5.23 The results in exercise 5.22 stay the same, we just add 3 new cases.

First we calculate $\dfrac{\partial E_n}{\partial w_{ki}}$.

Using 5.50:

$$\frac{\partial E_n}{\partial w_{ki}} = \left(\frac{\partial E_n}{\partial a_k}\right)\left(\frac{\partial a_k}{\partial w_{ki}}\right) = \delta_k x_i$$

1. Calculating $\dfrac{\partial^2 E_n}{\partial w_{ki}\,\partial w_{k'i'}}$

$$\begin{aligned}
\frac{\partial^2 E_n}{\partial w_{ki}\,\partial w_{k'i'}} &= \frac{\partial}{\partial w_{k'i'}}(\delta_k x_i) \\
&= x_i\left(\frac{\partial \delta_k}{\partial w_{k'i'}}\right) \\
&= x_i\left(\frac{\partial^2 E_n}{\partial w_{k'i'}\,\partial a_k}\right) \\
&= x_i\left(\frac{\partial}{\partial a_k}\,\frac{\partial E_n}{\partial w_{k'i'}}\right) \\
&= x_i\left(\frac{\partial}{\partial a_k}\,\left(\frac{\partial E_n}{\partial a_{k'}}\,\frac{\partial a_{k'}}{\partial w_{k'i'}}\right)\right) \\
&= x_i\left(\frac{\partial}{\partial a_k}\,\left(\frac{\partial E_n}{\partial a_{k'}}\,x_{i'}\right)\right) \\
&= x_i\,x_{i'}\left(\frac{\partial^2 E_n}{\partial a_{k'}\,\partial a_k}\right) \\
&= x_i\,x_{i'}\,M_{kk'}
\end{aligned}$$

2. Calculating $\dfrac{\partial^2 E_n}{\partial w^{(1)}_{ji'}\,\partial w_{ki}}$

$$\frac{\partial^2 E_n}{\partial w^{(1)}_{ji'}\,\partial w_{ki}} = \frac{\partial}{\partial w^{(1)}_{ji'}}(\delta_k x_i)$$

$$= x_i \left(\frac{\partial \delta_k}{\partial w_{ji'}^{(1)}} \right)$$

$$= x_i \left(\frac{\partial^2 E_n}{\partial a_k \; \partial w_{ji'}^{(1)}} \right)$$

$$= x_i \left(\frac{\partial}{\partial a_k} \; \frac{\partial E_n}{\partial w_{ji'}^{(1)}} \right)$$

$$= x_i \left(\frac{\partial}{\partial a_k} \; \left(\frac{\partial E_n}{\partial a_j} \; \frac{\partial a_j}{\partial w_{ji'}^{(1)}} \right) \right)$$

$$= x_i \left(\frac{\partial}{\partial a_k} \; \left(\frac{\partial E_n}{\partial a_j} \; x_{i'} \right) \right)$$

$$= x_i \, x_{i'} \left(\frac{\partial}{\partial a_k} \frac{\partial E_n}{\partial a_j} \right)$$

Applying 5.55, $\frac{\partial E_n}{\partial a_j} = \sum_{k'} \frac{\partial E_n}{\partial a_{k'}} \; \frac{a_{k'}}{\partial a_j}$

$$= x_i \, x_{i'} \left(\frac{\partial}{\partial a_k} \left(\sum_{k'} \frac{\partial E_n}{\partial a_{k'}} \; \frac{\partial a_{k'}}{\partial a_j} \right) \right)$$

$$= x_i \, x_{i'} \left(\frac{\partial}{\partial a_k} \left(\sum_{k'} \frac{\partial E_n}{\partial a_{k'}} \; h'(a_j) w_{k'j}^{(2)} \right) \right)$$

$$= x_i \, x_{i'} \, h'(a_j) \sum_{k'} \frac{\partial^2 E_n}{\partial a_k \partial a_{k'}} \; w_{k'j}^{(2)}$$

$$= x_i \, x_{i'} \, h'(a_j) \sum_{k'} M_{kk'} \; w_{k'j}^{(2)}$$

3. Calculating $\frac{\partial^2 E_n}{\partial w_{k'j} \; \partial w_{ki}}$

$$\frac{\partial^2 E_n}{\partial w_{k'j}^{(2)} \; \partial w_{ki}} = \frac{\partial}{\partial w_{k'j}^{(2)}} (\delta_k x_i)$$

$$= x_i \left(\frac{\partial \delta_k}{\partial w_{k'j}^{(2)}} \right)$$

$$
\begin{aligned}
&= x_i \left(\frac{\partial^2 E_n}{\partial w^{(2)}_{k'j} \; \partial a_k} \right) \\
&= x_i \left(\frac{\partial}{\partial a_k} \; \frac{\partial E_n}{\partial w^{(2)}_{k'j}} \right) \\
&= x_i \left(\frac{\partial}{\partial a_k} \left(\frac{\partial E_n}{\partial a_{k'}} \; \frac{\partial a_{k'}}{\partial w^{(2)}_{k'j}} \right) \right) \\
&= x_i \left(\frac{\partial}{\partial a_k} \left(\frac{\partial E_n}{\partial a_{k'}} \; z_j \right) \right) \\
&= x_i \, z_j \left(\frac{\partial^2 E_n}{\partial a_k \; \partial a_{k'}} \right) \\
&= x_i \, z_j \, M_{kk'}
\end{aligned}
$$