

2.31 In $p(\mathbf{y}|\mathbf{x})$, \mathbf{x} is fixed and \mathbf{z} is unknown. So, the mean of $p(\mathbf{y}|\mathbf{x})$ is given by $(\mathbf{x} + \boldsymbol{\mu}_{\mathbf{z}})$.

The variance is accounted for solely by \mathbf{z} here, so the variance of $p(\mathbf{y}|\mathbf{x})$ is $\boldsymbol{\Sigma}_{\mathbf{z}}$.

$$\implies p(\mathbf{y}|\mathbf{x}) = \mathcal{N}(\mathbf{y} | (\mathbf{x} + \boldsymbol{\mu}_{\mathbf{z}}), \boldsymbol{\Sigma}_{\mathbf{z}})$$

Using 2.109 and 2.110, we can see that here,

\mathbf{A} is \mathbf{I}

\mathbf{x} is \mathbf{x}

\mathbf{b} is $\boldsymbol{\mu}_{\mathbf{z}}$

$\boldsymbol{\Lambda}^{-1}$ is $\boldsymbol{\Sigma}_{\mathbf{x}}^{-1}$

\mathbf{L}^{-1} is $\boldsymbol{\Sigma}_{\mathbf{z}}^{-1}$

Which gives us:

$$\mathbb{E}[\mathbf{y}] = \boldsymbol{\mu}_{\mathbf{x}} + \boldsymbol{\mu}_{\mathbf{z}}$$

$$\text{var}[\mathbf{y}] = \boldsymbol{\Sigma}_{\mathbf{x}} + \boldsymbol{\Sigma}_{\mathbf{z}}$$