

5.39 Using 5.162,

$$p(\mathbf{w}|\alpha) = \mathcal{N}(\mathbf{w}|\mathbf{0}, \alpha^{-1}\mathbf{I})$$

And using 5.163,

$$p(\mathcal{D}|\mathbf{w}, \beta) = \prod_{n=1}^N \mathcal{N}(t_n|y(\mathbf{x}, \mathbf{w}), \beta^{-1})$$

Assuming that the mode is at \mathbf{w}_{MAP} , and identifying that

$$f(\mathbf{w}) = \left(\prod_{n=1}^N \mathcal{N}(t_n|y(\mathbf{x}, \mathbf{w}), \beta^{-1}) \right) (\mathcal{N}(\mathbf{w}|\mathbf{0}, \alpha^{-1}\mathbf{I}))$$

and

$$Z = p(\mathcal{D}|\alpha, \beta)$$

Applying 4.135, we get:

$$p(\mathcal{D}|\alpha, \beta) \simeq \left(\prod_{n=1}^N \mathcal{N}(t_n|y(\mathbf{x}, \mathbf{w}_{MAP}), \beta^{-1}) \right) (\mathcal{N}(\mathbf{w}_{MAP}|\mathbf{0}, \alpha^{-1}\mathbf{I})) \frac{(2\pi)^{W/2}}{|\mathbf{A}|^{1/2}}$$

$$\begin{aligned} \implies \ln p(\mathcal{D}|\alpha, \beta) &\simeq \ln \left(\left(\prod_{n=1}^N \mathcal{N}(t_n|y(\mathbf{x}, \mathbf{w}_{MAP}), \beta^{-1}) \right) \right. \\ &\quad \left. (\mathcal{N}(\mathbf{w}_{MAP}|\mathbf{0}, \alpha^{-1}\mathbf{I})) \frac{(2\pi)^{W/2}}{|\mathbf{A}|^{1/2}} \right) \end{aligned}$$

$$= \left(\sum_{n=1}^N \ln \mathcal{N}(t_n|y(\mathbf{x}, \mathbf{w}_{MAP}), \beta^{-1}) \right)$$

$$+ (\ln \mathcal{N}(\mathbf{w}_{MAP}|\mathbf{0}, \alpha^{-1}\mathbf{I})) + \frac{W}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{A}|$$

$$= \sum_{n=1}^N \ln \left(\frac{1}{(2\pi)^{1/2}(\beta^{-1})^{1/2}} \exp \left\{ -\frac{\beta (t_n - y(\mathbf{x}, \mathbf{w}_{MAP}))^2}{2} \right\} \right)$$

$$\begin{aligned}
& + \ln \left(\frac{1}{(2\pi)^{W/2} |\alpha^{-1} \mathbf{I}|^{1/2}} \exp \left\{ -\frac{\alpha \mathbf{w}_{MAP}^T \mathbf{w}_{MAP}}{2} \right\} \right) + \frac{W}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{A}| \\
& = \sum_{n=1}^N \left(-\frac{N}{2} \ln(2\pi) + \frac{1}{2} \ln \beta - \frac{\beta (t_n - y(\mathbf{x}, \mathbf{w}_{MAP}))^2}{2} \right) \\
& - \frac{W}{2} \ln(2\pi) + \frac{W}{2} \ln \alpha - \frac{\alpha \mathbf{w}_{MAP}^T \mathbf{w}_{MAP}}{2} + \frac{W}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{A}| \\
& = -\frac{N}{2} \ln(2\pi) + \frac{N}{2} \ln \beta - \sum_{n=1}^N \frac{\beta (t_n - y(\mathbf{x}, \mathbf{w}_{MAP}))^2}{2} \\
& + \frac{W}{2} \ln \alpha - \frac{\alpha \mathbf{w}_{MAP}^T \mathbf{w}_{MAP}}{2} - \frac{1}{2} \ln |\mathbf{A}| \\
& - \left(\frac{\beta}{2} \sum_{n=1}^N (t_n - y(\mathbf{x}, \mathbf{w}_{MAP}))^2 + \frac{\alpha}{2} \mathbf{w}_{MAP}^T \mathbf{w}_{MAP} \right) \\
& - \frac{N}{2} \ln(2\pi) + \frac{N}{2} \ln \beta + \frac{W}{2} \ln \alpha - \frac{1}{2} \ln |\mathbf{A}|
\end{aligned}$$

which is the same as the result in 5.175.