Lagrangian mechanics → Find torques and forces on a joint.

If we have a revolute joint → we find torque. See diagram.

Prismatic joint → we find ~~force~~ force

θ → position of robot manipulator $\dot{\theta}$ → velocity of robot manipulator.

q_j = generalized coordinates

Q_j = " forces.

$T = L - V$

KE − PE

$\frac{1}{2}mv^2$ mgh.

Analyse robot system on robot manipulator and find

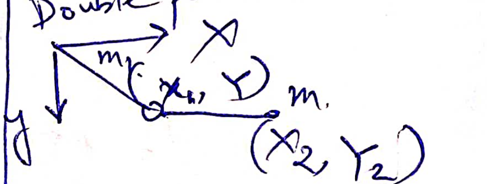
position vector (θ)

Differentiate to find velocity vector.

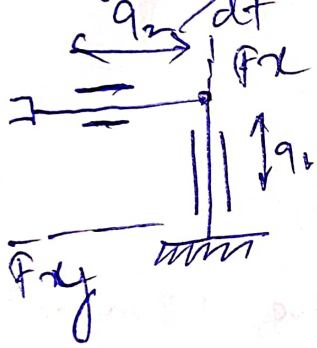
Position and motion.

$\tau = \frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) - \left(\frac{dL}{d\theta}\right)$

Double pendulum.



X and Y are not independent.



Two prismatic joints P_1 and P_2

Masses of links m and m_2

displacement of joints q_1 and q_2.

Input force → F_1 and F_2.

Position vectors of these two joints

Position vector → 0 to x movement.

$P_{m1} = \begin{bmatrix} 0 \\ q_1 \end{bmatrix}$ $P_{m2} = \begin{bmatrix} q_2 \\ q_1 \end{bmatrix}$ $V_m = \frac{dP_m}{dt}$

$V_{m_1} = \begin{bmatrix} 0 \\ \dot{q}_1 \end{bmatrix}$ $V_{m_2}\begin{bmatrix} \dot{q}_2 \\ \dot{q}_1 \end{bmatrix}$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}m\,v_y^2$

$KE = \frac{1}{2}m\,(v_x^2 + v_y^2)$

$KE_{m_1} = \frac{1}{2}m_1(\dot{q}_1^2)$ $KE_{m_2} = \frac{1}{2}m_2(\dot{q}_2^2 + \dot{q}_1^2)$

Potential energy of the system → mgh. (vertical component)

$PE_{m_1} = m_1 g q_1$ $\quad$ $PE_{m_2} = m_2 g q_1.$

$\mathcal{L} = \Sigma KE - \Sigma PE.$

$\mathcal{L} = KE_{m_1} + KE_{m_2} - PE_{m_2} - PE_{m_2}$

$= \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 (\dot{q}_2 + \dot{q}_1^2 - g q_1)$

$\mathcal{L} = m_1 (\frac{1}{2} \dot{q}_1^2 - g q_1) + m_2 (\frac{1}{2} \dot{q}_1^2 + \frac{1}{2} \dot{q}_2^2 - g q_1)$

$dL/d\dot{q}_1 = m_1 \dot{q}_1 + m_2 \dot{q}_1$

$dL/dq_1 = -m_1 g - m_2 g.$

$\delta L/\delta \dot{q}_2 = m_2 \dot{q}_2$

$\delta L/\delta q_2 = 0.$

For prismatic.

$f_{m_1} = \delta/\delta t \left(\frac{\delta L}{\delta \dot{q}_1}\right) - \frac{\delta L}{\delta q_1} - f_1(\dot{q}_1).$

$f_{m_2} = \delta/\delta t \left(\frac{\delta L}{\delta \dot{q}_2}\right) - \frac{\delta L}{\delta q_2} - f_2(\dot{q}_2)$

$= \delta/\delta t\, (m_2 \dot{q}_2)$

$= m_2 \ddot{q}_2$