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$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$x_1, x_2 \ldots x_n \Rightarrow$ sample size of n

$$L(x_1, x_2 \ldots x_n) = f(x_1) \cdot f(x_2) \cdots f(x_n)$$

$$\Rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) \ldots$$

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{x_i - \mu}{2\sigma^2}\right)^2 \quad \text{— } 1$$

taking partial derivative w.r.t μ

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i - \mu)}{2\sigma^2}\right)$$

$$= \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{x} - n\mu = 0$$
$$\bar{x} = \mu$$

$\theta_1 = \bar{x} \Rightarrow$ sample mean

Taking partial derivative of eqn 1 w.r.t σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum -\frac{(x_i - \mu)^2}{2(\sigma^2)} = 0$$

$$-n + \sum -\frac{(x_i - \mu)^2}{\sigma^2} = 0$$

$$n = \sum \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} (x_i - \mu)^2\right)$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

② <u>Binomial distribution</u>

$$\rightarrow {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

Take log on both sides

$$\log L = \sum_{i=1}^{n} \left(\log ({}^{n}C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i}\right)$$

differentiate wrt θ

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\sum x_i}{n^2}}$$