Lab 4

Question:

Write a python program to build a neural network model (with 1-2-1 layer architecture) with backpropagation training algorithm to approximate a function

$g(p) = sin(\frac{\pi}{4} \times p)$

Where, $-2 \leq p \leq 2$

Transfer function for layer 1: Log-sigmoidal

Transfer function for layer 2: Pure-limit

Learnign Rate: 0.1

Try changing architecture of the network and learning rate to demonstrate the changes of Mean Square Error w.r.t Epoch for the Nural Network.

Solution:

In [ ]:
```
# Import Libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from sympy import *
```

In [ ]:
```
# Net Calculation
def Z_op(X,W,B):
    Z = np.dot(X,W.T) + B
    return Z

# Actual Output Calculation
def Y_op(Tf,Z):
    Yout = []
    for z in Z:
        y = Tf(z)
        Yout.append(y)
    return np.array(Yout)

# Error Calculation
def Cost(Y,T):
    errors = T - Y
    # Sum of sq err
    SSE = 0
    for error in errors:
        SSE = SSE + error*error
    # Mean sq err
    MSE = SSE/len(errors)
    return MSE
```

Froward propagation:

$$\mathbf{a^{m+1} = f^{m+1}(W^{m+1}a^m + b^{m+1})}$$

For $m = 0, 1, \ldots, M-1$

Where,

f^i is transfer function of i th layer

a^i is actual output of i th layer

W^i is weight matrix of i th layer

b^i is bias of i th layer

In [ ]:
```
def feed_forward(W,B,X,Tf_list):
    Z_list = []
    X_list = []
    X_list.append(X)
    for i,j,k in zip(range(len(W)),range(len(B)),range(len(Tf_list))):
        z = Z_op(X,W[i],B[j])
        Z_list.append(z)
        # Using Transfer Function from the TF list
        y = Y_op(Tf_list[k],z)
        # Equating actual output to next neuron input
        X = y
        X_list.append(X)
    # Return actual output and net
    return Z_list,X_list
```

Sensivity:

The sensivity index represents,

$$S^m \equiv \frac{\partial \mathbf{F}}{\partial \mathbf{n^m}} = \begin{bmatrix} \frac{\partial F}{\partial n_1^m} \\ \frac{\partial F}{\partial n_2^m} \\ . \\ . \\ . \\ \frac{\partial F}{\partial n_{S^m}^m} \end{bmatrix}$$

Sensivity index for all the layer except last layer, (i.e. backpropagating through sensitivities):

$$S^m = \mathbf{F^m}(\mathbf{n^m})(\mathbf{W^{m+1}})^T \mathbf{s^{m+1}}$$

Sensivity of last layer:

$$S^M = -2\mathbf{F^M}(\mathbf{n^M})(\mathbf{t} - \mathbf{a})$$

Where,

$$\mathbf{F^m}(\mathbf{n^m}) = \begin{bmatrix} \dot{f}^m(n_1^m) & 0 & \ldots & 0 \\ 0 & \dot{f}^m(n_2^m) & \ldots & 0 \\ . & . & & . \\ . & . & & . \\ 0 & 0 & \ldots & \dot{f}^m(n_{S^m}^m) \end{bmatrix}$$

here,

$$f^m(n_j^m) = \frac{\partial f(n_j^m)}{\partial n_j^m}$$

i.e. derivative of transfer function,

and n^i is i th layer net output

In [ ]:
```
def sensitivity(Z_list,X_list,F_prime,Target,Weights):
    F_list = []
    Sn_list = []
    # To change the shape of vector from (n,) -> (n,1)
    def reverse_reshaping(vector):
      if vector.shape == (len(vector),):
        return vector.reshape(len(vector),1)
      else :
        return vector
    for i in range(len(Z_list)):
      # F dot diagonal matrics calculation
        D = []
        for k in Z_list[i]:
            d = F_prime[i](k)
            D.append(d)
        Dag = np.array(D)
        F_dot = np.diag(Dag)
        F_list.append(F_dot)
    # Assigning F_n to last layer F dot value
    F_n = F_list.pop(len(F_list)-1)
    A_n = X_list.pop(len(X_list)-1)
    # Calculation of Error for calculation of last layer senditivity
    error = Target - A_n
    # Calculation of sensitivity of last layer of neural network
    S_n = np.array(-2*np.dot(F_n,error.T))
    Sn_list.append(S_n)
    # Calculation of previous layer sensitivities
    for i in range(len(F_list),0,-1):
        # F_dot_W = multiplication of Fn and Wn+1 transpose
        F_dot_W = np.array(np.dot(F_list[i-1],Weights[i].T))
        S_n = np.dot(reverse_reshaping(F_dot_W),reverse_reshaping(S_n))
        Sn_list.append(S_n.T)
    # Return List of Sensitivity indices
    return Sn_list
```

Weight and Bias updation:

$$\mathbf{W^m}(k+1) = \mathbf{W^m}(k) - \alpha\mathbf{s^m}(\mathbf{a^{m-1}})^T$$

$$\mathbf{b^m}(k+1) = \mathbf{b^m}(k) - \alpha\mathbf{s^m}$$

Where, k is the epoch and α is the learning rate

In [ ]:
```
def weight_bias_updation(Weight,Bias,Alpha,Sensitivity,X_List):
    new_weights = []
    new_bias = []
    # To change the vector shape from (n,1) -> (n,)
    def reshaping(vector):
      if type(vector[0]) == np.float64:
        return vector
      else:
        if vector.shape == (len(vector),1):
          return vector.reshape(len(vector),)
```

```
        elif vector.shape == (1,len(vector[0])):
          return vector.reshape(len(vector[0]),)
        else :
          return vector
    # To change the shape of vector from (n,) -> (n,1)
    def reverse_reshaping(vector):
      if type(vector) == float:
        return vector
      else:
        if vector.shape == ():
          return vector.reshape(1,1)
        elif vector.shape == (len(vector),):
          return vector.reshape(len(vector),1)
        else :
          return vector
    # Reversing the Sensitivity List
    def Reverse(lst):
        return [ele for ele in reversed(lst)]
    sens = Reverse(Sensitivity)
    # Weight Updation
    for i in range(len(Weight)):
        # Using reshapeing and reverse reshaping for safe check the (n,1) and (n,) ca
        delta_w = reshaping(Alpha*sens[i]*reverse_reshaping(X_List[i]))
        w = Weight[i] - delta_w.T #As a(1) = X[0], The loop will run for w[0],w[1]...
        new_weights.append(w)
    # Bias Updation
    for i in range(len(Bias)):
        b = Bias[i] - reshaping(Alpha*sens[i].T)
        new_bias.append(b)
    return new_weights,new_bias
```

In [ ]:
```
def Error_Collection(Target,Output,All_Errors):
  Error = Cost(Output,Target)
  All_Errors.append(Error)
```

In [ ]:
```
def backpropagation(Weight,Bias,Pattern,Target,Alpha,Transfer_Function,F_Prime,Epoch)
  all_errors = []
  epoch_list = []
  # Loop through number of epochs
  for i in range(Epoch):
      # Feed Forward
      zl,xl = feed_forward(Weight,Bias,Pattern,Transfer_Function)
      actual_output = xl[len(xl)-1]
      # Error Calculation
      Error_Collection(Target,actual_output,all_errors)
      # Sensitivity
      sen = sensitivity(zl,xl,F_Prime,Target,Weight)
      # Weight Bias Updation
      Weight,Bias = weight_bias_updation(Weight,Bias,Alpha,sen,xl)
      epoch_list.append(i+1)
  # Plot Epoch vs MSE diagram
  plt.scatter(epoch_list,all_errors)
  plt.xlabel('Number of Epoch')
  plt.ylabel('Mean Square Error')
```

In [ ]:
```
def input_velues():
  # Getting user input
  print('Enter number of layers :')
  num_layers = int(input())
  num_nurons = []
  Tf_list = []
  F_prime = []
  W = []
  B = []
```

```
  X = []
  T = []
  print('\n')
  for num in range(num_layers):
    print('Enter numbers of neurons in layer {number} :'.format(number = num))
    num_nuron = int(input())
    num_nurons.append(num_nuron)
  print('Enter mathematical expression with respect to variable x.\n The variable sho
  for num in range(num_layers - 1):
    # Converting Tf expression into function
    f_exp = input('Enter Expression for transfer function of layer {number} :'.format
    x = Symbol('x')
    f = lambdify(x,f_exp)
    # Differentiation of the Given Function
    f_prime_exp = diff(f_exp)
    f_prime = lambdify(x,f_prime_exp)
    # Appending to the Transfer function and Derivative of Tf list
    Tf_list.append(f)
    F_prime.append(f_prime)

  def reshaping(vector):
    # To change the vector shape from (n,1) -> (n,)
    if type(vector[0]) == np.float64:
      return vector
    else:
      if vector.shape == (len(vector),1):
        return vector.reshape(len(vector),)
      elif vector.shape == (1,len(vector[0])):
        return vector.reshape(len(vector[0]),)
      else :
        return vector
  # Generating the random weight and bias
  for i in range(len(num_nurons)-1):
    w = np.random.rand(num_nurons[i+1],num_nurons[i])
    b = np.random.rand(num_nurons[i+1])
    W.append(w)
    B.append(b)
  new_w = []
  new_b = []
  # Reshaping the weight and bias matrics
  for w in W:
    w = reshaping(w)
    new_w.append(w)
  for b in B:
    b = reshaping(b)
    new_b.append(b)
  W = new_w
  B = new_b
  print('\n')
  print('Enter the list of patterns:')
  # Converting input string to point the input variable present in the environment
  pattern_input = input()
  X = globals()[pattern_input]
  print('Enter the target:')
  # Converting input string to point the target variable present in the environment
  target_input = input()
  T = globals()[target_input]
  X = np.array(X)
  T = np.array(T)
  # Input of Learning Rate and Epoch
  alpha = float(input('Enter the learning rate :'))
  epoch = int(input('Enter the number of epoch :'))
  return W,B,Tf_list,F_prime,X,T,alpha,epoch
```

In [ ]:

```
def main():
  # Getting the input
```

```
    W,B,Tf_list,F_prime,X,Target,alpha,epoch = input_velues()
    # Print initial Weight and Bias
    print('\nInitial Weights:')
    print(W)
    print('\nInitial Biases:')
    print(B)
    print('\n')
    # Backpropagetion
    backpropagation(W,B,X,Target,alpha,Tf_list,F_prime,epoch)
```

In [ ]:
```
def g(p):
    return 1 + np.sin(np.pi /4 * p)
```

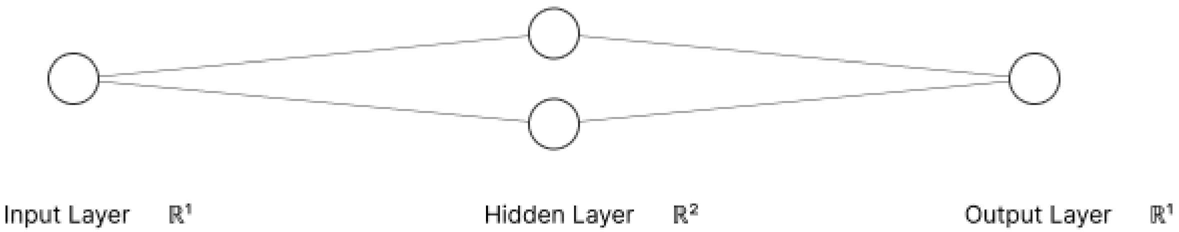
In [ ]:
```
inp = np.random.uniform(-2,2,)
inp
```

Out[ ]:
```
1.8846600863469622
```

In [ ]:
```
out = g(inp)
out
```

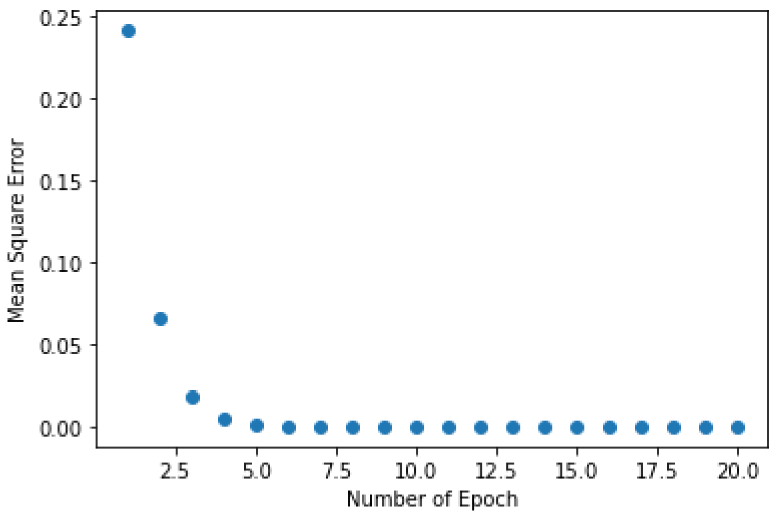
Out[ ]:
```
1.9958997342973805
```

Architecture of nural network:

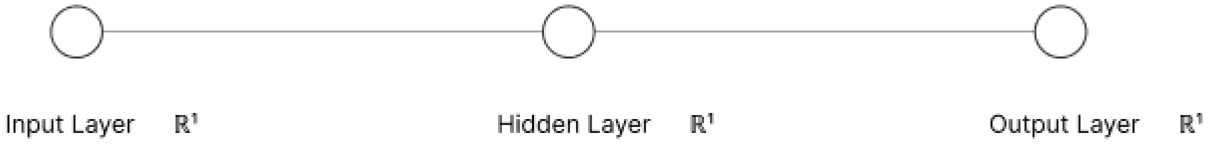


In [ ]:
```
main()
```

```
Enter number of layers :
3


Enter numbers of neurons in layer 0 :
1
Enter numbers of neurons in layer 1 :
2
Enter numbers of neurons in layer 2 :
1
Enter mathematical expression with respect to variable x.
 The variable should be continous, differntiable and use the proper Latex notation fo
r each expression.

Enter Expression for transfer function of layer 1 :1/(1+exp(-x))
Enter Expression for transfer function of layer 2 :x


Enter the list of patterns:
inp
Enter the target:
out
Enter the learning rate :0.1
Enter the number of epoch :20

Initial Weights:
[array([0.10287018, 0.89164604]), array([0.15541074, 0.96535242])]

Initial Biases:
[array([0.8999683 , 0.22554195]), array([0.54766327])]
```



Other architecture of the network:




```
In [ ]: main()
```

```
Enter number of layers :
3


Enter numbers of neurons in layer 0 :
1
Enter numbers of neurons in layer 1 :
1
Enter numbers of neurons in layer 2 :
1
Enter mathematical expression with respect to variable x.
 The variable should be continous, differntiable and use the proper Latex notation fo
r each expression.

Enter Expression for transfer function of layer 1 :1/(1+exp(-x))
Enter Expression for transfer function of layer 2 :x


Enter the list of patterns:
inp
Enter the target:
out
Enter the learning rate :0.2
Enter the number of epoch :20

Initial Weights:
[array([0.68158676]), array([0.20736426])]

Initial Biases:
[array([0.79287951]), array([0.75262676])]
```