4.21

$$\mathbf{\Phi}(a) = \int_{-\infty}^{a} \mathcal{N}(\theta|0,1)\, d\theta$$

$$= \int_{-\infty}^{0} \mathcal{N}(\theta|0,1)\, d\theta + \int_{0}^{a} \mathcal{N}(\theta|0,1)\, d\theta$$

$$= \frac{1}{2} + \int_{0}^{a} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\theta^2\right\}\, d\theta$$

$$= \frac{1}{2}\left(1 + \int_{0}^{a} \frac{2}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\theta^2\right\}\right)\, d\theta$$

$$= \frac{1}{2}\left(1 + \int_{0}^{a} \frac{1}{\sqrt{2}}\frac{2}{\sqrt{\pi}} \exp\left\{-\frac{1}{2}\theta^2\right\}\right)\, d\theta$$

$$= \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}} erf(a)\right)$$

which is the same as 4.116.