1.18 Equation 1.142 is:

$$\prod_{i=1}^{D} \int_{-\infty}^{\infty} e^{-x_i^2} dx_i = S_D \int_0^{\infty} e^{-r^2} r^{D-1} dr$$

Equation 1.124 gives us:

$$\int_{-\infty}^{\infty} exp\left(-\frac{1}{2\sigma^2}x^2\right) dx = (2\pi\sigma^2)^{1/2}$$

If $\sigma^2 = 1/2$, this becomes:

$$\int_{-\infty}^{\infty} e^{-x^2} dx = (\pi)^{1/2}$$

Therefore, the L.H.S of equation 1.142 becomes:

$$\prod_{i=1}^{D} \int_{-\infty}^{\infty} e^{-x_i^2} dx_i = \prod_{i=1}^{D} (\pi)^{1/2} = (\pi)^{D/2}$$

The integral in the R.H.S of equation 1.142 can be evaluated as:

$$\int_0^{\infty} e^{-r^2} r^{D-1} dr$$

$$= \int_0^{\infty} e^{-s} s^{(D-1)/2} \left(\frac{dr}{ds}\right) ds \qquad \text{where } s = r^2$$

$$= \int_0^{\infty} e^{-s} s^{(D-1)/2} \left(\frac{1}{2r}\right) ds$$

$$= \int_0^{\infty} e^{-s} s^{(D-1)/2} \left(\frac{1}{2s^{1/2}}\right) ds$$

$$= \frac{1}{2} \int_0^{\infty} e^{-s} s^{((D-1)/2-1/2)} ds$$

$$= \frac{1}{2} \int_0^{\infty} e^{-s} s^{(D/2-1)} ds$$

Applying 1.141 here, we get:

$$= \frac{1}{2}\Gamma(D/2)$$

Substituting the results into 1.142, we get:

$$(\pi)^{D/2} = S_D \frac{1}{2} \Gamma(D/2)$$

$$\implies S_D = \frac{2(\pi)^{D/2}}{\Gamma(D/2)}$$

The volume of a unit sphere in D dimensions is given by Source1 Source 2:

$$V_D = \int_0^1 S_D r^{D-1} dr$$

$$= \left[S_D (1/D) r^D \right]_0^1$$

$$= \frac{S_D}{D}$$

For D=2, and radius r = 1,

$$S_2 = \frac{2(\pi)^{2/2}}{\Gamma(2/2)} = \frac{2\pi}{\Gamma(1)} = 2\pi = 2\pi r$$

$$V_2 = \frac{S_2}{2} = \frac{2\pi}{2} = \pi = \pi r^2$$

For D=3,

$$S_3 = \frac{2(\pi)^{3/2}}{\Gamma(3/2)} = \frac{2\pi^{3/2}}{(\sqrt{\pi}/2)} = 4\pi = 4\pi r^2$$

$$V_3 = \frac{S_3}{3} = \frac{4\pi}{3} = \frac{4}{3} \pi r^3$$