Statistics:

Variables

- Numerical
  - conti.
  - disc. [count data]
- categorical
  - Reg. Cat.
  - ordinal [order matter]

Study

- observational [Doesn't interfer with how data arrived]
  - → Retrospective
  - → prospective
- experimental [Randomly assign subjects to treatments]

Sampling Method:

i) SRS (Simple Random Sampling)

ii) Stratified Sampling.

iii) Cluster Sampling.

iv) Multistage Sampling.

Placebo → Fake treatment.

Placebo Effect → Showing change despite being of placebo.

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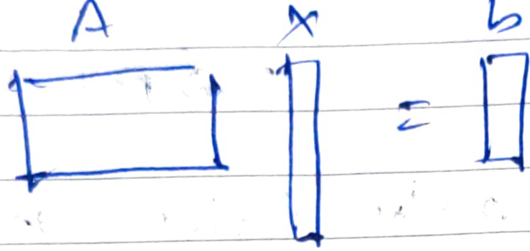
Least Sq., Regression, and pseudo inv.

$Ax = b$

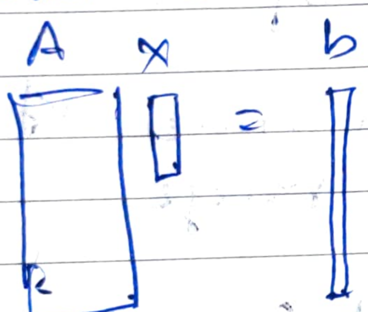
Underdetermined: $n < m$ (short-fat A)



∞ many soln.

$\min \|\tilde{x}\|_2$ s.t. $A\tilde{x} = b$
(min norm soln)

Overdetermined. $n > m$ (tall skinny A)



0 soln.

$\min \|A\tilde{x} - b\|_2$ (least sq soln)

$$A = U\Sigma V^T \Rightarrow A^+ = \underbrace{V\Sigma^{-1}U^T}_{\text{M. P. Pseudo inv. (left)}}$$

$$Ax = b$$

$$\Rightarrow U\Sigma V^T x = b$$

[V, U are orthogn. are normal ie $UU^T = I$]

$$\Rightarrow V\Sigma^{-1}U^T U\Sigma V^T x = \underbrace{V\Sigma^{-1}U^T}_{A^+} b := A^+ b$$

$$\tilde{x} = V\Sigma^{-1}U^T b$$

$\Rightarrow$

$$A\hat{x} = \hat{U}\hat{\Sigma}\underbrace{\hat{V}^T\hat{V}\hat{\Sigma}^{-1}}\hat{U}^T b$$

$$= \underbrace{\hat{U}\hat{U}^T b}$$

project of b into span$(\hat{U})$

$= \text{Span}(A)$

Sol^n of $Ax = b$ only exist if b is in column space of A.

* col(A) Column space of A
  $= \text{col}(\hat{U})$ — range.

~~kernal~~

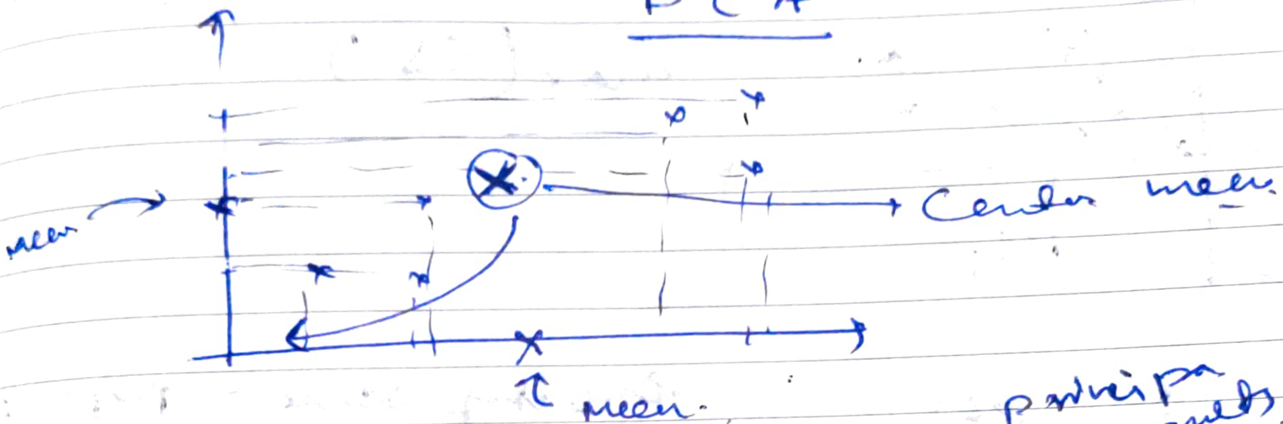
* ker(A^T) ortogonal complement to col(A)
  kernel of A^T.
* row(A) row space of A
  $= \text{col}(V) = \text{row}(V^T)$
* ker(A). null space
  ↳ set of all vector x s.t. $Ax = 0$

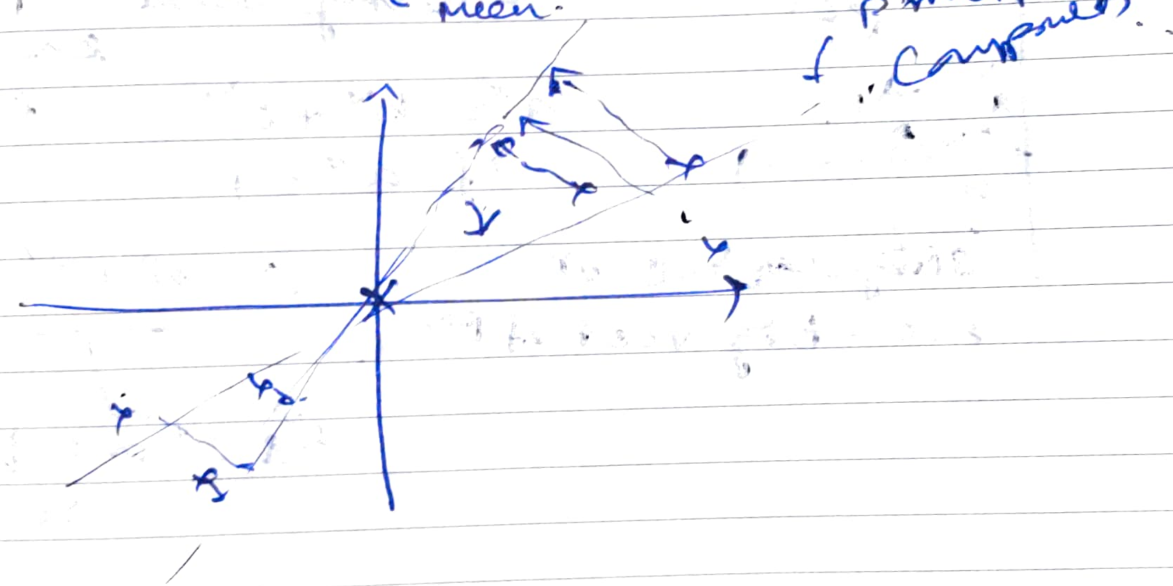
if dim (Ker. (A)) $\neq 0$. then ∞. soln.

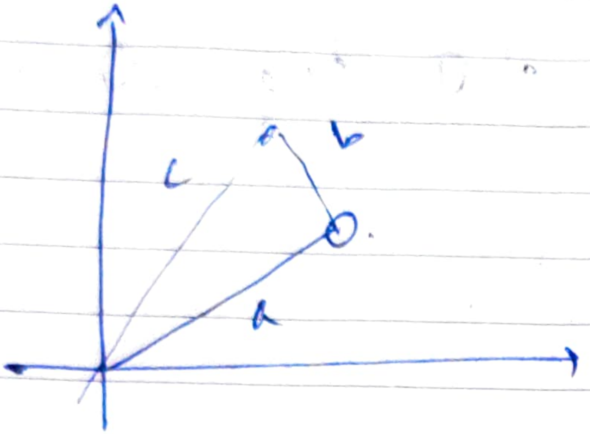
$$\underline{A(X + X_{null})} = b.$$

also soln.

PCA





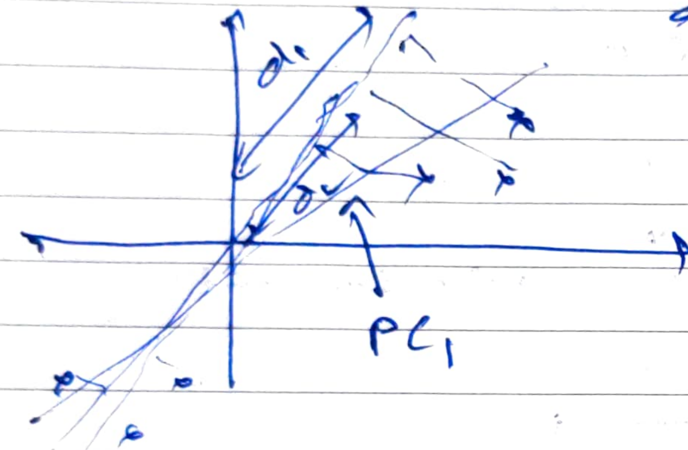


$a^2 = b^2 + c^2$.

if a is const, b & c are inv prop

$d_1^2 + d_2^2 + \dots + d_6^2 = SS$

max(SS)



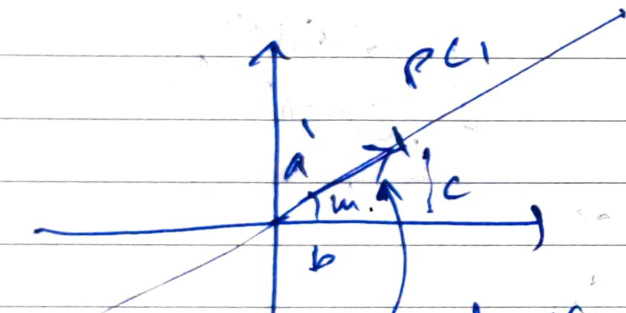
Loading Score = prop of each component in PC1.

$a^2 = b^2 + c^2$.

if $a^2 = 1$,

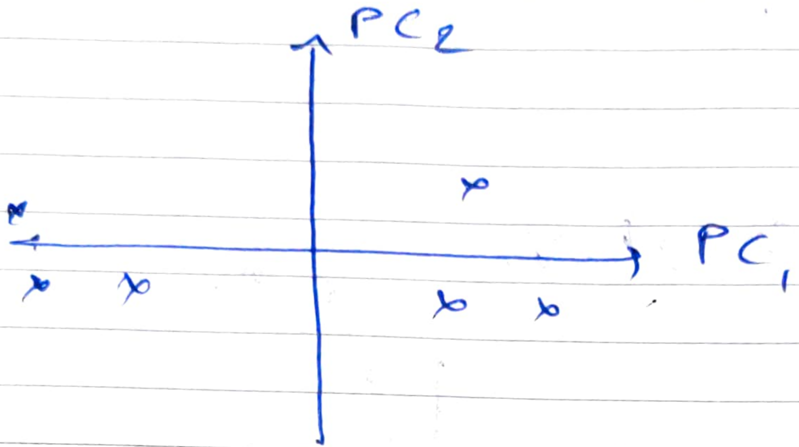
$\therefore b^2 + c^2 = 1$

b^2 & c^2 are loading score.



singular vec or the eig. vect of PC1.

PC_2 will be $\perp$ to PC_1 and goes through origin.



$X = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$ n sample

m = measurement

eigen decomposition of

$\underset{m \times m}{(X^T X)} \rightarrow \underset{?}{w}$.

$\underset{n \times m}{T} = \underset{n \times m}{X} \, \underset{m \times m}{W}$.

"score"

$col(W)$ = loadings.

each col. of W is a PC.

→ ordered col. by value of λ.

$W_p = \begin{bmatrix} | & & | \\ w_1 & \cdots & w_p \\ | & & | \end{bmatrix}$

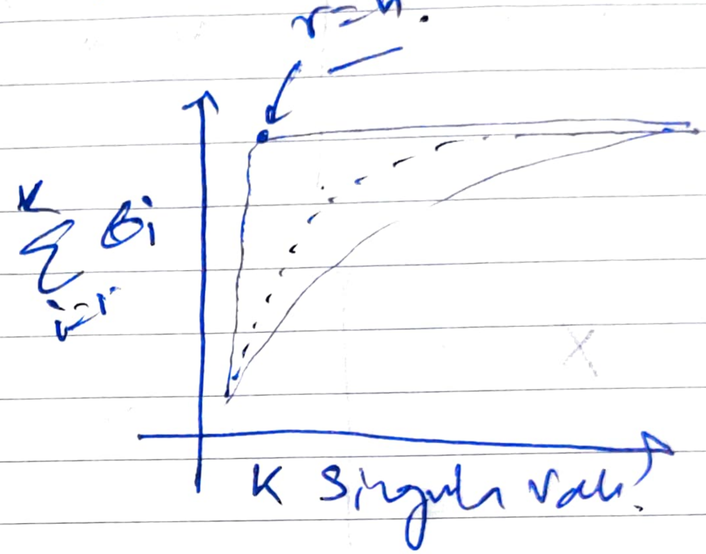
$$\boxed{T_r = X W_r}$$

$n \times r$ $\quad$ $n \times m$ $\quad$ $m \times r$

$$T = XW$$

$$XV = U \Sigma V^T . V$$

$$T = U \Sigma$$



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System of linear Eq:

i) inconsistant → No Soln.

ii) consintant → One or many Soln.

Soln. Techniques:

(I). Gauss Elem. → augmentar matrix $(A|B)$ → row echelon form → back substitution.

(II). Gauss Jordan → $(A|B)$ → make $(A|B)$ reduced row echelon. or make $A \to I$

over determined Sys.

no. of Eq. > no. of unknown.
[usually inconsistent]

$m > n$

under determine Sys:

no. of Eq. < no of unknown. $(n > m)$
[usually Consistent].

Homogenious $\rightarrow AX = O$. (always Consistant)

if $n > m \rightarrow$ exist a non trivial sol^n

Non mongenous $\rightarrow AX = B$ $(B \neq 0)$

Elementary Matrices:

Obtained from of I by using row operator.

Type-I $\rightarrow$ row exchange

Type-II $\rightarrow$ ~~row~~ scalar multiple of a row.

Type-III $\rightarrow$ multiplicas and addtn. of row

$$X \cdot W^T = \begin{pmatrix} \vdots \end{pmatrix}_{4\times 6} \times \begin{pmatrix} \vdots & \vdots \end{pmatrix}_{6\times 2}$$

$W^T \longrightarrow W = \underline{2\times 6}$

$\underline{X^T W} = (\vdots \ldots)_{6\times 1}$

$(n\times m)$

$N(A) = \{ x \in \mathbb{R}^n \mid Ax = 0 \}$

$N(A \in \mathbb{R}^n$

$A(\alpha x) = \alpha A x = \alpha \cdot 0 = 0$

$\alpha x \in N(A)$

$A(x+y) = Ax + Ay \qquad x, y \in N(A)$

$= 0 + 0 = 0$

Null Subspaces :

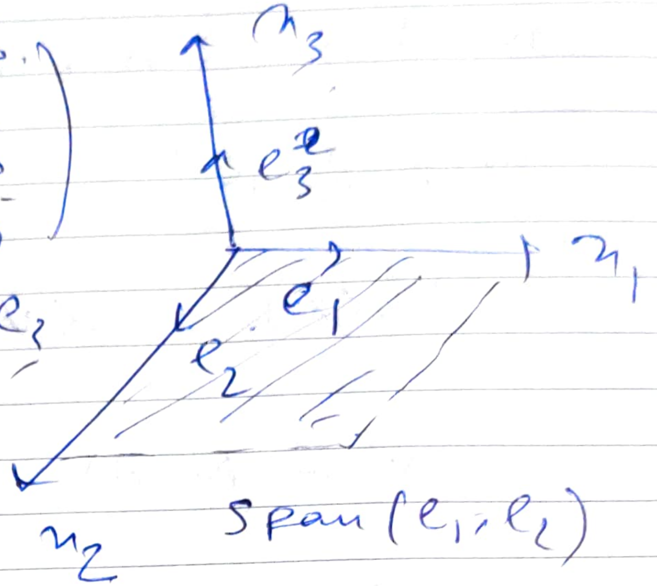
$N(A)$ is the Null subspace of A.

If $N(A) = \{ x \in \mathbb{R}^n \mid Ax = 0 \}$

Span : Set of all linear combinat of Vector space is the span of a vector.

Span of a Vector space:

$$\alpha e_1 + \beta e_2 = \begin{pmatrix} \alpha \\ \beta \\ 0 \end{pmatrix}$$



Span of R^3 ie, e_1, e_2, e_3

$\to \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3$

$$= \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}.$$

span(e_1, e_2)

$e_1, e_2 \ldots e_n$

$\to$ ~~an~~ unit basis vector.

$v_1 + v_2 + \ldots v_n \in V.$

Span$(v_1 + v_2 + \ldots, v_n) \in V \Rightarrow$ proof.

$\alpha(4)$

$\to \alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = V.$

we have to proof closure prop in addition and multiplication.

$\beta v = (\beta\alpha_1) v_1 + (\beta\alpha_2) v_2 + \ldots + (\beta\alpha_n) v_n$

$\in$ Span of $(v_1, v_2 + \ldots v_n)$

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n$$
$$w = \beta_1 v_1 + \beta_2 v_2 + \cdots + \beta_n v_n$$

$$v + w = (\alpha_1 + \beta_1) v_1 + (\alpha_2 + \beta_2) v_2 + \cdots$$
$$(\alpha_n + \beta_n) v_n$$
$$\in \text{Span}(v_1, v_2, \cdots v_n)$$

<u>spanning set:</u>

$\{v_1, v_2 \cdots v_n\}$ spanning set of V.

iff: $\text{Span}(v_1, v_2 \cdots v_n) = V$.

a) $\{(x_1, x_2)^T \mid x_1 + x_2 = 0\} \in \mathbb{R}^2$

$$x_1 = -x_2.$$

$$\therefore x = (a, -a)^T \in S$$

then, $\alpha x = (\alpha a, -\alpha a)^T \in S$

$x + y = (a, -a) + (b, -b) = \{(a+b), -(a+b)\} \in S$

Linear Independence

$\text{Span}(V_1, V_2 \dots V_n) \in V$

if $c_1 V_1 + c_2 V_2 + \dots + c_n V_n = 0$

$\Rightarrow c_1 = c_2 = \dots = c_n = 0$

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$c_1 x_{11} + c_2 x_{12} + \dots + c_n x_{1n} = 0$

$c_1 x_{21} + c_2 x_{22} + \dots + c_n x_{2n} = 0$

$\vdots \qquad \vdots \qquad \vdots$

$c_1 x_{n1} + c_2 x_{n2} + \dots + c_n x_{nn} = 0$

Vector space $C^{(n-1)}[a,b]$:

let, $f_1, f_2 \ldots f_n \in C^{(n-1)}[a,b]$.

$\exists$ $c_1, c_2 \ldots c_n \neq 0$ such that,

$c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) + \ldots + c_n f_n(x) = 0$.

$\forall x \in [a,b]$,

$c_1 f_1'(x) + c_2 f_2'(x) + \ldots + c_n f_n'(x)$

if f is continuous differentiable, $= 0$

$c_1 f_1(x) + c_2 f_2(x) + \ldots + c_n f_n(x) = 0$

$c_1 f_1'(x) + c_2 f_2'(x) + \ldots + c_n f_n'(x) = 0$.

$\vdots \qquad \vdots \qquad \vdots$

$c_1 f_1^{(n-1)}(x) + c_2 f_2^{(n-1)}(x) + \ldots + c_n f_n^{(n-1)}(x) = 0$.

for all fixed $x \in [a,b]$ the eq.

becomes:

$$\begin{pmatrix} f_1(x) & f_2(x) & \cdots & f_n(x) \\ f_1'(x) & f_2'(x) & \cdots & f_n'(x) \\ \vdots & \vdots & & \vdots \\ f_1^{(n-1)}(x) & f_2^{(n-1)}(x) & & f_n^{(n-1)}(x) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

will have the same nontrivial soln.

iff $\{f_1, f_2 \cdots f_n\}$ is linearly dependent in $C^{(n-1)}[a,b]$

<u>Wronskian:</u>

$$W\{f_1, f_2 \cdots f_n\}(x) = \begin{vmatrix} f_1(x) & \cdots & f_n(x) \\ f_1'(x) & \cdots & f_n'(x) \\ \vdots & & \vdots \\ f_1^{n-1}(x) & \cdots & f_n^{n-1}(x) \end{vmatrix}$$

if $W \neq 0 \rightarrow f_1 \cdots f_n$ linearly independent

but $W = 0$ doesn't mean $f_1, f_2 \cdots f_n$ linearly dependent.