

2.4

$$\begin{aligned}
& \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} = 1 \\
& \frac{\partial}{\partial \mu} \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} = 0 \\
& \sum_{m=0}^N \binom{N}{m} \frac{\partial (\mu^m (1-\mu)^{N-m})}{\partial \mu} = 0 \\
& \sum_{m=0}^N \binom{N}{m} (m \mu^{m-1} (1-\mu)^{N-m} - (N-m) \mu^m (1-\mu)^{N-m-1}) = 0 \\
& \sum_{m=0}^N \binom{N}{m} \mu^{m-1} (1-\mu)^{N-m-1} (m(1-\mu) - (N-m)\mu) = 0 \\
& \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} \left(\frac{m}{\mu} - \frac{(N-m)}{(1-\mu)} \right) = 0 \\
& \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} \frac{(m - m\mu - N\mu + m\mu)}{\mu(1-\mu)} = 0 \\
& \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} \frac{(m - N\mu)}{\mu(1-\mu)} = 0 \\
& \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} (m - N\mu) = 0 \\
& \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} m = \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} N\mu \\
& \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} m = N\mu \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} \\
& \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} m = N\mu \\
& \sum_{m=0}^N \text{Bin}(m|N, \mu) m = N\mu \\
& \mathbb{E}[m] = N\mu
\end{aligned}$$

Differentiating (2.264) twice with respect to μ , we can do it more simply by differentiating the result of the first derivative,

$$\frac{\partial}{\partial \mu} \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} m = \frac{\partial}{\partial \mu} N\mu$$

We can get the derivative on the L.H.S using the result we got above:

$$\sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} \frac{(m - N\mu)}{\mu(1-\mu)} m = N$$

$$\sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} (m - N\mu) m = N\mu(1-\mu)$$

$$\sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} (m^2 - Nm\mu) = N\mu - N\mu^2$$

$$\left(\sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} m^2 \right) - \left(\sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} Nm\mu \right) = N\mu - N\mu^2$$

$$\left(\sum_{m=0}^N \text{Bin}(m|N, \mu) m^2 \right) - \left(N\mu \sum_{m=0}^N \binom{N}{m} \mu^m (1-\mu)^{N-m} m \right) = N\mu - N\mu^2$$

$$\mathbb{E}[m^2] - N\mu\mathbb{E}[m] = N\mu - N\mu^2$$

$$\mathbb{E}[m^2] - \mathbb{E}[m]\mathbb{E}[m] = N\mu - N\mu^2$$

$$\mathbb{E}[m^2] - \mathbb{E}[m]^2 = N\mu(1-\mu)$$

$$\text{var}[m] = N\mu(1-\mu)$$