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# Modeling and control of underwater vehicle:Sparus

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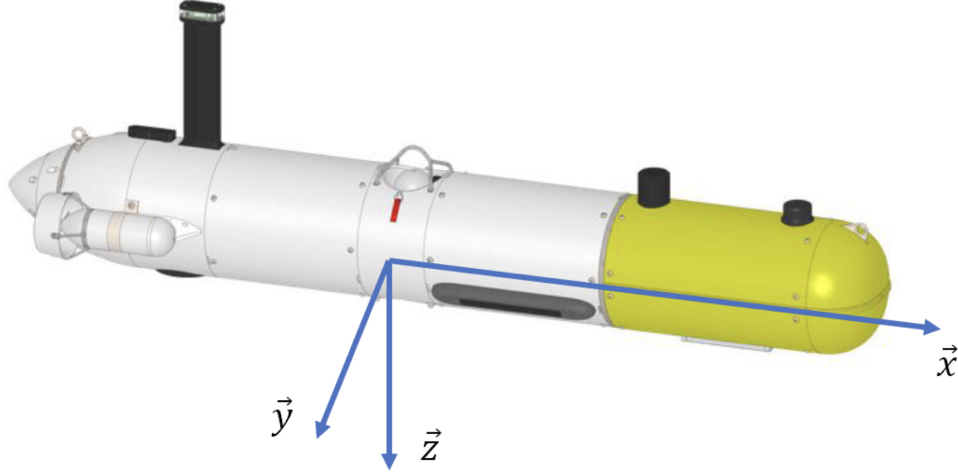


Figure 1: Sparus AUV with the considered axis

## 1 Introduction:

Sparus, the latest autonomous underwater vehicle (AUV) developed by IQUA Robotics in Girona, Spain, is a versatile platform capable of conducting a wide range of underwater operations. This report presents a modelling of the "Sparus". The model is based on the theory of added mass and drag matrix and was constructed using mathematical equations and estimations. Dynamic modelling of underwater vehicles involves considering the effects of added mass, damping, and stiffness. Added mass arises from the inertia of the surrounding water that is set in motion by the motion of the vehicle. Drag, on the other hand, represents the resistance encountered by the vehicle as it moves through the water. Stiffness, in this context, refers to the resistance to changes in the orientation of the vehicle.

We consider that the origin of the body lies over the gravity centre of the Sparus. It is placed at the x-position of the central thruster and at the middle of the cylinder.

$$\vec{r}_g = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Moreover, the coordinates of the buoyancy centre are:

$$\vec{r}_b = \begin{bmatrix} 0 \\ 0 \\ -0.02m \end{bmatrix}$$



### Problem 1

With the figure 1, compute all the dimension of the different bodies

**Solution.**

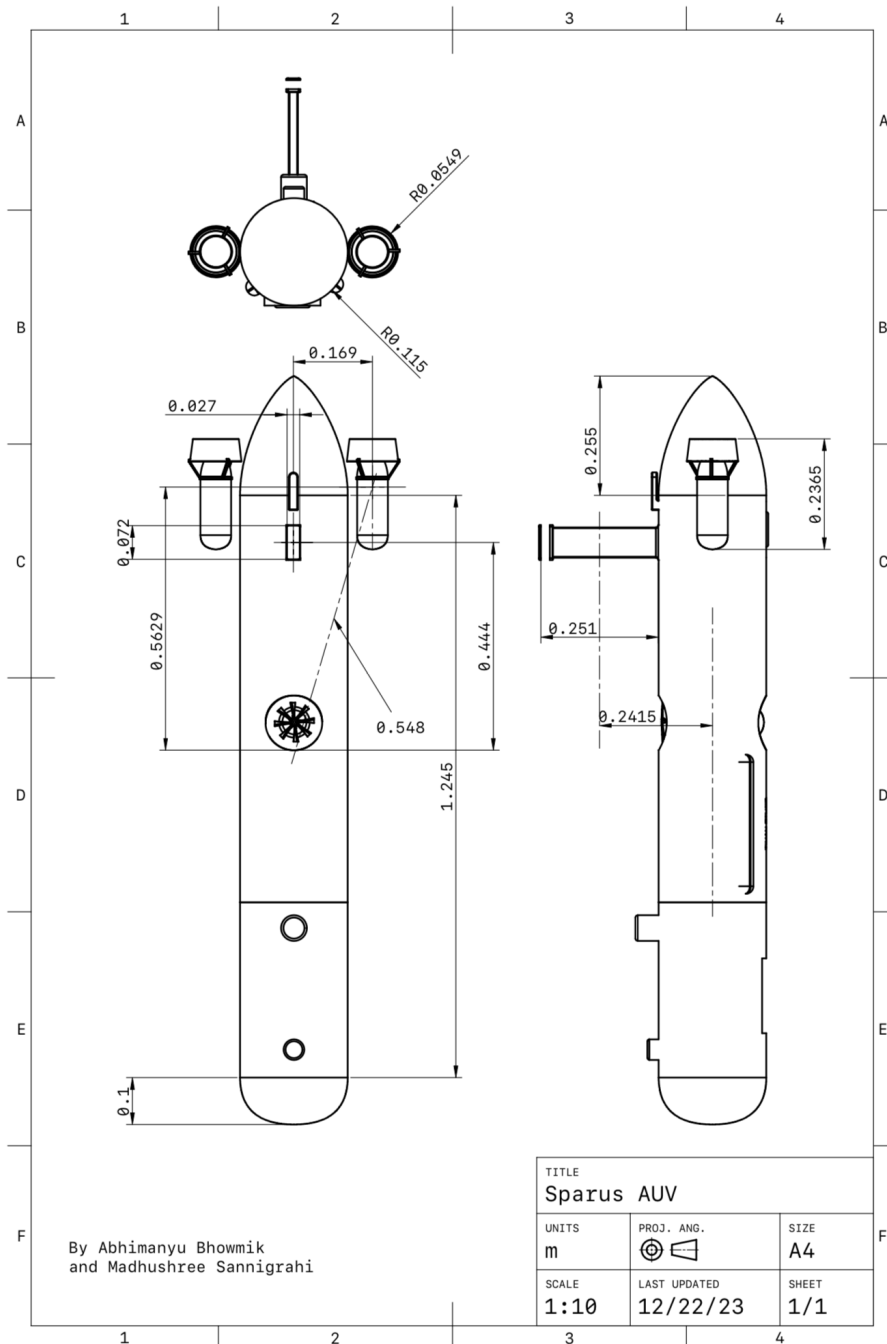


Figure 2: The Overall dimensions of the Sparus AUV



### Problem 2

Considering the given global real mass matrix explains all the terms: from which part the various terms in the matrix originate.

Also, analyze the orders of magnitude of the various terms and conclude.

**Solution.** The entire global real mass matrix can be broken up into 4 smaller matrices:

$$M_{RB}^{CO} = \begin{bmatrix} \begin{matrix} \text{green box} \\ \begin{matrix} 52 & 0 & 0 \\ 0 & 52 & 0 \\ 0 & 0 & 52 \end{matrix} \\ \text{green label } m \end{matrix} & \begin{matrix} \text{red box} \\ \begin{matrix} 0 & -0.1 & 0 \\ 0.1 & 0 & -1.3 \\ 0 & 1.3 & 0 \end{matrix} \\ \text{red label } -mS(r) \end{matrix} \\ \begin{matrix} \text{red box} \\ \begin{matrix} 0 & 0.1 & 0 \\ -0.1 & 0 & 1.3 \\ 0 & -1.3 & 0 \end{matrix} \\ \text{red label } mS(r) \end{matrix} & \begin{matrix} \text{blue box} \\ \begin{matrix} 0.5 & 0 & 0 \\ 0 & 9.4 & 0 \\ 0 & 0 & 9.5 \end{matrix} \\ \text{blue label } I_b \end{matrix} \end{bmatrix}$$

Figure 3: Global mass matrix

- the mass matrix  $m$
- $mS(r)$
- $I_b = I_g - S^2 m$

Let's discuss them one by one, starting with the  $mS(r)$  matrix. Here,  $m$  = mass,  $S(r)$  = Skew matrix of  $r$ . This matrix informs us about the impact of linear motion over the angular motion of the body. In simple words, this matrix tells us about the roll, pitch and yaw caused by the linear movement of the body along the x,y, and z-axis.

Overall, there has been no motion(roll, pitch, yaw) in the direction of the movement(x,y and z respectively) since the AUV is moving in a straight line. When an AUV moves in a straight line, there is no net force acting on it in the direction perpendicular to its movement. This is because the forces acting on the AUV, perpendicular to its motion are equal and opposite, so they cancel each other out.

For the linear motion along the x-axis (surge),

- There is a slight pitch (0.1) caused by motion along the x-axis due to the protruding antennae.



caused by linear motion:

along x →

along y →

along z →

roll	pitch	yaw
0	0.1	0
-0.1	0	1.3
0	-1.3	0

=  $\frac{m}{m_{body}} S(\sigma)$

skew matrix

(due to coriolis)

- Pitch is positive since the rear of the AUV faces downward motion due to the antennae. With respect to the y-axis, the motion is anti-clockwise (right-hand rule)
- Thrusters could have caused yaw but they are symmetric so they balance each other (thus, yaw = 0).

For the linear motion along the y-axis (sway),

- There is a slight roll caused due to the antennae.
- The antenna encounters a backward (clockwise) roll when moving in y-direction. Thus, it is positive.
- Yaw is much more than the roll because of the heavy thrusters + light antenna combination, multiplied with a large radius ( $r_y$ ).
- The heavy thrusters lie on the rear end of the main body which faces an anti-clockwise moment. Thus, the yaw is positive.

For the linear motion along the z-axis (heave),

- There is no roll since all the antennae and other protrusions are in the direction of motion.
- Pitch is caused due to the heavy metal thrusters and uneven mass distribution in the body, multiplied with a large radius ( $r_z$ ).
- The heavy thrusters lie on the main body's rear end, which faces a clockwise moment. Thus, the pitch is negative.

The mass matrix shows the body's actual mass, which remains constant at 52kg. Coming to the inertial matrix with respect to the body.

- Yaw and pitch are quite similar because of similar radius  $r_y$  and  $r_z$ .
- The difference between yaw and pitch is the additional attachments with the AUV like the thrusters(made of metal) and antennae(made of plastic). Here,  $m_{metal} \gg m_{plastic}$



$$m = \begin{bmatrix} 52 & 0 & 0 \\ 0 & 52 & 0 \\ 0 & 0 & 52 \end{bmatrix}$$

Figure 4: Mass matrix

$$\begin{array}{c} \text{row} \quad \text{pitch} \quad \text{yaw} \\ \begin{array}{c} \text{row} \\ \text{pitch} \\ \text{yaw} \end{array} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 9.4 & 0 \\ 0 & 0 & 9.5 \end{bmatrix} \\ I_b = I_g - S^2 m \end{array}$$

Figure 5: Inertial matrix

- The roll is much smaller since the radius  $r_x$  is very very small when compared to  $r_y$  and  $r_z$ . Also,  $m_{plastic}$  of the antennae is negligible.

### Problem 3

Compute each added mass matrix at the buoyancy centre of the Sparus. Except for the main body, the CG and CB of the other bodies are at the same point.

**Solution.** This section details the computation of added mass matrices for each body of the Sparus AUV. The specific calculations are provided in the accompanying MATLAB file. Two approaches are employed for added mass estimation:

- Prolate Spheroid theory (employing Lamb's K factors for spheroids) and
- Slender Body theory.

To determine the overall added mass of the vehicle, Lamb's K factor is used to compute coefficient  $m_{11}$ , while Slender Body theory is applied for all other coefficients. Notably, coefficient  $m_{44}$  is set to zero due to the rotation around the x-axis. Values of  $m_{22}$  and  $m_{33}$



	1	2	3	4	5	6
1						
2		$m_{22} = \int_L a_{22} dx$				$m_{26} = \int_L x a_{22} dx$
3			$m_{33} = \int_L a_{33} dx$		$m_{35} = -\int_L x a_{33} dx$	
4				$m_{44} = \int_L a_{44} dx$		
5			$m_{35} = -\int_L x a_{33} dx$		$m_{55} = \int_L x^2 a_{33} dx$	
6		$m_{26} = \int_L x a_{22} dx$				$m_{66} = \int_L x^2 a_{22} dx$

Figure 6: Added mass by Slender body theory

are similar due to axis symmetry with respect to the x-axis.

$$M_{hull}^b = \begin{bmatrix} 1.6038 & 0 & 0 & 0 & 0 & 0 \\ 0 & 57.3527 & 0 & 0 & 0 & 3.4036 \\ 0 & 0 & 57.3527 & 0 & -3.4036 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.4036 & 0 & 9.4499 & 0 \\ 0 & 3.4036 & 0 & 0 & 0 & 9.4499 \end{bmatrix}$$

Similar to the approach for the entire body, added mass matrices for the thrusters are also computed using Prolate Spheroid theory (Lamb's K factors for spheroids) for  $m_{11}$  and Slender Body theory for the remaining coefficients.

$$M_{thruster}^b = \begin{bmatrix} 0.0507 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7967 & 0 & 0 & 0 & 0.0936 \\ 0 & 0 & 0.7967 & 0 & -0.0936 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0936 & 0 & 0.0147 & 0 \\ 0 & 0.0936 & 0 & 0 & 0 & 0.0147 \end{bmatrix}$$

For the antenna, if we consider it as a rectangular parallelepiped. So we consider the body to be symmetric in all three planes (XY, XZ, YZ). Thus, only the six coefficients on the diagonal are non-zero, and there is no coupling between different degrees of freedom.

Lamb's K factor is employed to determine coefficient  $m_{33}$ , while Slender Body theory is applied for the remaining coefficients. Additionally, coefficient  $m_{66}$  is set to zero due to the



rotation around the z-axis.

$$M_{antennae}^b = \begin{bmatrix} 0.0518 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3305 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5798 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0072 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0011 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The Added mass concerning the Centre of Gravity and centre of Buoyancy of all the individual bodies are the same except the hull. For the hull, the Added mass matrix with respect to the centre of buoyancy is:

$$M_{hull}^{CB} = \begin{bmatrix} 1.6038 & 0 & 0 & 0 & -0.0321 & 0 \\ 0 & 57.3527 & 0 & 1.1471 & 0 & 3.4036 \\ 0 & 0 & 57.3527 & 0 & -3.4036 & 0 \\ 0 & 1.1471 & 0 & 0.0229 & 0 & 0.0681 \\ -0.0321 & 0 & -3.4036 & 0 & 9.4505 & 0 \\ 0 & 3.4036 & 0 & 0.0681 & 0 & 9.4499 \end{bmatrix}$$

To get the Added mass matrix at the Centre of Buoyancy of the Sparus, we need to transform the individual mass matrices to the buoyancy centre of the Sparus.

$$M_A^{CB} = H^T(r_{CG}^{CB})M_A^{CG}cH(r_{CG}^{CB}) \quad (1)$$

where,

$$H(r_{CG}^{CB}) = \begin{bmatrix} I_{3 \times 3} & S^T(r_{CG}^{CB}) \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}$$

$$S(\lambda) = \begin{bmatrix} 0 & -\lambda_1 & \lambda \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix}$$

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

where  $\lambda$  is the displacement from the centre of gravity of each shape to the centre of origin of the Sparus in x,y, and z directions.

The Added mass matrices of all the bodies for the buoyancy centre of the Sparus are as





follows:

$$M_{antennae}^{CB} = \begin{bmatrix} 0.0518 & 0 & 0 & 0 & 0.0115 & 0 \\ 0 & 0.3305 & 0 & -0.0735 & 0 & 0.1467 \\ 0 & 0 & 0.5798 & 0 & -0.2574 & 0 \\ 0 & -0.0735 & 0 & 0.0235 & 0 & -0.0326 \\ 0.0115 & 0 & -0.2574 & 0 & 0.1180 & 0 \\ 0 & 0.1467 & 0 & -0.0326 & 0 & 0.0651 \end{bmatrix}$$

$$M_{thrusterLeft}^{CB} = \begin{bmatrix} 0.0507 & 0 & 0 & 0 & -0.0010 & -0.0086 \\ 0 & 0 & 0 & 0 & 0 & 0.0936 \\ 0 & 0 & 0.7967 & 0.1354 & -0.5637 & 0 \\ 0 & 0 & 0.1354 & 0.0230 & -0.0958 & 0.0019 \\ -0.0010 & 0 & -0.5637 & -0.0958 & 0.4025 & 0.0002 \\ -0.0086 & 0.0936 & 0 & 0.0019 & 0.0002 & 0.1266 \end{bmatrix}$$

$$M_{thrusterRight}^{CB} = \begin{bmatrix} 0.0507 & 0 & 0 & 0 & -0.0010 & 0.0086 \\ 0 & 0 & 0 & 0 & 0 & 0.0936 \\ 0 & 0 & 0.7967 & -0.1354 & -0.5637 & 0 \\ 0 & 0 & -0.1354 & 0.0230 & 0.0958 & 0.0019 \\ -0.0010 & 0 & -0.5637 & 0.0958 & 0.4025 & -0.0002 \\ 0.0086 & 0.0936 & 0 & 0.0019 & -0.0002 & 0.1266 \end{bmatrix}$$

The absolute values of the Right and Left thrusters remain the same but the sign changes in  $m_{16}$ ,  $m_{34}$ ,  $m_{43}$ ,  $m_{45}$ ,  $m_{54}$ ,  $m_{56}$ ,  $m_{61}$  and  $m_{65}$ . This change is due to the positioning of the right and left thruster on the positive and negative direction of the y-axis respectively.

Now, to calculate the total Added Mass over Sparus with respect to the Center of Buoyancy, we add up individual added masses.

$$M^{CB} = M_{hull}^{CB} + M_{antennae}^{CB} + M_{thrusterLeft}^{CB} + M_{thrusterRight}^{CB}$$

$$\therefore M^{CB} = \begin{bmatrix} 1.7570 & 0 & 0 & 0 & -0.0226 & 0 \\ 0 & 57.6831 & 0 & 1.0735 & 0 & 3.7376 \\ 0 & 0 & 59.5259 & 0 & -4.7884 & 0 \\ 0 & 1.0735 & 0 & 0.0925 & 0 & 0.0392 \\ -0.0226 & 0 & -4.7884 & 0 & 10.3735 & 0 \\ 0 & 3.7376 & 0 & 0.0392 & 0 & 9.7682 \end{bmatrix}$$

#### Problem 4

Compare this matrix at CG and CO. Is it important to take into account the distance between the two points?

**Solution.** For our case, we have considered the origin of the base "body" (CO) in the



gravity centre of the Sparus(CG). To get the Added mass matrix at the Centre of Origin of the Sparus, we need to transform the mass matrices to the centre of the Sparus using Equation 1. The added mass matrices for individual parts in the gravity centre of the Sparus(CG) are:

$$M_{hull}^{CO} = \begin{bmatrix} 1.6038 & 0 & 0 & 0 & 0 & 0 \\ 0 & 57.3527 & 0 & 0 & 0 & 3.4036 \\ 0 & 0 & 57.3527 & 0 & -3.4036 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.4036 & 0 & 9.4499 & 0 \\ 0 & 3.4036 & 0 & 0 & 0 & 9.4499 \end{bmatrix}$$

$$M_{antennae}^{CO} = \begin{bmatrix} 0.0518 & 0 & 0 & 0 & 0.0126 & 0 \\ 0 & 0.3305 & 0 & -0.0801 & 0 & 0.1467 \\ 0 & 0 & 0.5798 & 0 & -0.2574 & 0 \\ 0 & -0.0801 & 0 & 0.0266 & 0 & -0.0356 \\ 0.0126 & 0 & -0.2574 & 0 & 0.1185 & 0 \\ 0 & 0.1467 & 0 & -0.0356 & 0 & 0.0651 \end{bmatrix}$$

$$M_{thrusterLeft}^{CO} = \begin{bmatrix} 0.0507 & 0 & 0 & 0 & 0 & -0.0086 \\ 0 & 0 & 0 & 0 & 0 & 0.0936 \\ 0 & 0 & 0.7967 & 0.1354 & -0.5637 & 0 \\ 0 & 0 & 0.1354 & 0.0230 & -0.0958 & 0 \\ 0 & 0 & -0.5637 & -0.0958 & 0.4025 & 0 \\ -0.0086 & 0.0936 & 0 & 0 & 0 & 0.1266 \end{bmatrix}$$

$$M_{thrusterRight}^{CO} = \begin{bmatrix} 0.0507 & 0 & 0 & 0 & 0 & 0.0086 \\ 0 & 0 & 0 & 0 & 0 & 0.0936 \\ 0 & 0 & 0.7967 & -0.1354 & -0.5637 & 0 \\ 0 & 0 & -0.1354 & 0.0230 & 0.0958 & 0 \\ 0 & 0 & -0.5637 & 0.0958 & 0.4025 & 0 \\ 0.0086 & 0.0936 & 0 & 0 & 0 & 0.1266 \end{bmatrix}$$

After calculating all the individual Added mass matrices in the CO of the individual bodies and summing them together, we get the following Added mass matrix for the entire AUV.

$$M^{CO} = M_{hull}^{CO} + M_{antennae}^{CO} + M_{thrusterLeft}^{CO} + M_{thrusterRight}^{CO}$$

$$\therefore, M^{CO} = \begin{bmatrix} 1.7570 & 0 & 0 & 0 & 0.0126 & 0 \\ 0 & 57.6831 & 0 & -0.0801 & 0 & 3.7376 \\ 0 & 0 & 59.5259 & 0 & -4.7884 & 0 \\ 0 & -0.0801 & 0 & 0.0726 & 0 & -0.0356 \\ 0.0126 & 0 & -4.7884 & 0 & 10.3733 & 0 \\ 0 & 3.7376 & 0 & -0.0356 & 0 & 9.7682 \end{bmatrix}$$



<b>Problem 5</b>
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Compare the values of the main solid with the others and conclude
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**Solution.**

- The hull of the AUV has the largest added mass coefficients because it is the largest and most important part of the body.
- The other solids on the AUV, such as the antennae, thruster, and USBL, have much smaller added mass coefficients because they are much smaller in size.
- The added mass matrices of the right and left thrusters have the same absolute value but different signs due to their placements on either side (positive or negative) of the y-axis.
- The hull and the Thrusters possess a symmetry in XY and XZ-axis. The antenna has 3 symmetrical planes in XY, XZ, and YZ planes.
- the USBLs have a very small dimension, almost negligible when compared to the hull. Thus we have not considered its added mass.

<b>Problem 6</b>
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Compare the added and real mass matrix and conclude.
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**Solution.** The added mass matrix is a very important factor in the dynamics of AUV swimming in water. It is responsible for the inertial forces that act on the AUV, which can have a significant impact on the AUV's swimming performance. The added mass matrix is also responsible for the stabilizing forces that act on the AUV, which help to prevent the AUV from becoming unstable.

- The added mass matrix is much larger than the real mass matrix. This is because the added mass represents the inertial forces that act on the AUV due to the surrounding water.
- The diagonal elements of the added mass matrix are much larger than the off-diagonal elements. This means that the added mass effects are most significant for the linear translations of the AUV.
- The added mass matrix is symmetric, which means that the added mass coefficients for the linear translations and rotations are equal.
- The added mass matrix is positive definite, which means that it has all positive eigenvalues. This means that the added mass effects are always stabilizing.



### Problem 7

Estimate all drag matrices.

#### Solution.

The objective is to calculate the drag matrices for each component of an AUV in its centre of gravity. This involves computing individual drag matrices for each component initially expressed in the component's buoyancy centre and then transferring them to the AUV's centre of gravity. The AUV, named Sparus, has been divided into 3 parts namely hull, antennae, and thrusters. Additionally, simple estimations of hull shapes have been made for these sections. With these estimations, we can find the drag matrix  $D$ , where the

Sections	3D Drag Shape	2D Drag Shape
Hull	Ellipsoid ( $L/D = 8$ )	Circular rod (Cylinder)
Thrusters	Ellipsoid ( $L/D = 4$ )	Circular rod (Cylinder)
Antenna	-	Rectangular rod

values of the constants are formulas from the parameter estimations of simple hull shapes.

$$D_{\text{Hull}} = \begin{bmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{66} \end{bmatrix}$$

The estimated shapes are given as:

$$D_{\text{Hull}} = \begin{bmatrix} 2.0774 & 0 & 0 & 0 & 0 & 0 \\ 0 & 55.2000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55.2000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.0656 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7.0656 \end{bmatrix}$$

$$D_{\text{Antenna}} = \begin{bmatrix} 6.9041 & 0 & 0 & 0 & 0 & 0 \\ 0 & 17.4420 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0036 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0090 \end{bmatrix}$$



$$D_{\text{Thruster}} = \begin{bmatrix} 0.1695 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.3159 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.3159 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0009 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0009 \end{bmatrix}$$

Incorporating negative values into the drag matrices is crucial for accurate computations. Drag force acts as a damping force opposing the movement of the Sparus, hence the need for negative values in the drag matrices to represent this opposing direction of force.

### Problem 8

Complete the simulator with some simple experiments to validate it.

**Solution.** The Sparus has three motors; one vertical and two horizontal. Because of this we see that there are only 3 DOFs that could be controlled: Surge, Heave and Yaw. To validate the model, we will use various cases and see the corresponding effect on the modelled AUV.

In our simulation, we modified the thruster forces to analyze their impact on the position, velocity, and acceleration of the craft over 100 seconds.

#### 1.1 CASE 1: Surge motion (along x-axis)

Surge motion, a fundamental movement, should exhibit stability during execution. By activating the right and left thrusters at 30% power in the positive x-axis direction, the craft moved forward. Upon analyzing the plots, we noticed that the surge velocity peaked

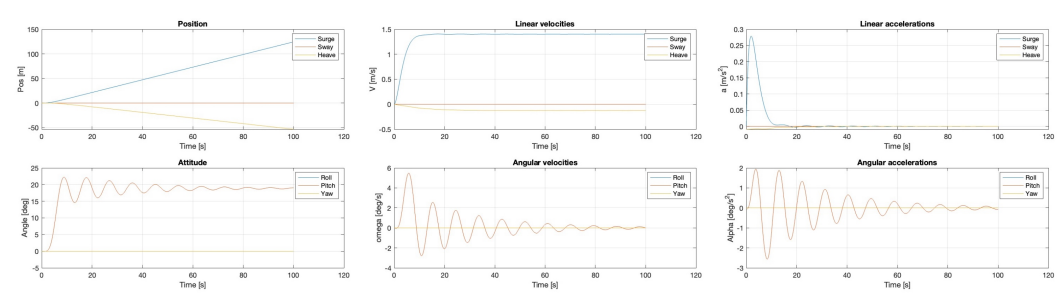


Figure 7: Simulation Results for Case I

and sustained a constant value. However, due to the inability to control the craft's angular velocity in the y-direction (pitch), we observed oscillations in that aspect.

The Sparus accelerates, aiming to return to its mean position, causing the surge acceleration to eventually reach zero. However, the inability to control angular acceleration in the y-direction leads to substantial oscillations in the body due to angular pitch acceleration.



## 1.2 CASE 2: Heave motion (along z-axis)

During our analysis, heave motion, considered a fundamental movement of the craft, should ideally demonstrate stability during execution. By activating the vertical thruster at 30% power in the positive z-axis direction, the craft dived into the water.

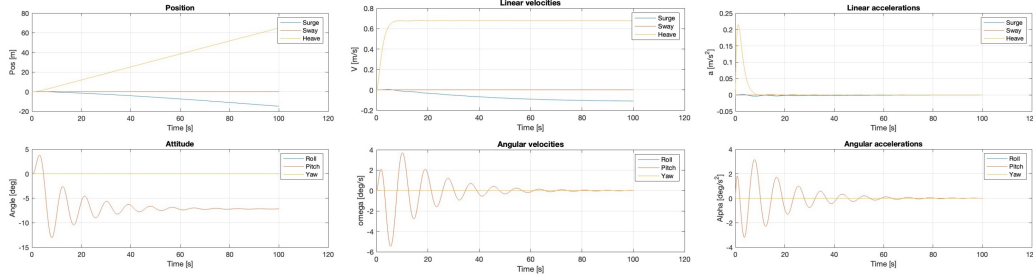


Figure 8: Simulation Results for Case II

From the velocity graph, it's evident that the AUV achieves its maximum velocity in the heave direction and maintains that constant velocity. This heave motion influences the angular velocity, and due to the inability to control the robot's pitch, the effect persists.

## 1.3 CASE 3: Yaw motion (around z-axis)

Executing yaw motion, another fundamental movement, should ideally demonstrate stability. By activating the right thruster at 5% power in the positive x-axis direction and the left thruster at 10% power, the craft initiates a left turn. Upon reviewing the plots, it's

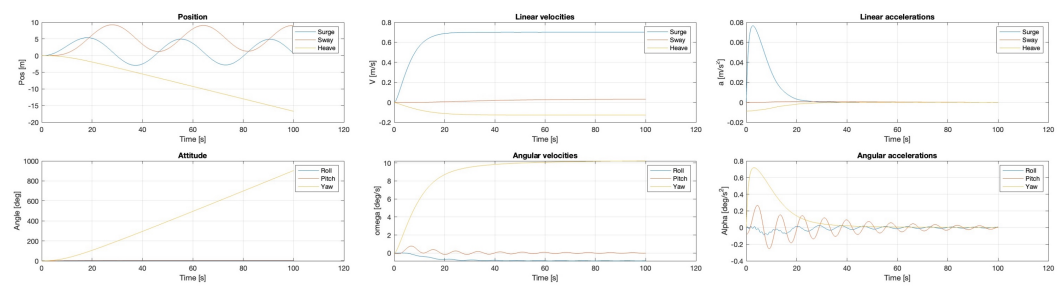


Figure 9: Simulation Results for Case III

evident that the yaw velocity reached a specific value and continued with a constant angular velocity. However, adjusting the periodic depth requires caution when considering pure yaw motion due to the craft's buoyancy.

### Problem 9

Find some simulations to highlight the impact of the different coefficients in the global mass matrix. (Impose linear accelerations)

**Solution.** To validate the impact of the different coefficients in the global mass matrix, we



can assume that some of the shapes considered during the analysis are negligible and we observed the effect of the negligence on the global mass matrix and hence on the simulation.

#### 1.4 Checking for the Effect of the Antenna

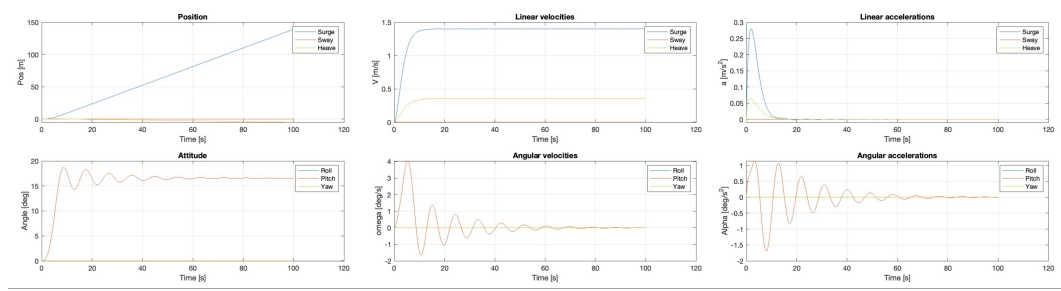


Figure 10: Simulation Results with no Antenna Estimation

We observe that the antenna estimation has little effect on the estimation of the Forces and moments on the linear and angular accelerations that affect the Sparus. Hence, could have been neglected in the analysis.

#### 1.5 Checking for the Effect of the Thrusters

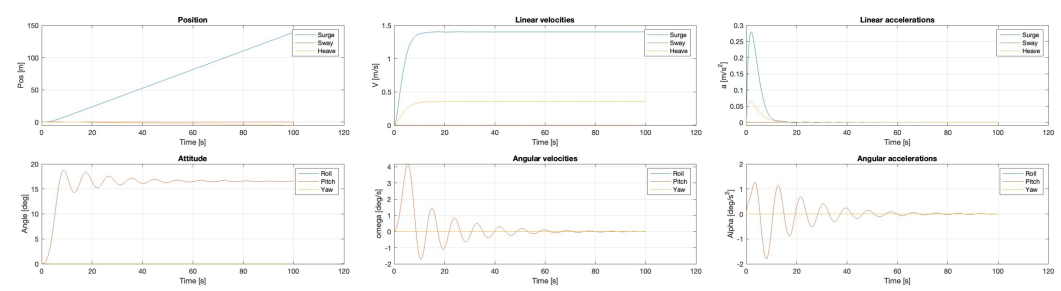


Figure 11: Simulation Results with no Thrusters Estimation

We observe that the added mass estimation of the thrusters has a minuscule effect on the estimation of the forces and moments on the linear and angular accelerations that affect the Sparus. Hence, they could have been neglected in the analysis.

Also, the global mass matrix after removing the added mass of the two thrusters or the antenna is almost the same.

$$Globalmassmatrix = \begin{bmatrix} 53.7052 & 0 & 0 & 0 & -0.1000 & 0 \\ 0 & 109.3527 & 0 & 0.1000 & 0 & 2.2908 \\ 0 & 0 & 110.9461 & 0 & -3.2310 & 0 \\ 0 & 0.1000 & 0 & 0.5460 & 0 & 0 \\ -0.1000 & 0 & -3.2310 & 0 & 19.6549 & 0 \\ 0 & 2.2908 & 0 & 0 & 0 & 19.2031 \end{bmatrix}$$



### Problem 10

Find some simulations to highlight the impact of the drag forces of the different bodies (Impose constant linear speed)

#### Solution.

To check for the impact of the drag which is responsible for the damping of the Sparus, we have made the values of the drag coefficients to be zero to observe the response.

In our simulation, we kept all the thrusters at 30% and provided it with an initial velocity along x-direction at  $10 \text{ ms}^{-1}$ , positioned at the origin. The simulation was for 100 seconds.

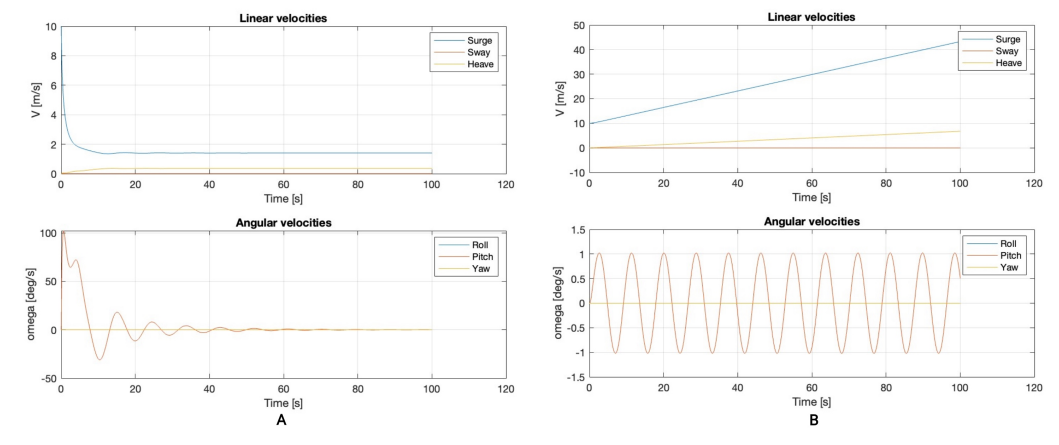


Figure 12: Simulation Results (A) with Drag Coefficients and (B) without Drag Coefficients

In the presence of Drag coefficients, if we provide an initial constant linear speed, it was observed that the speed decreases exponentially due to damping forces and stabilises once it balances out the drag(reaches equilibrium).

In a viscous fluid, the existence of damping forces leads to a non-conservative system regarding energy, known as interference drag. This drag arises due to the vortex sheet shedding at sharp edges, resulting in a damping force. Removing the drag coefficient from a body submerged in the fluid causes the system to oscillate, making it uncontrollable.