2.5 We are given:

$$\Gamma(a)\Gamma(b) = \int_0^\infty exp(-x)x^{a-1}dx \int_0^\infty exp(-y)y^{b-1}dy$$

Bringing the integral over y inside the integrand of the integral over x:

$$\Gamma(a)\Gamma(b) = \int_0^\infty \int_0^\infty exp(-x)x^{a-1}\, exp(-y)y^{b-1}dx\, dy$$

$$\Gamma(a)\Gamma(b) = \int_0^\infty \int_0^\infty exp(-(x+y))x^{a-1}\, y^{b-1}dx\, dy$$

Making the change of variable t = y + x where x is fixed:

$$\Gamma(a)\Gamma(b) = \int_0^\infty \int_0^\infty exp(-t)\, x^{a-1}\, (t-x)^{b-1}dx\, dt$$

Interchanging the order of the x and t integrations:

$$\Gamma(a)\Gamma(b) = \int_0^\infty \int_0^\infty exp(-t)\, x^{a-1}\, (t-x)^{b-1}dt\, dx$$

Making the change of variable x = tμ where t is fixed:

$$\Gamma(a)\Gamma(b) = \int_0^\infty \int_0^1 exp(-t)\, (t\mu)^{a-1}\, (t-t\mu)^{b-1}dt\, td(\mu)$$

$$\Gamma(a)\Gamma(b) = \int_0^\infty \int_0^1 exp(-t)\, t^a \mu^{a-1}\, t^{b-1}(1-\mu)^{b-1}dt\, d\mu$$

$$\Gamma(a)\Gamma(b) = \int_0^\infty \int_0^1 exp(-t)\, t^{a+b-1}\mu^{a-1}\, (1-\mu)^{b-1}dt\, d\mu$$

$$\Gamma(a)\Gamma(b) = \int_0^\infty exp(-t)\, t^{a+b-1}\, dt\, \int_0^1 \mu^{a-1}\, (1-\mu)^{b-1}\, d\mu$$

$$\Gamma(a)\Gamma(b) = \Gamma(a+b)\, \int_0^1 \mu^{a-1}\, (1-\mu)^{b-1}\, d\mu$$

$$\implies \int_0^1 \mu^{a-1}\, (1-\mu)^{b-1}\, d\mu = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$