2.1

$$\sum_{x=0}^{1} p(x|\mu) = \sum_{x=0}^{1} \mu^x (1-\mu)^{1-x}$$

$$= \mu^0 (1-\mu)^{1-0} + \mu^1 (1-\mu)^{1-1}$$

$$= (1)(1-\mu)^1 + \mu(1-\mu)^0$$

$$= 1 - \mu + \mu$$

$$= 1$$

$$\mathbb{E}[x] = \sum_{x=0}^{1} p(x)x$$

$$= \sum_{x=0}^{1} \mu^x (1-\mu)^{1-x} x$$

$$= \mu^0 (1-\mu)^{1-0} 0 + \mu^1 (1-\mu)^{1-1} 1$$

$$= 0 + \mu(1-\mu)^0$$

$$= \mu$$

$$var[x] = E[x^2] - E[x]^2$$

$$E[x^2] = \sum_{x=0}^{1} \mu^x (1-\mu)^{1-x} x^2$$

$$= \mu^0 (1-\mu)^{1-0} 0^2 + \mu^1 (1-\mu)^{1-1} 1^2$$

$$= 0 + \mu(1-\mu)^0$$

$$= \mu$$

$$\implies var[x] = \mu - \mu^2 = \mu(1-\mu)$$

$$H[x] = -\sum_{x=0}^{1} p(x) \ln p(x)$$

$$= -\sum_{x=0}^{1} \mu^x (1-\mu)^{1-x} \ln \left(\mu^x (1-\mu)^{1-x} \right)$$

$$= -\left(\mu^0 (1-\mu)^{1-0} \ln \left(\mu^0 (1-\mu)^{1-0} \right) + \mu^1 (1-\mu)^{1-1} \ln \left(\mu^1 (1-\mu)^{1-1} \right) \right)$$

$$= -((1-\mu) \ln(1-\mu) + \mu \ln(\mu))$$

$$= -\mu \ln(\mu) - (1-\mu) \ln(1-\mu)$$