1.37

$$H[\mathbf{y}|\mathbf{x}] = -\int\int p(\mathbf{y},\mathbf{x})\,\ln p(\mathbf{y}|\mathbf{x})\,d\mathbf{y}\,d\mathbf{x}$$

$$H[\mathbf{x}] = -\int p(\mathbf{x})\,\ln p(\mathbf{x})\,d\mathbf{x}$$

We can add $\mathbf{y}$ to the definition of $H[\mathbf{x}]$ in the following way, as $\mathbf{y}$ would just get integrated out to give the same result:

$$H[\mathbf{x}] = -\int\int p(\mathbf{y},\mathbf{x})\,\ln p(\mathbf{x})\,d\mathbf{y}\,d\mathbf{x}$$

Adding the two, we get:

$$H[\mathbf{y}|\mathbf{x}] + H[\mathbf{x}] = -\int\int p(\mathbf{y},\mathbf{x})\,\ln p(\mathbf{y}|\mathbf{x})\,d\mathbf{y}\,d\mathbf{x} - \int\int p(\mathbf{y},\mathbf{x})\,\ln p(\mathbf{x})\,d\mathbf{y}\,d\mathbf{x}$$

$$= -\int\int p(\mathbf{y},\mathbf{x})\,\ln(p(\mathbf{y}|\mathbf{x})p(\mathbf{x}))\,d\mathbf{y}\,d\mathbf{x}$$

$$= -\int\int p(\mathbf{y},\mathbf{x})\,\ln p(\mathbf{y},\mathbf{x})\,d\mathbf{y}\,d\mathbf{x}$$

$$= H[\mathbf{y},\mathbf{x}] = H[\mathbf{x},\mathbf{y}]$$