6.6 Verifying 6.15:

For $k(\mathbf{x}, \mathbf{x'}) = q(k_1(\mathbf{x}, \mathbf{x'}))$ where q is a polynomial of order D with non-negative coefficients, the Gram matrix $\mathbf{K}$ corresponding to kernel function k becomes:

$$\mathbf{K} = \sum_{i=1}^{D} c_i \, (\mathbf{K}_1)^i$$

where $(\mathbf{K}_1)^i$ is the Hadamard product of $\mathbf{K}_1$ with itself $(i-1)$ times.

As per the Schur product theorem, the Hadamard product of two positive definite matrices is also a positive definite matrix.

Then, $(\mathbf{K}_1)^i$ is also positive semidefinite, and since c_i is non-negative, $c_i \, \mathbf{K}_1^i$ is also positive semidefinite. Since the sum of positive semidefinite matrices is positive semidefinite, $\mathbf{K}$ is positive semidefinite and the kernel function k is valid.

Verifying 6.16:

For the kernel function $k(\mathbf{x}, \mathbf{x'}) = \exp(k_1(\mathbf{x}, \mathbf{x'}))$, we can use the Expansion of exponential function which says that:

$$e^z = \sum_{k=0}^{\infty} \frac{1}{k!} z^k$$

This expansion turns the kernel function k into another polynomial with nonnegative coefficients. Using the same reasoning as above, we can conclude that $\mathbf{K}$ is positive semidefinite and the kernel function k is valid.