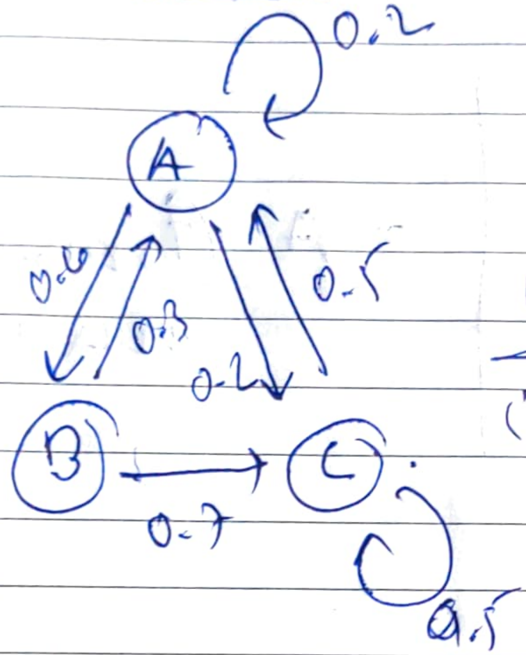
Markov Chain :



$$P(X_{n+1} = x \mid X_n = x_n)$$

Markov Property :

(i) State only depends on perious state.

(ii) sum of outgoing weights = 1 ; $\sum_i a_{in} = 1$

A Random walk generates prob :

$P(A) = \frac{4}{10}$ $P(B) = \frac{2}{10}$ $P(C) = \frac{4}{10}$

for 10 steps

for steps $\to \infty$.

$$\left.\begin{aligned} P(A) &\approx 0.3591 \\ P(B) &\approx 0.21245 \\ P(C) &\approx 0.43564 \end{aligned}\right\}$$ Stationary Prob dist.

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0.2 & 0.6 & 0.2 \\ B & 0.3 & 0 & 0.7 \\ C & 0.5 & 0 & 0.5 \end{array}$$

$$\begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix} = A \text{ (transition matrix)}$$

$$\pi_0 = [0 \quad 1 \quad 0]$$

$$\pi_0 A = [0 \quad 1 \quad 0] \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix} = [0.3 \quad 0 \quad 0.7]$$

$$\pi_1 A = [0.3 \quad 0 \quad 0.7] A = [0.41 \quad 0.18 \quad 0.41]$$

$$\pi_2 A = [0.34 \quad 0.25 \quad 0.41]$$

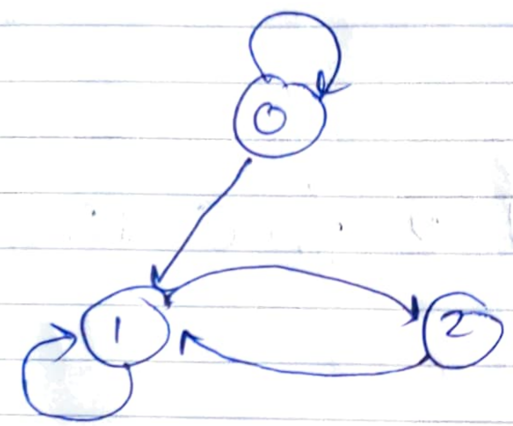
$\vdots \qquad \vdots$

for stationary state, π,

$\pi A = \pi \rightarrow$ eigen vector eq.

* $\pi \rightarrow$ is the left eigen vector of A with eigen value = 1

* $\pi[1] + \pi[2] + \dots \pi[n] = 1$

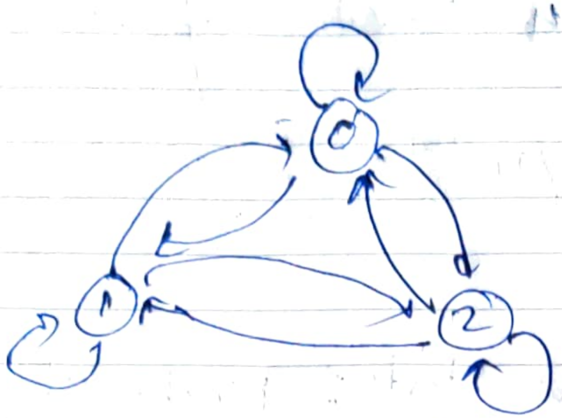


<u>Transient state</u>: Prob. of comming back to a state is < 1
eg. here stat 0 a transient stat.

<u>Recurrent State</u>: Prob of comming back to a state is = 1. eg. 1 & 2 are recurrent.

Markov chain contains transient stat → <u>Reducible</u>

Markov chain doesn't contain transient state → Irreducible.



$$A = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$$

Prob of reaching stat 2 fr 0 in exactly 2 steps.

$$P_{02}(2)$$

$$= A_{01} \times A_{12} + A_{00} \times A_{02} + A_{02} \times A_{22}$$

$$= 0.2 \times 0.2 + 0.5 \times 0.3 + 0.3 \times 0.1$$

$$= 0.22$$

12 Sunday

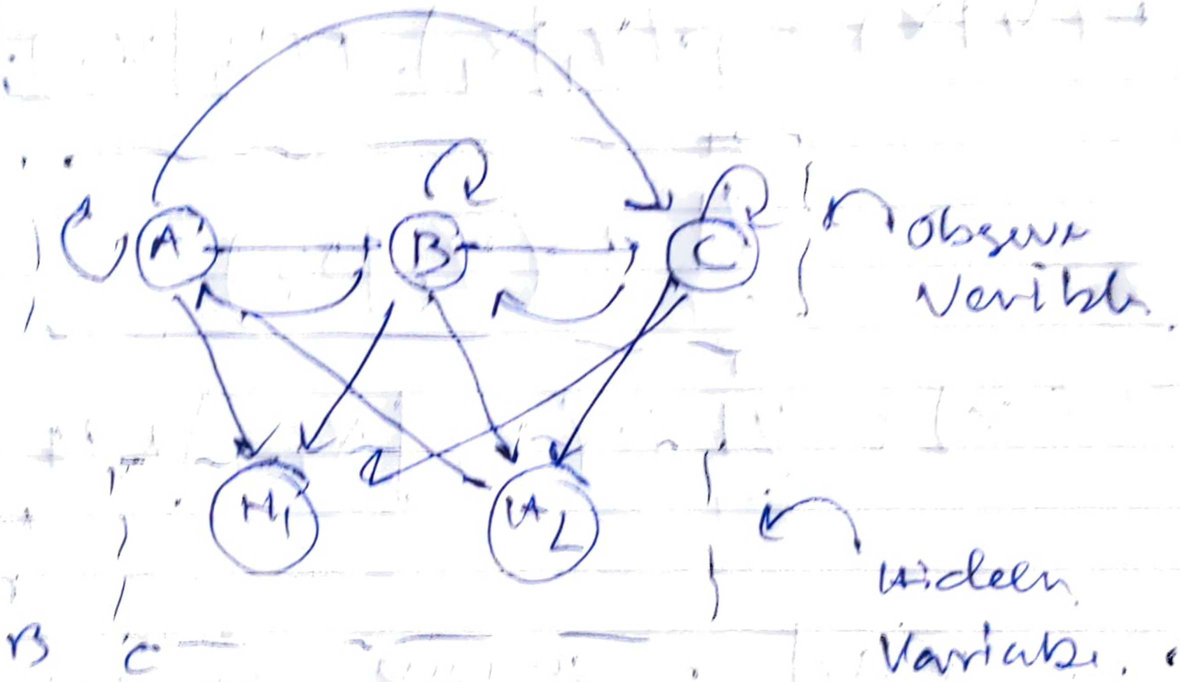
$$= \begin{bmatrix} A_{00} & A_{01} & A_{02} \end{bmatrix} \times \begin{bmatrix} A_{02} \\ A_{12} \\ A_{22} \end{bmatrix} = P_{02}(2)$$

$$A^2 = \begin{bmatrix} 0.4 & 0.38 & 0.22 \\ 0.44 & 0.32 & 0.24 \\ 0.54 & 0.26 & 0.20 \end{bmatrix} \quad P_{02}(2)$$

$$P_{ij}(2) = A^2_{ij} \Rightarrow P_{ij}(n) = A^n_{ij}$$

$$A^\infty_{ij} = P_{ij}(\infty) \rightarrow \text{stationary state } (\pi)$$

HMM:



$$\begin{array}{c|ccc} & A & B & C \\ \hline A & & & \\ B & & A & \\ C & & & \end{array} \qquad \begin{array}{c|cc} & H_1 & H_2 \\ \hline A & & \\ B & & E \\ C & & \end{array}$$

Transition Matrix

Emission Matrix

$$\underset{x = x_1, x_2 \ldots x_n}{\operatorname{argmax}} \underbrace{P(X = x_1, x_2 \ldots x_n \mid Y = y_1, y_2 \ldots y_n)}_{P(X|Y)}$$

$$= \underset{x = x_1, x_2 \ldots x_n}{\operatorname{argmax}} \frac{P(Y|X)\, P(X)}{P(Y)}$$

$$P(Y|X) = P(y_1|x_1) \cdot P(y_2|x_2) \ldots P(y_n|x_n)$$

$$= \prod_{i=1}^{n} P(y_i|x_i)$$

$$P(X) = \prod_{i=1}^{n} P(x_i|x_{i-1})$$ From Markov property

$$\therefore P(X|Y) = \underset{x = x_1 x_2 \ldots x_n}{\operatorname{argmax}} \prod P(y_i|x_i)\, P(x_i|x_{i-1})$$