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ScienceDirect

Procedia Computer Science 178 (2020) 311–319

Procedia Computer Science

www.elsevier.com/locate/procedia

9th International Young Scientist Conference on Computational Science (YSC 2020)

Mathematical Modeling of the Dynamics of 3-DOF Robot-Manipulator with Software Control

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Abstract

The paper presents a general method for constructing a mathematical model of multi-link robot-manipulators. For the 3-DOF manipulator, kinematic equations and differential equations of dynamics are obtained. To study mathematical models of the dynamics of robotic manipulators and applications in software control systems, it is necessary to develop special analytical methods for solving systems of differential equations. An analytical method of transformations applied for the study of mathematical models of the dynamics of 3-DOF robot-manipulators. The method allows to obtain a solution in an analytical form, taking into account all nonlinear components of the system of differential equations. The software package for study of nonlinear mathematical models was developed and implemented. The problem of software control of electric drives of 3-DOF robot-manipulator is considered and control actions are determined that ensure the fulfillment of a given trajectory by the manipulator. The method for constructing and analyzing a mathematical model of manipulators presented in this work can be used to study a wide class of multi-link manipulators with many degrees of freedom.

Keywords: mathematical modeling; 3-DOF robot-manipulator; kinematic equations; models of dynamics; differential equations; analytical method; multi-link manipulators;

1. Introduction

In modern automated production, economical robot-manipulators with three degrees of freedom (3-DOF) are increasingly used. Conventional operations such as feeding blanks and the automatic process of capturing and

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10.1016/j.procs.2020.11.033

placing blanks on a conveyor belt require only three degrees of freedom. The tasks of mathematical modeling, the study of the kinematics and dynamics of multi-link robotic manipulators, and manipulator control are relevant and important. Many scientific papers are devoted to methods of analysis of kinematics, dynamics, and modeling of multi-link robot-manipulators. For example, in [1], a method for analyzing the kinematic equations of a 3-DOF robot-manipulator was presented and modeling was performed in the Solid Edge. To increase production efficiency, a high-speed 3-DOF parallel manipulator was studied in [2]. In addition to kinematic analysis, a static analysis of 3-DOF robot-manipulator is also used [3]. In [4], mathematical modeling of the kinematics and dynamics of the Gantry-Tau 3-DOF manipulator is performed. In addition to the analysis of the kinematics of manipulator, the problems of path planning with the transformation of the operating mode of the planar parallel manipulator 3-DOF are investigated [5]. The article [6] explores the kinematics of a three-link manipulator used in the automotive industry. Various methods are used to study the kinematics of manipulators, for example [7]: the genetic method, the particle swarm optimization method, and the gravitational search method. To study the dynamics of the 3-DOF manipulator, the generally accepted matrix Lagrange equations are used [8].

In [9], the dynamic model of the 3-DOF manipulator for the design of the sliding mode controller is investigated. Industrial robot-manipulators are also being designed using machine learning and artificial intelligence methods [10], which increases the complexity of the robotic system. In [11, 12, 13], the 4-DOF parallel manipulator was studied, a prototype of manipulator was designed, and a control architecture was developed [14]. Also, in [15] examines hybrid robot-manipulators using the Monte Carlo method to evaluate the work area. To control manipulators a reverse kinematics and fuzzy logic controller is used for accurate and smooth movement [16, 17]. In [18], the dynamics of the 5-DOF manipulator is analyzed. A method for optimizing the movement of an industrial robot manipulator is proposed [19].

To study mathematical models of the dynamics of robot-manipulators and applications in control systems, it is necessary to develop special analytical methods for solving differential systems of equations. In this paper, we present a general method for determining the motion of a manipulator with many degrees of freedom. Using the method, we analyze the kinematics, dynamics of the 3-DOF manipulator. Let us present an analytical method of transformations for studying mathematical models of the dynamics of robotic manipulators. The method allows to obtain a solution in an analytical form, taking into account all components of nonlinear system of differential equations.

2. Method for determining dynamics of a three-link manipulator

Matrix Lagrange equations of the second kind are widely used in the study of the dynamics of a robotic manipulator [19]. In kinematics, transition matrices are used to describe the transition from a coordinate system associated with a manipulator link to a subsequent coordinate system of another link. In the general case, the spatial transition from one coordinate system to another is described by six parameters. In [20] it is shown that the number of parameters for the transition can be reduced to four parameters.

We denote the origin O_i of the local coordinate system for the i-link of manipulator. The origin of the fixed global coordinate system is denoted by O_0.

In general form, the transition matrix is presented:

$$A_{i,i+1}=\begin{bmatrix}\cos(\alpha) & -\cos(\beta)\sin(\alpha) & \sin(\alpha)\sin(\beta) & b\cos(\alpha)\\ \sin(\alpha) & \cos(\alpha)\cos(\beta) & -\cos(\alpha)\sin(\beta) & b\sin(\alpha)\\ 0 & \sin(\beta) & \cos(\beta) & a\\ 0 & 0 & 0 & 1\end{bmatrix}$$

To determine the transition matrix from the global to the local coordinate system, it is necessary to multiply the matrices: $A_{0i}=A_{01}A_{12}\ldots A_{i-1\,i}$

To compile a mathematical model of manipulator dynamics, matrix Lagrange equations of the form: $LT+DP=Q$, where

$L=\left[\frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}_1}\right)-\frac{\partial}{\partial q_1},\ldots,\frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}_i}\right)-\frac{\partial}{\partial q_i}\right]$ - row vector of Lagrange operators,

q_i - generalized coordinates for manipulator links,

$T = \frac{1}{2}\sum_{i=1}^{n} tr\left(\frac{dA_{0i}}{dt} H_i \frac{dA_{0i}}{dt}^T \right)$ - kinetic energy of all links of manipulator,

$Q = [Q_1,...,Q_i]$ - generalized forces for manipulator links,

$D = \left[\frac{\partial}{\partial q_1},...,\frac{\partial}{\partial q_i} \right]$ - row vector of derivatives, H_i - link inertia matrix.

$P = -\sum_{i=1}^{n} m_i \begin{bmatrix} 0 & 0 & g & 0 \end{bmatrix} A_i \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T$ - potential energy for the gravity forces of all links of manipulator.

One of the most difficult tasks is the construction of an analytical solution to the system of equations for the manipulator dynamics. If it is possible to construct an analytical solution, then the task of research and design of manipulator is greatly simplified. The analytical solution can be constructed by various methods: linearization, averaging, Krylov-Bogolyubov, Van der Pol, small parameter, harmonic balance, Poincare perturbations, the method of differential inequalities. If it is not possible to construct an analytical solution to a nonlinear system with acceptable accuracy, then the numerical Runge-Kutta method is used.

In this work, we develop a mathematical model of a three-link manipulator for the study of which we use the method of transformations presented in the authors' work [21]. Having constructed an analytical solution, we obtain the dependences for the generalized coordinates, speeds and accelerations on the generalized forces of manipulator's electric drives. Using analytical dependences, we find the values of the generalized forces of electric drives, necessary for moving the gripper of manipulator along a given trajectory. To control the movement of manipulator along a given trajectory, it is necessary to solve the problem of the dynamics of manipulator.

Let us investigate the motion of a three-link manipulator with control. We will construct a mathematical model of manipulator in the form of a system of nonlinear differential equations of motion.

We represent the control action in the form of a piecewise-smooth function and control signals in the electric drive, providing uniform movement of manipulator grip at a constant height with a constant working speed.

Consider a three-link manipulator with cylindrical

The kinematic diagram of manipulator consists of three rotational kinematic pairs (Figure 1).

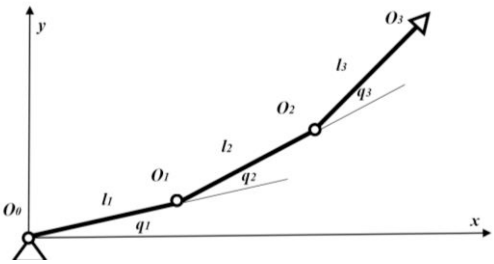

Fig. 1. Kinematic diagram of a three-link manipulator.

Let us denote the relative coordinate systems of the robot links in points O_1, O_2, O_3.

Define the angles of rotation of the links q_1, q_2, q_3 three-link manipulator as a function of time measured in radians.

Transition matrices are:

$$A_{01} = \begin{bmatrix} cos(q_1) & -sin(q_1) & 0 & l_1 cos(q_1) \\ sin(q_1) & cos(q_1) & 0 & l_1 sin(q_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_{12} = \begin{bmatrix} cos(q_2) & -sin(q_2) & 0 & l_2 cos(q_2) \\ sin(q_2) & cos(q_2) & 0 & l_2 sin(q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{13}=\begin{bmatrix} cos(q_3) & -sin(q_3) & 0 & l_3cos(q_3)\\ sin(q_3) & cos(q_3) & 0 & l_3sin(q_3)\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$A_{02}=A_{01}A_{12}=\begin{bmatrix} C_{12} & -S_{12} & 0 & l_1C_1+l_2C_{12}\\ S_{12} & C_{12} & 0 & l_1S_1+l_2S_{12}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},\ A_{03}=A_{01}A_{12}A_{23}=\begin{bmatrix} C_{123} & -S_{123} & 0 & l_1C_1+l_2C_{12}+l_3C_{123}\\ S_{123} & C_{123} & 0 & l_1S_1+l_2S_{12}+l_3S_{123}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Here, for brevity denoted: $C_k=cos(q_k),S_k=sin(q_k)$,

$C_{12}=\cos(q_1+q_2),S_{12}=\sin(q_1+q_2),C_{123}=\cos(q_1+q_2+q_3),S_{123}=\sin(q_1+q_2+q_3)$,

The equations of the kinematics of the 3-DOF manipulator are obtained.

The absolute coordinates of the grip of manipulator are determined as a function of the generalized coordinates:

$x_{03}=l_1cos(q_1)+l_2cos(q_1+q_2)+l_3cos(q_1+q_2+q_3)$, $y_{03}=l_1sin(q_1)+l_2sin(q_1+q_2)+l_3sin(q_1+q_2+q_3)$, $z_{03}=0$.

From the first two equations of the system, we determine the dependences of the generalized coordinates q_1,q_2 on the coordinates of the grip of manipulator under the assumption $q_3=0$.

Denoting $x_{03}=x,y_{03}=y$, we write the generalized coordinates:

$$q_1(x,y)=\tan^{-1}(\frac{1}{2l_1^2(x^2+y^2)}(l_1x\left(l_1^2-(l_2+l_3)^2+x^2+y^2\right)\pm$$
$$\sqrt{l_1^2(-y^2)\left(-(-l_1+l_2+l_3)^2+x^2+y^2\right)\left(-(l_1+l_2+l_3)^2+x^2+y^2\right)}))$$
$$q_2(x,y)=\tan^{-1}\left(\frac{-l_1^2-(l_2+l_3)^2+x^2+y^2}{2l_1(l_2+l_3)}\right)$$

Under the condition of uniform movement of manipulator grip at a constant working speed, the equalities are fulfilled $x=v_xt,y=v_yt$.

Substituting equalities in the expressions for the generalized coordinates, we obtain the generalized coordinates as a function of time:

$$q_1(t)=\tan^{-1}(\frac{1}{2l_1^2t^2(v_x^2+v_y^2)}(\sqrt{l_1^2(-t^2)v_y^2\left((-l_1+l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)\left((l_1+l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)}+$$
$$l_1tv_x\left(l_1^2-(l_2+l_3)^2+t^2(v_x^2+v_y^2)\right)))\ ,\ q_2(t)=-\tan^{-1}\left(\frac{l_1^2+(l_2+l_3)^2+t^2\left(-(v_x^2+v_y^2)\right)}{2l_1(l_2+l_3)}\right).$$

Generalized speeds are determined as a function of time:

$$\dot{q}_1(t)=q_1'=(\left(-l_1^2+(l_2+l_3)^2+t^2(v_x^2+v_y^2)\right)(-tl_1^3v_y^2+tl_1v_y^2\left((l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)+$$
$$v_x\sqrt{\left(-t^2l_1^2v_y^2\left((-l_1+l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)\left((l_1+l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)\right)}))/(2t^2l_1(v_x^2+v_y^2)\cdot$$
$$\sqrt{\left(-t^2l_1^2v_y^2\left((-l_1+l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)\left((l_1+l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)\right)}(1+$$
$$(\sqrt{\left(-t^2l_1^2v_y^2\left((-l_1+l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)\left((l_1+l_2+l_3)^2-t^2(v_x^2+v_y^2)\right)\right)}+$$
$$tl_1v_x\left(l_1^2-(l_2+l_3)^2+t^2(v_x^2+v_y^2)\right))^2/\left(4t^4l_1^4(v_x^2+v_y^2)^2\right)))$$

$$\dot{q}_2(t) = q_2' = t\left(v_x^2 + v_y^2\right) / l_1\left(l_2 + l_3\right)\left(1 + \frac{\left(l_1^2 + \left(l_2 + l_3\right)^2 - t^2\left(v_x^2 + v_y^2\right)\right)^2}{4l_1^2\left(l_2 + l_3\right)^2}\right).$$

Matrix Lagrange equations are used to study the dynamics of a three-link manipulator. We assume that the friction in the joints is small and is not taken into account when deriving the differential equations of manipulator motion. As a result of the application of the software package developed by the authors, a mathematical model of manipulator in the form of nonlinear differential equations was obtained.

$$\begin{aligned}
&3gl_3m_3\cos\left(q_1+q_2+q_3\right)+3gl_2\left(m_2+2m_3\right)\cos\left(q_1+q_2\right)+\\
&3gl_1\left(m_1+2\left(m_2+m_3\right)\right)\cos\left(q_1\right)-2l_3m_3(l_1\left(q_2'+q_3'\right)\left(2q_1'+q_2'+q_3'\right)\sin\left(q_2+q_3\right)+\\
&l_2q_3'\left(2\left(q_1'+q_2'\right)+q_3'\right)\sin\left(q_3\right))-3l_1l_2\left(m_2+2m_3\right)q_2'\left(2q_1'+q_2'\right)\sin\left(q_2\right)+\\
&l_3m_3q_3''\left(3l_1\cos\left(q_2+q_3\right)+3l_2\cos\left(q_3\right)+2l_3\right)+\\
&6q_1''\left(l_2l_1\left(m_2+2m_3\right)\cos\left(q_1\right)+l_3m_3\left(l_1\cos\left(q_2+q_3\right)+l_2\cos\left(q_3\right)+\frac{l_3}{3}\right)+l_1^2\left(\frac{m_1}{3}+m_2+m_3\right)+l_2^2\left(\frac{m_2}{3}+m_3\right)\right)+\\
&6q_2''\left(l_1l_2\left(\frac{m_2}{2}+m_3\right)\cos\left(q_2\right)+\frac{1}{6}l_3m_3\left(3l_1\cos\left(q_2+q_3\right)+6l_2\cos\left(q_3\right)+2l_3\right)+l_2^2\left(\frac{m_2}{3}+m_3\right)\right)=6M_1,\\
&3gl_3m_3\cos\left(q_1+q_2+q_3\right)+3gl_2\left(m_2+2m_3\right)\cos\left(q_1+q_2\right)+3l_1l_2\left(m_2+2m_3\right)\left(q_1'\right)^2\sin\left(q_2\right)-\\
&3l_3m_3\left(l_2q_3'\left(2q_1'+3q_3'\right)\sin\left(q_3\right)-l_1\left(q_1'\right)^2\sin\left(q_2+q_3\right)\right)+6q_1''(l_1l_2\left(\frac{m_2}{2}+m_3\right)\cos\left(q_2\right)+\\
&\frac{1}{6}l_3m_3\left(3l_1\cos\left(q_2+q_3\right)+6l_2\cos\left(q_3\right)+2l_3\right)+l_2^2\left(\frac{m_2}{3}+m_3\right))+2q_2''\left(3l_3l_2m_3\cos\left(q_3\right)+l_2^2\left(m_2+3m_3\right)+l_3^2m_3\right)+\\
&l_3m_3q_3''\left(3l_2\cos\left(q_3\right)+2l_3\right)=6M_2,\\
&l_3m_3\left(3g\cos\left(q_1+q_2+q_3\right)+12l_3q_3''+6q_1''\left(3l_1\cos\left(q_2+q_3\right)+3l_2\cos\left(q_3\right)+2l_3\right)+6q_2''\left(3l_2\cos\left(q_3\right)+2l_3\right)\right)=6M_3
\end{aligned}$$

Here m_1, m_2, m_3 - the masses and l_1, l_2, l_3 - lengths of manipulator links.

On the right side of the equations $M_1 = h_1F_1, M_2 = h_2F_2, M_3 = h_3F_3$, are the moments for the control forces F_1, F_2, F_3.

The solution to the system is constructed by the fourth-order Runge-Kutta numerical method. Figure 2 shows the generalized coordinates q_i and speeds $w_i = q_i'$.

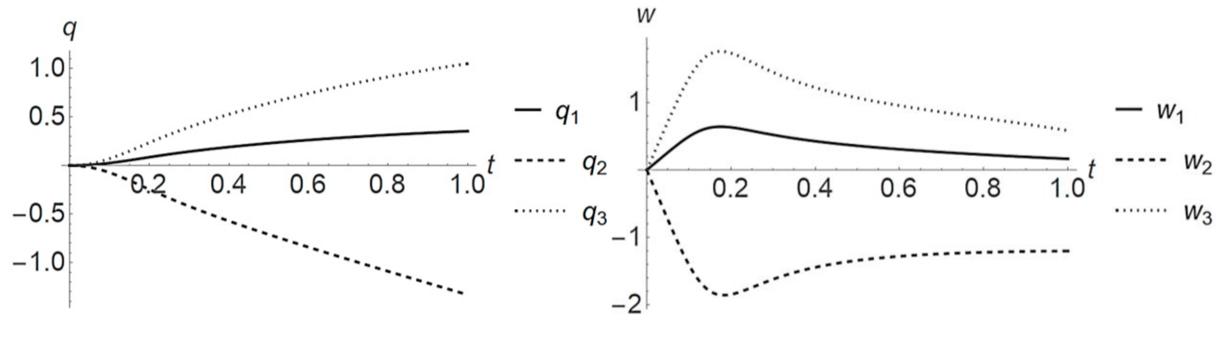

Fig. 2. Generalized coordinates and speeds of a three-link manipulator.

Figure 3 shows the gripping trajectory of a three-link manipulator

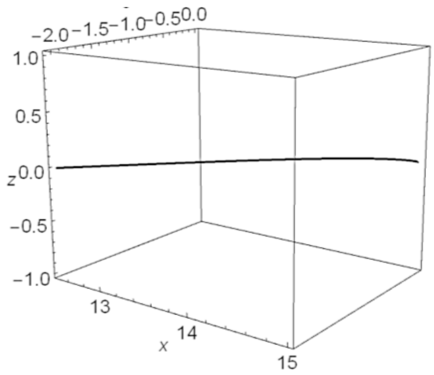

Fig. 3. The gripping path of the three-link manipulator.

The analytical solution of three differential equations can be constructed by various methods: linearization, averaging, Krylov-Bogolyubov, Van der Pol, small parameter, harmonic balance, Poincare perturbations, fitting method, transformation method [21].

The complexity of the calculations by the transformation method requires the use of computer tools. To solve systems of nonlinear differential equations by analytical and numerical methods, the author developed and registered a software package.

3. A software package for study of nonlinear mathematical models

The software package for study of nonlinear mathematical models uses new multi-language development technologies and uses three modern programming languages: the object-oriented C# language for implementing graphical module interfaces, the SQL structured query language and the PL / SQL procedural extension for working with the DBMS and the Mathematica functional programming language for ease of use users with mathematical models. The software package implements the basic methods for solving nonlinear systems. The software package allows to calculate nonlinear mathematical models, present the calculation results in graphical form. Models are saved in a single library of models in the DBMS with the possibility of further editing.

All mathematical models are implemented as a library for the software package. The developed program package consists of modules with graphical user interfaces: a module for calculating and analyzing the considered mathematical models loaded into the DBMS library, a module for numerically calculating nonlinear differential equations using numerical methods, and a module for calculating using analytical methods.

The state registration of the program package has been completed.

The calculation modules are an application running on a 64-bit Windows 7,8,10 operating system, which is developed on the Net Framework 4. Net is a new platform for developing Windows programs and replaces everything that came before it, including the whole A set of previous programming technologies, such as MFC, COM, ActiveX, ATL, ASP, ADO. The Net platform is a modern Windows programming technology. Net is based on a runtime engine that downloads and executes bytecode compiled programs that the Common Language Runtime (CLR) understands. Widely used compilers with Net support for C #, VisualBasic.Net, which provides Microsoft.

The developed model library is a set of data, model parameters and calculation methods stored in the DBMS.

The client – server architecture of the software package allows multiple researchers to work simultaneously with a single library of models. The researcher can contribute to the library of models.

The software package, through the Net data provider, connects to the database server service and gains access to the model library.

The Net Data Provider provides access to the database through the Native API. Data is passed using the classes contained in the Data Access assembly.

Figure 4 shows a diagram of the architecture components of the developed software package.

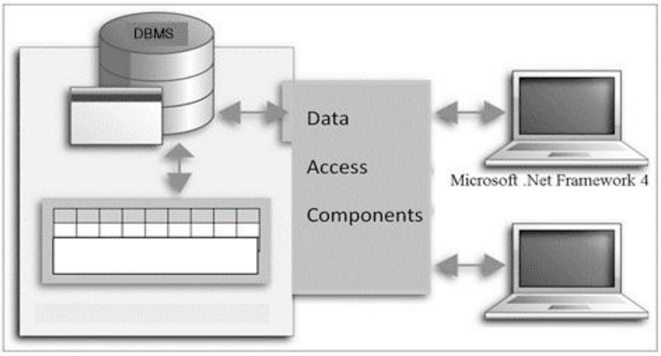

Fig. 4. Software architecture.

The user interface of the software package allows the researcher to select models in the library, load the model and change parameters, calculate and build graphs, and add new models to the library.

For the numerical calculation of mathematical models and their dynamic characteristics, a module of a complex of mathematical modeling has been developed. The user enters the mathematical equation under study, the initial conditions, the de-sired function, the range of parameters and selects the numerical method. As a result of calculation by the selected numerical method, the user receives graphs of the de-sired function and derivative. The module allows to select a numerical method for calculation from the proposed list.

The module implements numerical methods of the Runge–Kutta family of methods. The user can calculate the mathematical model by the following methods: Runge–Kutta fourth order, Runge–Kutta third order, Runge–Kutta–Felberg, Euler, mid-points, Heun method, Dormand–Prens method, inverse Euler method, implicit mid-point method, Bogaski–Champaigne method

To calculate mathematical models using analytical methods, a module of a complex of mathematical modeling has been developed. The user determines the coefficients of the nonlinear differential equation of the second order, the amplitude and frequency of the external periodic force. For the method of transformations, the number of parameters of the studied nonlinear model is optimized. As a result of polynomial transformations, the number of nonzero coefficients is significantly reduced, which greatly simplifies the analysis of transient and steady-state modes of the studied system. A simplified system of equations is the result of polynomial transformations. As a result of the calculation by the selected analytical method, the user receives an approximate analytical solution, graphs of the desired function and derivative. The module allows to select the following methods for calculation: the method of transformations, the small Poincare parameter, harmonic balance, and series expansion.

The complex of mathematical modeling can be widely used for nonlinear mathematical models of dynamical systems.

The analytical solution obtained by the method of transformations has the form:

$$q_1(t) = -0.181458 sin(0.403824t) + 0.436787 sin(1.333t) - 0.000802306 cos(0.403824t) - 0.00930515 cos(1.333t),$$

$$q_2(t) = -1.34404 sin(0.58023t) - 0.372256 sin(1.62503t) - 0.236023 cos(0.58023t) + 0.263701 cos(1.62503t),$$

$$q_3(t) = 0.0859091 sin(1.16811t) + 0.898713 sin(1.40356t) + 0.347838 cos(1.16811t) - 0.374483 cos(1.40356t).$$

The method of transformations allows the analysis of dynamic systems taking into account all nonlinear components.

4. The manipulator control

Having built an analytical solution, we solve the problem of manipulator control. The shaft of each electric drive for the link creates a control action - force, moment. The electric drive control system controls the moment and speed of the engine using signals from the master and feedback sensors. The input control signal for the electric drive depends on the gain for the power converter and provides the necessary control effect. At the output of the electric power converter, a voltage of adjustable amplitude and frequency is supplied to the stator windings of the motor. For software control of the electric drive, it is necessary to determine the control action that ensures the fulfillment of a given trajectory by the manipulator. Substituting the functions of generalized coordinates, velocities, and accelerations into the system of equations of motion, we obtain the functions of the control forces F_1, F_2, F_3, which ensure uniform movement of the grip of manipulator.

The control action F_1, F_2, F_3 in the form of a piecewise-smooth function created by electric drives is determined, which ensures the manipulator grip movement with a constant working speed.

$$F_1(t)=\begin{cases}\alpha_1+\beta_1 t, t\in[0,\tau_1]\\ \gamma_1, t\in[\tau_1,t_1]\end{cases},\ F_2(t)=\begin{cases}\alpha_2+\beta_2 t, t\in[0,\tau_1]\\ \gamma_2, t\in[\tau_1,t_1]\end{cases},\ F_3(t)=\begin{cases}\alpha_3+\beta_3 t, t\in[0,\tau_1]\\ \gamma_3, t\in[\tau_1,t_1]\end{cases}$$

Here, the coefficients $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2, \alpha_3, \beta_3, \gamma_3$ are control parameters.

For the manipulator with: $\tau_1 = 1, m_1 = 10, m_2 = 10, m_3 = 10, l_1 = 5, l_2 = 5, l_3 = 5$, control parameters are determined that ensure uniform movement of manipulator grip:

$$\alpha_1 = 763.42, \beta_1 = 22.19, \gamma_1 = 785.61, \alpha_2 = 765.11, \beta_2 = 7.45, \gamma_2 = 772.56, \alpha_3 = 191.27, \beta_3 = 1.86, \gamma_3 = 193.13,$$

The maximum values for the control forces of the electric drives are defined:

$$F_1^{max} = 0.79kN, F_2^{max} = 0.77kN, F_3^{max} = 0.19kN .$$

To verify the control action, a numerical solution is obtained for the system of differential equations of motion of manipulator using the Runge-Kutta method for given control parameters.

The presented method allows modeling the dynamics of multi-link manipulators with control.

5. Conclusions

As a result, the presented general method for constructing a mathematical model of multi-link robotic manipulators is applied to the 3-DOF manipulator. The kinematics and dynamics models of the 3-DOF manipulator are obtained and investigated. It is proposed to apply the analytical method of transformations to study the dynamics models of robotic manipulators. A software package has been developed for solving nonlinear models. Using the method of transformations, we constructed a solution in the analytical form of the dynamics model of the 3-DOF manipulator taking into account all components of nonlinear system of differential equations. For programmed control of electric drives of the 3-DOF manipulator, control actions are defined.

The proposed general method for constructing and analyzing the mathematical model of manipulators with the developed software package can be used to study a wide class of multi-link manipulators with many degrees of freedom.

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