3.14 Using the new basis,

$$\mathbf{S}_N = (\alpha\mathbf{I} + \beta\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1}$$

$$= (0\mathbf{I} + \beta\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1} \qquad \text{Since } \alpha \text{ is } 0.$$

$$= \frac{1}{\beta}\mathbf{I} \qquad \text{Since } \boldsymbol{\Psi} \text{ is orthonormal.}$$

Applying this to 3.62, we get

$$k(\mathbf{x}, \mathbf{x}') = \beta\boldsymbol{\psi}(\mathbf{x})^T\mathbf{S}_N\boldsymbol{\psi}(\mathbf{x}')$$

$$= \beta\boldsymbol{\psi}(\mathbf{x})^T\left(\frac{1}{\beta}\mathbf{I}\right)\boldsymbol{\psi}(\mathbf{x}')$$

$$= \boldsymbol{\psi}(\mathbf{x})^T\boldsymbol{\psi}(\mathbf{x}')$$

Now to prove that the kernel satisfies the summation constraint:

$$\sum_{n=1}^{N} k(\mathbf{x}, \mathbf{x}_n) = \sum_{n=1}^{N} \boldsymbol{\psi}(\mathbf{x})^T\boldsymbol{\psi}(\mathbf{x}_n)$$

$$= \sum_{n=1}^{N}\sum_{m=0}^{M-1} \psi_m(\mathbf{x})\psi_m(\mathbf{x}_n)$$

$$= \sum_{m=0}^{M-1}\sum_{n=1}^{N} \psi_m(\mathbf{x})\psi_m(\mathbf{x}_n)$$

$$= \sum_{m=0}^{M-1} \psi_m(\mathbf{x})\sum_{n=1}^{N} \psi_m(\mathbf{x}_n)$$

$$= \sum_{m=0}^{M-1} \psi_m(\mathbf{x})\sum_{n=1}^{N} \psi_m(\mathbf{x}_n)\psi_0(\mathbf{x}_n)$$

$$= \psi_0(\mathbf{x})\sum_{n=1}^{N} \psi_0(\mathbf{x}_n)\psi_0(\mathbf{x}_n) + \sum_{m=1}^{M-1} \psi_m(\mathbf{x})\sum_{n=1}^{N} \psi_m(\mathbf{x}_n)\psi_0(\mathbf{x}_n)$$

$$= 1.1 + \sum_{m=1}^{M-1} \psi_m(\mathbf{x}).0$$

$$= 1.$$