

2.24 Multiplying both sides of equation 2.76 by the given matrix:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = \begin{pmatrix} \mathbf{M} & -\mathbf{M}\mathbf{B}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{M} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}$$

$$R.H.S = \begin{pmatrix} \mathbf{M}\mathbf{A} - \mathbf{M}\mathbf{B}\mathbf{D}^{-1}\mathbf{C} & \mathbf{M}\mathbf{B} - \mathbf{M}\mathbf{B}\mathbf{D}^{-1}\mathbf{D} \\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{A} + (\mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1})\mathbf{C} & -\mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B} + (\mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1})\mathbf{D} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{M}\mathbf{A} - \mathbf{M}\mathbf{B}\mathbf{D}^{-1}\mathbf{C} & \mathbf{M}\mathbf{B} - \mathbf{M}\mathbf{B} \\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{A} + \mathbf{D}^{-1}\mathbf{C} + \mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1}\mathbf{C} & -\mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B} + \mathbf{D}^{-1}\mathbf{D} + \mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1}\mathbf{D} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{M}(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}) & \mathbf{0} \\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{M}(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}) + \mathbf{D}^{-1}\mathbf{C} + & -\mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B} + \mathbf{I} + \mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{M}\mathbf{M}^{-1} & \mathbf{0} \\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{M}(\mathbf{M}^{-1}) + \mathbf{D}^{-1}\mathbf{C} + & \mathbf{I} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{D}^{-1}\mathbf{C} + \mathbf{D}^{-1}\mathbf{C} & \mathbf{I} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}$$

$$= \mathbf{I}$$

It's trivial to see that L.H.S is also = \mathbf{I} .