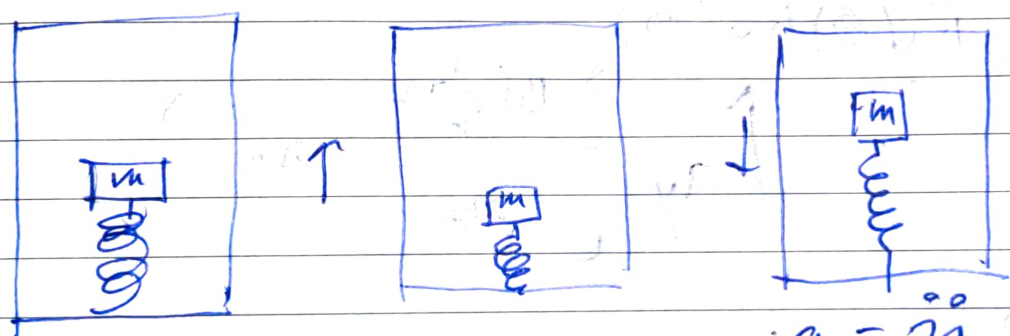
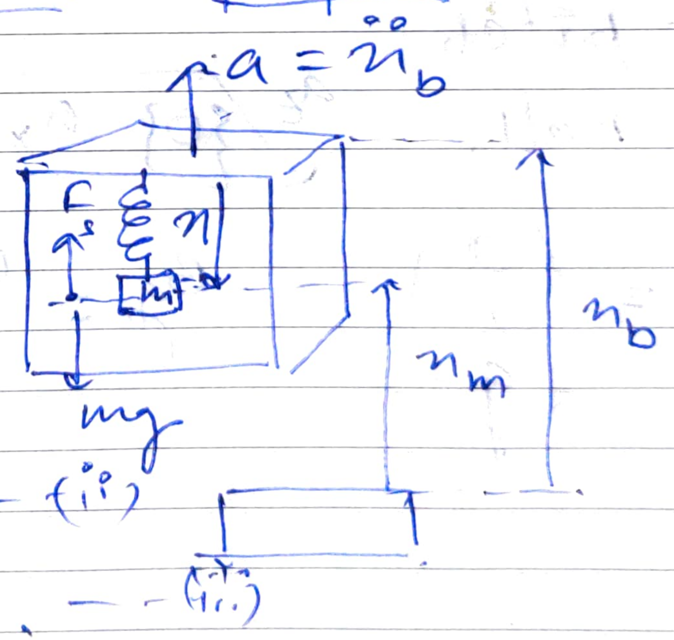
Inertial Meserement System:

IMU Mesures:

i) Acceleration (3-axis)
ii) Magnetic field (3-axis)
iii) Angular Velocity (3-axis)



Accelerometer:



$x_m = x_b - x$ --- (i)

$m\ddot{x}_m = F_s - mg$ --- (ii)

for spring, $F = kx$. --- (iii)

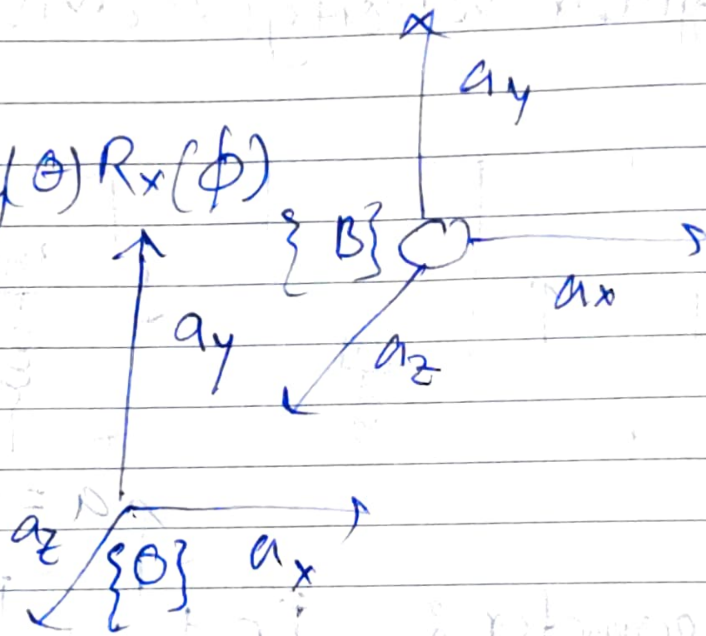
from (i), (ii) & (iii) →

$$m(\ddot{x}_{0} - \ddot{x}) = kx - mg$$

$$\Rightarrow m(a - \ddot{x}) = kx - mg$$

$$\Rightarrow x = \frac{m}{k}(a+g)$$

$${}^{0}R_{B} = R_z(\psi) R_y(\theta) R_x(\phi)$$



ψ = yaw

θ = pitch

ϕ = roll.

$${}^{0}g = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} \qquad {}^{B}g = {}^{B}R_{0}\,{}^{0}g$$

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \begin{pmatrix} -g\sin\theta \\ g\cos\theta\sin\phi \\ g\cos\theta\cos\phi \end{pmatrix}$$

$$\sin\theta = \frac{-a_x}{g}$$

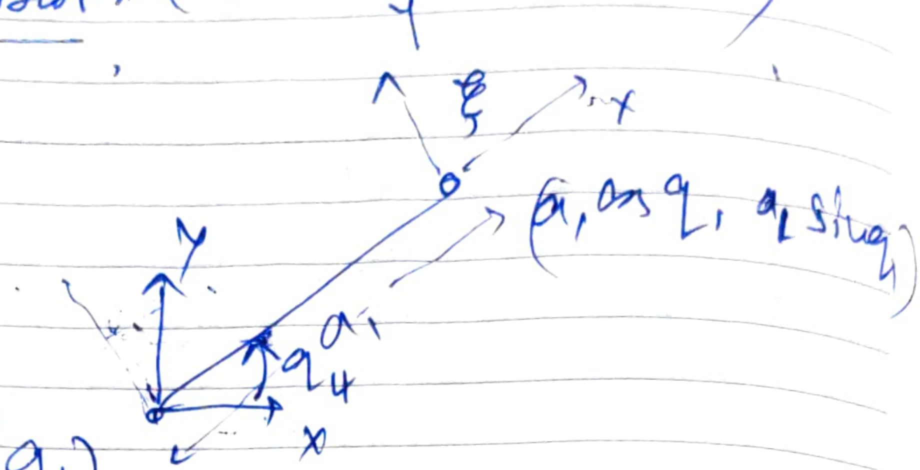
$$\tan\theta = \frac{a_y}{a_z}$$

$$^{B}B = {}^{B}R_{0}\,{}^{0}B = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} \quad {}^{0}B = \begin{pmatrix} \cos I \\ 0 \\ \sin I \end{pmatrix}$$

$$\psi = \tan^{-1} \frac{\cos\theta\,(b_z\sin\phi - b_y\cos\phi)}{b_x + B\sin I\sin\theta}$$

Spatial Link Robots (Forward kinematics)

Single Links:



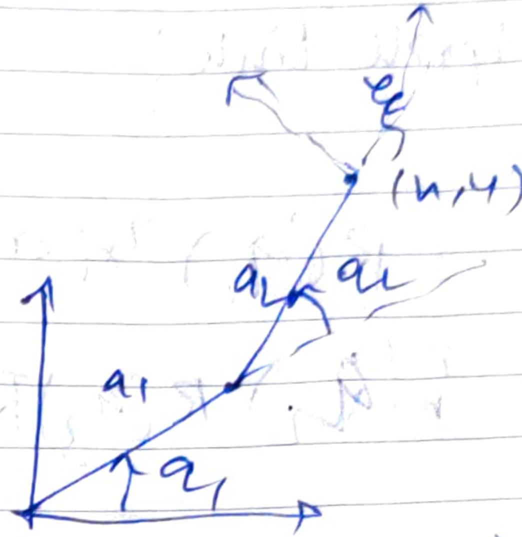
$E = R(q_1) T_x(a_1)$

$$E = \begin{pmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & a_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos q_1 & -\sin q_1 & a_1\cos q_1 \\ \sin q_1 & \cos q_1 & a_1\sin q_1 \\ 0 & 0 & 1 \end{pmatrix}$$

Double link:

$$\underline{E} = R_1(q_1)\, T_x(a_1)\, R(q_2)\, T_x(a_2)$$

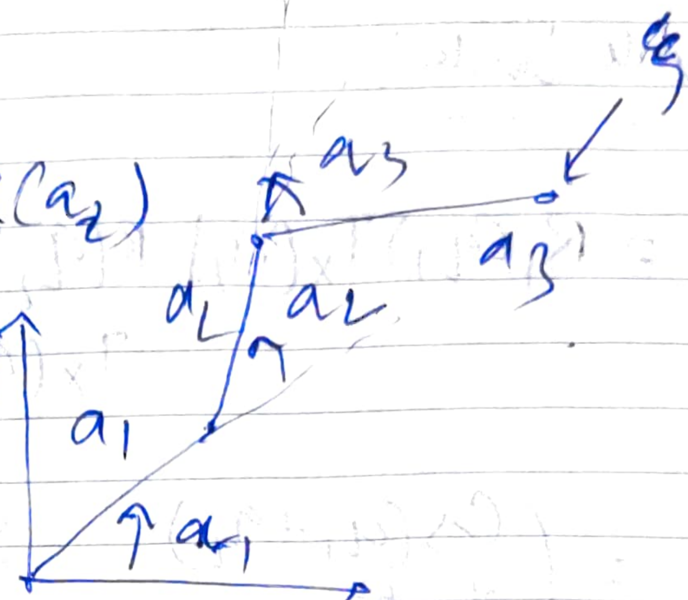


$$\underline{E} = \begin{pmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & a_2\cos(q_1+q_2) + a_1\cos q_1 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & a_2\sin(q_1+q_2) + a_1\sin q_1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_2\cos(q_1+q_2) + a_1\cos q_1 \\ a_2\sin(q_1+q_2) + a_1\sin q_1 \end{pmatrix}.$$

Tripple Link:

$$E = R(q_1)\, T_x(a_1)\, R(q_2)\, T_x(a_2)\, R(q_3)\, T_x(a_3)$$

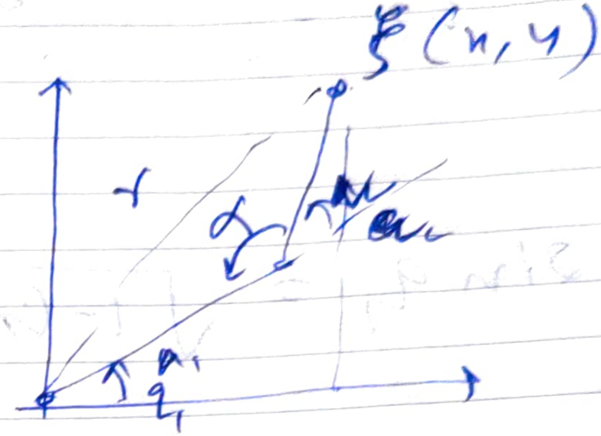


$$x = a_1 \cos q_1 + a_2 \cos(q_1 + q_2) + a_3 \cos(q_1 + q_2 + q_3)$$

$$y = a_1 \sin q_1 + a_2 \sin(q_1 + q_2) + a_3 \sin(q_1 + q_2 + q_3)$$

02 Sunday

Inverse Kinematics:



2-link:

$$r = \sqrt{x^2+y^2} \quad \text{---(i)}$$

From Cosine Rule

we know, $A^2 = B^2 + C^2 - 2BC\cos a$.

$$r^2 = a_1^2 + a_2^2 - 2a_1a_2\cos\alpha \quad \text{---(ii)}$$

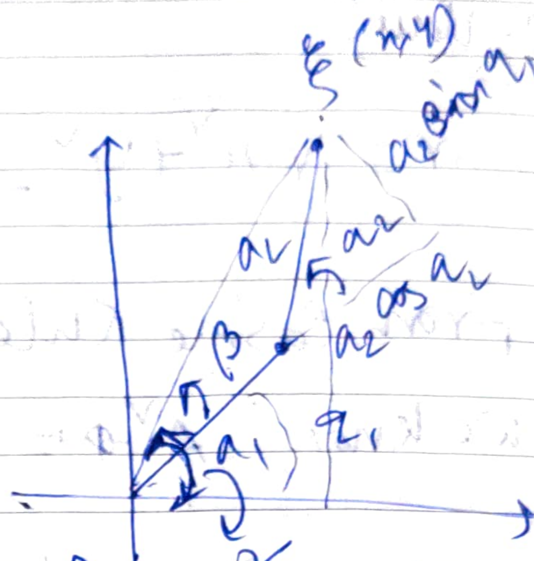
$$\cos\alpha = \frac{a_1^2 + a_2^2 - r^2}{2a_1a_2}$$

$$= \frac{a_1^2 + a_2^2 - x^2 - y^2}{2a_1a_2} \quad \text{[from (i)]} \quad \text{---(iii)}$$

$$q_2 = \pi - \alpha \Rightarrow \cos q_2 = \cos(\pi - \alpha)$$

$$= -\cos\alpha$$

$$\cos q_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2} \quad \text{---} \; (iv)$$



$$\sin q_2 = \sqrt{1 - \cos^2 q_2}$$

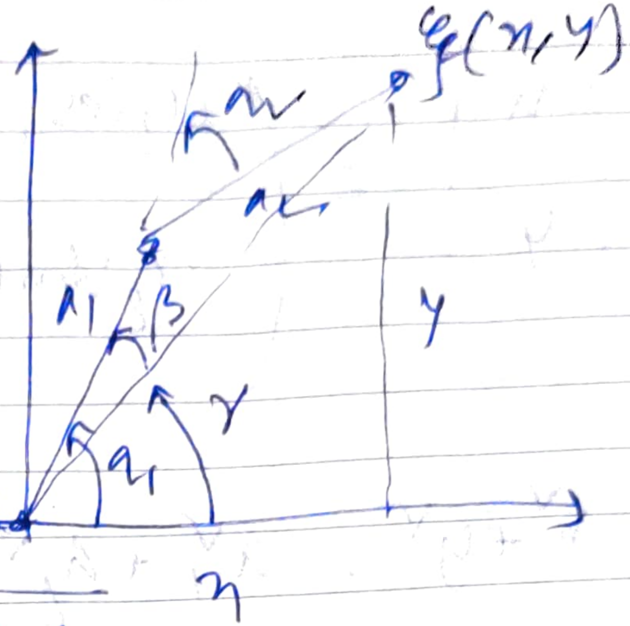
$$\beta = \tan^{-1} \frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2} \quad \text{---} \; (v)$$

$$\gamma = \tan^{-1} \frac{y}{x} \qquad q_1 = \gamma - \beta$$

$$q_1 = \tan^{-1} \frac{y}{x} - \tan^{-1} \frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}$$

$$q_2 = \cos^{-1} \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$\gamma = \tan^{-1}\frac{y}{x}$$



$$q_1 = \gamma + \beta$$

$$q_1 = \tan^{-1}\frac{y}{x} + \tan^{-1}\frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}$$

$$q_2 = -\cos^{-1}\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}$$

<u>Using Linear Algebra:</u>

$$E = R(q_1)\,T_x(a_1)\,R(q_2)\,T_x(a_2)$$

$$E = \begin{pmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & a_2\cos(q_1+q_2)+a_1\cos q_1 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & a_2\sin(q_1+q_2)+a_1\sin q_1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_2 \cos(q_1 + q_2) + a_1 \cos q_1 \\ a_2 \sin(q_1 + q_2) + a_1 \sin q_1 \end{pmatrix}$$

$$x^2 + y^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos q_2 \quad \text{--- (i)}$$

$$\cos q_2 = \frac{x_0^2 + y_0^2 - a_1^2 - a_2^2}{2 a_1 a_2} \quad \text{--- (ii)}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (a_1 + a_2 C_2)\cos q_1 - a_2 S_2 \sin q_1 \\ (a_1 + a_2 C_2)\sin q_1 + a_2 S_2 \cos q_1 \end{pmatrix} \quad \text{--- (iii)}$$

by substituting, $\cos q_2 \rightarrow C_2$, $\sin q_2 \rightarrow S_2$

we know that

$$a\cos\theta + b\sin\theta = c$$

$$\theta = \tan^{-1}\frac{c}{\pm\sqrt{a^2+b^2-c^2}} - \tan^{-1} a/b$$

Comparing with eqn

$\therefore$ $a = a_2 S_2$ $b = a_1 + a_2 C_2$ $c = y$

$$a_1 = \tan^{-1}\frac{y}{\sqrt{a_1^2+a_2^2+2a_1a_2C_2-y^2}} - \tan^{-1}\frac{a_2 S_2}{a_1+a_2 C_2}$$

$$\left[a_s\ a_2^2 S_2^2 + a_2^2 C_2^2 = a_2^2\right]$$

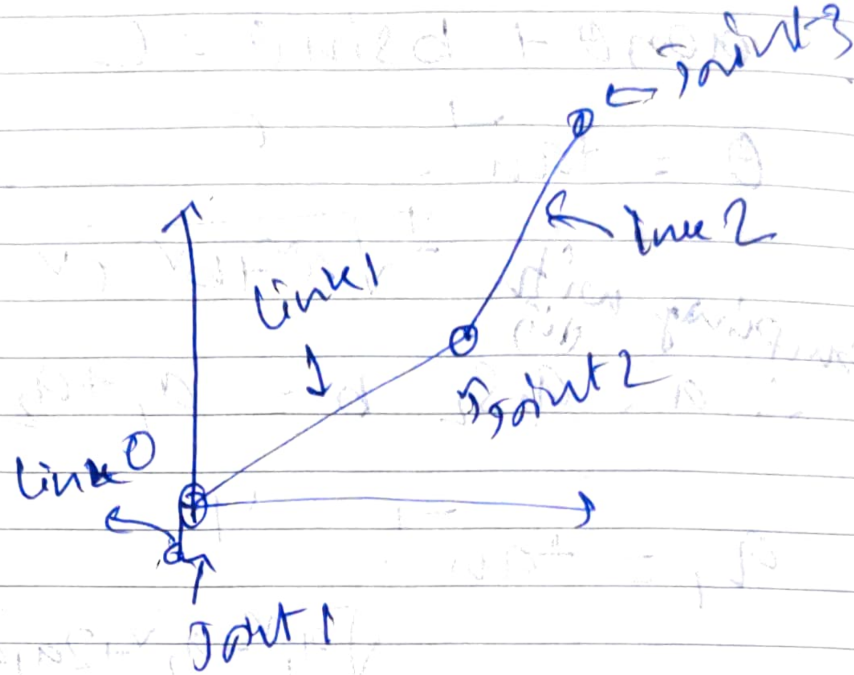
from, (i)

$$a_1 = \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{a_2 S_2}{a_1 + a_2 C_2}.$$

Denavit Hartenberg:

N Joints

N+1 Links

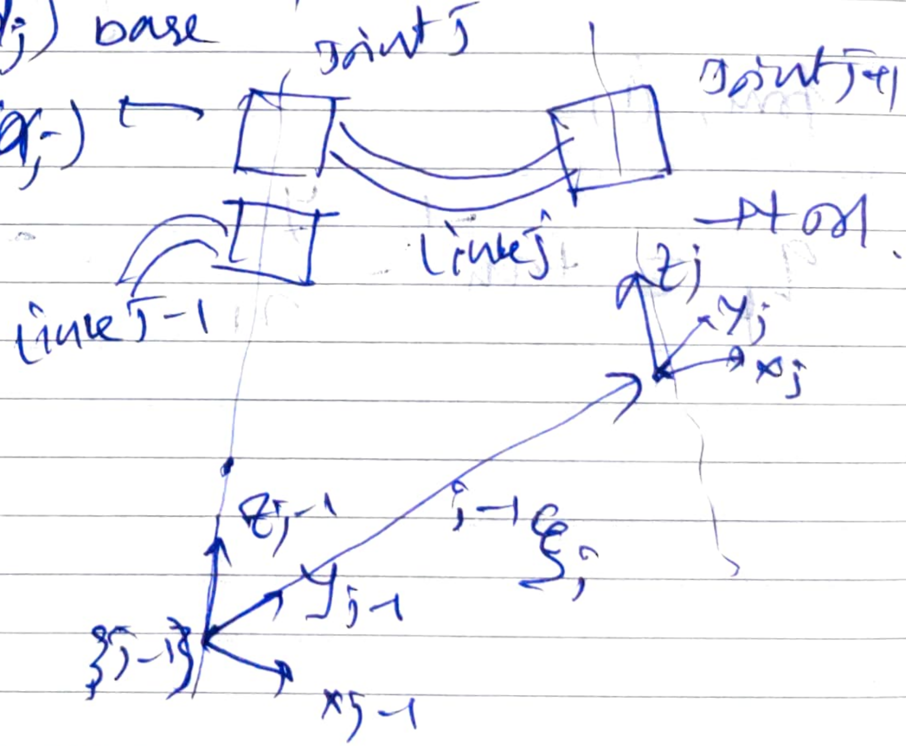


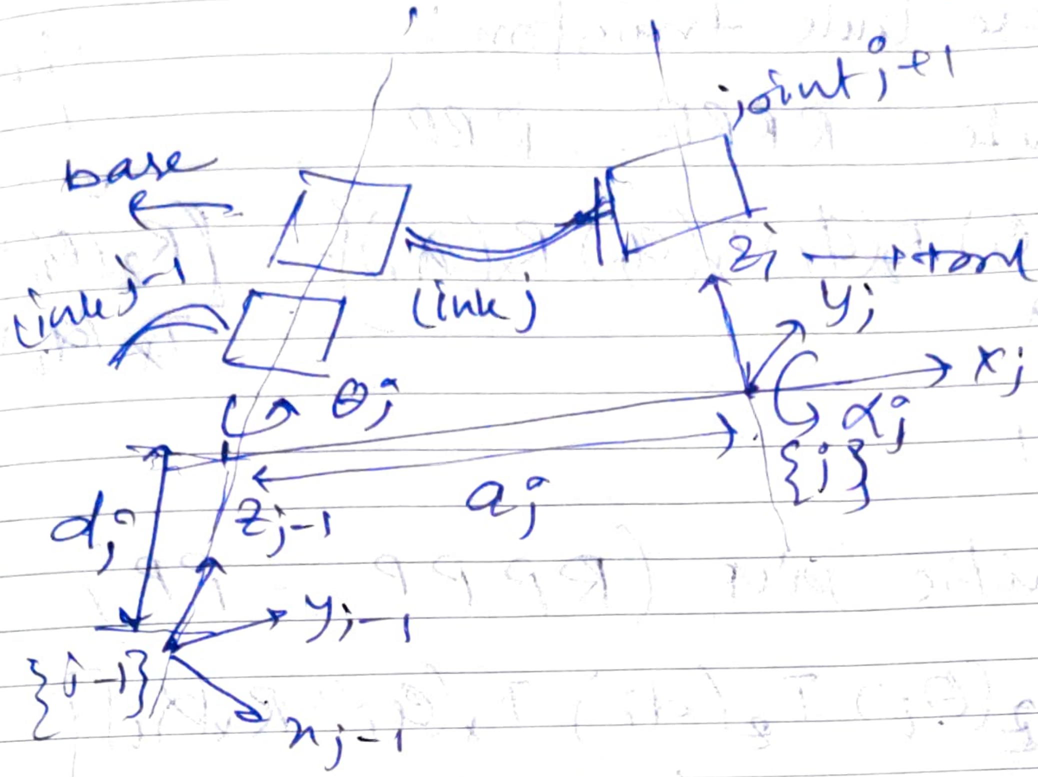
$^{j-1}\xi_j \sim A$

$A = R_z(\theta_j)\, T_z(d_j)$ base

$T_x(a_j)\, R_x(\alpha_j)$

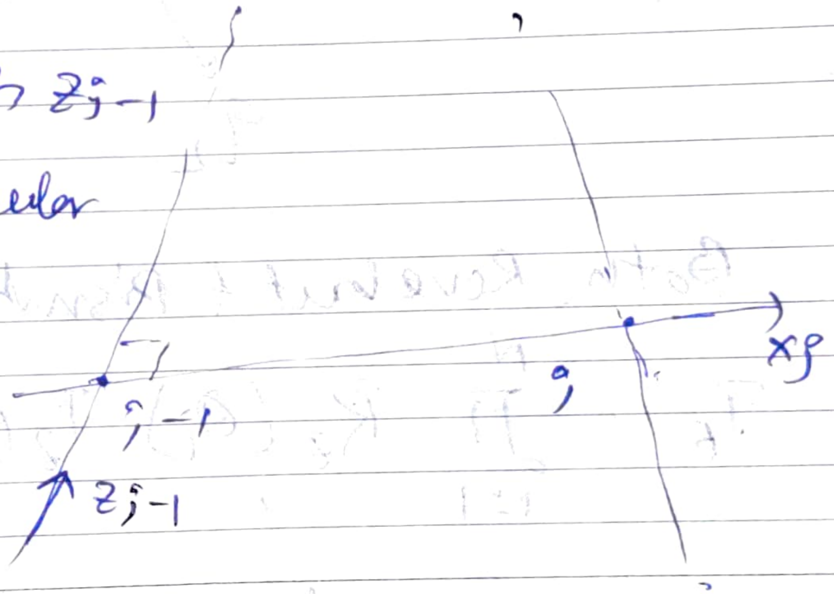
09 Sunday





* Axis x_j intersects z_{j-1}

* Axis x_j is prpnicular to axis z_{j-1}



Stack the link transform :

All Revolute, RRRR ... RRR $\longrightarrow$

$$T_E = \left[R_z(\theta_1)\, T_z(d_1)\, T_x(a_1)\, R_x(\alpha_1)\right] \cdots \left[R_z(\theta_n)\, T_z(\theta_n)\, T_x(a_n)\, R_x(\alpha_n)\right]$$

q_1 q_n

~~All~~ Prismatic Joint - (RPPP ... PP)

$$T_E = \prod_{i=1}^{N}\left[R_z(\theta_i)\, T_z(d_i)\, T_x(a_i)\, R_x(\alpha_i)\right]$$

q_2

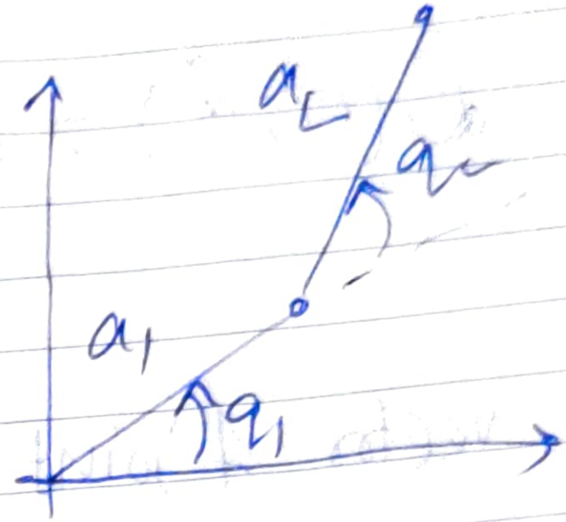
Both Revolut & Prismatic (RPRRP ... PPRR)

$$T_E = \prod_{i=1}^{N} R_z(\theta_i)\, T_z(d_i)\, T_x(a_i)\, R_x(\alpha_i).$$

q_1 q_2

D-H Parameters:

θ_j	d_j	a_j	α_j
q_1	0	a_1	0
q_2	0	a_2	0



General Form:

$$\xi_N = \mathcal{K}(q : \theta, d, a, \alpha, \sigma)$$

$\xi \rightarrow$ Pose

$q \rightarrow$ Joint Configuration.

$\theta \rightarrow$ Joint angle $d \rightarrow$ link offset

$a \rightarrow$ link length $\alpha \rightarrow$ link twist

$\sigma \rightarrow$ Joint type. $\sigma_j = \begin{cases} R \rightarrow \theta_j = q_j \\ P \rightarrow d_j = q_j \end{cases}$

Lagrangian Formulation of dynamics:

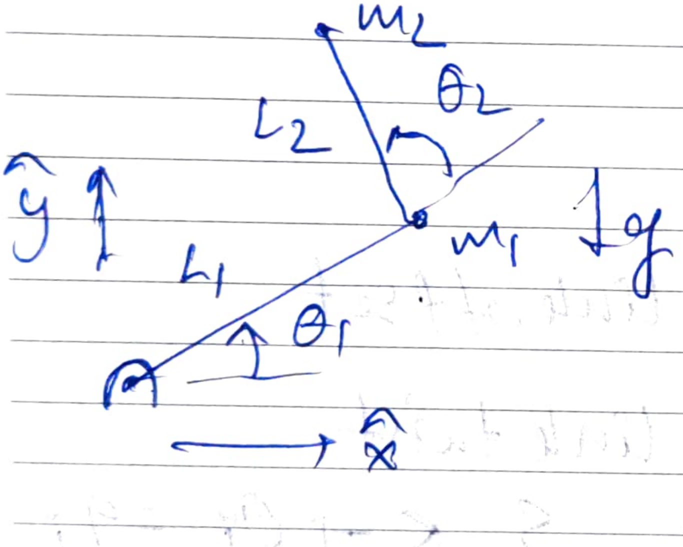
$$\mathcal{L}(\theta,\dot{\theta}) = K(\theta,\dot{\theta}) - P(\theta)$$

KE PE

vector of joint forces and torques

$$\tau = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta}$$

$$\tau_i = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} - \frac{\partial \mathcal{L}}{\partial \theta_i}$$



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} L_1 \cos\theta_1 \\ L_1 \sin\theta_1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -L_1 \sin\theta_1 \\ L_1 \cos\theta_1 \end{bmatrix} \dot{\theta}_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) \\ L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1+\theta_2) \\ L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1+\theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\frac{\partial \theta}{\partial \theta_1}$$

$$\begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial x_2}{\partial \theta_1} & \frac{\partial x_2}{\partial \theta_2} \\ \frac{\partial y_2}{\partial \theta_1} & \frac{\partial y_2}{\partial \theta_2} \end{bmatrix} \begin{bmatrix} \frac{\partial \theta_1}{\partial t} \\ \frac{\partial \theta_2}{\partial t} \end{bmatrix}$$

$$k_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_2^2)$$

$$= \frac{1}{2} m_1 L_1^2 \dot{\theta}_1^2$$

$$k_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$$

$$= \frac{1}{2} m_2 \left[(L_1^2 + 2L_1 L_2 \cos\theta_2 + L_2^2) \dot{\theta}_1^2 + 2(L_2^2 + L_1 L_2 \cos\theta_2) \dot{\theta}_1 \dot{\theta}_2 + L_2^2 \dot{\theta}_2^2 \right]$$

$$P_1 = m_1 g_0 y_1 = m_1 g_0 L_1 \sin\theta_1$$

$$P_2 = m_2 g y_2 = m_2 g (L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2))$$

$$\mathcal{L}(\theta, \dot{\theta}) = \sum_{i=1}^{2} (k_i - P_i)$$

$$\tau_i = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} - \frac{\partial \mathcal{L}}{\partial \theta_i}$$

16 Sunday

$$\mathcal{L}_{comp} = m_2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 \cos\theta_2$$

$$\tau_{2,comp} = \frac{d}{dt} \frac{\partial \mathcal{L}_{comp}}{\partial \dot{\theta}_2} - \frac{\partial \mathcal{L}_{comp}}{\partial \theta_2}$$

$$= \frac{d}{dt}\left(m_2 L_1 L_2 \dot{\theta}_1 \cos\theta_2\right) + m_2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2$$

$$= m_2 L_1 L_2 \ddot{\theta}_1 \cos\theta_2 - \cancel{m_2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2}$$

$$+ \cancel{m_1 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2}$$

$\left\{\text{using } \frac{d}{d\theta} u.v = \frac{du}{d\theta}v + u\frac{dv}{d\theta}\right\}$

$$= m_2 L_1 L_2 \ddot{\theta}_1 \cos\theta_2$$

For generalized systems,

$$\tau = \underbrace{M(\theta)}_{\text{mass matrix}}\ddot{\theta} + \underbrace{C(\theta, \dot{\theta})}_{\text{vel product term}} + \underbrace{g(\theta)}_{\text{gravity term}} + \underbrace{J^T(\theta) f_{tip}}_{\text{Term to create wrench at the bend}}$$

$F =$ "ma" + gravity.