6.8 Verifying 6.19:

It's already given that it's a valid kernel.

Verifying 6.20:

The Gram matrix corresponding to the kernel function $k(\mathbf{x}, \mathbf{x'}) = \mathbf{x}^T \mathbf{A} \mathbf{x'}$ is given by:

$$\mathbf{K} =$$

$$\begin{bmatrix} --- & \mathbf{x}_1^T & --- \\ --- & \mathbf{x}_2^T & --- \\ --- & | & --- \\ --- & \mathbf{x}_N^T & --- \end{bmatrix} \begin{bmatrix} \mathbf{A} \end{bmatrix} \begin{bmatrix} | & | & | & | \\ \mathbf{x}_1 & \mathbf{x}_2 & | & \mathbf{x}_N \\ | & | & | & | \end{bmatrix}$$

$$= \mathbf{X}^T \mathbf{A} \mathbf{X}$$

Since $\mathbf{A}$ is symmetric and positive semidefinite,

$$\mathbf{y}^T \mathbf{A} \mathbf{y} \geq 0, \forall \mathbf{y} \in \mathbb{R}^N$$

Now we consider

$$\mathbf{y}^T \mathbf{X}^T \mathbf{A} \mathbf{X} \mathbf{y}$$

$$= (\mathbf{X}\mathbf{y})^T \mathbf{A} (\mathbf{X}\mathbf{y})$$

$(\mathbf{X}\mathbf{y})^T \mathbf{A} (\mathbf{X}\mathbf{y}) \geq 0 \forall \mathbf{y} \in \mathbb{R}^N$, since $(\mathbf{X}\mathbf{y}) \in \mathbb{R}^N \forall \mathbf{y} \in \mathbb{R}^N$.

Therefore, $\mathbf{K}$ is positive semidefinite and k is a valid kernel.