CS648 : Randomized Algorithms
Semester I, 2011-12, CSE, IIT Kanpur

Assignment - 3 (due on 17th October, 9AM)

Note: The first two problems employ a novel application of backward analysis. To solve these problems, it will be better if you revise the sampling lemma for MST which we proved using backward analysis. You must provide concrete details for your solutions.

1. **Random points in an interval**

   We select n points uniformly and independently at random from $[0, 1]$. This splits the interval into $n + 1$ intervals. Let X be the random variable for the length of the shortest interval. Show that $E(X) = O(\frac{1}{n^2})$.

2. **Smallest Enclosing Circle**

   There is a set P of n points in a plane. We select a uniformly random sample of $\frac{n}{2}$ points from these. Let C be the smallest enclosing circle for the sampled points. Calculate the expected number of unsampled points which lie outside C.

3. **Convex Hull problem**

   Recall the convex hull problem discussed in class. Show that the time taken by the algorithm is concentrated around $O(n \log n)$ with high probability using the following two approaches. **Make sure you provide complete details of the proofs.**

   (a) Backward analysis

   (b) The approach we took to prove that the running time of randomized quick sort is concentrated around $O(n \log n)$ (see Lecture 8 on the course website).

4. **Analysing the duration of a randomized experiment by partitioning it into stages**

   We have been given a graph on n vertices which is initially empty, that is, there are no edges in the graph. There is a bag which contains all $\binom{n}{2}$ possible edges for the graph. We add edges one by one into the initially empty graph. At each step, we sample a random edge from the bag and add it to the graph. The corresponding sampling is without replacement. We stop as soon as the graph becomes connected. Give the best possible bound on the expected number of edges sampled in this process.

5. **Online sampling**

   You enter a shopping mall where you receive a sequence of m apples. You have a bag where you may keep at most $n \leq m$ apples. You have to do online sampling : On seeing a new apple, you may either replace some apple from your bag with this new apple or you may discard the new apple. Once you discard an apple, you will not be able to retrieve that again in future. Design the sampling strategy so that when you come out of the mall, your bag has a uniformly random sample of n apples from the mall. You must prove correctness of your strategy.