# CS648 : Randomized Algorithms
# Semester I, 2011-12, CSE, IIT Kanpur

## (LAST) Practice Sheet
Probabilistic Methods

1. If $\binom{n}{k}2^{-\binom{k}{2}+1} < 1$, then it is possible to color the edges of $K_n$ with two colors so that it has no monochromatic $K_k$ subgraph.

2. We have shown using probabilistic method that, if a graph $G$ has $n$ vertices and $m$ edges, then there exists a cut of size at least $m/2$. This bound can be further improved. To show this, prove the following results.

   (a) if $n$ is even, then there exists a cut of value at least $\frac{n}{2(n-1)}m$

   (b) if $n$ is odd, then there exists a cut of value at least $\frac{n+1}{2n}m$.

   **Hint:** Partition the vertices into two sets of equal sizes uniformly randomly.

3. A Boolean variable takes value either TRUE or FALSE. A literal is either a Boolean variable or a negation of it. A clause is a disjunction (OR) of 3 or more literals. For example, if $x_1, x_2, x_3$ are Boolean variables then $(x_1 \vee \bar{x_2} \vee x_3)$ is a clause. A clause is said to be satisfied under a given assignment of values TRUE and FALSE to the variables, if at least one of its literals is TRUE.

   We are given a collection of $n$ Boolean variables and $m$ clauses. Consider the problem of determining whether there exists an assignment of variables to the values so that all clauses is satisfied. This problem is one of the famous problems in theoretical computer science and is called SAT.

   Show that there always exists an assignment of values to the variables so that at least $7/8m$ clauses are satisfied.

4. Let $G = (V, E)$ be a graph on $n$ vertices and $nd/2$ edges. An independent set $S$ is a set of vertices so that for each $x, y \in S$, there is no edge between $x$ and $y$ in the graph $G$. Finding an independent set of large size is an important problem with many applications.

   Using the technique of "random sampling followed by alteration" prove that there exists a an independent set of size at least $\frac{n}{2d}$.

   **Hint:** Select each vertex of $G$ independently with probability $1 - 1/d$; delete each selected vertex along with all its edges. ...