4.24 From 4.145, we have:

$$p(\mathcal{C}_1|\boldsymbol{\phi}, \mathbf{t}) = \int p(\mathcal{C}_1|\boldsymbol{\phi}, \mathbf{w}, \mathbf{t})\, p(\mathbf{w}|\mathbf{t})\, d\mathbf{w}$$

$$\simeq \int \sigma(\mathbf{w}^T \boldsymbol{\phi})\, q(\mathbf{w})\, d\mathbf{w}$$

Using the Gaussian approximation for $q(\mathbf{w})$ from 4.144, this becomes:

$$R.H.S = \int \sigma(\mathbf{w}^T \boldsymbol{\phi})\, \mathcal{N}(\mathbf{w}|\mathbf{w}_{MAP}, \mathbf{S}_N)\, d\mathbf{w}$$

Substituting for $\sigma(\mathbf{w}^T \boldsymbol{\phi})$ using 4.146, we get:

$$= \int \left(\int \delta(a - \mathbf{w}^T \boldsymbol{\phi})\, \sigma(a)\, da \right) \mathcal{N}(\mathbf{w}|\mathbf{w}_{MAP}, \mathbf{S}_N)\, d\mathbf{w}$$

$$= \int \left(\int \delta(a - \mathbf{w}^T \boldsymbol{\phi})\, \mathcal{N}(\mathbf{w}|\mathbf{w}_{MAP}, \mathbf{S}_N)\, d\mathbf{w} \right) \sigma(a)\, da$$

The Dirac delta function can be written as a Gaussian (source):

$$\delta(x) = \lim_{\sigma \to 0} \delta_\sigma(x) = \lim_{\sigma \to 0} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2} \right) = \lim_{\sigma \to 0} \mathcal{N}(x|0, \sigma^2)$$

$$\implies R.H.S = \int \left(\int \lim_{\eta \to 0} \mathcal{N}((a - \mathbf{w}^T \boldsymbol{\phi})|0, \eta^2)\, \mathcal{N}(\mathbf{w}|\mathbf{w}_{MAP}, \mathbf{S}_N)\, d\mathbf{w} \right) \sigma(a)\, da$$

$$= \int \left(\int \lim_{\eta \to 0} \mathcal{N}(a|\mathbf{w}^T \boldsymbol{\phi}, \eta^2)\, \mathcal{N}(\mathbf{w}|\mathbf{w}_{MAP}, \mathbf{S}_N)\, d\mathbf{w} \right) \sigma(a)\, da$$

Using this result from 2.3.3:

$$\int_x \mathcal{N}(\mathbf{y}|\mathbf{A}\mathbf{x}+\mathbf{b},\mathbf{L}^{-1})\,\mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Lambda}^{-1}) = \mathcal{N}(\mathbf{y}|\mathbf{A}\boldsymbol{\mu}+\mathbf{b},\mathbf{L}^{-1}+\mathbf{A}\boldsymbol{\Lambda}^{-1}\mathbf{A})$$

where,

$$\mathbf{y} = a$$
$$\mathbf{A} = \boldsymbol{\phi}^T$$
$$\mathbf{x} = \mathbf{w}$$
$$\mathbf{b} = \mathbf{0}$$
$$\boldsymbol{\mu} = \mathbf{w}_{MAP}$$
$$\mathbf{L}^{-1} = \eta^2$$
$$\boldsymbol{\Lambda}^{-1} = \mathbf{S}_N$$

we get the following marginal distribution:

$$= \int \lim_{\eta\to 0} \mathcal{N}\left(a \,|\, \left(\mathbf{w}_{MAP}^T\boldsymbol{\phi}\right),\, \left(\eta^2 + \boldsymbol{\phi}^T\mathbf{S}_N\boldsymbol{\phi}\right)\right)\, \sigma(a)\, da$$
$$= \int \mathcal{N}\left(a \,|\, \left(\mathbf{w}_{MAP}^T\boldsymbol{\phi}\right),\, \left(\boldsymbol{\phi}^T\mathbf{S}_N\boldsymbol{\phi}\right)\right)\, \sigma(a)\, da$$
$$= \int \mathcal{N}(a|\mu_a, \sigma_a^2)\, \sigma(a)\, da$$

where $\mu_a = \mathbf{w}_{MAP}^T\boldsymbol{\phi}$ and $\sigma_a^2 = \boldsymbol{\phi}^T\mathbf{S}_N\boldsymbol{\phi}$

which is the same as the result in 4.151.

PS : I have used results from Section 2.3.3 to derive this result. The book asked for Section 2.3.2 to be used. I am not sure how to do that, this section seemed more appropriate to me.