5.4 This exercise seems similar to Exercise 4.16.

$$p(\text{True label is 1}) = p(\text{Correct class label is set} \mid \text{label in dataset is 1})$$

$$+p(\text{Incorrect class label is set} \mid \text{label in dataset is 0})$$

$$= (1-\epsilon)t_n + \epsilon(1-t_n)$$

$$p(\text{True label is 0}) = p(\text{Correct class label is set} \mid \text{label in dataset is 0})$$

$$+p(\text{Incorrect class label is set} \mid \text{label in dataset is 1})$$

$$= (1-\epsilon)(1-t_n) + \epsilon t_n$$

Using cross entropy, we get:

$$-\ln p(\mathbf{t}|\mathbf{X},\mathbf{w}) = -\sum_{n=1}^{N}\{(t_n(1-\epsilon) + (1-t_n)\epsilon)\ln y(\mathbf{x}_n,\mathbf{w})$$
$$+((1-t_n)(1-\epsilon) + t_n\epsilon)\ln(1-y(\mathbf{x}_n,\mathbf{w}))\}$$

Note : The official solution manual is incorrect here. Let's assume that $\epsilon = 1/2$. In that case, according to the official solution, the cross entropy becomes:

$$E(\mathbf{w}) = -\sum_{n=1}^{N}\{t_n\ln[(1-\epsilon)y(\mathbf{x}_n,\mathbf{w}) + \epsilon(1-y(\mathbf{x}_n,\mathbf{w}))]$$
$$+(1-t_n)\ln[1-(1-\epsilon)y(\mathbf{x}_n,\mathbf{w}) - \epsilon(1-y(\mathbf{x}_n,\mathbf{w}))]\}$$

$$= -\sum_{n=1}^{N}\{t_n\ln[(1/2)y(\mathbf{x}_n,\mathbf{w}) + (1/2)(1-y(\mathbf{x}_n,\mathbf{w}))]$$
$$+(1-t_n)\ln[1-(1/2)y(\mathbf{x}_n,\mathbf{w}) - (1/2)(1-y(\mathbf{x}_n,\mathbf{w}))]\}$$

$$
\begin{aligned}
&= - \sum_{n=1}^{N} \{ t_n \ln[(1/2) y(\mathbf{x}_n, \mathbf{w}) + (1/2) - (1/2) y(\mathbf{x}_n, \mathbf{w})] \\
&+ (1 - t_n) \ln[1 - (1/2) y(\mathbf{x}_n, \mathbf{w}) - (1/2) + (1/2) y(\mathbf{x}_n, \mathbf{w})] \}
\end{aligned}
$$

$$
= - \sum_{n=1}^{N} \{ t_n \ln[(1/2)] + (1 - t_n) \ln[1/2] \}
$$

$$
= - \sum_{n=1}^{N} \{ t_n \ln[(1/2)] + \ln[1/2] - t_n \ln[1/2] \}
$$

$$
= - \sum_{n=1}^{N} \{ \ln[1/2] \}
$$

$$
= N \ln[2]
$$

This would imply that for $\epsilon = 1/2$, the cross-entropy does not depend on the function $y(\mathbf{x}_n, \mathbf{w})$ at all. This is not correct.