6.24 To show that $\mathbf{W}$ is positive definite, we need to prove that

$$\mathbf{x}^T\mathbf{W}\mathbf{x} > 0 \forall \mathbf{x} \neq \mathbf{0}$$

Here $1 > W_{ii} > 0$, and $\mathbf{W}$ is a diagonal matrix.

Therefore, $\mathbf{W}\mathbf{x}$ is a vector with the i^{th} element given by :

$$W_{ii} * x_i$$

and

$$\mathbf{x}^T\mathbf{W}\mathbf{x} = \sum_i x_i W_{ii} x_i = \sum_i W_{ii} x_i^2$$

Since $\mathbf{x}$ is not a zero vector, this term is > 0 and $\mathbf{W}$ is positive definite.

To show that the sum of two positive definite matrices is itself positive definite, let's take $\mathbf{A}$ and $\mathbf{B}$ as Positive definite matrices.

$$\mathbf{x}^T(\mathbf{A} + \mathbf{B})\mathbf{x}$$

$$= \left(\mathbf{x}^T\mathbf{A}\mathbf{x}\right) + \left(\mathbf{x}^T\mathbf{B}\mathbf{x}\right)$$

Both the values being > 0, their sum is also > 0 and $\mathbf{A} + \mathbf{B}$ is also a positive definite matrix.