1.39

(a)
$$p(x = 0) = 1/3 + 1/3 = 2/3$$
$$p(x = 1) = 0 + 1/3 = 1/3$$

$$\implies \mathbf{H}(x) = -(2/3\ln(2/3) + 1/3\ln(1/3))$$
$$= 2/3\ln 3/2 + 1/3\ln 3$$
$$= 0.636$$

(b)
$$p(y = 0) = 1/3 + 0 = 1/3$$
$$p(y = 1) = 1/3 + 1/3 = 2/3$$

$$\implies \mathbf{H}(y) = -(1/3\ln(1/3) + 2/3\ln(2/3))$$
$$= 1/3\ln 3 + 2/3\ln 3/2$$
$$= 0.636$$

(c)
$$p(y = 0|x = 0) = (1/3)/(2/3) = 1/2$$
$$p(y = 1|x = 0) = (1/3)/(2/3) = 1/2$$
$$p(y = 0|x = 1) = (0)/(1/3) = 0$$
$$p(y = 1|x = 1) = (1/3)/(1/3) = 1$$

Using 1.111,

$$\implies \mathbf{H}(y|x) = -(1/3\ln 1/2 + 1/3\ln 1/2 + 0\ln 0 + 1/3\ln 1)$$
$$= 2/3\ln 2$$

(d)

$$p(x = 0|y = 0) = (1/3)/(1/3) = 1$$
$$p(x = 1|y = 0) = (0)/(1/3) = 0$$
$$p(x = 0|y = 1) = (1/3)/(2/3) = 1/2$$
$$p(x = 1|y = 1) = (1/3)/(2/3) = 1/2$$

Using 1.111,

$$\implies \mathbf{H}(y|x) = -(1/3\ln 1 + 0\ln 0 + 1/3\ln 1/2 + 1/3\ln 1/2)$$
$$= 2/3\ln 2$$

(e)

$$\mathbf{H}(x, y) = \mathbf{H}(y|x) + \mathbf{H}(x)$$

$$= 2/3\ln 2 + 1/3\ln 3 + 2/3\ln 3/2$$

$$= \ln 3$$

(f)

$$\mathbf{I}(x, y) = \mathbf{H}(y) - \mathbf{H}(y|x)$$

$$= 1/3\ln 3 + 2/3\ln 3/2 - 2/3\ln 2$$

$$= \ln 3 - 4/3\ln 2$$