_k \mathcal{N}_{nk} \left(-\frac{1}{2\sigma_k^2} \left(-2t_l + 2\mu_{kl}\right)\right)\right) \\
&= \frac{\pi_k \mathcal{N}_{nk}}{\left\{\sum_{k=1}^{K} \pi_k \, \mathcal{N}_{nk}\right\}} \left(\frac{\mu_{kl} - t_l}{\sigma_k^2}\right) \\
&= \gamma_{nk} \left(\frac{\mu_{kl} - t_l}{\sigma_k^2}\right)
\end{aligned}$$

which is the same as the result in 5.156.