CS648 : Randomized Algorithms
Semester I, 2011-12, CSE, IIT Kanpur

Practice problem sheet - 2

1. **Random walk on a line**

   A particle is performing a discrete random walk along the x-axis starting at the origin. At the beginning of each second the particle moves either 1 unit to the left or 1 unit to the right uniformly and independently at random. Prove that after n seconds the distance of the particle from origin is $O(\sqrt{n \log n})$ with high probability.

2. **Approximate median calculation**

   (a) Recall the Monte Carlo algorithm for finding an ϵ-approximate median from a set of elements in $O(\log n \log \log n)$ time. Analyze its success probability using Chernoff's bound.

   (b) Transform the above Monte Carlo algorithm into a Las Vegas algorithm to find an ϵ-approximate median in expected $O(n)$ time.

3. **Randomized load balancing**

   Suppose $O(n \log n)$ balls are placed uniformly, randomly and independently into n bins. Show that the load of the most heavily loaded bin is $\theta(\log n)$ with high probability.

4. **Randomized binary search tree**

   A randomized binary search tree is built by randomly permuting the given sequence of n elements uniformly, randomly and independently, and then inserting the elements in the order they appear in the permutation. Prove that, with high probability, the height of a randomized BST will be $O(\log n)$. Note that this problem is in some sense proves that the average height of a BST is $O(\log n)$.

5. **Random graph problem**

   Let $G(n, p)$ be the random graph on n vertices which is obtained by independently adding an edge between every pair of vertices with probability p. Let X be the random variable that denotes the number of distinct triangles in $G(n, p)$.

   (a) Calculate $E[X]$

   (b) Calculate $E[X^2]$.