CS648 : Randomized Algorithms
Semester I, 2011-12, CSE, IIT Kanpur

Practice problem sheet - 3

1. We toss a fair coin n times. A k-streak of flips is said to occur starting at toss i, if the outcome of all the k flips starting from i^{th} flip is the same. For example, for the sequence HTTTHH, there is a 2-streak occurring at $2nd$ toss, there is a 2-streak occurring at 3^{rd} toss, and there is a 2-streak occurring at 5^{th} toss. Here the total number of 2-streaks is 3 in the sequence HTTTHH. What is the expected number of k-streaks which you will see in n tosses of a fair coin ?

2. **Estimating all-pairs distances**

   Consider an undirected unweighted graph G on n vertices. For simplicity, assume that G is connected. We are also given a partial distance matrix M_c for some $c < 1$: For a pair of vertices i, j the entry $M_c[i, j]$ stores exact distance if i and j are separated by distance $\leq cn$, otherwise M_c stores a symbol # indicating that distance between vertex i and vertex j is greater than cn. Unfortunately, there are $\Theta(n^2)$ # entries in M_c , i.e., for $\Theta(n^2)$ pairs of vertices, the distance is not known. Design a Monte Carlo algorithm to compute exact distance matrix for G in $O(n^2 \log n)$ time. (Each entry of the distance matrix has to be correct with probability exceeding $1 - 1/n^2$).

   For this exercise, find a small set of witnesses using random sampling and proceed...

3. In the closest pair point problem we discussed in class, we had assumed that the distances between the points are unique. Perform the analysis without making this assumption.

4. **Backward Analysis**

   Given a set S of 2-D points, a point $(x_1, y_1) \in S$ is a dominating point if for every other point $(x_2, y_2) \in S$, at least one of $x_1 > x_2$ or $y_1 > y_2$ holds. Give an algorithm to compute the set of dominating points and analyze it using backward analysis.

   Hint: You may start by adding the point(s) with smallest and largest x co-ordinates to the dominating set. This creates a vertical strip which can be associated with all other points. In subsequent iterations, choosing further points might help further partitioning the space into vertical strips. You may look at this as similar to dividing the space into cones in the convex hull algorithm.