5.29 From 5.139, we have,

$$\tilde{E}(\mathbf{w}) = E(\mathbf{w}) + \lambda \mathbf{\Omega}(\mathbf{w})$$

$$\implies \frac{\partial \tilde{E}}{\partial w_i} = \frac{\partial E}{\partial w_i} + \frac{\partial \lambda \mathbf{\Omega}(\mathbf{w})}{\partial w_i}$$

$$R.H.S = \frac{\partial E}{\partial w_i} + \lambda \frac{\partial \mathbf{\Omega}(\mathbf{w})}{\partial w_i}$$

$$\frac{\partial \mathbf{\Omega}(\mathbf{w})}{\partial w_i} = \frac{\partial}{\partial w_i} \left(-\sum_i \ln \left(\sum_{j=1}^{M} \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right) \right)$$

Since the partial derivative is only w.r.t one i, this becomes:

$$= \frac{\partial}{\partial w_i} \left(-\ln \left(\sum_{j=1}^{M} \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right) \right)$$

$$= -\frac{1}{\left(\sum_{j=1}^{M} \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right)} \frac{\partial}{\partial w_i} \left(\sum_{j=1}^{M} \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right)$$

$$= -\frac{1}{\left(\sum_{j=1}^{M} \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right)} \frac{\partial}{\partial w_i} \left(\sum_{j=1}^{M} \frac{\pi_j}{(2\pi\sigma_j^2)^{(1/2)}} \exp \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \right)$$

$$= -\frac{1}{\left(\sum_{j=1}^{M} \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right)} \left(\sum_{j=1}^{M} \frac{\pi_j}{(2\pi\sigma_j^2)^{(1/2)}} \exp \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \frac{\partial \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right)}{\partial w_i} \right)$$

$$= -\frac{1}{\left(\sum_{j=1}^{M} \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right)} \left(\sum_{j=1}^{M} \frac{\pi_j}{(2\pi\sigma_j^2)^{(1/2)}} \exp \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \left(-\left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right) \right)$$

$$= \frac{1}{\left(\sum_{j=1}^{M} \pi_j \mathcal{N}(w_i|\mu_j, \sigma_j^2) \right)} \left(\sum_{j=1}^{M} \frac{\pi_j}{(2\pi\sigma_j^2)^{(1/2)}} \exp \left(-\frac{1}{2} \left(\frac{w_i - \mu_j}{\sigma_j} \right)^2 \right) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right)$$

$$= \frac{1}{\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)} \left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right) \right)$$

$$= \sum_{j=1}^{M} \frac{\pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)}{\left(\sum_{j=1}^{M} \pi_j \, \mathcal{N}(w_i|\mu_j, \sigma_j^2)\right)} \left(\frac{w_i - \mu_j}{\sigma_j^2} \right)$$

$$= \sum_{j=1}^{M} \gamma_j(w_i) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right)$$

Substituting this result in the partial derivative above, we get:

$$\frac{\partial \tilde{E}}{\partial w_i} = \frac{\partial E}{\partial w_i} + \lambda \sum_{j=1}^{M} \gamma_j(w_i) \left(\frac{w_i - \mu_j}{\sigma_j^2} \right)$$

which is the same as the result in 5.141.