2.56 Beta distribution. Already done in MML exercise 6.9.

Gamma distribution

$$Gam(\lambda|a,b) = \frac{1}{\Gamma(a)} b^a \lambda^{a-1} \exp(-b\lambda)$$

$$= \exp\left(\ln\left(\frac{1}{\Gamma(a)} b^a \lambda^{a-1} exp(-b\lambda)\right)\right)$$

$$= \exp\left(-\ln\Gamma(a) + a\ln b + (a-1)\ln\lambda - b\lambda\right)$$

$$= \frac{b^a}{\Gamma(a)} \exp((a-1)\ln\lambda - b\lambda)$$

Comparison with 2.194 gives us:

$$\mathbf{u}(\lambda) = [\ln\lambda \quad -\lambda]^T$$
$$\boldsymbol{\eta} = [(a-1) \quad b]^T$$
$$h(\lambda) = 1$$
$$g(\boldsymbol{\eta}) = \frac{(\boldsymbol{\eta}^T\mathbf{e}_2)^{(\boldsymbol{\eta}^T\mathbf{e}_1+1)}}{\Gamma(\boldsymbol{\eta}^T\mathbf{e}_1+1)}$$

where $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

von Mises distribution

$$p(\theta|\theta_0, m) = \frac{1}{2\pi I_0(m)} \exp\{m\,\cos(\theta-\theta_0)\}$$

$$= \frac{1}{2\pi I_0(m)} \exp\{m\,(\cos\theta\,\cos\theta_0 + \sin\theta\,\sin\theta_0)\}$$

Comparison with 2.194 gives us:

$$\mathbf{u}(\theta) = [\cos\theta \quad \sin\theta]^T$$
$$\boldsymbol{\eta} = m[\cos\theta_0 \quad \sin\theta_0]^T$$
$$h(x) = 1$$
$$g(\boldsymbol{\eta}) = \frac{1}{2\pi I_0\left(\sqrt{\boldsymbol{\eta}^T\boldsymbol{\eta}}\right)}$$