CS648 : Randomized Algorithms
Semester I, 2011-12, CSE, IIT Kanpur

Practice Sheet

Partitioning an experiment into stages and the method of bounded difference

1. Consider two complete graphs G_1 and G_2 on n vertices. Let $u \in G_1, v \in G_2$. Add an edge between u and v. Let $a \in G_1, a \neq u$ and $b \in G_2, b \neq v$ be any two vertices. In the resulting graph on $2n$ vertices, calculate the expected time of a random walk which originates at a and terminates at b.

2. There is a bag which contains n different types of coupons. Furthermore, the number of coupons of each type is infinite in the bag. We want to have at least one coupon for each of these types. To achieve this goal, we repeat the following sampling step : take out a coupon from the bag uniformly and randomly. So, during each sampling step, the coupon drawn is equally likely to be of any of the n types of coupons. What is the expected number of sampling steps required to collect $0.99n$ different types of coupons ?

3. Consider the following randomized process. There are n bins and n players, a nd each player has an infinite supply of balls. The bins are all initially empty . We have a sequence of rounds : in each round, each player throws a ball into a n empty bin chosen independently at random from all currently empty bins. What i s the expected number of rounds till every bin is non-empty ?

4. Prove using MOBD (Method of Bounded Difference) that after n steps of a uniform random walk on a line, with high probability, the particle will be within $O(\sqrt{n \log n})$ distance from the origin (the starting position). If you remember, a proof of this claim based on Chernoff bound was quite long and required manipulations of random variables. But using MOBD, it can be solved in just a few lines.

5. Suppose we toss a fair coin n times. Let X be the random variable for the number of times the pattern "HTTHTH" occurs in this sequence of tosses. Note that the occurrences of pattern may be overlapping. What is expected value of X ? Prove using method of bounded difference that $\mathbf{P}[X > \mathbf{E}[X] + c\sqrt{n \log n}]$ is very very low for some suitable constant c. Try to see why you could not apply Chernoff bound here.